2020/08/06

Econometrics I's Final Exam.

Deadline: August 6, 2020, AM10:30:00

- The answer should be written in English or Japanese.
- Your **name** and **student ID** number should be included in your **answer sheet**.
- You do not have to answer in order. The question number should be made clear.
- Send your answer to the email address: `tanizaki@econ.osaka-u.ac.jp`.
  The file size should be less than **1MB**.
  (This is not strict, but the file size should be as small as possible.)
- The subject should be **Econome 1** or **計量 1**. Otherwise, your mail may go to the **trash box**.
- **Answers only are not allowed.**
  All the calculation processes have to be included in the answer sheet.
  It is very important to make me understand what you have studied in Econometrics I.

1 Suppose that u_1, u_2, $\cdots$, u_T are mutually independently distributed with $\mathrm{E}(u_t) = 0$ and $\mathrm{V}(u_t) = \sigma^2$ for all $t = 1, 2, \cdots, T$.

Consider the following regression model:

$$y = X\beta + u,$$

where y, X, β and u are $T\times 1$, $T\times k$, $k\times 1$ and $T\times 1$ matrices or vectors. X is assmued to be nonstochastic and β is an unknown parameter. Answer the following questions. (5 points $\times$ 8)

(1) Derive OLSE of β, denoted by $\hat{\beta}$.

(2) Obtain mean and variance of $\hat{\beta}$.

(3) Show that $\hat{\beta}$ has the smallest variance among all the linear unbiased estimators.

(4) As T goes to infinity, derive an asymptotic distribution of $\sqrt{T}(\hat{\beta} - \beta)$.

(5) Let $s^2 = \dfrac{1}{T-k}(y - X\hat{\beta})'(y - X\hat{\beta})$ be an estimator of σ^2. Show that s^2 is an unbiased estimator of σ^2.

(6) Let R and r be $G \times k$ matrix and $G \times 1$ vector for $G \leq k$. Under the condition $R\beta = r$, derive OLSE of β, denoted by $\tilde{\beta}$.

(7) Derive the distribution of $\dfrac{(\tilde{u}'\tilde{u} - \hat{u}'\hat{u})/G}{\hat{u}'\hat{u}/(T-k)}$ for $\hat{u} = y - X\hat{\beta}$ and $\tilde{u} = y - X\tilde{\beta}$, where $\hat{\beta}$ and $\tilde{\beta}$ are obtained in ((1) and (4), respectively.

(8) Let R_1^2 and R_4^2 be coefficients of determination obtained in (1) and (4), respectively. Derive the distribution of $\dfrac{(R_1^2 - R_4^2)/G}{(1-R_1^2)/(T-k)}$.

2 Suppose that X_i takes 0 or 1. $X_1, X_2, \cdots, X_n$ are assumed to be mutually independently distributed with Bernoulli distribution $f(x;p) = p^x(1-p)^{1-x}$ for $x = 0, 1$, where p is an unknown parameter vector to be estimated. (6 points $\times$ 7)

(9) Derive the maximum likelihood estimator of p, denoted by $\hat{p}$.

(10) Obtain mean and variance of $\hat{p}$.

(11) Obtain Cramer-Rao lower bound. Show that $\hat{p}$ has the smallest variance within a class of unbiased estimators of p.

(12) Show that $\hat{p}$ is a consistent estimator of p, using Chebyshev's inequality. You have to explain how you apply Chebyshev's inequality.

(13) Derive an asymptotic distribution of $\sqrt{n}(\hat{p}-p)$, using the central limit theorem. You have to explain how you apply the central limit theorem.

(14) We want to test $H_0:\ p = 0.5$ against $H_1:\ p \neq 0.5$. Using the Wald test, explain the testing procedure.

(15) We want to test $H_0:\ p = 0.5$ against $H_1:\ p \neq 0.5$. Using the likelihood ratio test, explain the testing procedure.

3 Consider the same regression model as **1**. However, X is assmued to be stochastic. (6 points $\times$ 3)

(16) When X is correlated with u, show that $\hat{\beta}$ is a biased estimator and an inconsistent estimator, where $\hat{\beta}$ is the OLSE obtained in **1** (1).

(17) Derive a consistent estimator of β, denoted by β_{iv}, using an instrument variable Z ($T \times k$ matrix). Which assumptions do we need for Z?

(18) As T goes to infinity, derive an asymptotic distribution of $\sqrt{T}(\beta_{iv} - \beta)$, using the central limit theorem. You have to explain how you apply the central limit theorem.