Econometrics I's Homework

Deadline: April 22, 2020, PM23:59:59

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: `tanizaki@econ.osaka-u.ac.jp`.
- The subject should be Econome 1 or   1. Otherwise, your mail may go to the **trash box**.

Consider the following regression model:

$$y_t = \alpha + \beta X_t + u_t, \qquad t = 1, 2, \cdots, T,$$

where y_t and X_t denote dependent and independent variables, respectively. T is the sample size. u_1, u_2, $\cdots$, u_T are mutually independently distributed with mean zero and variance σ^2. α and β are unknown parameters to be estimated. Let $\hat{\beta}$ be the ordinary least squares estimator of β.

(1) Prove that $\hat{\beta}$ is a linear estimator of β.

(2) Prove that $\hat{\beta}$ is a linear unbiased estimator of β.

(3) Prove that $\hat{\beta}$ has minimum variance within a class of linear unbiased estimators

(4) Prove that $\hat{\beta}$ is a consistent estimator of β.

(5) Derive an asymptotic distribution of $\sqrt{T}(\hat{\beta} - \beta)$.