

# Econometrics I's Homework

**Deadline: April 29, 2020, PM23:59:59**

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: `tanizaki@econ.osaka-u.ac.jp`.
- The subject should be Econome 1 or 計量 1. Otherwise, your mail may go to the **trash box**.

1 Consider the following regression model:

$$y_t = \alpha + \beta X_t + u_t, \quad t = 1, 2, \dots, T,$$

where  $y_t$  and  $X_t$  denote dependent and independent variables, respectively.  $T$  is the sample size.  $u_1, u_2, \dots, u_T$  are mutually independently and **normally** distributed with mean zero and variance  $\sigma^2$ .  $\alpha$  and  $\beta$  are unknown parameters to be estimated. Let  $\hat{\beta}$  be the ordinary least squares estimator of  $\beta$ .

(1) Derive an exact distribution of  $\hat{\beta}$ , using the moment-generating function.

(2) Show that  $\frac{\hat{\beta} - \beta}{s\sqrt{\sum_{t=1}^T \omega_t^2}}$  is a  $t$  distribution with  $T - 2$  degrees of freedom, where  $s^2 = \frac{1}{T - 2} \sum_{t=1}^T (y_t - \hat{\alpha} - \hat{\beta}X_t)^2$ . We may use the fact that the degree of freedom is  $T - 2$ .