

# Econometrics I's Homework

**Deadline: May 20, 2020, PM23:59:59**

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: `tanizaki@econ.osaka-u.ac.jp`.
- The subject should be Econome 1 or 計量 1. Otherwise, your mail may go to the **trash box**.

1 Consider the following regression model:

$$y = X\beta + u$$

where  $y$ ,  $X$ ,  $\beta$  and  $u$  denote  $T \times 1$ ,  $T \times k$ ,  $k \times 1$  and  $T \times 1$  matrices.  $k$  and  $T$  are the number of explanatory variables and the sample size.  $u_1, u_2, \dots, u_T$  are mutually independently and **normally** distributed with mean zero and variance  $\sigma^2$ , i.e.,  $u \sim N(0, \sigma^2 I_T)$ .  $\beta$  is a vector of unknown parameters to be estimated. Let  $\hat{\beta}$  be the ordinary least squares estimator of  $\beta$ .

- (1) Show that  $s^2 = \frac{1}{T-k}(y - X\hat{\beta})'(y - X\hat{\beta})$  is an unbiased estimator of  $\sigma^2$ .
- (2) Show that  $\frac{(T-k)s^2}{\sigma^2}$  is distributed as a  $\chi^2$  random variable with  $T-k$  degrees of freedom.
- (3) Show that  $\hat{\beta}$  is a best linear unbiased estimator.