2020/05/21

Econometrics I's Homework

Deadline: May 27, 2020, PM23:59:59

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: `tanizaki@econ.osaka-u.ac.jp`.
- The subject should be Econome 1 or 計量 1. Otherwise, your mail may go to the **trash box**.

1 Consider the following regression model:

$$y = X\beta + u$$

where y, X, β and u denote $T\times 1$, $T\times k$, $k\times 1$ and $T\times 1$ matrices. k and T are the numberof explanatory variables and the sample size. u_1, u_2, $\cdots$, u_T are mutually independently and **normally** distributed with mean zero and variance σ^2, i.e., $u \sim N(0, \sigma^2 I_T)$. β are a vector of unknown parameters to be estimated. Let $\hat\beta$ be the ordinary least squares estimator of β.

(1) $\hat\beta$ is normally distributed with mean β and variance $\sigma^2(X'X)^{-1}$.
Then, show that $\dfrac{(\hat\beta-\beta)'X'X(\hat\beta-\beta)}{\sigma^2} \sim \chi^2(k)$.
Especially, why is the degree of freedom equal to k?

(2) Show that $\hat\beta$ is independent of $s^2 = \dfrac{1}{T-k}(y-X\hat\beta)'(y-X\hat\beta)$.

(3) Show that $\dfrac{(\hat\beta-\beta)X'X(\hat\beta-\beta)/k}{(y-X\hat\beta)'(y-X\hat\beta)/(T-k)} \sim F(k, T-k)$.

(4) Show that $\displaystyle\sum_{i=1}^{T}(y_i-\overline{y})^2 = y'(I_T - \frac{1}{T}ii')y$, where $y=(y_1, y_2, \cdots, y_T)'$ and $i=(1,1,\cdots,1)'$.

(5) Show that $I_T - \dfrac{1}{T}ii'$ is symmetric and idempotent.

2 $X_1,\ X_2,\ \cdots,\ X_n$ are assumed to be mutually independently, identically and normally distributed with mean μ and variance σ^2. We want to prove $\dfrac{\sum_{i=1}^n (X_i - \overline{X})^2}{\sigma^2} \sim \chi^2(n-1)$, which is shown in standard statistics textbooks, where $\overline{X} = \dfrac{1}{n}\displaystyle\sum_{i=1}^n X_i$.

Define $X = (X_1,\ X_2,\ \cdots,\ X_n)'$ and $i = (1, 1, \cdots, 1)'$, which are $n \times 1$ vectors. Then, we can rewrite as follows:

$$X \sim N(\mu i, \sigma^2 I_n).$$

Answer the following questions.

(6) What is the distribution of $\dfrac{(X - \mu i)'(X - \mu i)}{\sigma^2}$?

(7) Show that

$$(X - \mu i)'(I_n - \frac{1}{n} i i')(X - \mu i) = \sum_{i=1}^n (X_i - \overline{X})^2.$$

(8) Show that $\dfrac{(X - \mu i)'(I_n - \frac{1}{n} i i')(X - \mu i)}{\sigma^2} \sim \chi^2(n-1)$. (That is, $\dfrac{\sum_{i=1}^n (X_i - \overline{X})^2}{\sigma^2} \sim \chi^2(n-1)$ is obtained.)