2020/05/28

Econometrics I's Homework

Deadline: June 3, 2020, PM23:59:59

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: `tanizaki@econ.osaka-u.ac.jp`.
- The subject should be Econome 1 or 計量 1. Otherwise, your mail may go to the **trash box**.

1 Consider the following regression model:

$$y = X\beta + u = i\beta_1 + X_2\beta_2 + u$$

where y, $X = (i, X_2)$, $\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$ and u denote $T \times 1$, $T \times k$, $k \times 1$ and $T \times 1$ matrices. k and T are the numberof explanatory variables and the sample size. Note that $i = (1, 1, \cdots, 1)'$. β_1 is a constant term. β_2 is a $(k-1) \times 1$ vector, and X_2 is a $T \times (k-1)$ matrix. u_1, u_2, $\cdots$, u_T are mutually independently and **normally** distributed with mean zero and variance σ^2, i.e., $u \sim N(0, \sigma^2 I_T)$. β are a vector of unknown parameters to be estimated. Let $\hat\beta$ be the ordinary least squares estimator of β. We define a vector of residuals as $e = y - X\hat\beta$.

(1) Show that $Mi = 0$ and $Me = e$, where $M = I_T - \frac{1}{T}ii'$.

(2) Show that $e'e = y'My - \hat\beta' X' M X \hat\beta = y'My - \hat\beta_2' X_2' M X_2 \hat\beta_2$, where $\hat\beta = \begin{pmatrix} \hat\beta_1 \\ \hat\beta_2 \end{pmatrix}$.

(3) The coefficient of determination is defined as: $R^2 = 1 - \dfrac{e'e}{y'My}$, where $M = I_T - \dfrac{1}{T}ii'$.

Show that $R^2 = \dfrac{\hat\beta_2' X_2' M X_2 \hat\beta_2}{y'My}$

(4) Let R be an $G \times k$ vector with $G \le k$. Derive the distribution of $R\hat\beta$.

(5) The null hypothesis is given by $H_0: \ R\beta = r$.

Under the null hypothesis, derive the distribution of $R\hat\beta$.

(6) We want to test $\beta_2 = 0$. What are G, R and r?

(7) We know that $\dfrac{(R\hat{\beta}-r)'(R(X'X)^{-1}R')^{-1}(R\hat{\beta}-r)/G}{e'e/(T-k)} \sim F(G,T-k)$.

Show that $\dfrac{\hat{\beta}_2' X_2' M X_2 \hat{\beta}_2/G}{e'e/(T-k)} \sim F(G,T-k)$.

(8) When we test $H_0:\ \beta_2 = 0$, the test statistic is given by $\dfrac{R^2/(k-1)}{(1-R^2)/(T-k)}$.

Show that $\dfrac{R^2/(k-1)}{(1-R^2)/(T-k)} \sim F(k-1,T-k)$.

(9) Suppose that we have the following estimation result:

$$y_i = \underset{(3.163)}{0.3} + \underset{(0.240)}{0.65}\ X_i, \qquad s = 1.07, \quad R^2 = 0.786$$

where the sample size is $T = 4$, ( ) indicates the standard error of the corresponding coefficient estimate, s denote the standard error of regression, and R^2 represents the coefficient of determination.

Let β be the coefficient of X_i.

We want to test $\beta = 0$ for the regression model: $y_i = \alpha + \beta X_i + u_i$.

First, using the estimate 0.65 and its standard error of coefficient (i.e., 0.240), test $\beta = 0$.

Second, using R^2, test $\beta = 0$.

Compare both test statistics and make sure $t^2(T-k) = F(1,T-k)$

I think that you can find various distribution tables in standard introductory statistics textbooks.

For example, see `http://www2.econ.osaka-u.ac.jp/~tanizaki/cv/books/stat3.pdf`, pp.251 –.