2020/06/04

Econometrics I's Homework

Deadline: June 10, 2020, PM23:59:59

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: `tanizaki@econ.osaka-u.ac.jp`.
- The subject should be Econome 1 or 計量 1. Otherwise, your mail may go to the **trash box**.

1 Consider the regression model:

$$y = X\beta + u, \qquad u \sim N(0, \sigma^2 I_T),$$

where y, X, β and u are $T \times 1$, $T \times k$, $k \times 1$ and $T \times 1$.

Let $\hat\beta$ be the ordinary least squares estimator, and $\tilde\beta$ be the ordinary least squares estimator restricted to $R\beta = r$, where R and r are $G \times k$, $G \times 1$ and $G \leq k$. $\hat u$ and $\tilde u$ are defined as the OLS residual and the restricted OLS residual, respectively.

$$y = X\hat\beta + \hat u$$
$$y = X\tilde\beta + \tilde u, \qquad R\tilde\beta = r$$

(1) Derive the restricted OLS $\tilde\beta$.

(2) Show the following:

$$\frac{(\tilde u'\tilde u - \hat u'\hat u)/G}{\hat u'\hat u/(T-k)} \;\sim\; F(G, T-k).$$

2 We consider estimating the following three production functions.

$$\log(Y_t) = \alpha_0 + \alpha_1 \log(K_t) + \alpha_2 \log(L_t) + u_t \qquad (1)$$

$$\log(y_t) = \beta_0 + \beta_1 \log(k_t) + u_t \qquad (2)$$

$$\log(Y_t) = \gamma_0 + \gamma_1 \log(K_t) + \gamma_2 \log(L_t) + \gamma_3 D_t + \gamma_4 D_t \log(K_t) + \gamma_5 D_t \log(L_t) + u_t \qquad (3)$$

The estimation period is 1969 – 1997 (it's too old!). Let Y_t be GDP (10 billion yen, 1992 price), K_t be the net worth (10 billion yen, deflated by the GDP deflator), L_t be the number of employees, D_t be the dummy variable, which is one after 1991 and zero before 1991, y_t be the per capita GDP (10 billion yen, 1992 price, $y_t = Y_t/L_t$), and k_t be the per capita net worth (10 billion yen, deflated by the GDP deflator, $k_t = K_t/L_t$). The error terms $u_1, u_2, \cdots, u_T$ are mutually independently, identically and normally distributed.

The following estimation results are obtained.

$$\log(Y_t) = -\underset{(7.283)}{30.6242} + \underset{(5.054)}{.230042}\log(K_t) + \underset{(8.266)}{2.23565}\log(L_t)$$

$$R^2 = .986684, \quad \overline{R}^2 = .985659, \quad \widehat{\sigma}^2 = .00141869$$

$$\log(y_t) = -\underset{(41.08)}{3.53058} + \underset{(19.62)}{.504043}\log(k_t)$$

$$R^2 = .934448, \quad \overline{R}^2 = .932020, \quad \widehat{\sigma}^2 = .00354801$$

$$\log(Y_t) = -\underset{(3.630)}{34.6168} + \underset{(2.588)}{.204302}\log(K_t) + \underset{(4.155)}{2.48045}\log(L_t)$$
$$-\underset{(1.090)}{54.8287}D_t + \underset{(.4665)}{.243766}D_t\log(K_t) + \underset{(1.134)}{2.84275}D_t\log(L_t)$$

$$R^2 = .987960, \quad \overline{R}^2 = .985342, \quad \widehat{\sigma}^2 = .00145010$$

Note that the values in the parentheses denote the t values, R^2 is the coefficient of determination, $\overline{R}^2$ is the adjusted R^2, and $\widehat{\sigma}^2$ is the variance estimate of regression.

Answer the following questions.

(3) Test $H_0: \ \alpha_1 = \alpha_2 = 0$.

(4) Test whether the production function is homogeneous.

(5) Test whether the structural change occurred after 1991.

For each question, show the testing procedure in detail.