

Econometrics I's Homework

Deadline: June 17, 2020, PM23:59:59

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: tanizaki@econ.osaka-u.ac.jp.
- The subject should be Econome 1 or 計量 1. Otherwise, your mail may go to the **trash box**.

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(1) Show that there exists P such that $\Omega = PP'$ when Ω is a positive definite matrix.

(3) Consider the following regression model:

$$y_t = x_t\beta + u_t, \quad u_t \sim N(0, \sigma^2 z_t^2), \quad t = 1, 2, \dots, T,$$

where u_1, u_2, \dots, u_T are mutually independent.

What is the variance-covariance matrix, denoted by $\sigma^2\Omega$, of $u = (u_1, u_2, \dots, u_T)'$.

(2) Consider the following regression model:

$$y_t = x_t\beta + u_t, \quad u_t = \rho u_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2), \quad t = 1, 2, \dots, T,$$

where $\epsilon_1, \epsilon_2, \dots, \epsilon_T$ are mutually independent.

What is the variance-covariance matrix, denoted by $\sigma^2\Omega$, of $u = (u_1, u_2, \dots, u_T)'$.

(3) Let b be the best linear unbiased estimator of β under the following regression model:

$$y = X\beta + u, \quad u \sim N(0, \sigma^2\Omega),$$

where y, X, β and u are $T \times 1, T \times k, k \times 1$ and $T \times 1$.

Derive b .

(4) Consider the regression model in (3). We have two estimators, $\hat{\beta}$ and b , to estimate β .

Obtain $E(\hat{\beta})$ and $V(\hat{\beta})$.

Show that $V(\hat{\beta}) - V(b)$ is a positive definite matrix.