2020/06/18

Econometrics I's Homework

Deadline: June 24, 2020, PM23:59:59

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: `tanizaki@econ.osaka-u.ac.jp`.
- The subject should be Econome 1 or 計量 1. Otherwise, your mail may go to the **trash box**.

1 Suppose that $X_1, X_2, \cdots, X_n$ are mutually independently and normally distributed with $\mathrm{E}(X_i) = \mu$ and $\mathrm{V}(X_i) = \sigma^2$ for all $i = 1, 2, \cdots, n$.

That is, the density function of X_i is given by:

$$f(x) = (2\pi\sigma^2)^{-1/2} \exp\Big(-\frac{1}{2\sigma^2}(x-\mu)^2\Big)$$

(1) Obtain the likelihood function of μ and σ^2.

(2) Obtain the maximum likelihood estimators of μ and σ^2, denoted by $\tilde{\mu}$ and $\tilde{\sigma}^2$.

(3) Obtain the variances of $\tilde{\mu}$ and $\tilde{\sigma}^2$.

(4) Obtain Fisher's information matrix, denoted by $I(\theta)$ for $\theta = (\mu, \sigma^2)$.

(5) Compare the variances in (3) and $I(\theta)$ in (4).