

Econometrics I's Homework

Deadline: July 15, 2020, PM23:59:59

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: tanizaki@econ.osaka-u.ac.jp.
- The subject should be Econome 1 or 計量 1. Otherwise, your mail may go to the **trash box**.

1 Suppose that u_1, u_2, \dots, u_T are mutually independently distributed with $E(u_t) = 0$ and $V(u_t) = \sigma^2$ for all $t = 1, 2, \dots, T$.

Consider the following regression model:

$$y = X\beta + u,$$

where y , X , β and u are $T \times 1$, $T \times k$, $k \times 1$ and $T \times 1$ matrices or vectors. Answer the following questions.

- (1) Let $\hat{\beta}$ be the ordinary least squares estimator of β . Show that $\hat{\beta}$ is a consistent estimator of β . You have to make clear the underlying assumptions.
- (2) As T goes to infinity, what is the asymptotic distribution of $\frac{1}{\sqrt{T}}X'u$?
- (3) Obtain the asymptotic distribution of $\sqrt{T}(\hat{\beta} - \beta)$.

2 Suppose that X_1, X_2, \dots, X_T are mutually independently distributed with the density functions $f(x_i; \theta)$, $i = 1, 2, \dots, T$.

- (4) Let $\hat{\theta}$ be the maximum likelihood estimator of θ . Show that $\hat{\theta}$ is a consistent estimator of θ . You have to make clear the underlying assumptions.
- (5) As T goes to infinity, what is the asymptotic distribution of $\frac{1}{\sqrt{T}} \sum_{i=1}^T \frac{\partial \log f(X_i; \theta)}{\partial \theta}$?
- (6) Obtain the asymptotic distribution of $\sqrt{T}(\hat{\theta} - \theta)$.