2020/07/16

# Econometrics I's Homework

## Deadline: July 22, 2020, PM23:59:59

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: `tanizaki@econ.osaka-u.ac.jp`.
- The subject should be Econome 1 or 計量 1. Otherwise, your mail may go to the **trash box**.

**1** Using annual data from 1982 to 2004, we estimate a demand function of bread. The notations are as follows:

$Q_{1t}$ : Purchase volume of bread at time $t$ (1g)
$Y_t$ : Income at time $t$ (Japanese yen, and 2000 base year)
$P_{1t}$ : Price of bread at time $t$ (Japanese yen per 100g, and 2000 base year)
$P_{2t}$ : Price of rice at time $t$ (Japanese yen per 1kg, and 2000 base year)

We have estimated the following demand function:

$$\log Q_{1t} = \underset{(2.90)}{5.899} + \underset{(4.132)}{0.644} \log Y_t - \underset{(13.19)}{1.205} \log P_{1t} + \underset{(0.242)}{0.00756} \log P_{2t}, \qquad (1)$$

$$R^2 = 0.925, \quad \overline{R}^2 = 0.913, \quad \hat{\sigma}^2 = 0.016564^2, \quad DW = 1.212, \quad \log L = 63.87,$$

Estimation period: 1982 to 2004,

where the values in the parentheses denote the $t$-values, $R^2$ represents the coefficient of determination, $\overline{R}^2$ indicates the adjusted coefficient of determination, $\hat{\sigma}$ is the standard error in the regression equation, $DW$ denotes the Durbin-Watson statistic, and $\log L$ represents the estimate of the log-likelihood function.

Using the residuals in Eq. (1), denoted by $\widehat{u}_t$, we have estimated the following.

$$\widehat{u}_t = \underset{(1.79)}{0.366} \ \widehat{u}_{t-1}, \qquad (2)$$

$$R^2 = 0.131, \quad \overline{R}^2 = 0.131, \quad \hat{\sigma}^2 = 0.014363^2, \quad DW = 1.793, \quad \log L = 62.64,$$

Estimation period: 1983 to 2004.

Moreover, assuming that the error term in Eq. (1) is the first-order autocorrelated and using the maximum likelihood method, we have obtained the following estimation results:

$$\log Q_{1t} = \underset{(2.62)}{6.33} + \underset{(3.38)}{0.613} \log Y_t - \underset{(12.7)}{1.223} \log P_{1t} + \underset{(0.615)}{0.0263} \log P_{2t} \tag{3}$$

$$R^2 = 0.936, \quad \overline{R}^2 = 0.922, \quad \hat{\sigma}^2 = 0.015769^2, \quad DW = 1.739, \quad \log L = 65.58,$$

$$\hat{\rho} = \underset{(1.90)}{0.402}, \quad \text{Estimation period: 1982 to 2004},$$

where $\hat{\rho}$ denotes the first-order autocorrelation coefficient estimate in the error term, and the value in the parenthesis of $\hat{\rho}$ indicates its $t$-value.

(1) Using the Lagrange multiplier test, explain how to test the first-order autocorrelation in the error term.

(2) Using the likelihood ratio test, explain how to test the first-order autocorrelation in the error term.

(3) Using the Wald test, explain how to test the first-order autocorrelation in the error term.