Econometrics I: Solutions of the homework #14

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July 21, 2020

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1 Question 1

1.1 (1)

When X and u are correlated that is to say $E(X'u) \neq 0$, Thus the OLSE can be written as:

$$\hat{\beta} = (\frac{1}{n}X'X)^{-1}\frac{1}{n}X'u \longrightarrow \beta + M_{xx}^{-1}M_{xu} \neq 0$$

where:

$$\frac{1}{n}X'X \longrightarrow M_{xx}, \quad \frac{1}{n}X'u \longrightarrow M_{xu}$$

Which imply that $\hat{\beta} \neq \beta$. In such case OLSE is inconsistent

*If you have any errors in handouts and materials, please contact me via lvang12@hotmail.com

1.2 (2)

the matrix Z is not correlated with X and u which satisfies the orthogonal condition $\frac{1}{n}Z'u \longrightarrow M_{zu} = 0$. Multiplying Z' on both sides of the regression model $y = X\beta + u$, we obtain:

$$Z'y = Z'X\beta + Z'u$$

Dividing n on both sides of the above equation and take plim on both sides. Then we obtain the following:

$$plim(\frac{1}{n}Z'y) = plim(\frac{1}{n}Z'X)\beta + plim(\frac{1}{n}Z'u) = plim(\frac{1}{n}Z'X)\beta$$

Accordingly, we obtain:

$$\beta = (plim(\frac{1}{n}Z'X))^{-1}plim(\frac{1}{n}Z'y)$$

Therefore:

$$\beta_{IV} = (Z'X)^{-1}Z'y$$

1.3 (3)

Assume the followings:

$$\frac{1}{n}Z'X \longrightarrow M_{zx}, \quad \frac{1}{n}Z'Z \longrightarrow M_{zz}, \quad \frac{1}{n}Z'u \longrightarrow 0$$

Asymptotic Distribution of β_{IV}:

$$\beta_{IV} = (Z'X)^{-1}Z'y = (Z'X)^{-1}Z'(X\beta + u) = \beta + (Z'X)^{-1}Z'u$$

which is rewritten as:

$$\sqrt{n}(\beta_{IV} - \beta) = (\frac{1}{n}Z'X)^{-1}(\frac{1}{\sqrt{n}}Z'u)$$

Applying the Central Limit Theorem to $\frac{1}{\sqrt{n}}Z'u$, we have the following result:

$$\frac{1}{\sqrt{n}}Z'u \to N(0, \sigma^2 M_{zz})$$

Therefore:

$$\sqrt{n}(\beta_{IV} - \beta) = (\frac{1}{n}Z'X)^{-1}(\frac{1}{\sqrt{n}}Z'u) \to N(0, \sigma^2 M_{zx}^{-1} M_{zz} {M'}_{zx}^{-1})$$