Econometrics I: Solutions of Homework #15

Hiroki Kato *

August 10, 2020

Contents

1 Solutions

Throughout questions, we will test the first-order autocorrelation:

$$H_0 : \rho = 0, \quad H_1 : \rho \neq 0. \tag{1}$$

Let θ be a vector of ML estimate. A vector of null hypothesis is $h(\theta) = \rho$, which is 1×1 vector. Thus, all of three test statistics described below are asymptotically distributed over $\chi^2(1)$.

Note that $h(\tilde{\theta}) = 0$ where $\tilde{\theta}$ be a vector of ML estimate with restrictions of null hypothesis. A vector $\hat{\theta}$ denotes ML estimate without restrictions of null hypothesis.

1.1 Question 1: Lagrangian multiplier test

Consider the following regression equation:

$$\hat{u}_t = \phi \hat{u}_{t-1} + \epsilon_t, \tag{2}$$

*e-mail: vge008kh@student.econ.osaka-u.ac.jp. Room 503. If you find any errors in handouts and materials, please contact me via e-mail.

where $\hat{u}_t$ is residual at period t, and $\epsilon_t \sim \text{iid}N(0, \sigma^2)$. The null hypothesis is equivalent to $\phi = 0$. The LM test for autocorrelation is asymptotically equal to the squared t-value of coefficient ϕ under the null hypothesis. That is, $t^2 \to \chi^2(1)$, where t is t value of coefficient ϕ.

By empirical results shown in question, we obtain

$$1.79^2 = 3.20. \tag{3}$$

The 5% upper probability point of $\chi^2(1)$ is 3.84, and the 10% upper probability point of $\chi^2(1)$ is 2.710. Thus, the LM test rejects the null hypothesis $\rho = 0$ at 10% significance level.

1.2 Question 2: Likelihood ratio test

Under the null hypothesis $h(\theta) = 0$, a test statistics of LR test is given by

$$-2(\log L(\theta) - \log L(\hat{\theta})) \to \chi^2(1). \tag{4}$$

Since $h(\theta) = h(\tilde{\theta}) = 0$, we can replace θ with $\tilde{\theta}$,

$$-2(\log L(\tilde{\theta}) - \log L(\hat{\theta})) \to \chi^2(1). \tag{5}$$

Note that $\log L(\tilde{\theta})$ is the estimate of log-likelihood function without the first-order autocorrelation, while $\log L(\hat{\theta})$ is the estimate of log-likelihood function assuming the error term is the first-order autocorrelated.

By empirical results shown in question, we obtain

$$-2(63.87 - 65.58) = 3.42. \tag{6}$$

The 5% upper probability point of $\chi^2(1)$ is 3.84, and the 10% upper probability point of $\chi^2(1)$ is 2.710. Thus, the LR test rejects the null hypothesis $\rho = 0$ at 10% significance level.

1.3 Question 3: Wald test

Under the null hypothesis $h(\theta) = 0$, the test statistics of Wald test is given by

$$h(\hat{\theta})(R_{\hat{\theta}} I(\hat{\theta})^{-1} R'_{\hat{\theta}})^{-1} h'(\hat{\theta}) \to \chi^2(1), \tag{7}$$

where $R_{\hat{\theta}} = \partial h(\hat{\theta})/\partial \hat{\theta}'$, which is $G \times k$ matrix. Since $h(\theta)$ is a single linear restriction, this test statistics is simply rewritten as follows: [1]

$$W = \frac{\hat{\rho}^2}{\text{AsyVar}(\hat{\rho})} \to \chi^2(1). \tag{8}$$

The test statistics W has a chi-squared distribution with one degree of freedom, which is the distribution of the square of the standard normal test statistics. Hence, the square of t test statistics is asymptotically equal to the Wald test statistics. To implement the Wald test, we use t statistics of coefficient $\hat{\rho}$.

By empirical results, we obtain the Wald test statistics:

$$1.90^2 = 3.61,$$

which is compared with $\chi^2(1)$. The 5% upper probability point of $\chi^2(1)$ is 3.84, and the 10% upper probability point of $\chi^2(1)$ is 2.710. Thus, the Wald test rejects the null hypothesis $\rho = 0$ at 10% significance level.

[1] Suppose that θ contains two parameters, α and ρ. Then, $R_{\hat{\theta}} = (0, 1)$. Since $I(\hat{\theta})^{-1}$ is asymptotically equal to a variance-covariance matrix, that is,

$$I(\hat{\theta})^{-1} \to \begin{pmatrix} V(\hat{\alpha}) & Cov(\hat{\alpha}, \hat{\rho}) \\ Cov(\hat{\alpha}, \hat{\rho}) & V(\hat{\rho}) \end{pmatrix}.$$

Thus, we have $R_{\hat{\theta}} I(\hat{\theta})^{-1} R'_{\hat{\theta}} = \text{AsyVar}(\hat{\rho})$.