7.2 Granger Cuasality Test (グレンジャー因果性テスト)

Consider the bivariate case:

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \phi_{11,1} & \phi_{12,1} \\ \phi_{21,1} & \phi_{22,1} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \cdots + \begin{pmatrix} \phi_{11,p} & \phi_{12,p} \\ \phi_{21,p} & \phi_{22,p} \end{pmatrix} \begin{pmatrix} y_{1,t-p} \\ y_{2,t-p} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix}$$

Take an example of the first equation.

- Unrestricted Model (Sum of Squared Residuals, denoted by SSR_1):

$$\begin{aligned} y_{1,t} = \mu_1 &+ \phi_{11,1} y_{1,t-1} + \phi_{12,1} y_{2,t-1} \\ &+ \phi_{11,2} y_{1,t-2} + \phi_{12,2} y_{2,t-2} \\ &\quad \cdots \\ &+ \phi_{11,p} y_{1,t-p} + \phi_{12,p} y_{2,t-p} + \epsilon_{1,t} \end{aligned}$$

$$\longrightarrow \hat{\epsilon}_{1,t} \quad \longrightarrow \text{Unrestricted VAR}(p) \longrightarrow \mathrm{SSR}_1 = \sum_{t=p+1}^{T} \hat{\epsilon}_{1,t}^2$$

Test $H_0: \ \phi_{12,1} = \phi_{12,2} = \ \cdots \ = \phi_{12,p} = 0.$

When H_0 is correct, we say there is no causality from y_2 to y_1.

$\Longrightarrow$ **Granger Causality Test**

- Restricted Model (Sum of Squared Residuals, denoted by SSR_0):

Under $H_0: \ \phi_{12,1} = \phi_{12,2} = \ \cdots \ = \phi_{12,p} = 0$, we estimate the following regression:

$$
\begin{aligned}
y_{1,t} &= \mu_1 + \phi_{11,1} y_{1,t-1} + 0 \times y_{2,t-1} \\
&\quad + \phi_{11,2} y_{1,t-2} + 0 \times y_{2,t-2} \\
&\qquad \cdots \\
&\quad + \phi_{11,p} y_{1,t-p} + 0 \times y_{2,t-p} + \epsilon_{1,t} \\
&= \mu_1 + \phi_{11,1} y_{1,t-1} \\
&\quad + \phi_{11,2} y_{1,t-2} \\
&\qquad \cdots
\end{aligned}
$$

$$+ \phi_{11,p} y_{1,t-p} + \epsilon_1$$

$$\longrightarrow \ \tilde{\epsilon}_{1,t} \quad \longrightarrow \ \text{Restricted VAR}(p) \ \longrightarrow \ \text{SSR}_0 = \sum_{t=p+1}^{T} \tilde{\epsilon}_{1,t}^2$$

The number of parameters to be estimated:

- Unestricted Model: $2p + 1$

  $\longrightarrow$ VAR(p), p lagged coefficients, one constant term $\longrightarrow$ $2p + 1$

- Restricted Model: $p + 1$

  $\longrightarrow$ The number of restrictions = p (= G)

Therefore, asymptotically we have the following distribution:

$$F = \frac{(\text{SSR}_0 - \text{SSR}_1)/p}{\text{SSR}_1/(T - 2p - 1)} \ \sim \ F(p, T - 2p - 1),$$

or

$$pF \ \sim \ \chi^2(p).$$

In general, we consider testing the Granger causality from y_j to y_i.

VAR(p) model:

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \epsilon_t,$$

where $y_t : k \times 1, \quad \mu : k \times 1, \quad \phi_p : k \times k, \quad \epsilon_t : k \times 1.$

ϕ_p is given by:

$$\phi_p = \begin{pmatrix} \phi_{11,p} & \cdots & \phi_{1j,p} & \cdots & \phi_{1k,p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{i1,p} & \cdots & \phi_{ij,p} & \cdots & \phi_{ik,p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{k1,p} & \cdots & \phi_{kj,p} & \cdots & \phi_{kk,p} \end{pmatrix}$$

Test the Granger causality from y_j to y_i.

$\longrightarrow$ Focus on the ith equation.

$$\begin{aligned} y_{i,t} = \mu_i &+ \phi_{i1,1} y_{1,t-1} + \cdots + \phi_{ij,1} y_{j,t-1} + \cdots + \phi_{ik,1} y_{k,t-1} \\ &+ \phi_{i1,2} y_{1,t-2} + \cdots + \phi_{ij,2} y_{j,t-2} + \cdots + \phi_{ik,2} y_{k,t-2} \\ &\quad \cdots \\ &+ \phi_{i1,p} y_{1,t-p} + \cdots + \phi_{ij,p} y_{j,t-p} + \cdots + \phi_{ik,p} y_{k,t-p} + \epsilon_{i,t} \end{aligned}$$

Supose that the null hypothesis is: $H_0: \phi_{ij,1} = \phi_{ij,2} = \cdots = \phi_{ij,p} = 0$, while the alternative hypothesis is: $H_1:$ not H_0.

$$\begin{aligned} \text{SSR}_0 &= \text{Sum of Squared Residuals under } H_0 \\ \text{SSR}_1 &= \text{Sum of Squared Residuals under } H_1 \end{aligned}$$

Under H_0, the asymptotic distribution is given by:

$$F = \frac{(\mathrm{SSR}_0 - \mathrm{SSR}_1)/p}{\mathrm{SSR}_1/(T-kp-1)} \sim F(p, T-kp-1),$$

or

$$pF \sim \chi^2(p).$$

Example: VAR(p): Data: 1994 年第一四半期〜2014 年第一四半期

`gdp` = GDP (実質，10 億円，季調済，内閣府 HP から取得)

`def` = GDP デフレータ (季調済，内閣府 HP から取得)

`r` = 貸出約定平均金利 (%，新規，総合・国内銀行，日銀 HP から取得)

`m` = 通貨流通高（平均発行高，億円，季調済，日銀 HP から取得)

```
. gen time=_n                                  → time のデータ作成

. tsset time                                   → time が時間データとする
        time variable:  time, 1 to 81
                delta:  1 unit

. gen lgdp=log(gdp)                            → gdp の対数変換

. gen lm=log(m/(def/100))                      → マネーサプライ m を実質化で，対数変換

. varsoc d.lgdp d.r d.lm                       → 各変数階差を取って，ラグ次数を決める

   Selection-order criteria
   Sample:  6 - 81                              Number of obs      =        76
  +---------------------------------------------------------------------------+
  |lag |    LL      LR      df    p      FPE       AIC       HQIC      SBIC    |
  |----+----------------------------------------------------------------------|
  |  0 |  541.22                        1.4e-10  -14.1637  -14.1269  -14.0717  |
  |  1 | 571.181  59.923*   9  0.000  8.2e-11* -14.7153* -14.5682* -14.3473* |
  |  2 | 575.715  9.0675    9  0.431  9.2e-11  -14.5978  -14.3404  -13.9537  |
  |  3 |  579.55  7.6704    9  0.568  1.1e-10  -14.4619  -14.0942  -13.5418  |
  |  4 | 583.767  8.4328    9  0.491  1.2e-10   -14.336   -13.858  -13.1399  |
  +---------------------------------------------------------------------------+
   Endogenous:  D.lgdp D.r D.lm
    Exogenous:  _cons
```

```
. var d.lgdp d.r d.lm, lags(1)               → 各変数階差を取って，3 変数 VAR(1)

Vector autoregression

Sample:  3 - 81                                   No. of obs      =        79
Log likelihood =  592.2334                        AIC             = -14.68945
FPE            =  8.38e-11                        HQIC            = -14.54526
Det(Sigma_ml)  =  6.18e-11                        SBIC            = -14.32954

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
D_lgdp               4     .010717   0.0422   3.480972   0.3232
D_r                  4     .087186   0.2553    27.0782   0.0000
D_lm                 4     .009434   0.2903   32.30929   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_lgdp       |
        lgdp |
         LD. |   .2031129   .1119361     1.81   0.070    -.0162778    .4225037
             |
           r |
         LD. |   .0045431   .0120151     0.38   0.705    -.0190061    .0280922
             |
          lm |
         LD. |   .0152162   .1086739     0.14   0.889    -.1977807     .228213
             |
       _cons |   .0019504   .0019124     1.02   0.308    -.0017978    .0056986
-------------+----------------------------------------------------------------
```

D_r						
lgdp						
LD.	.4341641	.9106374	0.48	0.634	-1.350652	2.218981
r						
LD.	.5085677	.0977469	5.20	0.000	.3169874	.7001481
lm						
LD.	.1845222	.8840978	0.21	0.835	-1.548278	1.917322
_cons	-.0202984	.0155578	-1.30	0.192	-.0507912	.0101943
D_lm						
lgdp						
LD.	-.1972406	.098541	-2.00	0.045	-.3903774	-.0041037
r						
LD.	-.029395	.0105773	-2.78	0.005	-.0501261	-.0086639
lm						
LD.	.4472679	.0956691	4.68	0.000	.2597599	.634776
_cons	.0071036	.0016835	4.22	0.000	.0038039	.0104033

. vargranger → Granger Cuasality Test (グレンジャー因果性テスト)

Granger causality Wald tests

Equation	Excluded	chi2	df	Prob > chi2
D_lgdp	D.r	.14297	1	0.705
D_lgdp	D.lm	.0196	1	0.889
D_lgdp	ALL	.15705	2	0.924
D_r	D.lgdp	.22731	1	0.634
D_r	D.lm	.04356	1	0.835
D_r	ALL	.3039	2	0.859
D_lm	D.lgdp	4.0064	1	0.045
D_lm	D.r	7.7232	1	0.005
D_lm	ALL	10.798	2	0.005

→ gdp から m への因果関係（p 値 0.045）
r から m への因果関係（p 値 0.005）

→ 学部の教科書レベルの貨幣需要関数
m=f(gdp,r)，すなわち，m は gdp と r の関数

7.3 Impulse Response Function (インパルス応答関数):

$$\frac{\partial y_{i,t+m}}{\partial \epsilon_{j,t}}, \qquad m = 1, 2, \cdots,$$

where $i, j = 1, 2, \cdots, k$.

Example: AR(p) Process:

When y_t is stationary, we obtain:

$$\begin{aligned} y_t &= (I_k - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p)^{-1} \epsilon_t \\ &= \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots \end{aligned}$$

The impulse response function is:

$$\frac{\partial y_{i,t+k}}{\partial \epsilon_{j,t}} = \theta_{ij,k}, \qquad k = 1, 2, \cdots,$$

where $\theta_{ij,k}$ denotes the (i, j)th element of θ_k.

$$\begin{aligned} y_t &= \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots \\ &= PP^{-1}\epsilon_t + \theta_1 PP^{-1}\epsilon_{t-1} + \theta_2 PP^{-1}\epsilon_{t-2} + \cdots \\ &= \Omega_0 \eta_t + \Omega_1 \eta_{t-1} + \Omega_2 \eta_{t-2} + \cdots, \end{aligned}$$

where $\mathrm{V}(\eta_t) = I_k$, and $\Omega_i = \theta_i P$ for $i = 0, 1, 2, \cdots$ and $\Omega_0 = P$.

$$\frac{\partial y_{i,t+m}}{\partial \eta_{j,t}}, \qquad m = 1, 2, \cdots,$$

where $i, j = 1, 2, \cdots, k$.

$\Longrightarrow$ **Orthogonalized Impulse Response Function (直交化インパルス応答関数)**

Example:

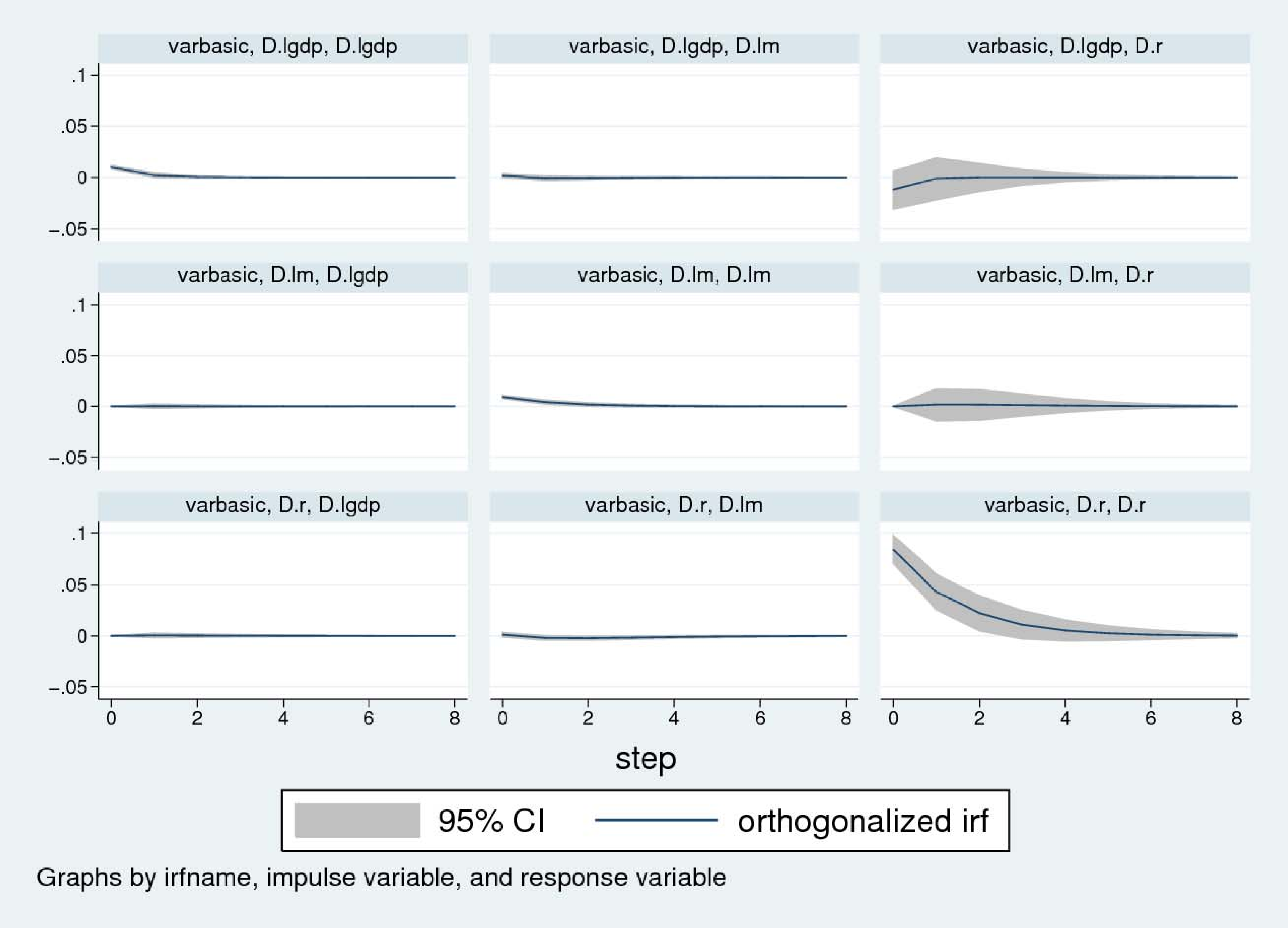
```
. varbasic d.lgdp d.r d.lm, lags(1)
Vector autoregression
```

```
Sample:  3 - 81                                    No. of obs      =        79
Log likelihood =  592.2334                         AIC             = -14.68945
FPE            =  8.38e-11                         HQIC            = -14.54526
Det(Sigma_ml)  =  6.18e-11                         SBIC            = -14.32954

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
D_lgdp               4     .010717   0.0422   3.480972   0.3232
D_r                  4     .087186   0.2553    27.0782   0.0000
D_lm                 4     .009434   0.2903   32.30929   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_lgdp       |
        lgdp |
         LD. |   .2031129   .1119361     1.81   0.070    -.0162778    .4225037
             |
           r |
         LD. |   .0045431   .0120151     0.38   0.705    -.0190061    .0280922
             |
          lm |
         LD. |   .0152162   .1086739     0.14   0.889    -.1977807     .228213
             |
       _cons |   .0019504   .0019124     1.02   0.308    -.0017978    .0056986
-------------+----------------------------------------------------------------
D_r          |
        lgdp |
         LD. |   .4341641   .9106374     0.48   0.634    -1.350652    2.218981
             |
           r |
```

LD.	.5085677	.0977469	5.20	0.000	.3169874	.7001481
lm						
LD.	.1845222	.8840978	0.21	0.835	-1.548278	1.917322
_cons	-.0202984	.0155578	-1.30	0.192	-.0507912	.0101943
D_lm						
lgdp						
LD.	-.1972406	.098541	-2.00	0.045	-.3903774	-.0041037
r						
LD.	-.029395	.0105773	-2.78	0.005	-.0501261	-.0086639
lm						
LD.	.4472679	.0956691	4.68	0.000	.2597599	.634776
_cons	.0071036	.0016835	4.22	0.000	.0038039	.0104033


varbasic, D.lgdp, D.lgdp
varbasic, D.lgdp, D.lm
varbasic, D.lgdp, D.r
varbasic, D.lm, D.lgdp
varbasic, D.lm, D.lm
varbasic, D.lm, D.r
varbasic, D.r, D.lgdp
varbasic, D.r, D.lm
varbasic, D.r, D.r
.1
.05
0
−.05
0
2
4
6
8
step
95% CI
orthogonalized irf
Graphs by irfname, impulse variable, and response variable