「Econometrics II」 Homework No.2

Deadline: January 24, 2022, PM23:59:59

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer to the email address: `tanizaki [at] econ.osaka-u.ac.jp`.
- The subject should be **Econome** or **計量**. Otherwise, your mail may go to the **trash box**.

1 Consider the following regression model:

$$y_i = x_i\beta + u_i$$

for $i = 1, 2, \cdots, n$. x_i and β are $1 \times k$ and $k \times 1$ vectors. u_i is mutually independently distributed with mean zero and variance σ^2.

(1) When x_i is correlated with u_i, show that the OLS estimator $\hat{\beta}$ is inconsistent.

(2) When x_i is not correlated with u_i, show that $\sqrt{n}(\hat{\beta} - \beta) \longrightarrow N(0, \sigma^2 M_{xx}^{-1})$, where the OLS estimator is $\hat{\beta}$, and $M_{xx} = \lim_{n\to\infty} \dfrac{1}{n}\sum_{i=1}^{n} x_i' x_i$.

(3) Suppose that we have another $1 \times k$ variable z_i, which is not correlated with u_i. Using z_i, construct a consistent estimator of β, denoted by $\tilde{\beta}$.

(4) Show that $\sqrt{n}(\tilde{\beta} - \beta) \longrightarrow N(0, \sigma^2 M)$. Obtain M, utilizing the followings:

$$M_{zx} = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} z_i' x_i \qquad M_{zz} = \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} z_i' z_i$$

(5) We consider testing whether x_i is correlated with u_i. Explain the testing procedure for choice of either $\hat{\beta}$ or $\tilde{\beta}$.

2 Consider the following regression:

$$y_{it} = x_{it}\beta + v_i + u_{it} \qquad i = 1, 2, \cdots, n \ \text{ and } \ t = 1, 2, \cdots, T$$

where v_i is the individual effect and the error term u_{it} is mutually independently distributed as $u_{it} \sim N(0, \sigma_u^2)$.

(6) Suppose that $v_i \sim N(0, \sigma_v^2)$ is independently distributed with each other. When x_{it} is not correlated with v_i and u_{it}, construct the likelihood function of y_{it} for $i = 1, 2, \cdots, n$ and $t = 1, 2, \cdots, T$. Obtain the maximum likelihood estimator of β, denoted by $\tilde{\beta}$.

(7) When v_i is correlated with x_{it}, we want to obtain a consistent estimator of β, which is denoted by $\hat{\beta}$. Derive $\hat{\beta}$.

(8) We need to choose $\tilde{\beta}$ or $\hat{\beta}$. Which estimator should be chosen? Explain the procedure.

3 Consider the following regression:

$$y_i = x_i\beta + u_i \qquad i = 1, 2, \cdots, n$$

where the error term u_i is correlated with x_i. Assume that u_i is independent of u_j for $i \neq j$.

(9) Consider another regression model: $x_i = z_i\Gamma + v_i$, where x_i, z_i, Γ and v_i are $1 \times k$, $1 \times p$, $p \times k$ and $1 \times k$ vectors or matries. Suppose that z_i is not correlated with u_i and v_i. Estimate Γ by OLS, which is denoted by $\hat{\Gamma}$. Utilizing $\hat{\Gamma}$, obtain a consistent estimator of β.

(10) Derive the aymptotic distribution of the consistent estimator of β given by (9).