TA session #4

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Theorem . Gamma distribution

$$f(x) = \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha\Gamma(\alpha)}, \qquad x > 0$$
$$= 0, \qquad x \leq 0 \tag{1}$$

Lemma . Gamma function

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx$$

*property: $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$

Theorem . χ^2 distribution when $\beta = 2$, $\alpha = \nu/2$

$$f(x) = \frac{x^{\frac{\nu}{2}-1}e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)}, \qquad x > 0$$
$$= 0, \qquad x \leq 0 \tag{2}$$

Theorem . Moment-generating function of χ^2 distribution

$$M_x(\theta) = (1-2\theta)^{-\nu/2}$$

Proof. The moment-generating function is

$$M_x(\theta) = E[e^{\theta x}] = \frac{1}{2^{\nu/2}\Gamma(\nu/2)}\int_0^\infty x^{\frac{\nu}{2}-1}e^{-\frac{x(1-2\theta)}{2}}dx\,,$$

let $z = \frac{x(1-2\theta)}{2}$, then

$$M_x(\theta) = \frac{2^{\nu/2}(1-2\theta)^{-\nu/2}}{2^{\nu/2}\Gamma(\nu/2)}\int_0^\infty z^{\frac{\nu}{2}-1}e^{-z}dz.$$
$$\Gamma(\nu/2) = \int_0^\infty z^{\frac{\nu}{2}-1}e^{-z}dz$$
$$M_x(\theta) = (1-2\theta)^{-\nu/2}$$

□

Lemma . Properties of moment

$$\mu_r = E(X^r)$$
$$M_x^{'}(0) = \mu_1, \quad M_x^{''}(0) = \mu_2, \quad ...$$

The mean and variance of $x \sim \chi^2$ are ν and 2ν.