2022/05/11

Econometrics I's Homework

Deadline: May 24, 2022, PM23:59:59

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- Send your answer (PDF or image file) to the email address: `tanizaki@econ.osaka-u.ac.jp`.
- The subject should be Econome 1 or 計量 1. Otherwise, your mail may go to the **trash box**.

1 Consider the following regression model:

$$y_t = \alpha + \beta X_t + u_t, \qquad t = 1, 2, \cdots, T,$$

where y_t and X_t denote dependent and independent variables, respectively. T is the sample size. u_1, u_2, $\cdots$, u_T are mutually independently distributed with mean zero and variance $\sigma^2 < \infty$. α and β are unknown parameters to be estimated.

(1) Derive the ordinary least squares estimators of α and β, which should be denoted by $\hat{\alpha}$ and $\hat{\beta}$.

(2) Obtain mean and variance of $\hat{\beta}$.

(3) Obtain mean and variance of $\hat{\alpha}$.

(4) Prove that $\hat{\beta}$ is a linear estimator of β.

(5) Prove that $\hat{\beta}$ is a linear unbiased estimator of β.

(6) Prove that $\hat{\beta}$ has minimum variance within a class of linear unbiased estimators

(7) Prove that $\hat{\beta}$ is a consistent estimator of β.

(8) Derive an asymptotic distribution of $\sqrt{T}(\hat{\beta} - \beta)$. Note that a distribution of u_t is not assumed.

(9) As an extra assumption, suppose that u_t is **normally** distributed for all t. Derive an exact distribution of $\hat{\beta}$, using the moment-generating function.

(10) In addition to (9), consider estimating σ^2. Show that $\dfrac{\hat{\beta} - \beta}{s\sqrt{\sum_{t=1}^{T} \omega_t^2}}$ is a t distribution with $T - 2$ degrees of freedom, where $s^2 = \dfrac{1}{T-2}\displaystyle\sum_{t=1}^{T}(y_t - \hat{\alpha} - \hat{\beta}X_t)^2$ and $\omega_t = \dfrac{X_t - \overline{X}}{\sum_{t=1}^{T}(X_t - \overline{X})^2}$. You may use the fact that the degree of freedom is $T - 2$.