2022/06/15

Econometrics I's Homework #3

Deadline: July 5, 2022, PM23:59:59 by the email, or PM18:00:00 in Tanizaki's mailbox

- The answer should be written in English or Japanese.
- Your name and student ID number should be included in your answer sheet.
- If the file size is less than **2.5MB**, send your answer to the email address: `tanizaki@econ.osaka-u.ac.jp` by PM23:59:59. The subject should be Econome or 計量. Otherwise, your mail may go to the **trash box**.
- If the file size is greater than **2.5MB**, print out your answer and put it in Tanizaki's mailbox (which is near Kyomu) by PM18:00:00.

1 Consider the regression model:

$$y = X\beta + u, \qquad u \sim N(0, \sigma^2 I_T),$$

where y, X, β and u are $T \times 1$, $T \times k$, $k \times 1$ and $T \times 1$.

Let $\hat{\beta}$ be the ordinary least squares estimator, and $\tilde{\beta}$ be the ordinary least squares estimator restricted to $R\beta = r$, where R and r are $G \times k$, $G \times 1$ and $G \leq k$. $\hat{u}$ and $\tilde{u}$ are defined as the OLS residual and the restricted OLS residual, respectively.

$$y = X\hat{\beta} + \hat{u}$$
$$y = X\tilde{\beta} + \tilde{u}, \qquad R\tilde{\beta} = r$$

(1) Derive the restricted OLS $\tilde{\beta}$.

(2) Show the following:

$$\frac{(\tilde{u}'\tilde{u} - \hat{u}'\hat{u})/G}{\hat{u}'\hat{u}/(T-k)} \sim F(G, T-k).$$

(3) Show the following:

$$\frac{(\hat{R}^2 - \tilde{R}^2)/G}{(1 - \hat{R}^2)/(T - k)} \sim F(G, T - k),$$

where $\hat{R}^2$ denotes the coefficient of determination from the unrestricted regression model and $\tilde{R}^2$ represents the coefficient of determination from the restricted regression model.

2 We consider estimating the following three production functions.

$$\log(Y_t) = \alpha_0 + \alpha_1 \log(K_t) + \alpha_2 \log(L_t) + u_t \tag{1}$$

$$\log(y_t) = \beta_0 + \beta_1 \log(k_t) + u_t \tag{2}$$

$$\log(Y_t) = \gamma_0 + \gamma_1 \log(K_t) + \gamma_2 \log(L_t) + \gamma_3 D_t + \gamma_4 D_t \log(K_t) + \gamma_5 D_t \log(L_t) + u_t \tag{3}$$

The estimation period is 1969 – 1997 (it's too old!). Let Y_t be GDP (10 billion yen, 1992 price), K_t be the net worth (10 billion yen, deflated by the GDP deflator), L_t be the number of employees, D_t be the dummy variable, which is one after 1991 and zero before 1991, y_t be the per capita GDP (10 billion yen, 1992 price, $y_t = Y_t/L_t$), and k_t be the per capita net worth (10 billion yen, deflated by the GDP deflator, $k_t = K_t/L_t$). The error terms $u_1, u_2, \cdots, u_T$ are mutually independently, identically and normally distributed.

The following estimation results are obtained.

$$\log(Y_t) = -\underset{(7.283)}{30.6242} + \underset{(5.054)}{.230042} \log(K_t) + \underset{(8.266)}{2.23565} \log(L_t)$$

$$R^2 = .986684, \quad \overline{R}^2 = .985659, \quad \hat{\sigma}^2 = .00141869$$

$$\log(y_t) = -\underset{(41.08)}{3.53058} + \underset{(19.62)}{.504043} \log(k_t)$$

$$R^2 = .934448, \quad \overline{R}^2 = .932020, \quad \hat{\sigma}^2 = .00354801$$

$$\log(Y_t) = -\underset{(3.630)}{34.6168} + \underset{(2.588)}{.204302} \log(K_t) + \underset{(4.155)}{2.48045} \log(L_t)$$

$$- \underset{(1.090)}{54.8287} D_t + \underset{(.4665)}{.243766} D_t \log(K_t) + \underset{(1.134)}{2.84275} D_t \log(L_t)$$

$$R^2 = .987960, \quad \overline{R}^2 = .985342, \quad \hat{\sigma}^2 = .00145010$$

Note that the values in the parentheses denote the t values, R^2 is the coefficient of determination, $\overline{R}^2$ is the adjusted R^2, and $\hat{\sigma}^2$ is the variance estimate of regression.

Answer the following questions.

(4) Test $H_0: \ \alpha_1 = \alpha_2 = 0$.

(5) Test whether the production function is homogeneous.

(6) Test whether the structural change occurred after 1991.

For each question, show the testing procedure in detail.

3

(7) Show that there exists P such that $\Omega = PP'$ when Ω is a positive definite matrix.

(8) Consider the following regression model:

$$y_t = x_t\beta + u_t, \qquad u_t \sim N(0, \sigma^2 z_t^2), \qquad t = 1, 2, \cdots, T,$$

where u_1, u_2, $\cdots$, u_T are mutually independent.

What is the variance-covariance matrix, denoted by $\sigma^2\Omega$, of $u = (u_1, u_2, \cdots, u_T)'$.

(9) Consider the following regression model:

$$y_t = x_t\beta + u_t, \qquad u_t = \rho u_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma^2), \qquad t = 1, 2, \cdots, T,$$

where ϵ_1, ϵ_2, $\cdots$, ϵ_T are mutually independent.

What is the variance-covariance matrix, denoted by $\sigma^2\Omega$, of $u = (u_1, u_2, \cdots, u_T)'$.

(10) Let b be the best linear unbiased estimator of β under the following regression model:

$$y = X\beta + u, \qquad u \sim N(0, \sigma^2\Omega),$$

where y, X, β and u are $T \times 1$, $T \times k$, $k \times 1$ and $T \times 1$.

Derive b.

(11) Consider the regression model in (3). We have two estimators, $\hat\beta$ and b, to estimate β.

Obtain $\mathrm{E}(\hat\beta)$ and $\mathrm{V}(\hat\beta)$.

Show that $\mathrm{V}(\hat\beta) - \mathrm{V}(b)$ is a positive definite matrix.