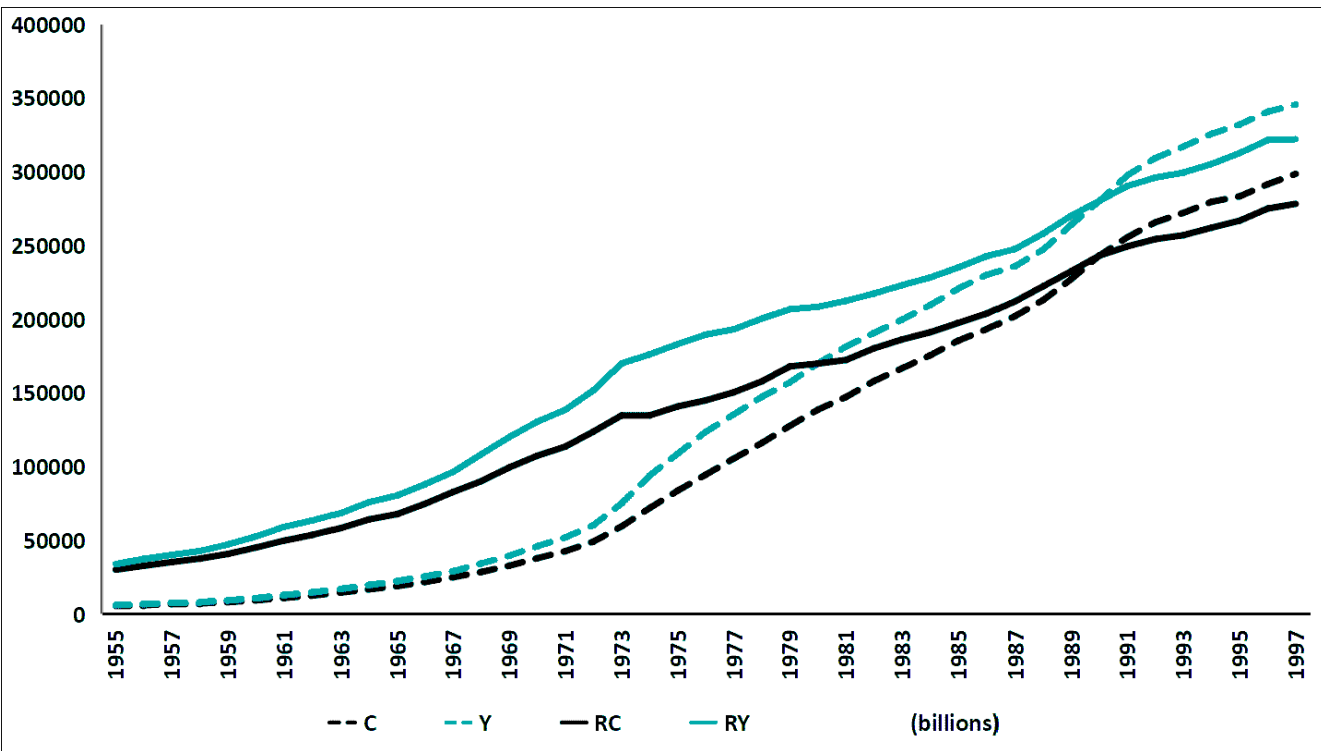


7 Example: F Distribution (Restricted OLS and Unrestricted OLS)

Date file $\Rightarrow$ cons99.txt (Next slide)

Each column denotes year, nominal household expenditures (家計消費, 10 billion yen), household disposable income (家計可処分所得, 10 billion yen) and household expenditure deflator (家計消費デフレーター, 1990=100) from the left.

year	cons	yd	price	year	cons	yd	price
1955	5430.1	6135.0	18.1	1977	105397.8	135318.4	70.1
1956	5974.2	6828.4	18.3	1978	115960.3	147244.2	73.5
1957	6686.3	7619.5	19.0	1979	127600.9	157071.1	76.0
1958	7169.7	8153.3	19.1	1980	138585.0	169931.5	81.6
1959	8019.3	9274.3	19.7	1981	147103.4	181349.2	85.4
1960	9234.9	10776.5	20.5	1982	157994.0	190611.5	87.7
1961	10836.2	12869.4	21.8	1983	166631.6	199587.8	89.5
1962	12430.8	14701.4	23.2	1984	175383.4	209451.9	91.8
1963	14506.6	17042.7	24.9	1985	185335.1	220655.6	93.9
1964	16674.9	19709.9	26.0	1986	193069.6	229938.8	94.8
1965	18820.5	22337.4	27.8	1987	202072.8	235924.0	95.3
1966	21680.6	25514.5	29.0	1988	212939.9	247159.7	95.8
1967	24914.0	29012.6	30.1	1989	227122.2	263940.5	97.7
1968	28452.7	34233.6	31.6	1990	243035.7	280133.0	100.0
1969	32705.2	39486.3	32.9	1991	255531.8	297512.9	102.5
1970	37784.1	45913.2	35.2	1992	265701.6	309256.6	104.5
1971	42571.6	51944.3	37.5	1993	272075.3	317021.6	105.9
1972	49124.1	60245.4	39.7	1994	279538.7	325655.7	106.7
1973	59366.1	74924.8	44.1	1995	283245.4	331967.5	106.2
1974	71782.1	93833.2	53.3	1996	291458.5	340619.1	106.0
1975	83591.1	108712.8	59.4	1997	298475.2	345522.7	107.3
1976	94443.7	123540.9	65.2				



Estimate using TSP 5.0.

```
LINE *****
|      1  freq a;
|      2  smpl 1955 1997;
|      3  read(file='cons99.txt') year cons yd price;
|      4  rcons=cons/(price/100);
|      5  ryd=yd/(price/100);
|      6  d1=0.0;
|      7  smpl 1974 1997;
|      8  d1=1.0;
|      9  smpl 1956 1997;
|     10  d1ryd=d1*ryd;
|     11  olsq rcons c ryd;
|     12  olsq rcons c d1 ryd d1ryd;
|     13  end;
*****
```

Equation 1

=====

Method of estimation = Ordinary Least Squares

Dependent variable: RCONS

Current sample: 1956 to 1997

Number of observations: 42

Mean of dependent variable = 149038.

Std. dev. of dependent var. = 78147.9

Sum of squared residuals = .127951E+10

Variance of residuals = .319878E+08

Std. error of regression = 5655.77

R-squared = .994890

Adjusted R-squared = .994762

Durbin-Watson statistic = .116873

F-statistic (zero slopes) = 7787.70

Schwarz Bayes. Info. Crit. = 17.4101

Log of likelihood function = -421.469

Estimated Standard

Variable	Coefficient	Error	t-statistic
C	-3317.80	1934.49	-1.71508
RYD	.854577	.968382E-02	88.2480

Read E+10, E+08 and E-02 as $\times 10^{10}$, $\times 10^8$ and $\times 10^{-2}$, respectively.

That is, .127951E+10 should be $.127951 \times 10^{10}$.

Equation 2

=====

Method of estimation = Ordinary Least Squares

Dependent variable: RCONS

Current sample: 1956 to 1997

Number of observations: 42

Mean of dependent variable = 149038.
Std. dev. of dependent var. = 78147.9
Sum of squared residuals = .244501E+09
Variance of residuals = .643423E+07
Std. error of regression = 2536.58
R-squared = .999024
Adjusted R-squared = .998946
Durbin-Watson statistic = .420979
F-statistic (zero slopes) = 12959.1
Schwarz Bayes. Info. Crit. = 15.9330
Log of likelihood function = -386.714

Estimated Standard

Variable	Coefficient	Error	t-statistic
C	4204.11	1440.45	2.91861
D1	-39915.3	3154.24	-12.6545
RYD	.786609	.015024	52.3561
D1RYD	.194495	.018731	10.3839

1. Equation 1

Significance test:

Equation 1 is:

$$RCONS = \beta_1 + \beta_2 RYD$$

$$H_0 : \beta_2 = 0$$

(No.1) t Test $\implies$ Compare 88.2480 and $t(42 - 2)$.

(No.2) F Test $\implies$ Compare $\frac{R^2/G}{(1 - R^2)/(n - k)} = \frac{.994890/1}{(1 - .994890)/(42 - 2)} = 7787.8$ and $F(1, 40)$. Note that $\sqrt{7787.8} = 88.2485$.

1% point of $F(1, 40) = 7.31$

$H_0 : \beta_2 = 0$ is rejected.

2. Equation 2:

$$\text{RCONS} = \beta_1 + \beta_2 \text{D1} + \beta_3 \text{RYD} + \beta_4 \text{RYD} \times \text{D1}$$

$$H_0 : \beta_2 = \beta_3 = \beta_4 = 0$$

$$F \text{ Test} \implies \text{Compare } \frac{R^2/G}{(1 - R^2)/(n - k)} = \frac{.999024/3}{(1 - .999024)/(42 - 4)} = 12965.5$$

and $F(3, 38)$.

$$1\% \text{ point of } F(3, 38) = 4.34$$

$$H_0 : \beta_2 = \beta_3 = \beta_4 = 0 \text{ is rejected.}$$

3. Equation 1 vs. Equation 2

Test the structural change between 1973 and 1974.

Equation 2 is:

$$\text{RCONS} = \beta_1 + \beta_2 \text{D1} + \beta_3 \text{RYD} + \beta_4 \text{RYD} \times \text{D1}$$

$$H_0 : \beta_2 = \beta_4 = 0$$

Restricted OLS $\implies$ Equation 1

Unrestricted OLS $\implies$ Equation 2

$$\frac{(\tilde{u}'\tilde{u} - e'e)/G}{e'e/(n-k)} = \frac{(.127951\text{E} + 10 - .244501\text{E} + 09)/2}{.244501\text{E} + 09/(42-4)} = 80.43$$

which should be compared with $F_{0.01}(2, 38)$.

1% point of F distribution with (2,38) degrees of freedom

$$\implies F_{0.01}(2, 38) = 5.211 < 80.43$$

$H_0 : \beta_2 = \beta_4 = 0$ is rejected. $\implies$ The structure was changed in 1974.

Or, using the coefficients of determination,

$$\begin{aligned}\frac{(\tilde{u}'\tilde{u} - e'e)/G}{e'e/(n-k)} &= \frac{(\hat{R}^2 - \tilde{R}^2)/G}{(1 - \hat{R}^2)/(n-k)} \\ &= \frac{(.999024 - .994890)/2}{(1 - .999024)/(42 - 4)} \\ &= 80.48 > 5.211 = F_{0.01}(2, 38)\end{aligned}$$

$\hat{R}^2$ and $\tilde{R}^2$ are the unrestricted R^2 and the restricted R^2 .

8 Generalized Least Squares Method (GLS, 一般化最小自乘法)

1. Regression model: $y = X\beta + u$, $u \sim N(0, \sigma^2\Omega)$

2. Heteroscedasticity (不等分散, 不均一分散)

$$\sigma^2\Omega = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_n^2 \end{pmatrix}$$

First-Order Autocorrelation (一階の自己相関, 系列相関)

In the case of time series data, the subscript is conventionally given by t , not i .

$$u_t = \rho u_{t-1} + \epsilon_t, \quad \epsilon_t \sim \text{iid } N(0, \sigma_\epsilon^2)$$

$$\sigma^2 \Omega = \frac{\sigma_\epsilon^2}{1 - \rho^2} \begin{pmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \cdots & \rho^{n-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & 1 \end{pmatrix}$$

$$V(u_t) = \sigma^2 = \frac{\sigma_\epsilon^2}{1 - \rho^2}$$

3. The Generalized Least Squares (GLS, 一般化最小二乘法) estimator of β ,

denoted by b , solves the following minimization problem:

$$\min_b (y - Xb)' \Omega^{-1} (y - Xb)$$

The GLSE of β is:

$$b = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} y$$

4. In general, when Ω is symmetric, Ω is decomposed as follows.

$$\Omega = A' \Lambda A$$

Λ is a diagonal matrix, where the diagonal elements of Λ are given by the eigen values.

A is a matrix consisting of eigen vectors.

When Ω is a positive definite matrix, all the diagonal elements of Λ are positive.

5. There exists P such that $\Omega = PP'$ (i.e., take $P = A' \Lambda^{1/2}$). $\implies P^{-1} \Omega P'^{-1} = I_n$

Multiply P^{-1} on both sides of $y = X\beta + u$.

We have:

$$y^{\star} = X^{\star} \beta + u^{\star},$$

where $y^{\star} = P^{-1}y$, $X^{\star} = P^{-1}X$, and $u^{\star} = P^{-1}u$.

The variance of $u^{\star}$ is:

$$V(u^{\star}) = V(P^{-1}u) = P^{-1}V(u)P'^{-1} = \sigma^2 P^{-1} \Omega P'^{-1} = \sigma^2 I_n.$$

because $\Omega = PP'$, i.e., $P^{-1} \Omega P'^{-1} = I_n$.

Accordingly, the regression model is rewritten as:

$$y^{\star} = X^{\star} \beta + u^{\star}, \quad u^{\star} \sim (0, \sigma^2 I_n)$$

Apply OLS to the above model.

Let b be as estimator of β from the above model.

That is, the minimization problem is given by:

$$\min_b (y^* - X^*b)'(y^* - X^*b),$$

which is equivalent to:

$$\min_b (y - Xb)' \Omega^{-1} (y - Xb).$$

Solving the minimization problem above, we have the following estimator:

$$\begin{aligned} b &= (X^{*'} X^*)^{-1} X^{*'} y^* \\ &= (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} y, \end{aligned}$$