Estimation of Regional Business Cycle in Japan using Bayesian Panel Spatial Autoregressive Probit Model

Kazuhiko Kakamu†  Hajime Wago†  Hisashi Tanizaki‡

Abstract

This paper considers the panel spatial autoregressive probit model from Bayesian point of view. The probit model is useful for qualitative (specifically, binary) data analyses. As it is well known, it is not easy to estimate the panel probit model by the maximum likelihood method. Therefore, in this paper, we consider the Markov chain Monte Carlo (MCMC) method to estimate the parameters as an alternative estimation method. Moreover, including the spatial autoregressive effect into the panel probit model we consider estimating the panel spatial autoregressive probit model. Our model (i.e., the panel spatial autoregressive probit model) is illustrated with simulated data set and it is compared with the panel probit model. Furthermore, taking into account the

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spatial interaction, we explore the regional business cycle across 47 prefecture data in Japan, where the estimation period is from 1979 to 2003. Using Japanese regional data, we conclude from the empirical results that the spatial interaction plays an important role in the regional business cycle.

**Key Words**: Business cycle; Markov chain Monte Carlo (MCMC); Panel spatial autoregressive probit model.

**JEL Classification**: C11, C15, C23, R11.

1 Introduction

The probit model has been widely used in qualitative data regression, especially in microeconometrics or business cycle analysis, and it has been extended to the panel analysis. Maddala (1987) gives excellent surveys of the panel probit model. Although the probit model is useful for qualitative data analysis, it is difficult to estimate the panel probit model by the maximum likelihood method, as pointed out by Bertschek and Lechner (1998). Therefore, alternative estimation methods such as the generalized method of moments (GMM) are proposed (see Bertschek and Lechner, 1998; Greene, 2004 and so on) for estimation of the panel probit model.

Since the seminal work by Anselin (1988), the spatial dependency becomes the concern of economic activity. Therefore, the probit model with the spatial dependency (i.e., the spatial autoregressive probit model) has been examined both analytically and empirically. For example, Holloway et al. (2002) apply the spatial probit model to high-yielding variety (HYV) adoption among Bangladeshi rice producers. Smith and LeSage (2004) also propose the spatial dependency with individuals in each location, and apply to the 1996 presidential election results for 3110 US counties. As it is difficult to estimate the spatial autoregressive probit model by the maximum likelihood method, all the past studies are estimated by the Markov chain Monte Carlo (MCMC) method (e.g., LeSage, 2000).

In the business cycle analysis, it is important to capture the turning point. Several models are proposed, i.e., probit and logit model (Maddala, 1992); sequential probability recursion model (Neftci, 1982); Markov switching model (Hamilton, 1989); dynamic Markov-switching factor model (Watanabe, 2003). However, they do not take into account the regional business cycle with the
spatial dependency. In this paper, incorporating the spatial dependency into the panel probit model, we consider the panel spatial autoregressive probit model as a regional business cycle model.

From Bayesian point of view, Kakamu and Wago (2008) examine the panel spatial autoregressive model. This paper considers the panel spatial autoregressive probit model in Bayesian framework, by extending LeSage (2000) and Kakamu and Wago (2008). Our model (i.e., the panel spatial autoregressive probit model) is illustrated with simulated data set and it is compared with the panel probit model, where the spatial autoregressive effect is ignored. We find from the simulation results that the bias appears in the parameter estimates when the estimated model is the panel probit model and the true model is the panel spatial autoregressive model. In addition, the bias increases as the spatial autoregressive effect is large, which implies that the spatial interaction plays an important role.

Furthermore, taking into account the spatial interaction, we also explore the regional business cycle in Japan, where 47 prefecture data during the periods from 1979 to 2003 are utilized. From the empirical results, (i) we confirm that the occurrence and collapse of the bubble economy is a common phenomena in Japan, i.e., the probability that the growth rate of regional IIP (Index of Industrial Production) is positive falls drastically in 1992-1993 and rises after 1993 (note that the bubble economy in Japan burst in 1991), (ii) the business cycle in Japan is very similar to the regional business cycle in each prefecture, and (iii) the posterior distributions of the spatial interaction do not include zero in the 95% credible intervals, when the business cycle phase (expansion or contraction) in most of the prefectures is similar to each other (i.e., 1979, 1984, 1988-1993, 1998 and 2001). Thus, we conclude from Japanese regional data that the spatial interaction plays an important role in the business cycle.

The rest of this paper is organized as follows. In the next section, we summarize the panel spatial autoregressive probit model. Section 3 discusses computational strategy of the MCMC methods. In Section 4 our approach is illustrated with simulated data set. Section 5 presents the empirical results based on the business index records in Japan, where 47 prefectures data from 1979 to 2003 are examined. Section 6 summarizes the results with concluding remarks.
2  Panel Spatial Autoregressive Probit Model

Let $y_{it}$ be a binary data and $x_{it}$ be a $1 \times k$ vector of exogenous variables, where $i$ indicates the region ($i = 1, 2, \cdots, n$) and $t$ represents time period ($t = 1, 2, \cdots, T$). Let $w_{ij}$ denote the spatial weight of the $j$th region in the $i$th region, which is given by: (i) $w_{ij} = 0$ for all $i = j$ and (ii) $w_{ij} = 1/m_i$ when the $j$th region is contiguous with the $i$th region and $w_{ij} = 0$ otherwise, where $m_i$ denotes the number of regions which is contiguous with the $i$th region. Note that we have $\sum_{j=1}^{n} w_{ij} = 1$ for all $i$. In the panel model, we consider that the unobservable component which is specific to the $i$th region affects the dependent variable (e.g., Kakamu and Wago, 2008). $\alpha_i$ is denoted by the unobservable component. Then, the panel spatial autoregressive probit model with parameters $\rho_t, \alpha_i$ and $\beta$ is written as follows:

$$y_{it} = \begin{cases} 1, & \text{if } z_{it} \geq 0, \\ 0, & \text{if } z_{it} < 0, \end{cases}$$

$$z_{it} = \rho_t \sum_{j=1}^{n} w_{ij} z_{jt} + \alpha_i + x_{it} \beta + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, 1),$$

(1)

where $z_{it}$ is a latent variable (see Tanner and Wang, 1987). In (1), $\rho_t$ represents the spatial interaction at time $t$. We may assume $\rho_t = \rho$. In the model above, however, it might be more plausible to consider that the spatial interaction varies over time. Let us define:

$$\rho = (\rho_1, \rho_2, \cdots, \rho_T)', \quad \alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)',$$

$$z_t = (z_{1t}, z_{2t}, \cdots, z_{nt})', \quad z = (z_1', z_2', \cdots, z_T'),$$

$$y_t = (y_{1t}, y_{2t}, \cdots, y_{nt})', \quad y = (y_1', y_2', \cdots, y_T'),$$

$$X_t = (x_{1t}', x_{2t}', \cdots, x_{nt}'), \quad X = (X_1', X_2', \cdots, X_T').$$

$W$ is called the spatial weight matrix (see, e.g., Anselin, 1988), where the element of $W$ in row $i$ and column $j$ is denoted by $w_{ij}$, defined above. Then, the likelihood function of the model (1) is given by:

$$L(y | \rho, \alpha, \beta, z, X, W) = \prod_{t=1}^{T} f(y_t | \rho_t, \alpha, \beta, z_t, X_t, W),$$

(2)

where

$$f(y_t | \rho_t, \alpha, \beta, z_t, X_t, W) = (2\pi)^{-\frac{n}{2}} |I_n - \rho_t W| \exp \left( -\frac{e_t' e_t}{2} \right)$$
\[
\times \prod_{i=1}^{n} \prod_{t=1}^{T} \left\{ y_{it} \mathbf{1}_{(0, \infty)}(z_{it}) + (1 - y_{it}) \mathbf{1}_{(-\infty, 0]}(z_{it}) \right\}.
\]

\(\mathbf{I}_n\) indicates the \(n \times n\) unit matrix, \(\mathbf{e}_t\) is given by \(\mathbf{e}_t = z_t - \rho_t W z_t - X_t \beta - \alpha\), and \(\mathbf{1}_{(a,b)}(x)\) denotes the indicator function, which takes 1 when \(x\) lies on the interval between \(a\) and \(b\).

### 3 Posterior Analysis

#### 3.1 Joint Posterior Distribution

Since we utilize Bayesian method for estimation, we adopt the following hierarchical prior:

\[
\pi(\rho, \alpha, \beta, \mu, \xi^2) = \left\{ \prod_{t=1}^{T} \pi(\rho_t) \right\} \left\{ \prod_{i=1}^{n} \pi(\alpha_i | \mu, \xi^2) \right\} \pi(\beta) \pi(\mu) \pi(\xi^2),
\]

which is used in Kakamu and Wago (2008), where \(\mu\) and \(\xi^2\) indicate the mean and variance of \(\alpha_i\), respectively.

Given the prior density \(\pi(\rho, \alpha, \beta, \mu, \xi^2)\) and the likelihood function (2), the joint posterior distribution can be expressed as:

\[
\pi(\rho, \alpha, \beta, \mu, \xi^2 | z, y, X, W) \propto \pi(\rho, \alpha, \beta, \mu, \xi^2) \prod_{t=1}^{T} f(y_t | \rho_t, \alpha, \beta, z_t, X_t, W). \tag{3}
\]

We assume the prior distributions, i.e., \(\pi(\rho_t), \pi(\alpha_i | \mu, \xi^2), \pi(\beta), \pi(\mu)\) and \(\pi(\xi^2)\), as follows:

\[
\rho_t \sim \mathcal{U}(-1, 1), \quad \alpha_i | \mu, \xi^2 \sim \mathcal{N}(\mu, \xi^2), \quad \beta \sim \mathcal{N}(\beta_0, \Sigma_0),
\]

\[
\mu \sim \mathcal{N}(\mu_0, \xi^2_0), \quad \xi^2 \sim \mathcal{IG}(\nu_0, \lambda_0^2),
\]

where \(\mathcal{IG}(a, b)\) denotes the inverse gamma distribution with scale parameter \(a\) and shape parameter \(b\).

#### 3.2 Posterior Simulation

From the joint posterior distribution (3), now we can implement the MCMC method. The Markov chain sampling scheme can be constructed from the full...
conditional distributions of \( \{z_t\}_{t=1}^T \), \( \{\rho_t\}_{t=1}^T \), \( \{\alpha_i\}_{i=1}^n \), \( \beta \), \( \mu \) and \( \xi^2 \), which are shown in Sections 3.2.1 – 3.2.3. Using the Gibbs sampler (e.g., see Gelfand and Smith, 1990), the random draws from the posterior distribution of \( \{z_t\}_{t=1}^T \), \( \{\rho_t\}_{t=1}^T \), \( \{\alpha_i\}_{i=1}^n \), \( \beta \), \( \mu \) and \( \xi^2 \) are generated.

3.2.1 Sampling \( \{z_t\}_{t=1}^T \)

In the case of the probit model, it is required to generate the latent variables \( \{z_t\}_{t=1}^T \). Let us define \( z_{-t} = \{z_1, \cdots, z_{t-1}, z_{t+1}, \cdots, z_T\} \), where \( z_t \) is excluded from \( \{z_t\}_{t=1}^T \). Tanner and Wang (1987) proposed the data augmentation method to generate the latent variables. We utilize Tanner and Wang (1987) in this paper. The full conditional distribution of \( z_t \) follows:

\[
z_t | \rho, \alpha, \beta, \mu, \xi^2, z_{-t}, y, X, W \sim MTN_{a_t \leq z_t \leq b_t}(\hat{z}_t, \hat{V}_t),
\]

(4)

with \( \hat{z}_t = \hat{V}_t^{-1}(I_n - \rho_t W)'(\alpha + X_t \beta) \), \( \hat{V}_t^{-1} = (I_n - \rho_t W)'(I_n - \rho_t W) \), \( a_t = (a_{1t}, a_{2t}, \cdots, a_{nt})' \) and \( b_t = (b_{1t}, b_{2t}, \cdots, b_{nt})' \). \( MTN_{a \leq z \leq b}(\mu, V) \) denotes the multivariate truncated normal distribution with mean \( \mu \) and scale matrix \( V \), where \( z \) is distributed between \( a \) and \( b \). In this paper, the truncation is set to be \( (a_{it}, b_{it}) = (0, \infty) \) when \( y_{it} = 1 \) and \( (a_{it}, b_{it}) = (-\infty, 0) \) when \( y_{it} = 0 \). The multivariate truncated normal random variable \( z_t \) is sampled by the Metropolis-Hastings algorithm, which is proposed by Miyawaki (2008).

3.2.2 Sampling \( \{\rho_t\}_{t=1}^T \)

Define \( \rho_{-t} = \{\rho_1, \cdots, \rho_{t-1}, \rho_{t+1}, \cdots, \rho_T\} \), where \( \rho_t \) is excluded from \( \rho \). From (3), the full conditional distribution of \( \rho_t \) is written as:

\[
p(\rho_t | \rho_{-t}, \alpha, \beta, \mu, \xi^2, z, y, X, W) \propto |I_n - \rho_t W| \exp \left( -\frac{e_t'e_t}{2} \right),
\]

(5)

with \( e_t = (I_n - \rho_t W)z_t - X_t \beta - \alpha \). Using the Metropolis algorithm (e.g., see Tierney, 1994), a random draw of \( \rho_t \) is sampled from the conditional distribution \( p(\rho_t | \rho_{-t}, \alpha, \beta, \mu, \xi^2, z, y, X, W) \).

The following Metropolis step is used: (i) sample \( \rho_t^{new} \) from:

\[
\rho_t^{new} = \rho_t^{old} + c_t \eta_t, \quad \eta_t \sim N(0, 1),
\]

(6)
where \( c_t \) is called the tuning parameter and \( \rho_t^{old} \) denotes the random draw previously sampled, (ii) evaluate the acceptance probability:

\[
\omega(\rho_t^{old}, \rho_{t}^{new}) = \min \left( \frac{p(\rho_{t}^{new}|\rho_{-t}, \alpha, \beta, \mu, \xi^2, z, y, X, W)}{p(\rho_{t}^{old}|\rho_{-t}, \alpha, \beta, \mu, \xi^2, z, y, X, W)}, 1 \right),
\]

and (iii) set \( \rho_t = \rho_{t}^{new} \) with probability \( \omega(\rho_t^{old}, \rho_{t}^{new}) \) and \( \rho_t = \rho_t^{old} \) otherwise. The proposal random draw of \( \rho_t \) generated from (6) takes the real value within the interval \((-\infty, \infty)\), although the prior distribution of \( \rho_t \) lies on the interval between \(-1\) and 1 (see Section 3.1). If the candidate of \( \rho_t \) does not lie on the interval \((-1, 1)\), the conditional posterior should be zero and accordingly the proposal value is rejected with probability one (see Chib and Greenberg, 1998).

We need the tuning parameter \( c_t \) in sampling \( \rho_t \). In the numerical example discussed below, we choose the tuning parameter such that the acceptance rate becomes between 0.4 and 0.6 (see Holloway et al., 2002).

### 3.2.3 Sampling the Other Parameters

The full conditional distribution of \( \beta \) is:

\[
\beta|\rho, \alpha, \mu, \xi^2, z, y, X, W \sim N(\hat{\beta}, \hat{\Sigma}),
\]

with \( \hat{\beta} = \hat{\Sigma} [X'(z - (\Psi \otimes W)z - \Delta \alpha) + \Sigma^{-1}_0 \beta_0] \), \( \Delta = (1_T \otimes I_n) \), \( 1_T \) is a \( T \times 1 \) unit vector, and \( \Sigma = (X'X + \Sigma^{-1}_0)^{-1} \). The element of \( \Psi \) in row \( t \) and column \( s \) is given by \( \rho_t \) for \( t = s \) and zero for \( t \neq s \), i.e., \( \Psi = \text{diag}(\rho) \).

Given \( z, \rho \) and \( \beta \), (1) is written as:

\[
z_{it} - \sum_{j=1}^n \rho_t w_{ij} z_{jt} - x_{it\beta} = \epsilon_{it}, \quad \epsilon_{it} \sim N(\alpha_i, 1).
\]

Let us define \( \alpha_{-i} = \{\alpha_1, \cdots, \alpha_{i-1}, \alpha_{i+1}, \cdots, \alpha_n\} \), where \( \alpha_i \) is excluded from \( \alpha \). The full conditional distribution of \( \alpha_i \) follows:

\[
\alpha_i|\rho, \alpha_{-i}, \beta, \mu, \xi^2, z, y, X, W \sim N(\hat{\alpha}_i, \hat{\xi}^2),
\]

with \( \hat{\alpha}_i = \hat{\xi}^2 \{\sum_{t=1}^T(z_{it} - \sum_{j=1}^n \rho_t w_{ij} z_{jt} - x_{it\beta}) + \xi^{-2} \mu \} \) and \( \hat{\xi}^2 = (T + \xi^{-2})^{-1} \).

Full conditional distributions of \( \mu \) and \( \xi^2 \) are given by:

\[
\mu|\rho, \alpha, \beta, \xi^2, z, y, X, W \sim N(\hat{\mu}, \hat{\sigma}^2),
\]

\[
\xi^2|\rho, \alpha, \beta, \mu, z, y, X, W \sim IG(\frac{\nu}{2}, \frac{\lambda}{2}),
\]
with \( \hat{\mu} = \hat{\sigma}^2 (\xi^{-2} \sum_{i=1}^{n} \alpha_i + \xi_0^{-2} \mu_0) \), \( \hat{\sigma}^2 = (\xi^{-2} n + \xi_0^{-2})^{-1} \), \( \hat{\nu} = n + \nu_0 \) and \( \hat{\lambda} = (\alpha - \mu)'(\alpha - \mu) + \lambda_0 \).

Thus, from (4), (5), (7) – (10), the random draws of \( (z, \rho, \alpha, \beta, \mu, \xi^2) \) are easily sampled from Gibbs sampler (e.g., Gelfand and Smith, 1990).

4 Numerical Example by Simulated Data

To illustrate the panel spatial autoregressive probit model using Bayesian estimation, \( y_{it} \) and \( z_{it} \) \( (i = 1, 2, \ldots, n \) and \( t = 1, 2, \ldots, T) \) are generated from:

\[
y_{it} = \begin{cases} 
1, & \text{if } z_{it} \geq 0, \\
0, & \text{if } z_{it} < 0, 
\end{cases}
\]

\[
z_{it} = \rho \sum_{j=1}^{n} w_{ij} z_{jt} + \alpha_i + 1.0 x_{1it} + 1.0 x_{2it} + \epsilon_{it}, \quad \epsilon_{it} \sim \mathcal{N}(0, 1),
\]

where \( \alpha_i \) is sampled from the normal distribution \( \mathcal{N}(0, 2) \), and \( x_{it} = (x_{1it}, x_{2it}) \) is generated as a vector of two standard normal variates, i.e., \( k = 2 \). In this numerical example, we construct the spatial weight matrix \( W \) as: (i) generate \( w^*_{ij} \) for \( i > j \) from Bernoulli distribution with probability of success 0.2, (ii) set \( w^*_{ji} = w^*_{ij} \) for \( i \neq j \) and \( w^*_{ij} = 0 \) for \( i = j \), and (iii) compute \( \sum_{j=1}^{n} w^*_{ij} = m_i \) and \( w_{ij} = w^*_{ij}/m_i \) for all \( i, j \). To see the effect of the spatial autocorrelation, we consider the cases of \( \rho = 0.3, 0.6, 0.9 \). In this simulation study, \( \rho \) is assumed to be constant over time. For the number of regions, \( n = 50 \) is taken. To see the effect of \( T \), we consider the cases of \( T = 5, 10, 15 \). For the prior distributions, we set the hyper-parameters as follows:

\[
\beta_0 = 0, \quad \Sigma_0 = 100 \times I_k, \quad \mu_0 = 0, \quad \xi_0^2 = 100, \quad \nu_0 = 2, \quad \lambda_0 = 0.01. \quad (11)
\]

Since it is interesting to see the cases where the spatial autocorrelation is ignored, we estimate the panel spatial autoregressive probit model for two cases: \( \rho = 0 \) and \( \rho \neq 0 \). Using the simulated data, we implement the MCMC algorithm, where 300,000 iterations are performed and the first 100,000 iterations are discarded as the initial burn-in period. Taking every ten draws out of the remaining iterations (i.e., 20,000 random draws are chosen), we obtain the posterior statistics for the parameters. Based on the convergence diagnostic test proposed by Geweke (1992), we make sure whether the 100,000 burn-in iterations are practically large enough. In the burn-in iterations, the tuning
Table 1: Simulated Data: Posterior Means and Standard Deviations

<table>
<thead>
<tr>
<th>$T$</th>
<th>True Value</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.6$</th>
<th>$\rho = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$\mu$</td>
<td>-0.131 (0.208)</td>
<td>-0.027 (0.263)</td>
<td>0.064 (0.194)</td>
</tr>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>0.773 (0.143)</td>
<td>1.290 (0.245)</td>
<td>1.136 (0.264)</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>1.063 (0.171)</td>
<td>1.368 (0.251)</td>
<td>1.254 (0.272)</td>
</tr>
<tr>
<td></td>
<td>$\xi^2$</td>
<td>1.674 (0.727)</td>
<td>2.908 (1.461)</td>
<td>1.584 (1.079)</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>0.290 (0.204)</td>
<td>0.342 (0.169)</td>
<td>0.900 (0.031)</td>
</tr>
<tr>
<td>10</td>
<td>$\mu$</td>
<td>-0.030 (0.177)</td>
<td>-0.067 (0.214)</td>
<td>-0.047 (0.214)</td>
</tr>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>0.847 (0.103)</td>
<td>1.132 (0.132)</td>
<td>1.140 (0.144)</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>0.870 (0.101)</td>
<td>1.151 (0.130)</td>
<td>0.985 (0.143)</td>
</tr>
<tr>
<td></td>
<td>$\xi^2$</td>
<td>1.372 (0.402)</td>
<td>2.163 (0.679)</td>
<td>2.121 (0.697)</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>0.329 (0.137)</td>
<td>0.561 (0.086)</td>
<td>0.887 (0.023)</td>
</tr>
<tr>
<td>15</td>
<td>$\mu$</td>
<td>-0.031 (0.203)</td>
<td>-0.018 (0.212)</td>
<td>-0.042 (0.194)</td>
</tr>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>0.921 (0.092)</td>
<td>0.983 (0.095)</td>
<td>1.025 (0.114)</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>0.917 (0.087)</td>
<td>0.942 (0.094)</td>
<td>1.074 (0.114)</td>
</tr>
<tr>
<td></td>
<td>$\xi^2$</td>
<td>1.841 (0.555)</td>
<td>2.133 (0.613)</td>
<td>1.784 (0.545)</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>0.169 (0.117)</td>
<td>0.584 (0.077)</td>
<td>0.898 (0.018)</td>
</tr>
</tbody>
</table>

(b) Panel Spatial Autoregressive Probit Model with Restriction ($\rho = 0$)

<table>
<thead>
<tr>
<th>$T$</th>
<th>True Value</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.6$</th>
<th>$\rho = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$\mu$</td>
<td>-0.167 (0.213)</td>
<td>-0.051 (0.277)</td>
<td>0.160 (0.090)</td>
</tr>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>0.754 (0.139)</td>
<td>1.237 (0.241)</td>
<td>0.451 (0.084)</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>1.036 (0.165)</td>
<td>1.350 (0.259)</td>
<td>0.560 (0.096)</td>
</tr>
<tr>
<td></td>
<td>$\xi^2$</td>
<td>1.523 (0.666)</td>
<td>2.887 (1.504)</td>
<td>0.025 (0.045)</td>
</tr>
<tr>
<td>10</td>
<td>$\mu$</td>
<td>0.057 (0.182)</td>
<td>-0.077 (0.211)</td>
<td>0.038 (0.113)</td>
</tr>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>0.846 (0.104)</td>
<td>1.066 (0.124)</td>
<td>0.620 (0.079)</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>0.863 (0.100)</td>
<td>1.035 (0.117)</td>
<td>0.476 (0.073)</td>
</tr>
<tr>
<td></td>
<td>$\xi^2$</td>
<td>1.335 (0.389)</td>
<td>1.871 (0.581)</td>
<td>0.411 (0.144)</td>
</tr>
<tr>
<td>15</td>
<td>$\mu$</td>
<td>-0.058 (0.202)</td>
<td>-0.105 (0.196)</td>
<td>-0.364 (0.102)</td>
</tr>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>0.913 (0.092)</td>
<td>0.905 (0.089)</td>
<td>0.526 (0.062)</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>0.899 (0.085)</td>
<td>0.831 (0.082)</td>
<td>0.558 (0.060)</td>
</tr>
<tr>
<td></td>
<td>$\xi^2$</td>
<td>1.781 (0.525)</td>
<td>1.688 (0.468)</td>
<td>0.365 (0.117)</td>
</tr>
</tbody>
</table>
parameter $c_t$ is chosen between 0.024 and 0.146. Ox version 4.1 (see Doornik, 2006) is used to obtain all the results in this paper.

Table 1 shows the posterior estimates of the parameters. (a) in Table 1 represents the panel spatial autoregressive probit model with no restriction (i.e., $\rho \neq 0$), while (b) in the table indicates the panel spatial autoregressive probit model with restriction (i.e., $\rho = 0$), which is equivalent to the panel probit model. The values in the parentheses give us the standard deviations based on the random draws generated from the posterior distributions. It is shown from Table 1 that the posterior estimates of (a) are not too different from those of (b) in the case of $\rho = 0.3$. However, as $\rho$ increases, distinct differences between (a) and (b) appear for all the estimates of $\mu$, $\beta$ and $\xi^2$. As it is easily expected, we can observe serious biases in the restricted model (b) as $\rho$ is large. In the case of the panel spatial autoregressive model (a), the mean estimates of $\beta$, $\xi^2$ and $\rho$ are not too different from the true values. That is, under the condition that there exists the spatial dependency in the true model, the biases of the estimated posterior means become serious if we estimate the panel probit model in which the spatial autocorrelation is ignored. In addition, we can see that all the posterior means go to the true values as $T$ is large (i.e., see the case of $T = 15$).

The posterior distributions of $\rho$ are displayed in Figure 1. We cannot find the features of the empirical distributions at a glance. However, it is shown
that the posterior distributions are skewed to the left. Note that in each figure
Skew indicates skewness of the empirical distribution based on the MCMC
draws. As $\rho$ is large, the skewness to the left becomes large in absolute value.

5 Regional Business Cycle in Japan

As an empirical application, we consider the regional business cycle across 47
prefectures in Japan (i.e., $n = 47$) during the periods from 1979 to 2003 (i.e.,
$T = 25$). Let $\text{IIP}_it$ be Index of Industrial Production by Region, which is taken
from Ministry of Economy, Trade and Industry. The dependent variable $y_{it}$
takes one if the growth rate of $\text{IIP}_it$ is positive and zero otherwise. The busi-
ness cycle is not observed in general. Therefore, when the growth rate of $\text{IIP}_it$
is positive, we consider that the $i$th region at time $t$ is in the expansion period
and then $y_{it} = 1$ is set as a proxy variable of the business index. Contrarily,
when the growth rate of $\text{IIP}_it$ is negative, the $i$th region at time $t$ is in the con-
traction period and accordingly $y_{it} = 0$ is set. As explanatory variables $x_{it}$, we
take Scheduled Cash Earnings for Male Workers (WAGE) and Job Offers-to-
Applicants Rate (JOB) from Ministry of Health Labor and Welfare, Loans and
Discounts Outstanding of Domestically Licensed Banks by Prefecture (BANK)
from Bank of Japan, and Index of Financial Potential (FINANCE) from Min-
istry of Internal Affairs and Communications. Note that FINANCE is given
by the average of the revenue-expenditure ratio data in the past three years
for each prefecture. FINANCE shows a large value when a prefecture is fi-
ancially generous. All the data are also available from Statistical Informa-
tion Institute for Consulting and Analysis (called Sinfornica, Japanese web-
site: http://www.sinfonica.or.jp/datalist/index.html). Furthermore,
all the data except for FINANCE are transformed into per capita data, and
all the data are converted into the percent change data. As discussed in Section
2, the weight matrix $W$ consists of contiguity dummy variables, proposed by
Kakamu et al. (2008),\(^1\) and the average number of dummy variables is about

\(^1\)Except for Okinawa prefecture, all Japanese prefectures are located in the four major
islands, i.e., Hokkaido, Honshu, Shikoku and Kyushu. We consider that these four islands
are connected with each other by trains and roads, despite the fact that the four islands
are geographically separated. For example, we consider that Hokkaido is contiguous with
Honshu through the Seikan railway tunnel. Honshu and Shikoku are contiguous with each
other through the Awaji and Seto Bridges, and Kyushu is also contiguous with Honshu
through the Kanmon Tunnel and Bridge. Okinawa is the only one prefecture which is
independent of all the other prefectures. For Japanese prefectures, note as follows. Shikoku
Table 2: Empirical Results

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>2.5%</th>
<th>97.5%</th>
<th>CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.156</td>
<td>0.073</td>
<td>0.015</td>
<td>0.301</td>
<td>0.770</td>
</tr>
<tr>
<td>WAGE</td>
<td>-0.170</td>
<td>1.303</td>
<td>-2.780</td>
<td>2.340</td>
<td>0.304</td>
</tr>
<tr>
<td>JOB</td>
<td>3.142</td>
<td>0.440</td>
<td>2.308</td>
<td>4.034</td>
<td>0.196</td>
</tr>
<tr>
<td>BANK</td>
<td>3.516</td>
<td>1.340</td>
<td>0.930</td>
<td>6.154</td>
<td>0.182</td>
</tr>
<tr>
<td>FINANCE</td>
<td>-0.362</td>
<td>1.629</td>
<td>-3.546</td>
<td>2.827</td>
<td>0.353</td>
</tr>
<tr>
<td>$\xi^2$</td>
<td>0.026</td>
<td>0.025</td>
<td>0.002</td>
<td>0.089</td>
<td>0.115</td>
</tr>
</tbody>
</table>

4. Using the same hyper-parameters as in Section 4, the MCMC algorithm is implemented, using 300,000 iterations and discarding the first 100,000 iterations as the initial burn-in period. Out of the remaining draws (i.e., 200,000 draws), we take every ten draws to obtain the posterior statistics for the parameters. The tuning parameters (i.e., $c_t$, $t = 1, 2, \ldots, T$) are chosen between 0.027 and 0.424 in the burn-in period.

Table 2 shows the estimation results, where Mean, SD and CD represent the posterior mean, the standard deviation and Geweke’s convergence diagnostics (Geweke, 1992), respectively. CD represents the $p$ value based on the test statistic on difference between two sample means (i.e., dividing all the generated random draws into three parts, we compute two sample means from the first 20% and last 50% of the random draws), where the test statistic is asymptotically distributed as a standard normal random variable. We judge that the random draws generated by MCMC do not converge to the random draws generated from the target distribution when CD is less than 0.05 or greater than 0.95. See Geweke (1992) for detail discussion on CD. From the estimation results in Table 2, we can conclude that the random draws generated by MCMC converge to the random draws generated from the target distribution, because CD is between 0.05 and 0.95.

It might be expected in Table 2 that all of JOB, BANK and FINANCE are positively correlated with the business cycle indicator $y_{it}$. That is, we consists of four prefectures, i.e., Tokushima, Kagawa, Ehime and Kochi prefectures (see Figure 9). Kyusyu includes seven prefectures, i.e., Fukuoka, Saga, Nagasaki, Kumamoto, Oita, Miyazaki and Kagoshima prefectures (see Figure 10). Honshu consists of 34 prefectures (i.e., all the prefectures except for Hokkaido in Figure 3 and all the prefectures in Figures 4–8).
can consider that a prefecture is in the expansion period when JOB, BANK or FINANCE is large. A large JOB indicates that there are a lot of job offers relative to applicants, i.e., JOB represents mobility in labor market. A large BANK represents that a bank can lend a lot of money to residents, i.e., BANK indicates liquidity in money market. A large FINANCE indicates that a prefecture is financially generous. As for WAGE, there are two aspects: (i) an increase in WAGE results in tight labor market and accordingly the business cycle phase switches from expansion to recession due to a decrease in profits, and (ii) the expansion in the business cycle continues because of an increase in WAGE. Therefore, WAGE has both positive and negative effects to the business cycle indicator $y_{it}$.

The estimation results in Table 2 are as follows. JOB and BANK affect the business cycle index (i.e., $y_{it}$) positively, because the posterior means of JOB and BANK are estimated positively and the 95% credible intervals do not include zero. The mobility in labor market and the liquidity in money market lead to expansion in the business cycle. Thus, from the results in Table 2, we can conclude that the business cycle index (i.e., $y_{it}$) positively depend on JOB and BANK. However, WAGE and FINANCE do not affect the business index, because the 95% credible intervals of WAGE and FINANCE include zero. As mentioned above, WAGE includes both positive and negative effects to the business cycle, and therefore we obtain the estimation results that WAGE is positive but insignificant. As for FINANCE, we expect that there is a positive effect to the business cycle, but we have the result that FINANCE does not affect the business cycle. For one reason, as mentioned above, the FINANCE data are constructed by the average of the revenue-expenditure ratio data in the past three years for each prefecture, and accordingly they show the smoothed movements, compared with the business cycle fluctuations. Therefore, FINANCE does not necessarily reflect the business cycle fluctuations.

Figure 2 describes the movements of $\rho_t$ (i.e., regional interdependency at time $t$), where the solid line represents the posterior means of $\rho_t$, $t = 1, 2, \cdots, T$, and the two dotted lines are given by 95% credible intervals. According to Cabinet Office, Government of Japan, the trough-to-peak (i.e., expansion) periods in Japan are given by October 1977 to February 1980, February 1983 to January 1985, November 1986 to February 1991, October 1993 to May 1997, and January 1999 to November 2000, which correspond to the shaded areas in Figure 2. We can find that the spatial dependencies in Japanese business cycle vary over time, which are positively estimated in 1979-1980, 1984, 1988-1993, 1997-1998 and 2001, because the 95% credible in-
Intervals of $\rho_t$ do not include zero for these periods. According to Figure 2, the significantly positive estimates of $\rho_t$ in 1979-1980, 1984, 1988-1991 and 1997 are observed in the expansion periods (i.e., the shaded areas), while those in 1992-1993, 1998 and 2001 are in the contraction periods. It is known that in Japan the bubble burst in 1991. Except for 1997, $\rho_t$ is significantly positive in the expansion periods before the bubble burst, but in the contraction periods after the bubble burst. The relationship between the spatial autoregressive parameter estimates (Figure 2) and the regional business cycles (Figures 3 – 10) will be discussed later.

Figures 3 – 10 show the probability that the business cycle in each prefecture is in the expansion periods, i.e., the probability that $z_{it} > 0$ occurs given prefecture $i$ and time $t$. There, the number of the random draws of $z_{it}$ greater than zero is divided by the number of all the generated random draws of $z_{it}$. Note that $z_{it}$ is generated from (4). We divide Japan into eight regions, following Kakamu and Fukushige (2005), and discuss their features for each region. The shaded areas in Figures 3 – 10 represent the trough-to-peak periods in Japan, which are shaded in the same areas as Figure 2. We can observe from Figures 3 – 10 that the business cycle in Japan is very similar to the regional business cycle in each prefecture, because a lot of the prefectures are in the shaded areas when the expansion probability is large.
Figure 3: Hokkaido and Tohoku Region

Figure 4: Kanto Region

Figure 5: Hokuriku and Koshinetsu Region

Figure 6: Tokai Region
Figure 7: Kinki Region

Figure 8: Chugoku Region

Figure 9: Shikoku Region

Figure 10: Kyushu and Okinawa Region
Hokkaido and Tohoku Region: Each line in Figure 3 shows the movements of the probability that the business cycle is in the expansion period. Hokkaido and Tohoku region is located in the most north of Japan. The business cycle in this region shows the following features. In 1979, 1984, 1988-1992, 1998 and 2000-2001, the expansion probabilities take almost the same values for all the prefectures in Hokkaido and Tohoku region. Note that the periods from 1988 to 1992 (around the 3rd shaded area in the figure) correspond to the bubble economy in Japan. On the other hand, the expansion probabilities are different for each prefecture in the other periods. Especially, the expansion probabilities between 1992 and 1997 are quite different for each prefecture.

Kanto Region: In Figure 4, the movements of the expansion probabilities in Kanto region are displayed. Note that Tokyo belongs to this region. From the figure, each prefecture in Kanto region takes almost the same expansion probabilities in 1979, 1984, 1988-1989, 1992-1993, 1995 and 1998. However, we obtain the results that Kanto region is different from Hokkaido and Tohoku region in 1990-1991 and 2000. The prefectures in Kanto region take different values in 1990-1991 and 2000, but all the prefectures in Hokkaido and Tohoku region (Figure 3) take similar values.

Hokuriku and Koshinetsu Region: Figure 5 displays the movements of the expansion probabilities in Hokuriku and Koshinetsu region. In 1979, 1983-1985, 1988-1993, 1998 and 2000-2001, we have similar expansion probabilities for all the prefectures in Hokuriku and Koshinetsu region.

Tokai Region: In Figure 6, the movements of the expansion probabilities in Tokai region are displayed. In 1979-1980, 1983-1985, 1987-1989, 1992-1993, 1995-1998, 2000 and 2003, all the expansion probabilities are similar for all the prefectures.

Kinki Region: Figure 7 shows the expansion probabilities in Kinki region. In 1979-1980, 1984, 1988-1990, 1992, 1997-1998 and 2001, all the expansion probabilities in Kinki region take similar values. The remarkable point is the movement of the expansion probabilities in Osaka. Osaka economy is the second largest in Japan, but the expansion probabilities after 1998 are less than 0.5. The recession in Osaka economy is serious after 1998.
Table 3: Business Cycle Pattern in Each Region

| Region | Year | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 00 | 01 | 02 | 03 |
|--------|------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| HT (Fig.3) | o | o | o | o | o | o | o | × | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Ka (Fig.4) | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| HK (Fig.5) | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| To (Fig.6) | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Ki (Fig.7) | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Ch (Fig.8) | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Sh (Fig.9) | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| KO (Fig.10)* | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |

(i) KT, Ka, HK, To, Ki, Ch, Sh and KO indicate Hokkaido and Tohoku, Kanto, Hokuriku and Koshinetsu, Tokai, Kinki, Chugoku, Shikoku, and Kyushu and Okinawa regions, respectively.

(ii) See Figures 3 – 10 for the prefectures included in each region.

(iii) * represents the results in which Okinawa prefecture is excluded.

Chugoku Region: Figure 8 displays the expansion probabilities in Chugoku region. In 1979, 1984, 1988-1989, 1993, 1998 and 2000-2001, all the expansion probabilities are similar. However, during the rest of the periods, the expansion probabilities are quite different for each prefecture.

Shikoku Region: Figure 9 indicates the expansion probabilities in Shikoku region. In 1979, 1984, 1988-1992, 1995, 1998 and 2001, we can observe that the expansion probabilities are similar for all the prefectures in Shikoku region.

Kyushu and Okinawa Region: Figure 10 displays the expansion probabilities in Kyushu and Okinawa region. Except for Okinawa, the expansion probabilities in 1984, 1990-1991, 1998 and 2003 are similar. For the rest of the periods, the expansion probabilities take very different values for all the prefectures in Kyushu and Okinawa region. In addition, Okinawa is very different from the other prefectures, because the expansion probabilities between 1991 and 2001 are less than 0.5 and the cases where the expansion probability is greater than 0.5 are only 9 out of 25 years. It might be plausible to conclude that the recession in Okinawa is very serious during the periods from 1979 to 2003.
Based on Figures 3 – 10, Table 3 summarizes the business cycle pattern within each region. ◦ indicates that all the prefectures within each region takes similar values in the expansion period, where all the expansion probabilities are greater than 0.5. × represents that all the prefectures within each region takes similar values in the recession period, where all the expansion probabilities are less than 0.5.

Compared with Figure 2 and Table 3, when most of the regions are in expansion periods (in Table, 3 see the columns where the number of ◦ is at least 5), the spatial autocorrelation estimates are significantly greater than zero (see Figure 2). That is, in Table 3 we have at least five ◦’s in 1979, 1984 and 1988 – 1990, and in Figure 2 the spatial autocorrelation estimates are significantly positive in the periods, judging from the 95% credible interval band. Contrarily, when most of the regions are in contraction periods (in Table 3, see the columns where the number of × is at least 5), the spatial autocorrelation estimates are significantly positive (see Figure 2). That is, in Table 3 we have at least five ×’s in 1992, 1998 and 2001, and in Figure 2 the spatial autocorrelation estimates are significantly positive in the periods. Thus, before the bubble burst the interdependency among 47 prefectures increases in the expansion periods, but after the bubble burst the interdependency increases in the contraction periods. Therefore, we can consider that a structural change occurred when the bubble burst in 1991. Kakamu and Fukushige (2005) found that (i) in the 1980’s the interregional income inequality increases and similarly the individual income inequality also increases and (ii) in the 1990’s the interregional income inequality decreases although the individual income inequality increases. In this paper, we can observe the structural change on the interdependency when the bubble burst in 1991. This result is consistent with Kakamu and Fukushige (2005).

6 Concluding Remarks

This paper has examined the panel spatial autoregressive probit model from Bayesian point of view. We have expressed the joint posterior distribution (3), and considered the MCMC methods to estimate the parameters of the model in Section 3.2. We have illustrated our approach using both simulated data and actual data. In Section 4, we find from the simulation results that (i) when there exists the spatial dependency, the biases of the estimated posterior means become serious if we ignore the spatial autocorrelation, and (ii) all the
posterior means go to the true values as $T$ is large. Thus, the simulation results imply that the spatial interaction plays an important role in regional economic models.

Moreover, in Section 5 we have considered the regional business cycle in Japan. We have the following findings: (i) we confirm that the occurrence and collapse of bubble economy is a common phenomena in Japan, i.e., the probability that the growth rate of IIP is positive falls drastically from 1992-1993 and rises after 1993 (note that the bubble economy burst in 1991), (ii) the business cycle in Japan is very similar to the regional business cycle in each prefecture, and (iii) the posterior distributions of the spatial interaction $\rho_t$ do not include zero in the 95% credible intervals when the business cycle phase (i.e., expansion or contraction) in most of the prefectures is similar to each other (see 1979, 1984, 1988-1990, 1992, 1998 and 2001 in Table 3), i.e., the spatial interaction effect is significantly positive in the expansion periods before the bubble burst and in the contraction periods after the bubble burst. Thus, we have observed the structural change on the regional interdependency when the bubble burst in 1991. We conclude from Japanese regional data that the spatial interaction plays an important role in the business cycle.

References


