Malas Notches*

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Abstract

This paper shows that the sufficient statistic approach to the welfare properties of income (and other) taxes does not extend to tax systems with notches, because with notches, changes in bunching induced by changes in tax rates have a first-order effect on tax revenues. In an income tax setting, we show that the marginal excess burden (MEB) and the welfare-maximizing top rate of tax are given by the relevant formulae for a proportional tax as in Feldstein (1999) plus a correction factor. The Feldstein formulae always underestimate the MEB and overestimate the revenue and welfare-maximizing rate of tax. Quantitatively, these mis-estimates can be very large; the MEB can be underestimated by an order of magnitude. An application to VAT is discussed; with a calibration to UK data, the MEB of the VAT is roughly three times what is would be if VAT was simply a proportional tax.

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1 Introduction

In a recent survey, Chetty (2009) argues that an important new development in public economics is the so-called sufficient statistic approach, which "derives formulas for the welfare consequences of policies that are functions of high-level elasticities rather than deep primitives" (Chetty (2009), p 451). In turn, this means that to assess the welfare properties of these policies, only these elasticities, rather than fully structural models, need to be estimated.[1]

The sufficient statistic approach originated in a seminal paper by Feldstein (1999), who showed that the marginal excess burden (MEB) of a proportional income tax only depends on the behavioral responses to the tax via a sufficient statistic, the personal elasticity of taxable income (ETI). Feldstein's paper has given rise to a large literature devoted to obtaining empirical estimates of the ETI (Gruber and Saez (2002), Saez, Slemrod, and Giertz (2012), Kleven and Schultz (2014), Weber (2014)).

Subsequently, Saez (2001) and Saez, Slemrod, and Giertz (2012) showed that the Feldstein formula for the MEB could be extended to the top rate of tax in a progressive piece-wise linear income tax system, and they also established formulae for the revenue and welfare-maximizing rate of tax. These formulae also have the sufficient statistic feature; specifically, they depend only on the elasticity of the ETI, a statistic of the income distribution, which is constant if the top tail of the income distribution is Pareto[2], and a possibly a welfare weight.

In this paper, we ask the question as to whether these sufficient statistic properties of key formulae also extend to tax systems with notches. Generally, a tax notch occurs when there is a discontinuous change in the tax liability as the tax base varies (Slemrod (2013), Kleven (2016)).

In practice, we do see notches in several major kinds of taxes, and these are being increasingly studied in the empirical literature. Significant notches in the personal income tax system are quite rare, although they do exist; for example, in Pakistan (Kleven and Waseem (2013)), there are notches where the tax on all income below the notch can rise by as much as 5%, and in Ireland, an emergency income levy after the financial crisis

[1] Chetty (2009) also argues that this sufficient statistic approach is also valuable in several other contexts, such as evaluating the welfare gain from social insurance programs, and the welfare effects of changes in taxes with optimization frictions.

[2] The formula is that the marginal excess burden equals $\frac{tea}{1-t-te}$, where t is the rate of tax, e is the personal elasticity of taxable income with respect to the net of tax rate $1-t$, and a is the Pareto parameter.

had a notch of up to 4% (Hargaden (2015))[3]. There are even small notches in the federal income tax in the US, and larger notches induced by income-dependent entitlement to tax credits (Slemrod (2013)).

Notches also exist in other major taxes. For example, notches are, or were until recently, present in housing transactions taxes in the UK and the US (Best and Kleven (2014), Kopczuk and Munroe (2014)). They also arise in the corporate income tax in Costa Rica (Bachas and Mauricio (2015)). Slemrod (2013) notes that there are many examples of commodity tax notches, where a marginal change in some characteristic can change the product classification so as to produce a discrete change in the tax liability.[4] Finally, as argued by Liu and Lockwood (2015), a VAT threshold can be thought of as a tax notch; a firm's VAT liability changes discontinuously when its sales go over the registration threshold. Indeed, given the importance and near-ubiquity of VAT, this is in fact the most important example of a tax notch.

We first study notches in the income tax setting of Saez (2010) and others, where households differ in ability or taste so that the disutility of generating taxable income varies across households. For simplicity, we assume a two-bracket tax i.e. a tax with a lower rate (which could be zero) below a threshold, and a higher rate above. In this setting, our first contribution is to derive exact formulae for the marginal excess burden of the higher rate of tax, and for the welfare-maximizing rate higher rate of tax. These formulae are the same as in Feldstein (1999) for a proportional income tax, with a correction factor that captures the effect of the *bunching response* to an increase in the top rate tax on tax revenue.

The bunching response measures the change in the *number* of households bunching at the threshold to avoid paying the top rate of tax, and is thus distinct from the intensive margin response of taxable income of given household to the tax rate; the latter has been the focus of the ETI literature. With a notch, (unlike the case of a kink), the bunching response affects tax revenue because with a notch, the tax schedule is discontinuous at the threshold.

Our second key finding is that the correction factor cannot be expressed as a simple function of the usual sufficient statistics i.e. the intensive margin elasticity of the ETI and the Pareto parameter. It does depend on these variables, but it also depends on the lower rate of tax and on the size of the bunching interval. So, the sufficient statistic approach

[3] From Table 1 of Hargaden (2015), in 2010, earnings of above 26000 Euro incurred a charge of 1040 Euro.

[4] For example, in the US, the Gas Guzzler Tax, under which high-performance cars are subject upon initial sale to a per-vehicle tax that is higher, the lower is the fuel economy of the car.

seems to break down with tax notches, unless the correction factor turns out to be small.

Out third contribution is to investigate this question. Qualitatively, ignoring the correction factor underestimates the marginal excess burden and overestimates the welfare-maximizing rate of tax. Calibrations show that the percentage error from using the Feldstein formulae can be very large. At baseline values, the marginal excess burden is underestimated by a factor of six, and the revenue-maximizing tax is overestimated by around half, and the errors can be much larger for some parameter values. So, the conclusion is that at least in the income tax setting, the sufficient statistic approach is not practical.

We then turn to apply our approach to the VAT, which is the most empirically important example of a tax notch. We present a simple model of small traders who differ in productivity, and are subject to VAT at rate t above a threshold level of sales. We show that this model is formally equivalent to our income tax model, in the sense that registered firms above the threshold face an effective rate of VAT t_R on value-added, and firms below the threshold face a lower rate t_N. It may seem counter-intuitive that non-registered firms face a positive rate of effective VAT; this is because non-registered firms cannot claim back VAT on inputs (so-called "embedded" VAT).

We then show that the MEB of an increase in the statutory rate of VAT is given by the Feldstein formula for a proportional tax plus a correction factor as in the income tax case. However, the details of the correction factor are more complex, because an increase in the statutory rate t increases both the effective rates t_R, t_N. A calibration of the model shows that the proportional tax formula for the MEB of the VAT underestimates the true MEB by a factor of up to three. This framework also allows us to evaluate the effect of increased compliance costs of VAT in the MEB via its impact on bunching; increased compliance costs increase bunching and thus increase the MEB, but the effect is quantitatively small.

The remainder of the paper is arranged as follows. After the literature review in Section 2, in Section 3, we set up the model. Section 4 has the main analytical results for the income tax, and Section 5 the simulations. Section 6 deals with the extension to the VAT, and Section 7 concludes.

2 Related Literature

This paper speaks to a number of related literatures. First, it is already known that due to externalities of one kind or another, the sufficient statistic approach has its limitations. Saez, Slemrod, and Giertz (2012) give the examples of deductibility from income tax of charitable giving and mortgage interest payments for residential housing. In these cases, an increase in the marginal rate of tax will boost charity income and home ownership respectively, which may be valuable objectives in themselves. Saez, Slemrod, and Giertz (2012) call these classical externalities[5].

Fiscal externalities, where the actions of the household generate additional revenue for the government and thus benefits other households, can also cause the sufficient statistic approach to fail, or at least require adjustment, but in these cases a simple change to the formula is sometimes possible. Chetty's (2010) analysis of income tax evasion is a case in point[6]. As Gillitzer and Slemrod (2016) show, in this case the standard formula for the marginal efficiency cost of funds can be adjusted in the same way it must be adjusted for any fiscal externality, i.e. whenever a change in tax rates induces taxpayers to shift income to another tax. Our results are rather different to these cases of both classical and fiscal externality. In our setting, there is no fiscal or other externality- rather, the sufficient statistic approach fails because the bunching response ha a first-order effect on tax revenue.

A second related literature is on VAT. Here, there are two distinct sets of related papers. First, there is a growing literature on the effect of VAT thresholds on firm behavior. Theoretical contributions include Keen and Mintz (2004), Kanbur and Keen (2014) and Liu and Lockwood (2015), and empirical studies include Liu and Lockwood (2015) and Harju, Matikka, and Rauhanen (2016). The theoretical work of Kanbur, Keen and Mintz focusses on the optimal threshold of the VAT, holding the rate of tax fixed, and is thus complementary to this paper, which characterizes the MEB of an increase in the rate, holding the threshold fixed. In fact, we effectively ask the question of whether it is legitimate to ignore the threshold altogether when calculating the MEB of the VAT. Therefore, our paper relates to as second literature on the marginal excess burden of indirect taxes, including VAT (e.g. Ballard, Shoven, and Whalley (1985), Rutherford, and Paltsev (1999)). In these papers, when the marginal excess burden of VAT is calculated, it is always assumed that the VAT is a proportional tax i.e. the VAT threshold is ignored. This paper shows that this simplifying assumption yields seriously biased estimates.

[5] See Doerrenberg, Peichl, and Siegloch (2015) for a more formal statement of this argument, and estimates of how deductions respond to tax rate changes for the case of Germany.

[6] Chetty shows that when the household can evade the personal income tax at a cost, if that cost is a pure transfer payment i.e. a fine times a probability of detection, there is effectively a positive fiscal externality of evasion - it generates additional revenue for the government and thus benefit for all households. In this case, as we might expect, we see that the elasticity of taxable income over-estimates the excess burden of the tax.

A third related literature is that on the MEB and welfare-maximizing taxes with kinks in the tax schedule. Here, we make a small contribution as a by-product of our main focus, which is on notches. In this case, it generally understood that the marginal excess burden of the top rate of income tax, and the welfare-maximizing top rate depends via simple formulae, only on the elasticity of the ETI, and the Pareto statistic of the income distribution. However, there seems to be some confusion about the conditions required for this result. Saez, Slemrod, and Giertz (2012) suggest that what is required is that assumption that "behavioral responses take place only along the intensive margin", or more precisely that the bunching response of an increase in the top rate of tax is of second order relative to the extensive margin response.[7] This assumption is very strong, as even with a kink, there is always a bunching response. Our Proposition 4.1 below shows that this assumption is *not* necessary, because no matter what the size of the bunching response, the response has no effect on tax revenue, to first order, as the tax schedule is continuous. All that is required is that the distribution of taxpayer types n is continuous, a standard assumption.

A final related literature is the small one on the design of piece-wise linear income taxes. In any early contribution Slemrod et. al. (1994) consider the design of a two-bracket income tax, and they explicitly take into account bunching responses in doing so. They did not obtain analytical results but their numerical simulations suggest that the tax schedule should be concave i.e. the higher tax should be below the lower tax. More recently, Apps, Long, and Rees (2014) have extended their work. This work is somewhat related to our finding that the intensive margin response should be adjusted in the case of the VAT, as explained below.

[7] Specifically, they say the following. "The change dt could induce a small fraction dN of the N taxpayers to leave (or join if $dt < 0$) the top bracket. As long as behavioral responses take place only along the intensive margin, each individual response is proportional to dt so that the total revenue effect of such responses is second order ($dN.dt$) and hence can be ignored in our derivation."

3 The Model and Preliminary Results

3.1 Set-Up

We follow Saez (2010) in our set-up. There are individual taxpayers indexed by a skill or taste parameter $n \in [\underline{n}, \overline{n}]$, distributed in the population with density $h(n)$. A type n individual has preferences over consumption c and taxable income z of the form

$$u(c, z; n) = c - \psi(z; n)$$

where $\psi(z; n)$ is the disutility of earning income z. So, in this specification of $u(c, z; n)$, we assume away income effects for convenience. We also assume:

A1. $\psi_z, \psi_{zz} > 0,\ \psi_n, \psi_{nz} < 0.$

So, A1 says that a higher n represents are higher skill level (i.e. higher wage), or a lower taste for leisure. Assumption A1 is satisfied for example, by the iso-elastic specification of Saez (2010):

$$\psi(z; n) = \frac{n}{1 + \frac{1}{e}} \left(\frac{z}{n}\right)^{1+\frac{1}{e}} \tag{1}$$

The budget constraint is $c = z - T(z)$, where $T(.)$ is the tax function. So, a household's utility over z is $u(z; n) = z - T(z) - \psi(z; n)$.

Finally, for future reference, define the optimal taxable income at tax rate t for a type n to be;

$$z(1 - t, n) \equiv \arg\max_z \{(1 - t)z - \psi(z; n)\}$$

Note from A1 that $z_{1-t}, z_n > 0$, where subscripts denote derivatives. So, z_{1-t} is the response of taxable income to the net-of-tax rate. Following Saez, Slemrod and Giertz (2012) we call this the intensive margin response.

3.2 Kinks and Notches

For simplicity, we focus on a two-bracket tax, although our arguments apply straightforwardly to the case of the highest tax in a piecewise-linear tax system with any number of brackets. We will assume that the tax system is progressive; that is, the tax rate on incomes in the higher income bracket is strictly greater than the tax on incomes in the lower income bracket.

So, with a two-bracket tax, for a kink, the tax function is

$$T_K(z) = \begin{cases} t_L z, & z \le z_0 \\ t_L z_0 + t_H(z - z_0), & z > z_0 \end{cases} \tag{2}$$

for $z_0 > 0,\ t_H > t_L \geq 0$; that is, all income below the kink point z_0 is taxed at the lower rate t_L, and all income in excess of the kink is taxed at the higher rate. For a notch, the tax function is

$$T_N(z) = \begin{cases} t_L z, & z \leq z_0 \\ t_H z, & z > z_0 \end{cases} \tag{3}$$

with $t_H > t_L \geq 0$. That is, when taxable income is below z_0, a tax at rate t_L is paid on all income, but when t_H is above z_0, a tax at rate t_H is paid on *all* income.

3.3 Bunching

With either a kink or a notch, all types in an interval $n \in [n_L, n_H]$ will bunch at taxable income z_0. In both cases, the lowest type who bunches is the one who is just willing to earn taxable income z_0 at the lower tax rate i.e. the critical n_L is defined by the condition

$$z(n_L, 1 - t_L) = z_0 \tag{4}$$

With a kink, the highest type who bunches, n_H, is defined by the condition that the optimal choice of taxable income of the n_H-type at tax t_H is just z_0 i.e.

$$z(1 - t_H; n_H) = z_0 \tag{5}$$

With a notch, n_H is defined by the condition that the n_H type must be indifferent between staying at the notch and paying tax t_L, and choosing z optimally, and paying t_H on *all income* (Kleven and Waseem (2013), Kleven (2016)). To write this indifference condition, we first define the indirect utility function

$$v(t; n) \equiv \max_z \{(1 - t)z - \psi(z; n)\}$$

Note that the derivative of v with respect to t, v_t, is $-z(1 - t; n)$. Then, the condition defining n_H can be written:

$$(1 - t_L)z_0 - \psi(z_0; n_H) = v(t_H; n_H) \tag{6}$$

The left-hand side of (6) is utility when taxable income is constrained to be at the notch value z_0. Note that this indifference condition implies $z(1 - t_H, n_H) > z_0$; because if $z(1 - t_H, n_H) < z_0$, the n_H-type could choose z optimally *and* stay below the notch. Note the difference between indifference condition (6) and the condition (5).

3.4 The Bunching Response

Here, we study the effect of a change in t_H on the mass of individuals who bunch i.e. on the size of the interval $[n_L, n_H]$. Note first from (4) that n_L is unaffected by t_H for both a kink and a notch. Next, in the kink case, note that

$$\frac{\partial n_H}{\partial t_H} = \frac{z_{1-t_H}}{z_n} > 0 \tag{7}$$

So, we have a bunching response to t_H: i.e. an increase in the tax rate above the kink makes going above the kink less attractive, and so more people bunch below the kink.

In the notch case, note from (6) that t_H does affect n_H, and in fact, using $v_t = -z$, we see that

$$\frac{\partial n_H}{\partial t_H} = \frac{z(1-t_H, n_H)}{\psi_n(z_0; n_H) - \psi_n(z(1-t_H, n_H); n_H)} \tag{8}$$

Also, as $\psi_{nz}(z;n) < 0$ and $z(1-t_H, n_H) > z_0$, we see that for any n :

$$\psi_n(z(1-t_H, n_H); n) < \psi_n(z_0; n)$$

and consequently from (8):

$$\frac{\partial n_H}{\partial t_H} > 0 \tag{9}$$

So, again we see that the bunching response to a change in t_H is intuitive; an increase in the tax rate above the notch makes going above the notch less attractive, and so more people bunch at the notch.

4 Main Results

4.1 The Effect of the Bunching Response on Tax Revenue

Here, we show that the effect of the bunching response on tax revenue with a kink and a notch are qualitatively different, being zero and negative respectively. With a kink, revenue can be written

$$R = t_L \left(\int_{\underline{n}}^{n_L} z(1-t_L; n)h(n)dn + \int_{n_L}^{\overline{n}} z_0 h(n)dn \right) + t_H \left(\int_{n_H}^{\overline{n}} (z(1-t_H; n) - z_0)h(n)dn \right) \tag{10}$$

Note that all households with $n \geq n_L$ pay tax at the lower rate on the first z_0 of earnings.

In the kink case, the bunching effect on tax revenue i.e. the effect of a change in t_H on R via a change in n_H in t_H is, from (12):

$$\frac{\partial R}{\partial n_H} = -t_H(z(1-t_H; n_H) - z_0)h(n_H) = 0 \tag{11}$$

So, overall, with a kink, the effect of the bunching response on tax revenue is zero.

With a notch, revenue is

$$R = t_L \left(\int_{\underline{n}}^{n_L} z(1-t_L;n)h(n)dn + \int_{n_L}^{n_H} z_0 h(n)dn \right) + t_H \left(\int_{n_H}^{\overline{n}} z(1-t_H;n)h(n)dn \right) \quad (12)$$

Comparing this to (10), we see two differences. Because the higher rate applies to *all* income for those earning above z_0, the threshold z_0 no longer enters into the the tax base for t_H, and so the upper limit of integration on z_0 in the tax base for t_L falls from $\overline{n}$ to n_H, reflecting the fact that now only individuals below n_H pay any tax at the lower rate.

Note from (12) that;

$$\frac{\partial R}{\partial n_H} = (t_L z_0 - t_H z(1-t_H;n_H))h(n_H) < 0 \quad (13)$$

This is strictly negative as $t_H > t_L,\ z(1-t_H;n_H) > z_0$. So, in contrast to the kink case, the bunching effect on tax revenue R from an increase in t_H is negative, as $\frac{\partial n_H}{\partial t_H} > 0$ from (9). This is because a small increase in n_H has two effects on revenue that are both negative. First, there is a discontinuity in the tax *base*; the earnings of these who now locate at the notch fall discontinuously from $z(1-t_H;n_H)$ to z_0. Second, there is a discontinuity in the tax *rate* applying to that base*;* all these earnings are taxed at a lower rate, t_L rather than t_H.

So, we conclude:

Proposition 1. *The effect of the bunching response on tax revenue is zero for a kink, but strictly negative for a notch.*

This result is the key one that drives the rest of the paper. The result that bunching response on tax revenue is zero for a kink also has a useful implication that helps to clarify some confusion in the literature. As already noted, Saez, Slemrod, and Giertz (2012) argue that for sufficient statistic formulae to apply in the kink case, what is required is that assumption that "behavioral responses take place only along the intensive margin", or more precisely that the bunching response of an increase in the top rate of tax is of second order relative to the extensive margin response. Proposition 4.1 shows that this assumption is not required, because no matter how large $\frac{\partial n_H}{\partial t_H}$, $\frac{\partial R}{\partial n_H} = 0$ in the kink case.

4.2 The Marginal Excess Burden

Here, we derive a formula for the marginal excess burden (MEB) of t_H when there is a notch and show that it can be written as the MEB of a proportional tax plus a correction factor. To define the MEB, note that due to quasi-linearity, the natural measure of welfare is the integral of indirect utilities, say W, plus revenue R, which is assumed to be redistributed as a lump-sum back to households when calculating the MEB. So,

$$MEB = -\frac{d(W+R)/dt_H}{dR/dt_H} \tag{14}$$

The minus sign ensures that the marginal excess burden is measured as a positive number.

From (12), we see that the effect of an increase in t_H on tax revenue is:

$$\frac{dR}{dt_H} = B_H + \underbrace{t_H \left.\frac{\partial B_H}{\partial t_H}\right|_{n_H \text{ const}}}_{\text{intensive-margin}} + \underbrace{\frac{\partial R}{\partial n_H}\frac{\partial n_H}{\partial t_H}}_{\text{bunching}} \tag{15}$$

Here

$$B_H = \int_{n_H}^{\overline{n}} z(1-t_H;n)h(n)dn \tag{16}$$

is the base in which the higher rate of tax is levied.

So, (15) is composed of three terms, the mechanical effect B_H, and two behavioral effects on tax revenue, the intensive-margin and bunching effects. The intensive-margin effect on tax revenue is standard; it describes how tax revenue changes because of changes in earnings, conditional on the taxpayer staying the same tax bracket. The bunching effect on tax revenue and its impact on the marginal excess burden is the focus of our investigation.

To compute dW/dt_H, note first that the integral of indirect utilities is

$$W = \int_{\underline{n}}^{n_L} v(1-t_L;n)h(n)dn + \int_{n_L}^{n_H} (z_0(1-t_L) - \psi(z_0;n))h(n)dn + \int_{n_H}^{\overline{n}} v(1-t_H;n)h(n)dn \tag{17}$$

Note that by definition, a small change in n_H has no effect on welfare, because n_H is defined by (6) above. So, using $v_t = -z(1-t,n)$, we see that

$$\frac{dW}{dt_H} = -\int_{n_H}^{\overline{n}} z(1-t_H;n)h(n)dn = -B_H \tag{18}$$

So, plugging (15), (18) back into (14), dividing through by B_H, and multiplying by $1-t_H$, and noting that holding n_H constant, $\frac{\partial B_H}{\partial(1-t_H)} = -\frac{\partial B_H}{\partial t_H}$, we see that

$$MEB = \frac{t_H e + C}{1-t_H-t_H e + C},\ C = -\frac{1-t_H}{B_H}\frac{\partial R}{\partial n_H}\frac{\partial n_H}{\partial t_H} \tag{19}$$

Here,

$$e = \frac{1-t_H}{B_H}\frac{\partial B_H}{\partial(1-t_H)}\bigg|_{n_H \text{ const}} = \frac{1-t_H}{B_H}\int_{n_H}^{\overline{n}}\frac{\partial z(1-t_H;n)}{\partial(1-t_H)}h(n)dn \tag{20}$$

is the intensive-margin elasticity of the tax base B_H with respect to the net of tax rate $1-t_H$, and C is a correction factor, which captures the effect of a changing n_H, the bunching response, on the MEB, via its effect on revenue. Of course, given the specification (1), e is a constant independent of n_H. This formula is standard, except that it includes the effect of the bunching response on tax revenue in both numerator and denominator via C.

We then have;

Proposition 2. *Assume A1, and that the distribution of ability (and pretax-income) is Pareto, with shape and scale parameters $a, \underline{n}$. Then, the MEB with a notch is*

$$MEB = \frac{t_H e + C}{1 - t_H - t_H e - C}, \tag{21}$$

where

$$C = \frac{(1-t_H)(t_H(1-t_H)^e - t_L z_0/n_H)(a-1)(1+e)}{(1-t_H)^{1+e} - \left(\frac{z_0}{n_H}\right)^{(1+e)/e}} > 0. \tag{22}$$

Moreover, in (22), n_H is defined by (6).

Some comments are appropriate at this point. First, the MEB (21) is the formula for the marginal excess burden of a *proportional* income tax, as shown by Feldstein (1999), plus the correction factor C. This is intuitive; all households above n_H are paying tax at rate t_H on all their income, so for n_H *fixed*, t_H is indeed a proportional tax. So, as already remarked, the correction factor C just captures the effect of a changing n_H, the bunching response, on the MEB, via its effect on revenue.

Second, we can ask how the MEB compares to the MEB in a kinked tax system. As shown for example, by Saez (2001), the latter is

$$MEB_K = \frac{t_H e a}{1 - t_H - t_H e a}$$

Clearly, MEB_K depends only on simple sufficient statistics; other than the tax rate t_H, it depends only on e, the intensive-margin elasticity of taxable income, and a, the shape parameter of the income distribution.

By contrast, from (22) that C is a complex object. It depends not only on sufficient statistics e, a, and the top rate of tax, t_H, but also on other parameters of the tax system

t_L, z_0, and finally, it also depends on n_H. In turn, generally, n_H cannot be solved for in closed form from *(6)*. Given this, one question is whether we can get a good approximation to MEB by setting $C = 0$. This is a question addressed in Section 5 below, where we will see that the approximation is generally very inaccurate.

4.3 The Welfare-Maximizing Rate of Tax

In his well-known article, Saez (2001) derived a formula for the welfare-maximizing rate of tax for a one-bracket tax system.[8] He showed that this tax depended only on e and a, plus a parameter he called $\overline{g}$, which is the "ratio of of social marginal utility for top bracket taxpayers to the marginal value of public funds for the government." In the special case where $\overline{g} = 0$, this gives the revenue-maximizing rate of tax.

Here, we develop a similar formula for the optimal t_H. We will show that it is equal to the formula for the welfare-maximizing proportional tax, plus a term in the correction factor C above. To do this, we assume now, following Saez, that the government's objective is not the integral of indirect utilities as in (17), but the integral of a strictly increasing, concave transformation $G(.)$ of utilities. The function G captures social aversion to inequality in the usual way. Also, we suppose that the government has a revenue requirement E. Also, we assume following Saez (2001) that the welfare weight $g = G'$ is constant above n_H at some $\overline{g}$; if this is not the case, the optimal tax has an additional term in the covariance of g and z. Finally, normalize the Lagrange multiplier on the government revenue constraint $R \leq E$ to unity. Then we can show:

Proposition 3. *The welfare-maximizing level of t_H is;*

$$t^* = \frac{1 - \overline{g} - C}{1 - \overline{g} + e} \tag{23}$$

with C defined in (22) above.

To interpret this, note first that there is a direct connection of (23) to the formula derived by Saez (2001) for the optimal linear tax on top earners, which is equation (9) in his paper. He allowed for an income effect in labour supply in his setting, so setting this equal to zero, his equation, in our notation, reduces to

$$t^* = \frac{1 - \overline{g}}{1 - \overline{g} + ea} \tag{24}$$

[8] This is given in equation (9) of Saez (2001).

Moreover, it is easily checked that if there is no exempt income in the one-bracket tax system, so it becomes a proportional tax, $a = 1$ in the above formula, so

$$t^* = \frac{1-\overline{g}}{1-\overline{g}+e} \tag{25}$$

Comparing (23) to (25), we see that the former is equal to the optimal proportional tax minus a correction factor $\frac{C}{1-\overline{g}+e}$. This factor reflects the fact that with a notch, there is an additional cost to taxation because of the bunching response.

As already remarked, C is a complex object; it depends not only on sufficient statistics e, a, and the top rate of tax, t_H, but also on other parameters of the tax system t_L, z_0, and finally, it also depends on n_H. In turn, generally, n_H cannot be solved for in closed form from *(6).* Given this, one question is whether we can get a good approximation to t_H^* by setting $C = 0$. This is a question addressed in Section 5 below.

5 Simulations

We have seen that the MEB of an increase in t_H and the optimal t_H are given by the corresponding formulae for a proportional tax t_H plus a correction factor. Moreover, the formulae for a proportional tax are very simple, depending only on the intensive-margin elasticity e, and thus can easily be calculated. This raises the question of whether the MEB and optimal tax, calculated assuming that t_H is a proportional tax, are good approximations to the true MEB and optimal tax. To investigate this, we calibrate the model.

We require values for e, a, t_H, t_L, and z_0. First, we assume that the intensive-margin elasticity (20) is constant as in (1), so we only need to calibrate the parameter e. Our baseline parameter values are chosen as follows. Following Piketty and Saez (2003), we set $a = 1.5$, and following Saez, Slemrod, and Giertz (2012) and Kleven and Schultz (2014), we set $e = 0.25$. Regarding the tax rates, we first set $t_L = 0.2$, which is broadly in line with the average income and payroll tax paid by US households[9]. It is also the basic rate of income tax in the UK. For the notch, we use the fact that notches in personal income tax, where they exist, are small. For example, Kleven and Waseem (2013) show that in the Pakistani income tax, the notch ranges between 2 and 5 percentage points. So, we will take our baseline notch $t_H - t_L = \Delta t = 0.03$.

[9] "Overview Of The Federal Tax System As In Effect For 2015", Joint Committe on Taxation, Congtress of the United States.

To choose $\underline{n}$, z_0 we assume that only the top 20% of the population pay a higher rate of income tax, roughly the proportion in the UK. Define n_0 to be the skill level corresponding to taxable income just at the notch i.e. $n_0(1-t_L)^e = z_0$. This requires that 80% of the population have skills below n_0 i.e. $H(n_0) = 1 - \left(\frac{\underline{n}}{n_0}\right)^{\alpha} = 0.8$, or $\frac{\underline{n}}{n_0} = (0.2)^{1/1.5} = 0.342$. Given that only the ratio $\frac{\underline{n}}{n_0}$ is determined, we set $\underline{n} = 1$, so $n_0 = 2.924$. But then $z_0 = 2.924(0.8)^{0.25} = 2.168$.

Finally, from (22), we need a value for n_H. Under the assumption (1), the indifference condition (6) reduces to

$$e(n_H)^{-1/e}(z_0)^{1+\frac{1}{e}} + n_H(1-t_H)^{1+e} - (1-t_L)z_0(1+e) = 0 \tag{26}$$

Equation (26) has two roots, and we take the larger root to ensure that $n_H(1-t_L)^e > z_0$. Finally, parameter values are chosen so that the denominator in (21) is positive, which is equivalent to $dR/dt_H > 0$ i.e. that the tax rate is on the right side of the Laffer curve. This requires simply that the notch is greater than 0.0015.[10]

Figures 1(a)-(c) show both the true MEB, as given by (21), and the approximation, treating t_H as a proportional tax i.e. setting $C = 0$ in (21). The former is denoted by MEB in the Figures, and the latter by MEB_A.

- Figure 1 in here -

The error in using MEB_A at the baseline values can be read off from Figure 1(a), setting $e = 0.25$. It can be seen that true MEB is about 0.6, whereas the approximation is about 0.1. So, the error in using the proportional formula is about a factor of six. Figure 1(a) also shows that MEB is increasing in e, at a faster rate than MEB_A, so when $e = 0.4$ for example, the error in using MEB_A is almost an order of magnitude.

Figure 1 (b) shows that MEB is also increasing in a, the Pareto parameter which measures (inversely) the size of the tail of the income distribution. As MEB_A is independent of a, this means that the the error in using MEB_A is increasing in a.

Finally, Figure 1(c) shows MEB, MEB_A as the size of the tax notch varies. We can see that as the notch becomes very small, the true MEB becomes very large. This is because $C \to \infty$ as $t_H \to t_L$. While this cannot be proved analytically, the intuition is clear from (22). As $t_H \to t_L$, then $n_H \to n_L = z_0(1-t_H)^{-e}$, or $(1-t_H)^e \to z_0/n_H$. So, both numerator and denominator in (22) tends to zero, but the denominator does so faster

[10] For the denominator in (21) to be positive, we require $1 - t_H(1+e) > C$, which is satisfied for $t_H - t_L > 0.0015$.

Figures 2 and 3 show the optimal tax t^* for the cases where first $\overline{g} = 0$ (revenue-maximization), and $\overline{g} = 0.25$ (welfare-maximization). Again, we show t^* as defined in (23), along with the approximation setting $C = 0$, which we denote by t^*_A. In each figure, we show both t^*, t^*_A as both e and a vary.

- Figures 2 and 3 in here -

Both of these taxes are decreasing in e, as we might expect. Also, both taxes are decreasing in a. The error in using t^*_A as an approximation for t^* is generally smaller than for the MEB. For example, at baseline parameter values, the true revenue-maximizing tax is about 0.55, whereas the approximation is 0.8.

6 An Application to VAT

6.1 The Set-Up

As remarked in the introduction, perhaps the most important example of a tax notch is the value-added tax. Then In this section, we present a simple model of value-added tax, based on Liu and Lockwood (2015), which mathematically, is equivalent to the model developed above. We then calibrate the model using UK data from Liu and Lockwood (2015), to estimate the MEB from the VAT, taking into account the welfare effects of bunching at the threshold.

Consider a single industry with a fixed, large number of small traders $a \in [\underline{a}, \overline{a}]$ producing a homogenous good. Small trader a combines his own labor input l with an intermediate input x to produce output y via a fixed coefficients technology

$$y = \min\left\{l, \frac{x}{\alpha}\right\}, \tag{27}$$

where α measures the the input requirement per unit of output. In particular, for all traders, one unit of output requires α units of input.

We assume that trader has an iso-elastic disutility of labor

$$\psi(l; a) = \frac{Aa}{1 + \frac{1}{e}} \left(\frac{l}{a}\right)^{1+\frac{1}{e}} \tag{28}$$

So, traders differ in their disutility of labour. This assumption is not essential, but facilitates comparison to the income tax case.[11]

[11] For example, a could enter into the production function, (27) instead, as in Liu and Lockwood (2016).

For simplicity, it is assumed that traders only sell to final consumers, who have perfectly elastic demand for the good at price $p = 1$. This is analogous to the assumption made in the taxable income literature that the wage is fixed, i.e. labor demand is perfectly elastic at a fixed wage. Finally, the intermediate input is produced only from labour supplied by non-trader households via a fixed-coefficients technology where one unit of labour are needed to produce one unit of the intermediate input. So, the tax-exclusive price of the output is w, the wage.

The traders face and the producers of the intermediate inputs face a VAT system. If the trader is registered, he must charge VAT on sales y at rate t, but can claim back any VAT paid on inputs. The trader must register for VAT if the value of sales y exceeds the threshold y_0, but can register voluntarily even if $y < y_0$.

Note that when not registered, the price of the input is $w(1+t)$. So, the profit for the non-registered trader is

$$\pi_N = (1 - \alpha w(1+t))y = (1 - \gamma(1+t))y, \ \gamma = \alpha w. \tag{29}$$

where γ is the cost of inputs relative to revenue per unit sold. For the registered trader, we reason as follows. This trader must charge VAT on his output. None of the output VAT can be passed on to the buyer, as he has perfectly elastic demand. So, revenue per unit sold is $p/(1+t)$. But, if the trader is registered, he can claim back VAT on the input use x, so the price of the input is w. So, overall, the profit for the registered trader is

$$\pi_R = \left(\frac{1}{1+t} - \alpha w\right) y = \left(\frac{1}{1+t} - \gamma\right) y. \tag{30}$$

We now assume, to make the analysis interesting, that $1 > \gamma(1+t)$. From (29), this ensures that non-registered firms make a positive profit. Also, it ensures that for a given value of sales y, $\pi_N > \pi_R$, so there is no voluntary registration. This is important because then the VAT threshold functions exactly like a tax notch.

6.2 Effective VAT Rates

Now define $n = a(1-\gamma)$. Then, after some rearrangement, we can show that the utility of trader n can be written as a function of value-added $z = y(1-\gamma)$ and the VAT system as follows;

$$u(z;n) = z - T(z) - \frac{A}{(1-\gamma)} \frac{n}{1+\frac{1}{e}} \left(\frac{z}{n}\right)^{1+\frac{1}{e}} \tag{31}$$

where

$$T(z) = \begin{cases} t_N z, & z \le z_0 \\ t_R z, & z > z_0 \end{cases}, \ t_R = \frac{t}{(1+t)(1-\gamma)}, t_N = \frac{\gamma t}{1-\gamma}. \tag{32}$$

As A is a free parameter, we set it equal to $1-\gamma$. Then, (32) is mathematically equivalent to (3).

Here, t_N, t_R are the *effective tax rates* faced by nonregistered and registered traders respectively on the value-added they generate. Obviously, both effective rates are increasing in the statutory rate, t. Also, note that both rates are increasing in input intensity γ. Moreover, from our assumption $1 > (1+t)\gamma$, $t_R > t_N$.

So, faced with the tax schedule (32), all traders in the interval $n \in [n_L, n_R]$ will bunch at the VAT threshold z_0. Moreover, $n_L = z_0/(1-t_N)^e$, and n_R solves (26) with t_H, t_L replaced by t_R, t_N.

Finally, letting $z(1-t;n)$ be the value-added chosen by an unconstrained firm facing tax t, it can be shown that the revenue from the VAT is as in (12), with t_H, t_L replaced by t_R, t_N i.e.

$$R = t_N \left(\int_{\underline{n}}^{n_N} z(1-t_N;n)h(n)dn + \int_{n_N}^{n_R} z_0 h(n)dn \right) + t_R \left(\int_{n_R}^{\overline{n}} z(1-t_R;n)h(n)dn \right) \quad (33)$$

In (33), the base on which t_N is levied is the value-added of non-registered traders, and the base of t_R is the value-added of registered traders.

6.3 The Marginal Excess Burden of the VAT

With the VAT, a change in the statutory rate t of VAT will change both effective tax rates t_N, t_R unless $\gamma = 0$ i.e. no intermediate inputs are used. This is of course, analogous to a reform that changes both t_H and t_L in the income tax model. So, for the VAT, the formula for the MEB becomes somewhat more complex. To present the formula for the MEB in this case, we need a few more definitions. First, note from (33), using $z(1-t);n) = (1-t)^e n$, the effective bases of t_N and t_R are

$$B_N = \int_{\underline{n}}^{n_N} (1-t_N)^e n h(n)dn + z_0(H(n_R) - H(n_N)), \; B_R = \int_{n_R}^{\overline{n}} (1-t_R)^e n h(n)dn \quad (34)$$

Then, from (34), the intensive-margin elasticities of B_R, B_N with respect to the net-of-tax rate are

$$\frac{1-t_R}{B_R}\frac{\partial B_R}{\partial t_R}\bigg|_{n_R \text{ const}} = e, \; \frac{1-t_N}{B_N}\frac{\partial B_N}{\partial(1-t_N)}\bigg|_{n_N \text{ const}} = e\phi, \quad (35)$$

where

$$\phi = \frac{\int_{\underline{n}}^{n_N} z(1-t_N;n)h(n)dn}{B_N} < 1 \quad (36)$$

The term ϕ captures a new effect of bunching; with bunching, the mass $H(n_R) - H(n_N)$ of the non-registered firms that are bunching are unresponsive to a change in the rate of VAT, which lowers the aggregate intensive-margin elasticity of the tax base B_N with respect to t_N.[12]

Moreover, recall that an increase in t causes both t_N and t_R to increase, so

$$\theta = \frac{\frac{B_R}{1-t_R}\frac{\partial t_R}{\partial t}}{\frac{B_R}{1-t_R}\frac{\partial t_R}{\partial t} + \frac{B_N}{1-t_N}\frac{\partial t_N}{\partial t}} \tag{37}$$

measures the importance of a change in t_R on revenue relative to t_N. Armed with these new definitions, we can state our result.

Proposition 4. *Assume A1, and that the distribution of sales (and pretax-income) is Pareto, with shape and scale parameters* $a, \underline{n}$. *Then, the MEB of the VAT is*

$$MEB = \frac{\tau\varepsilon + C}{1 - \tau(1+\varepsilon) - C} \tag{38}$$

where

$$\tau = (1-\theta)t_N + \theta t_R, \ \varepsilon = \frac{(1-\theta)t_N\phi + \theta t_R}{(1-\theta)t_N + \theta t_R}e \tag{39}$$

and finally the correction factor is

$$C = -\frac{\frac{\partial R}{\partial n_R}\left(\frac{\partial t_N}{\partial t}\frac{\partial n_R}{\partial t_N} + \frac{\partial t_R}{\partial t}\frac{\partial n_R}{\partial t_R}\right)}{\frac{B_R}{1-t_R}\frac{\partial t_R}{\partial t} + \frac{B_N}{1-t_N}\frac{\partial t_N}{\partial t}} \tag{40}$$

So, we note now that bunching impacts the calculation of the MEB in two ways. First, as before, there is a correction factor C in (38). The correction factor is more complex than in the income tax case. The reason for the additional complexity is clear from (40); an increase in t now increases both t_R, t_N and in turn, both of these effective taxes affect n_R, the top of the bunching interval, and thus revenue. An explicit formula for C in terms of parameters can be derived as in (22) above; this is done in the Appendix.

In addition, there is a second, new effect of bunching in (39). Bunching dampens the intensive-margin response to a change in t, because at a fixed n_N, n_R, firms in this interval will not adjust their sales in response to a change in t. This is captured by the term ϕ.

An interesting special case is where the small traders do not use any intermediate input, so i.e. $\gamma = 0$. Then $t_N = 0$, $t_R = \frac{t}{1+t} = \tau$, so (38) simplifies to

$$MEB = \frac{\frac{t}{1+t}e + C}{1 - \frac{t}{1+t}(1+e) - C} \tag{41}$$

[12] A similar point has been noted before by Slemrod et. al. (1994) and Apps, Long, and Rees (2014) who consider the design of a two-bracket income tax. Because the tax system studied is kinked, not notched, the formula for the optimal lower rate of tax depends only on the intensive margin elasticity, but this elasticity is dampened by the fact that taxpayers at the kink do not adjust their behavior in response to the tax.

It can be checked that in this case, C is given by the explicit formula (22), replacing t_H, t_L by $t_R, 0$ respectively.[13]

Finally, how realistic is it that the distribution of sales s (or value-added z) is Pareto? As already remarked, in the US, there is evidence that the size distribution of firms as measured by sales is Pareto (Luttmer (2007)). TO BE COMPLETED

6.4 Simulations

Here calibrate the VAT model, and plot the true MEB in (38) and an approximation to the MEB as parameters vary. The approximation is the one treating VAT as a proportional tax i.e. setting $C = 0$ in (41), which gives

$$MEB_A = \frac{\frac{t}{1+t}e}{1 - \frac{t}{1+t}(1+e)}$$

The parameters are calibrated as follows. In the UK, the statutory rate of VAT is 20%, so $t = 0.2$. Liu and Lockwood (2016) calculate that for the universe of firms in the UK that file a corporate tax return, $\gamma = 0.45$. This gives $t_N = 0.16$, $t_R = 0.30$. As already remarked, there is evidence that the size distribution of firms as measured by sales is Pareto; Luttmer (2007) has a value for the US of $a = 1.06$. [TO BE UPDATED].

Next, define n_0 to be the productivity level corresponding to turnover just at the threshold i.e. $n_0(1-t_L)^e = z_0$. From Liu and Lockwood (2016), 62.5% of firms are below the threshold. So, $\frac{\underline{n}}{n_0}$ must satisfy $H(n_0) = 1 - \left(\frac{\underline{n}}{n_0}\right)^{1.06} = 0.625$, or $\frac{\underline{n}}{n_0} = (0.375)^{1/1.06} = 0.396$. Given that only the ratio $\frac{\underline{n}}{n_0}$ is determined, we set $\underline{n} = 1$, so $n_0 = 2.53$. But then $z_0 = 2.53(0.84)^{0.25} = 2.422$.

Our results are given in Figures 4 and 5. Figure 4 shows the simpler case with no intermediate inputs i.e. $\gamma = 0$, in which case we know that formula (38) reduces to the formula with a notched income tax.

- Figure 4 in here -

We can see that at the baseline figures for the parameters e.g. $e = 0.25$ in Figure 4(a), the true MEB is about 50% higher than the approximation. This difference is much smaller than in the income tax case, and is driven partly by the lower value of a in the VAT case. Indeed, we can see in Figure 4(b) that the accuracy of the approximation MEB_A

[13] If there is no bunching i.e. if $t_N = t_R$, then $\phi = 1, C = 0$, so $MEB = \frac{\tau e}{1-\tau(1+e)}$. But this requires that $\tau = 1$, so in this case, $z \to 0$. Also, before this point, $1 - \tau(1+e) < 0$, so this case is not interesting.

falls rapidly as a rises, because MEB is increasing in a whereas MEB_A is independent of a.

Figure 5 shows the more realistic case with $\gamma = 0.45$. Here, we see that the difference between the true MEB and the approximation is somewhat higher; the true MEB is about 3 times higher than the approximation. As in the case with no inputs, the true MEB is increasing in both e and a.

- Figure 5 in here -

6.5 The Marginal Excess Burden and the Cost of VAT Compliance

In practice, there are significant compliance costs to being VAT-registered i.e. preparing and filing a tax return, and paying any tax owed. In the UK, these costs are relatively low as a proportion of turnover, even for firms at the threshold. For example, a recent literature review found that for the UK, at the registration threshold, these costs were around 1.5% of turnover, declining to 0.1% or less for large companies (Federation of Small Businesses (2010)). However, compliance costs can be much higher in other countries. For example, a report by PwC found that for a fictional small firm, the hours taken for compliance with VAT vary by region from an average 73 hours within the EU to 192 for Latin America, and even within the EU, there are substantial differences, with 22 hours in required in Finland to 288 in Bulgaria (PwC (2009)). So, it is definitely of interest to ask how the marginal excess burden varies with compliance costs.

We can model compliance costs as follows. Let k be the cost of compliance as a fraction of sales at the threshold. We assume a fixed cost ks_0, or $\frac{k}{1-\gamma}z_0$ of compliance if registered, so that net utility of the trader with registration is $u_R(z;a) - \frac{k}{1-\gamma}z_0$.

The MEB can then be calculated exactly as before, except that n_R now solves

$$(1-t_N)z_0(1+e) - e(n_R)^{-1/e}(z_0)^{1+\frac{1}{e}} - n_R(1-t_R)^{1+e} + \frac{(1+e)k}{1-\gamma}z_0 = 0$$

The results of variation in registration costs k on the MEB are shown in Figure 6. We allow k to vary between 0% and 5% of sales. All other parameters are at their baseline values, with $\gamma = 0.45$. We expect that an increase in k will increase bunching and thus increase the correction factor and the MEB, and this is exactly what happens.

- Figure 6 in here -

Figure 6 shows that the MEB of the VAT does increase with k, but the effect is very small. An increase of compliance costs of zero to 5% of turnover at the threshold only increases the MEB from 0.334 to 0.341, an increase of about 2%.

7 Conclusions

This paper has shown that the sufficient statistic approach to the welfare properties of income (and other) taxes does not extend to tax systems with notches, because with notches, changes in bunching induced by changes in tax rates have a first-order effect on tax revenues. In an income tax setting, we showed that the marginal excess burden (MEB) and the welfare-maximizing top rate of tax are given by the relevant formulae for a proportional tax as in Feldstein (1999) plus a correction factor. The Feldstein formulae always underestimate the MEB and overestimate the revenue and welfare-maximizing rate of tax. Quantitatively, these mis-estimates can be very large; the MEB can be underestimated by an order of magnitude, but the errors in calculating the welfare-maximizing tax are somewhat smaller.

An application to VAT was also studied. A simple model of small traders who differ in productivity, and are subject to VAT at rate t above a threshold level of sales was shown to be formally equivalent to the income tax model. We then show that the MEB of an increase in the statutory rate of VAT is given by the Feldstein formula for a proportional tax plus a correction factor as in the income tax case. A calibration of the model shows that the proportional tax formula for the MEB of the VAT underestimates the true MEB by a factor of up to three. This framework also allows us to evaluate the effect of increased compliance costs of VAT in the MEB via its impact on bunching; increased compliance costs increase bunching and thus increase the MEB, but the effect is quantitatively small.

8 References

Apps, P., Long, N., & Rees, R. (2014). Optimal piecewise linear income taxation. *Journal of Public Economic Theory*, 16(4), 523-545.

Bachas, P., and Mauricio S. Not (ch) Your Average Tax System: Corporate Taxation Under Weak Enforcement. Mimeo, 2015.

Ballard, C. L., Shoven, J. B., & Whalley, J. (1985). The total welfare cost of the United States tax system: a general equilibrium approach. National Tax Journal, 125-140.

Best, M. C., & Kleven, H. J. (2014). Housing Market Responses to Transaction Taxes: Evidence From Notches and Stimulus in the UK.

Chetty, R. (2009). Sufficient statistics for welfare analysis: a bridge between structural and reduced-form methods. *Annual Review of Economics* 1:451-488

Doerrenberg, P., Peichl, A., & Siegloch, S. (2015). The elasticity of taxable income in the presence of deduction possibilities. *Journal of Public Economics.*

Ebert, U. (1992). A reexamination of the optimal nonlinear income tax. *Journal of Public Economics*, 49(1), 47-73.

Feldstein, M. (1999). Tax avoidance and the deadweight loss of the income tax. *Review of Economics and Statistics* 81(4), 674-680.

Federation of Small Businesses (2010), *Impact of increasing VAT registration threshold for Small Businesses.*

Gruber, J., & Saez, E. (2002). The elasticity of taxable income: evidence and implications. *Journal of public Economics*, 84(1), 1-32.

Gillitzer, C. and J. Slemrod (2016), Does Evasion Invalidate the Welfare Sufficiency of the ETI? unpublished paper, University of Michigan

Hargaden, E. P. (2015). Taxpayer responses over the cycle: Evidence from Irish notches."

Harju, J., Matikka, T., & Rauhanen, T. (2016). The effects of size-based regulation on small firms: evidence from VAT threshold.

Kanbur, R., & Keen, M. (2014). Thresholds, informality, and partitions of compliance. *International Tax and Public Finance*, 21(4), 536-559.

Keen, M., & Mintz, J. (2004). The optimal threshold for a value-added tax. *Journal of Public Economics*, 88(3), 559-576.

Kleven, H. J. (2016). Bunching. *Annual Review of Economics*, 8(1).

Kleven, H. J., & Waseem, M. (2013). Using notches to uncover optimization frictions and structural elasticities: Theory and evidence from Pakistan. *The Quarterly Journal of Economics*, 128(2), 669-723.

Kleven, H. J., & Schultz, E. A. (2014). Estimating taxable income responses using Danish tax reforms. *American Economic Journal: Economic Policy*, 6(4), 271-301.

Kopczuk, W., & Munroe, D. J. (2014). Mansion tax: The effect of transfer taxes on the residential real estate market (No. w20084). *National Bureau of Economic Research.*

Liu, L. & Lockwood, B. (2015). VAT Notches, CEPR Discussion Paper 10606

Luttmer, E. G. (2007). Selection, growth, and the size distribution of firms. *The Quarterly Journal of Economics*, 1103-1144.

PwC (2009). *The impact of VAT compliance on business.* London

Piketty, T., & Saez, E. (2013). Optimal Labor Income Taxation. *Handbook of Public Economics.* Vol. 5, 391-474.

Rutherford, T., & Paltsev, S. (1999). From an input-output table to a general equilibrium model: assessing the excess burden of indirect taxes in Russia. Draft, University of Colorado.

Saez, E. (2001). Using elasticities to derive optimal income tax rates. *The Review of Economic Studies*, 68(1), 205-229.

Saez, E. (2010). Do taxpayers bunch at kink points?. *American Economic Journal: Economic Policy*, 180-212.

Saez, E., Slemrod, J., & Giertz, S. H. (2012). The elasticity of taxable income with respect to marginal tax rates: A critical review. *Journal of Economic Literature*, 3-50.

Slemrod, J. (2013). Buenas notches: lines and notches in tax system design. *eJournal of Tax Research*, 11(3), 259.

Slemrod, J., Yitzhaki, S., Mayshar, J., & Lundholm, M. (1994). The optimal two-bracket linear income tax. *Journal of Public Economics*, 53(2), 269-290.

Weber, C. E. (2014). Toward obtaining a consistent estimate of the elasticity of taxable income using difference-in-differences. *Journal of Public Economics*, 117, 90-103.

A Appendix

Proof of Proposition 1. It remains to derive a formula for C. From (8), noting that

$$\psi_n = -\frac{1}{e}\frac{n^{-(1+1/e)}}{1+\frac{1}{e}}(z)^{1+\frac{1}{e}} = -\frac{1}{1+e}\left(\frac{z}{n}\right)^{1+1/e}. \tag{42}$$

and $z(1-t;n) = (1-t)^e n$, we have

$$\frac{\partial n_H}{\partial t_H} = \frac{(1-t_H)^e n_H(1+e)}{(1-t_H)^{1+e} - \left(\frac{z_0}{n_H}\right)^{1+1/e}} \tag{43}$$

Next, from (13) and (16), using the fact that $z(1-t;n) = (1-t)^e n$, we have

$$\begin{aligned}\frac{1}{B_H}\frac{\partial R}{\partial n_H} &= \frac{(t_L z_0 - t_H z(1-t_H;n_H))h(n_H)}{\int_{n_H}^{\overline{n}} z(1-t_H;n)h(n)dn} \\ &= \frac{(t_L z_0 - t_H(1-t_H)^e n_H)h(n_H)}{(1-t_H)^e\int_{n_H}^{\overline{n}} nh(n)dn}\end{aligned} \tag{44}$$

So, plugging (43),(44) into (**??**), we have:

$$\begin{aligned}C &= \frac{(1-t_H)(t_H(1-t_H)^e n_H - t_L z_0)h(n_H)}{(1-t_H)^e\int_{n_H}^{\overline{n}} nh(n)dn}\frac{(1-t_H)^e n_H(1+e)}{(1-t_H)^{1+e} - \left(\frac{z_0}{n_H}\right)^{1+1/e}} \\ &= \frac{(1-t_H)(t_H(1-t_H)^e n_H - t_L z_0)}{(1-t_H)^e E[n\,|n\geq n_H]}\frac{h(n_H)}{(1-H(n_H))}\frac{n_H(1+e)}{(1-t_H)^{1+e} - \left(\frac{z_0}{n_H}\right)^{1+1/e}}\end{aligned} \tag{45}$$

where in the second line we have used $\int_{n_H}^{\overline{n}} nh(n)dn = E[n\,|n\geq n_H]\,(1-H(n_H))$.

Now, given that n follows a Pareto distribution with shape and scale parameters $a, \underline{n}$, we also know that

$$E[n\,|n\geq n_H] = \frac{an_H}{a-1}, \frac{h(n)}{1-H(n)} = \frac{a}{n} \tag{46}$$

Plugging (46) into (45), we get:

$$C = \frac{(1-t_H)(t_H(1-t_H)^e - t_L z_0/n_H)(a-1)(1+e)}{\left((1-t_H)^{1+e} - \left(\frac{z_0}{n_H}\right)^{1+1/e}\right)} \tag{47}$$

as required. □

Proof of Proposition 4.3. The government objective, written as a Lagrangean including the constraint $R = E$, is

$$\begin{aligned}W = &\int_{\underline{n}}^{n_L} G(v(1-t_L;n))h(n)dn + \int_{n_L}^{n_H} G((z_0(1-t_L) - \psi(z_0;n)))h(n)dn \\ &+ \int_{n_H}^{\overline{n}} G(v(1-t_H;n))h(n)dn + \lambda(R-E)\end{aligned}$$

So, the welfare-maximizing top rate of tax is defined by

$$\frac{\partial W}{\partial t_H} = -\int_{n_H}^{\overline{n}} g(n)z(1-t_H;n)h(n)dn + \lambda\frac{\partial R}{\partial t_H} = 0 \tag{48}$$

where $g = G'$. So, plugging (15), in (48), and rearranging, we get;

$$-cov(g,z)(1-H(n_H)) - \overline{g}B_H + \lambda\left[B_H + t_H\left.\frac{\partial B_H}{\partial t_H}\right|_{n_H \text{ const}} + \frac{\partial R}{\partial n_H}\frac{\partial n_H}{\partial t_H}\right] = 0 \tag{49}$$

where

$$cov(g,z) = \int_{n_H}^{\overline{n}} (g-\overline{g})zh^*(n)dn,\ h^* = h/(1-H(n_H),\ \overline{g} = \int_{n_H}^{\overline{n}} gh^*(n)dn.$$

Dividing though (49) by B_H and λ, and using (**??**), we get

$$-\theta + 1 - \frac{t_H e}{1-t_H} + \frac{1}{B_H}\frac{\partial R}{\partial n_H}\frac{\partial n_H}{\partial t_H} = 0 \tag{50}$$

with

$$\theta = \frac{cov(g,z)(1-H(n_H))}{B_H\lambda} + \frac{\overline{g}}{\lambda}$$

Multiplying (50) through by $1-t_H$ and using the definition of C in (**??**), we get

$$(1-t_H)(1-\theta) - t_H e - C = 0$$

Rearranging this last expression, assuming $cov(g,z) = 0$ and normalizing λ to 1 gives (23). □

Derivation of (31), (32), (33). We first derive (31), (32). Trader utility is profit minus the disutility of labour. So, combining (42), (29), (30) and using $n = a(1-\gamma),\ l = y$, get:

$$\begin{aligned} u_N &= (1-\gamma(1+t))y - \frac{A}{1-\gamma}\frac{n}{1+\frac{1}{e}}\left(\frac{y(1-\gamma)}{n}\right)^{1+\frac{1}{e}} \\ u_R &= \left(\frac{1}{1+t} - \gamma\right)y - \frac{A}{1-\gamma}\frac{n}{1+\frac{1}{e}}\left(\frac{y(1-\gamma)}{n}\right)^{1+\frac{1}{e}} \end{aligned} \tag{51}$$

Now, using $z = y(1-\gamma)$ in (51), we get

$$\begin{aligned} u_N &= \frac{1-\gamma(1+t)}{1-\gamma}z - \frac{A}{1-\gamma}\frac{n}{1+\frac{1}{e}}\left(\frac{z}{n}\right)^{1+\frac{1}{e}} \\ u_R &= \left(\frac{1}{(1+t)(1-\gamma)} - \frac{\gamma}{1-\gamma}\right)z - \frac{A}{1-\gamma}\frac{n}{1+\frac{1}{e}}\left(\frac{z}{n}\right)^{1+\frac{1}{e}} \end{aligned} \tag{52}$$

Finally, we note that for (52) to imply (32), we require

$$1-t_N=\frac{1-\gamma(1+t)}{1-\gamma},\ 1-t_R=\frac{1}{(1+t)(1-\gamma)}-\frac{\gamma}{1-\gamma} \tag{53}$$

But, solving (53) for t_N, t_R, we get (32) as required.

Now we derive (33). Let $y(n)$ be the sales of an $n-type$ trader. Then, revenue from the from the VAT is

$$R=\frac{t}{1+t}\int_{n_R}^{\overline{n}}y(n)h(n)dn+t\int_{\underline{n}}^{n_R}\gamma y(n)h(n)dn \tag{54}$$

The first term is revenue from VAT levied on the value of sales of registered firms, because the sale price is $1/(1+t)$, and the second term is revenue from inputs sold by the intermediate input producer to firms that do not register for VAT. Using $z(n)=y(n)(1-\gamma)$, we can write this as

$$R=\frac{t}{(1+t)(1-\gamma)}\int_{n_R}^{\overline{n}}z(n)h(n)dn+\frac{t\gamma}{1-\gamma}\int_{\underline{n}}^{n_R}z(n)h(n)dn \tag{55}$$

Finally, replacing $z(n)$ by $z(1-t_N;n), z_0$, or $z(1-t_R;n)$ where appropriate, we get (33) as required. □

Proof of Proposition 6.3. Let B_N, B_R be the bases of the effective taxes t_N, t_R defined in (34). Then from (17),(33), and remembering that a change in the statutory rate of VAT t changes t_N, t_R via (32), we have:

$$\frac{dW}{dt}=-\left(\frac{\partial t_N}{\partial t}B_N+\frac{\partial t_R}{\partial t}B_R\right) \tag{56}$$

$$\frac{dR}{dt}=\frac{\partial t_N}{\partial t}\left(B_N+t_N\left.\frac{\partial B_N}{\partial t_N}\right|_{n_R\text{ const}}\right)+\frac{\partial t_R}{\partial t}\left(B_R+t_R\left.\frac{\partial B_R}{\partial t_R}\right|_{n_R\text{ const}}\right)-C' \tag{57}$$

where

$$C'=-\frac{\partial R}{\partial n_R}\left(\frac{\partial t_N}{\partial t}\frac{\partial n_R}{\partial t_N}+\frac{\partial t_R}{\partial t}\frac{\partial n_R}{\partial t_R}\right) \tag{58}$$

So, plugging (56),(57) into (14), we have, after rearrangement

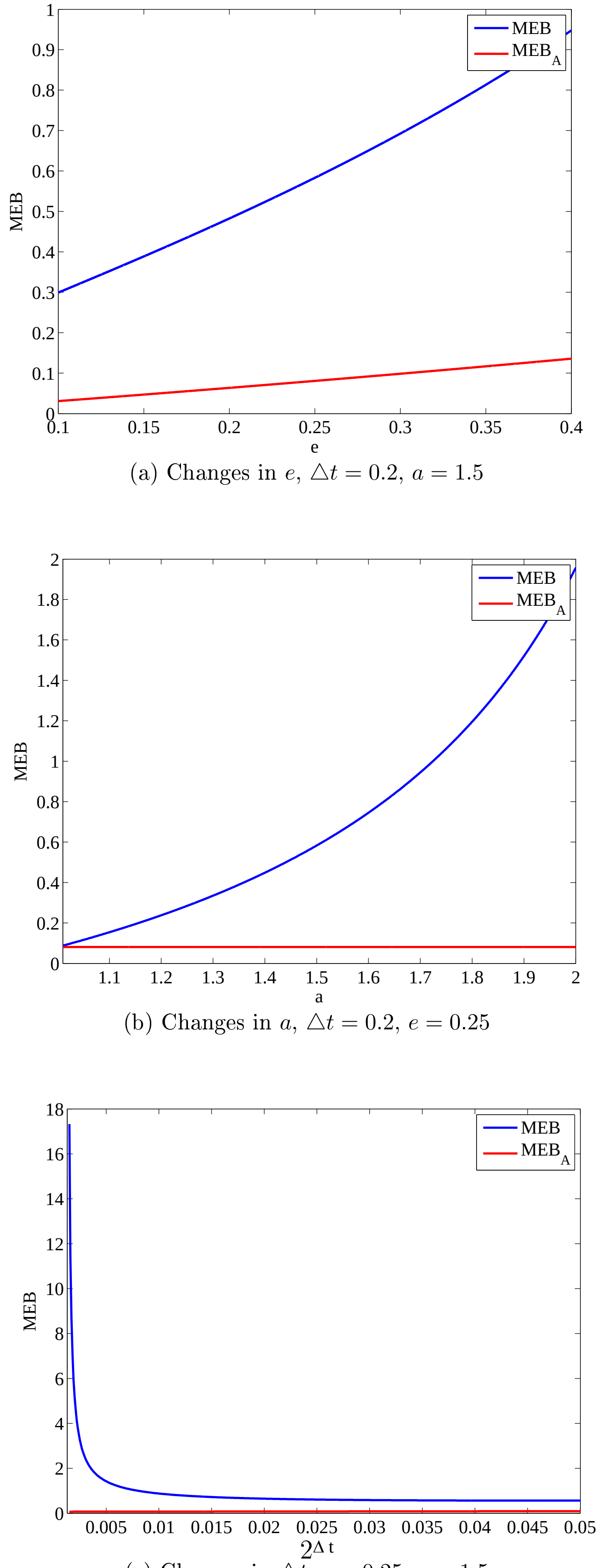
$$\begin{aligned} MEB &= -\frac{d(W+R)/dt}{dR/dt} \qquad (59)\\ &= \frac{\frac{B_N}{1-t_N}\frac{\partial t_N}{\partial t}t_N\left(\frac{1-t_N}{B_N}\left.\frac{\partial B_N}{\partial(1-t_N)}\right|_{n_R\text{ const}}\right)+\frac{B_R}{1-t_R}\frac{\partial t_R}{\partial t}t_R\left(\frac{1-t_R}{B_R}\left.\frac{\partial B_R}{\partial(1-t_R)}\right|_{n_R\text{ const}}\right)+C'}{\frac{B_N}{1-t_N}\frac{\partial t_N}{\partial t}\left(1-t_N-t_N\frac{1-t_N}{B_N}\left.\frac{\partial B_N}{\partial t_N}\right|_{n_R\text{ const}}\right)+\frac{B_R}{1-t_R}\frac{\partial t_R}{\partial t}\left(1-t_R-t_R\frac{1-t_R}{B_R}\left.\frac{\partial B_R}{\partial t_R}\right|_{n_R\text{ const}}\right)-C'}\\ &= \frac{\frac{B_N}{1-t_N}\frac{\partial t_N}{\partial t}e\phi+\frac{B_R}{1-t_R}\frac{\partial t_R}{\partial t}t_Re+C'}{\frac{B_N}{1-t_N}\frac{\partial t_N}{\partial t}(1-t_N(1+e\phi))+\frac{B_R}{1-t_R}\frac{\partial t_R}{\partial t}(1-t_R(1+e))-C'} \end{aligned}$$

where in the last line, we have used (35).So, dividing top and bottom of (59) by $\frac{B_R}{1-t_R}\frac{\partial t_R}{\partial t}+\frac{B_N}{1-t_N}\frac{\partial t_N}{\partial t}$ and using the definition of θ from (37), and the definition of C from (40), we get

$$MEB=\frac{(1-\theta)t_N e\phi+\theta t_R e+C}{1-(1-\theta)t_N(1+e\phi)-\theta t_R(1+e)-C} \tag{60}$$

Finally, using the definitions of $\tau=(1-\theta)t_N+\theta t_R$, $\varepsilon=\frac{(1-\theta)t_N\phi+\theta t_R}{(1-\theta)t_N+\theta t_R}e$, (60) can be rearranged to (39), as required. □

Figure 1: The Marginal Excess Burden

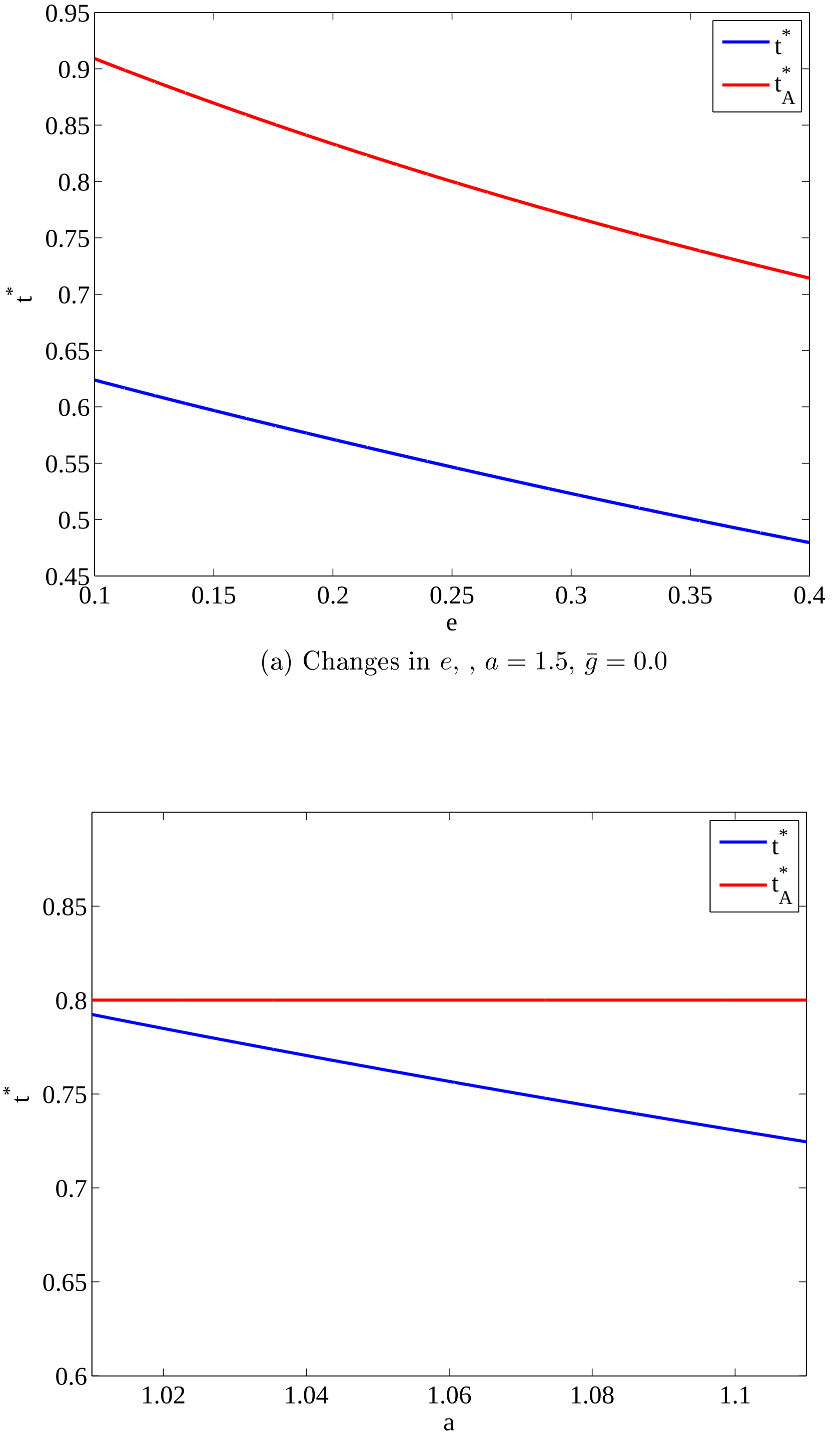


(a) Changes in e, $\triangle t = 0.2$, $a = 1.5$

(b) Changes in a, $\triangle t = 0.2$, $e = 0.25$

(c) Changes in $\triangle t$, $e = 0.25$, $a = 1.5$

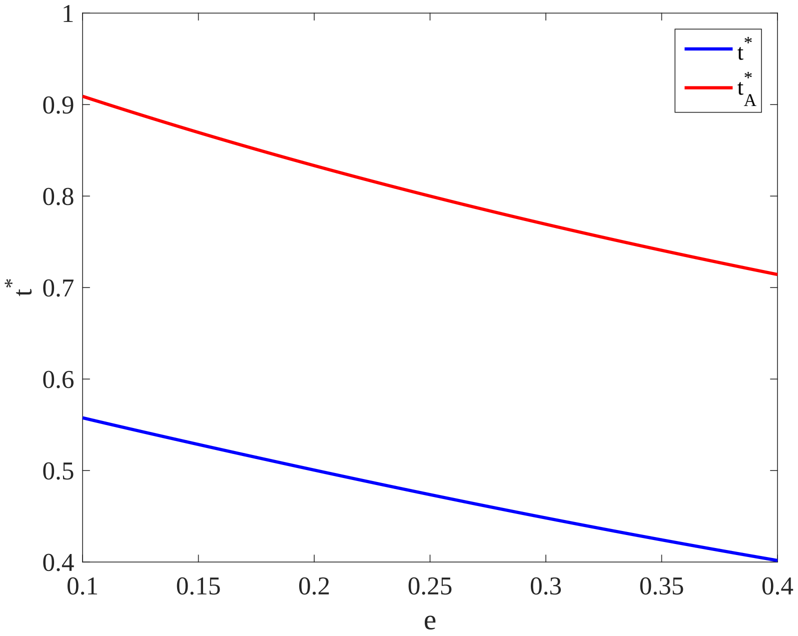
Figure 2: The Revenue-Maximising Top Rate of Tax



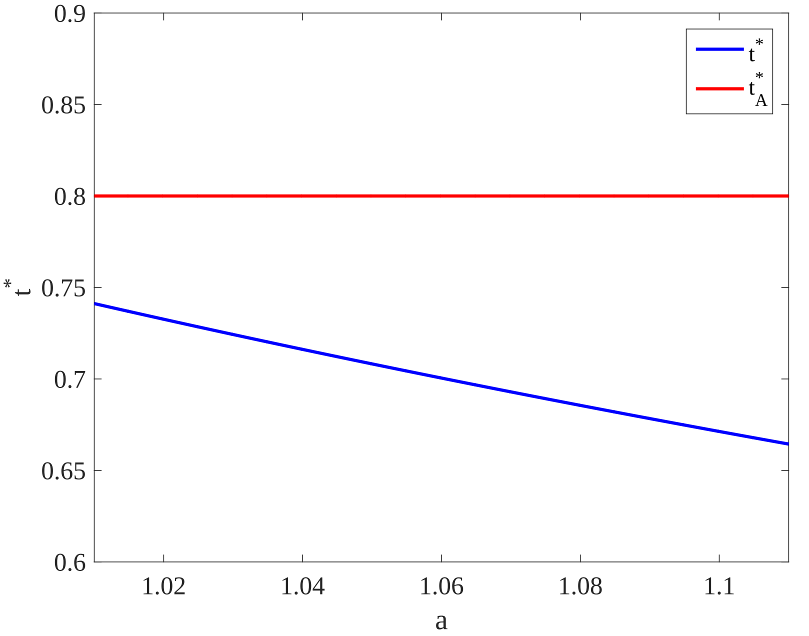
(a) Changes in e, , $a = 1.5$, $\bar{g} = 0.0$

(b) Changes in a, , $e = 0.25$, $\bar{g} = 0.0$

Figure 3: The Wefare-Maximising Top Rate of Tax

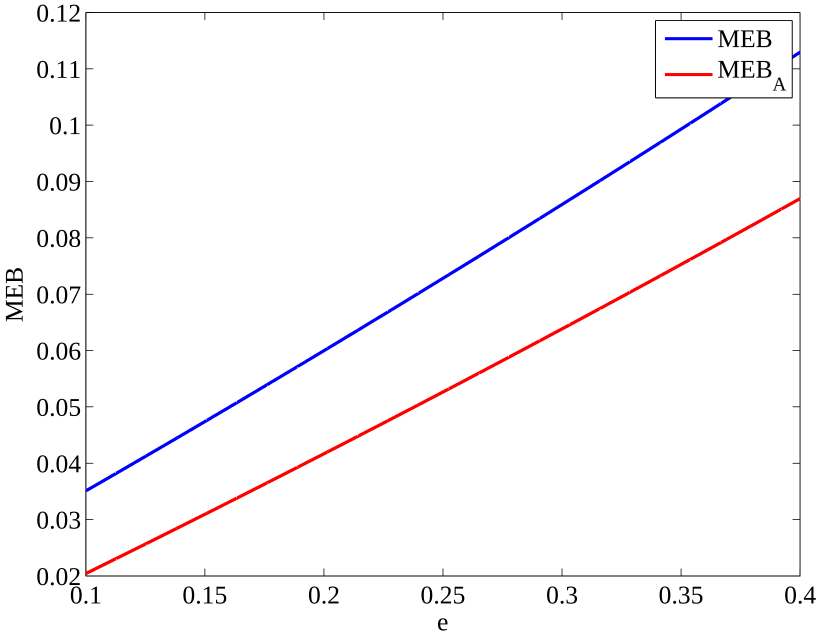


(a) Changes in e, , $a = 1.5$, $\bar{g} = 0.25$

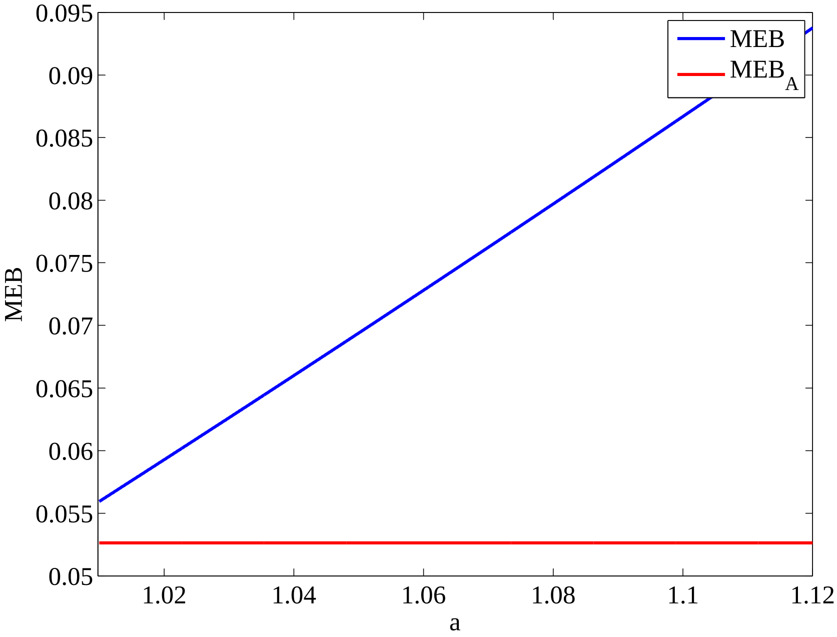


(b) Changes in a, $e = 0.25$, $\bar{g} = 0.25$

Figure 4: The Marginal Excess Burden of the VAT

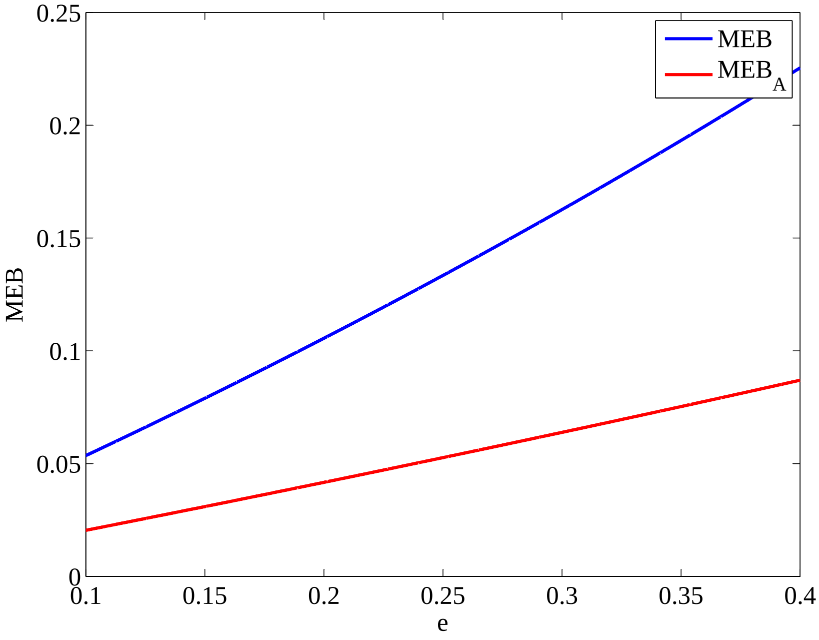


(a) Changes in e, $a = 1.06$, $s = 0.0$

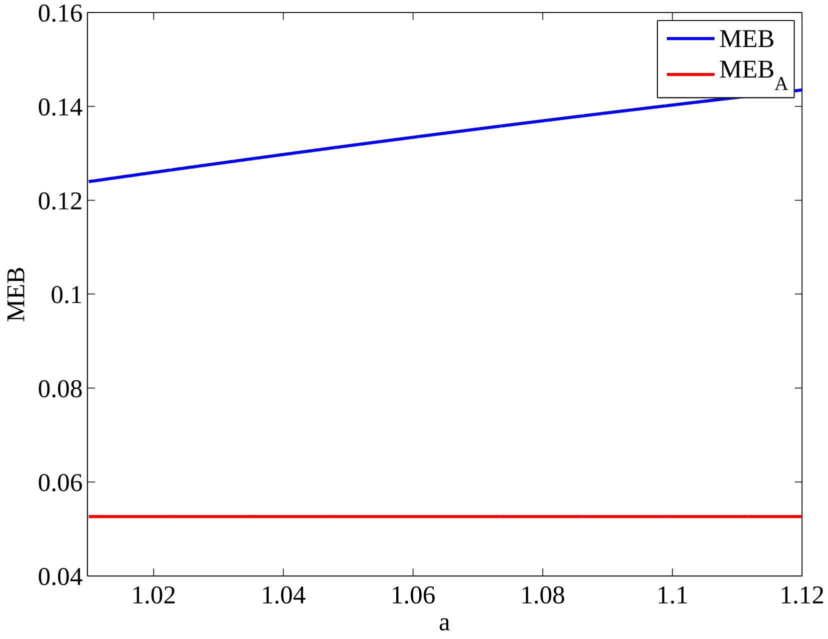


(b) Changes in a, $e = 0.25$,$s = 0.0$

Figure 5: The Marginal Excess Burden of the VAT

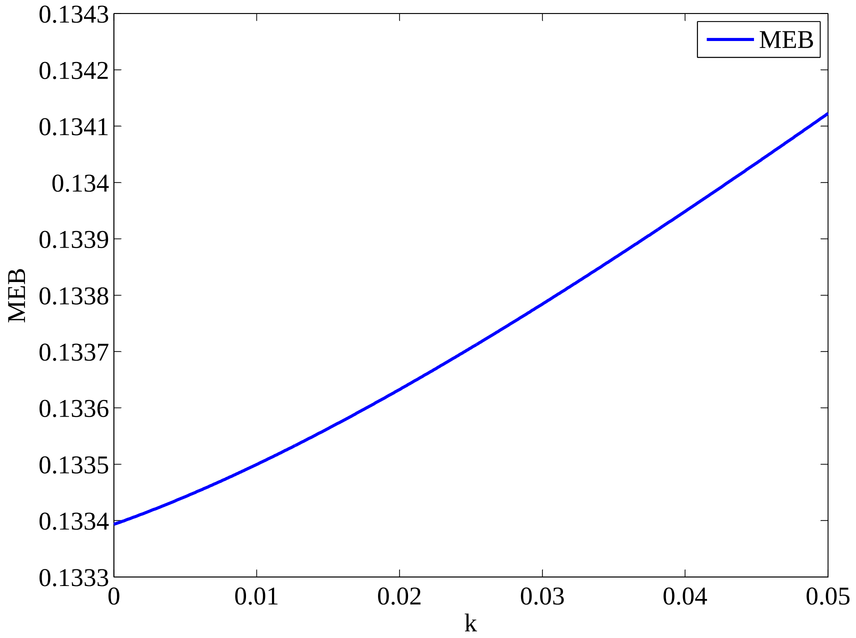


(a) Changes in e, $a = 1.06$, $s = 0.45$



(b) Changes in a, $e = 0.25$,$s = 0.45$

Figure 6: The Marginal Excess Burden of the VAT and Registration Costs



(a) Changes in k, $e = 0.25$,$a = 1.06$, $s = 0.45$

Malas Notches

Ben Lockwood

Not-For Publication Appendix

Details of MEB Simulation for the VAT Case. We need to express all the relevant elements of the MEB in terms of the parameters, t, γ, z_0,and n_R, n_N. In turn, we know that $n_N = z_0/(1-t_N)^e$ and that n_R is determined by

$$(1-t_N)z_0(1+e) - e(n_R)^{-1/e}\left(z_0\right)^{1+\frac{1}{e}} - n_R(1-t_R)^{1+e} = 0 \tag{1}$$

Assume that the distribution of firms is Pareto with shape and scale parameters $a, \underline{n}$. Without loss of generality, we assume $\underline{n} = 1$; so, the distribution and density of n is $H(n) = 1 - n^{-a},\ h(n) = \frac{a}{n^{a+1}}$. So, using these formulae and $z(1-t;n) = (1-t)^e n$, we have by routine calculation;

$$\begin{aligned} B_R &= (1-t_R)^e \int_{n_R}^{\overline{n}} nh(n)dn = (1-t_R)^e \frac{a}{a-1}(n_R)^{1-a} \qquad (2) \\ B_N &= (1-t_N)^e \int_{1}^{n_N} nh(n)dn + z_0(H(n_R) - H(n_N)) \\ &= (1-t_N)^e \frac{a}{a-1}(1-(n_N)^{1-a}) + z_0\left((n_N)^{-a} - (n_R)^{-a}\right) \end{aligned}$$

Moreover, from the formulae for t_N, t_R in the paper, we have:

$$\frac{\partial t_R}{\partial t} = \frac{1}{(1-\gamma)(1+t)^2},\ \frac{\partial t_R}{\partial t} = \frac{\gamma}{(1-\gamma)} \tag{3}$$

So, plugging (3) into the formula for θ in the paper, we can write

$$\theta = \frac{\frac{B_R}{1-t_R}}{\frac{B_R}{1-t_R} + \frac{B_N}{1-t_N}\gamma(1+t)^2} \tag{4}$$

Plugging (2) into (5) allows us to compute θ as a function of t, γ, z_0,and n_N, n_R.

Next, using $z(1-t;n) = (1-t)^e n$, and the properties of the Pareto distribution, we have;

$$\phi = \frac{\int_{\underline{n}}^{n_N} z(1-t_N;n)h(n)dn}{B_N} = \frac{(1-t_N)^e \frac{a}{a-1}(1-(n_N)^{1-a})}{B_N} \tag{5}$$

So, using (2), (5), ϕ can be computed as a function of t, γ, z_0,and n_N, n_R.

Finally, recalling the definition of C in the paper, we have:

$$\begin{aligned} C &= -\frac{\frac{\partial R}{\partial n_R}\left(\frac{\partial t_N}{\partial t}\frac{\partial n_R}{\partial t_N} + \frac{\partial t_R}{\partial t}\frac{\partial n_R}{\partial t_R}\right)}{\frac{B_N}{1-t_N}\frac{\partial t_N}{\partial t} + \frac{B_R}{1-t_R}\frac{\partial t_R}{\partial t}} \\ &= -\frac{\frac{\partial R}{\partial n_R}\left(\gamma\frac{\partial n_R}{\partial t_N} + \frac{1}{(1+t)^2}\frac{\partial n_R}{\partial t_R}\right)}{\frac{B_N}{1-t_N}\gamma + \frac{B_R}{1-t_R}\frac{1}{(1+t)^2}} \end{aligned} \tag{6}$$

where in the second line, we use (3).

It remains to calculate $\frac{\partial n_R}{\partial t_N}, \frac{\partial n_R}{\partial t_R}, \frac{\partial R}{\partial n_R}$. From (1), we have:

$$\begin{aligned} \frac{\partial n_R}{\partial t_R} &= \frac{(1-t_R)^e n_R(1+e)}{(1-t_R)^{1+e} - \left(\frac{z_0}{n_R}\right)^{1+1/e}} \\ \frac{\partial n_R}{\partial t_N} &= \frac{-z_0(1+e)}{(1-t_R)^{1+e} - \left(\frac{z_0}{n_R}\right)^{1+1/e}} \end{aligned} \tag{7}$$

Moreover, from the formula for $\frac{\partial R}{\partial n_R}$ in the paper, and the iso-elastic form of $z(1-t,n)$, we get

$$\frac{\partial R}{\partial n_R} = (t_N z_0 - t_R(1-t_R)^e n_R)h(n_R) \tag{8}$$

Plugging (7),(8) into (6), and using the formula for the density of the Pareto density to substitute out $h(n_R)$, we eventually get:

$$C = \frac{(t_R(1-t_R)^e n_R - t_N z_0)\left((1-t_R)^e n_R \frac{1}{(1+t)^2} - z_0\gamma\right)(1+e)}{\left((1-t_R)^{1+e} - \left(\frac{z_0}{n_R}\right)^{1+1/e}\right)\left(\frac{B_R}{1-t_R}\frac{1}{(1+t)^2} + \frac{B_N}{1-t_N}\gamma\right)} \frac{a}{(n_R)^{a+1}}$$

This expression for C is computable knowing t, γ, z_0,and n_R, n_L. Thus, all the components of MEB in the paper can be calculated. □