Optimal age-dependent income taxation in a dynamic extensive model: The case for negative participation tax to the young people*

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Abstract

We consider an optimal age-dependent income taxation in a dynamic model where labor-leisure choice is extensive margin in each period, each household faces idiosyncratic shocks of labor productivity and pecuniary cost to work, and there is no insurance market against the idiosyncratic shocks. We show the well known property of optimal participation tax rate in the static model continue to hold in our dynamic economy, and participation tax rates for some income groups with low consumptions are likely negative. In dynamic models, the optimal participation tax rate depends on age as well as labor income. Our numerical simulations suggest that negative participation tax should be restricted to young households.

JEL classification: H21, H23, J22

Keywords: Extensive margin; Age-dependent taxation; Labor income tax

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1 Introduction

The optimal income taxation literature has developed models to analyze the design of income tax/transfer programs. Following the seminal paper of Mirrlees (1971), many studies focus exclusively on the intensive labor margin, in other words, households choose hours of work or intensity of work. In that framework, negative marginal tax rates can never be optimal, ruling out in-work credits.

Much recent literature has emphasized the role of the extensive margin. In the extensive labor margin, households choose whether or not to participate in labor force with fixed hours of work. In this setting, it can be optimal to adopt wage subsidies or in-work credits with negative participation tax for low income households (see e.g., Diamond (1980), Saez (2002), Choné and Laroque (2005, 2011) and Jacquet et al. (2013)).

Although these studies have influential findings both theoretical and policy implications, a large part of the literature is static and does not directly address the intertemporal side of the problem. There are few studies treat dynamic model with extensive labor margin (see Laroque (2011) and Choné and Laroque (2017)). They investigate time invariant non-linear income tax/transfer in a stationary life-cycle model with perfect insurance market, and shows that a worker faces a negative participation tax rate if its social weight with respect to lifetime permanent incomes is larger than the average.[1] Under the stationary assumption, the distribution of the productivities and cost to work are constant overtime and the correlation between current and permanent income becomes a central problem. Because they assume perfect insurance market, personal event, such as illness or accident, are fully insured by private financial market. The main objective of the government in their model is the redistribution among households with different expected permanent income at birth, which solely determine their social weight. A perfect correlation between current and permanent income would make the static and the dynamic model equivalent. The policy instrument of Laroque (2011) and Choné and Laroque (2017) is age-independent. Thus, in the extensive labor margin, the design of an optimal tax scheme depending on age or whole history remains an unsettled question.

Recent dynamic Mirrlees literature (see Kocherlakota (2010)) examines intertemporal shocks about labor productivities and preferences in the intensive labor margin. With the extensive margin, Diamond and Mirrlees (1978) and Golosov and Tsyvinski (2006) treat the permanent disability shocks. It means that once disabled, labor status is no longer a matter of choice. In our dynamic extensive labor margin with shocks, a household can decide to participate or not in the labor market in each period. It is important to examine these shocks in the extensive labor margin since a household who decides not to work at some period for some reasons (e.g., bad health condition, involuntary unemployment, or lack of ability) may change its labor status and go back to work in another period. In adopting these settings, we can study a redistributive policy to avoid 'poverty trap'.

The results from dynamic Mirrlees literature show that the optimal income tax policy with shocks have to be history-dependent. The history-dependent nature is, however, complicated and rare in the real world. The implements of the optimal allocation could be combined labor income tax with other instruments. For examples, Grochulski and Kocherlakota (2010) show that, the optimal allocation can be implemented through a combination of three policy instruments that they term social security systems, namely a tax on current labor income until retirement,

[1] Laroque (2011) also examines the usefulness of a complemental positive linear tax on wealth.

a retrospective capital income tax only at retirement that is dependent on labor income history, and a history-contingent payment after retirement. Michau (2014) also show that the optimal allocation can be implemented by a history-dependent social security system in a life-cycle model with both intensive and extensive (retirement) margin and without skill shocks. On the contrary, Weinzierl (2011) examines partial tax reforms to age-dependent labor income taxation, and shows that there are small welfare losses from implementing not full optimal history-dependent income taxation, but age-dependent income taxation.[2] We adopt latter approach and focus solely on the role of age-dependent income taxation.

Thus, we investigate the age-dependent and age-independent optimal income tax schedule in a dynamic model with extensive labor margin in each period and idiosyncratic shocks of labor productivity and cost to work without perfect insurance market. The main purpose of the government here is the redistribution among households with different history of idiosyncratic shocks. We derived the following results. First, in the case of age-dependent labor income tax, we derive the optimal age-specific participation tax rates which is similar to that in static model. On the contrary to static model, negative participation tax rate for households with lower consumption apply in each age group. Second, in the case of age-independent labor income tax, we confirm the optimal participation tax rates are similar to the static model and the stationary life-cycle model. Finally, we numerically solve the optimal tax policies in some example two periods economies and suggest that the negative participation tax should be applied to low-productive young households only.

The remainder of the paper is organized as follows. In Section 2, we present a dynamic model with extensive labor margin and idiosyncratic shocks. In Section 3, we investigate the optimal income tax problem and derive the optimal participation tax rates both age-dependent case and age-independent case. In Section 4, we consider two periods example economies with various parameter and numerically solves the optimal income tax policies. In Section 5, we present our conclusions.

2 The model

The economy has a discrete time index $t \in \{0, 1, 2, \cdots, T\}$ and is populated by a government and one unit of continuous $T+1$ periods-lived households. Each household faces two dimensional idiosyncratic shock $\theta_t = (w_t, \delta_t)$ at period t, where $w_t \in \mathbb{R}_+$ and $\delta_t \in \mathbb{R}_+$ respectively represent labor productivity and pecuniary opportunity cost to work measured in units of consumption good at that period. There are $I \in \mathbb{N}$ possible levels of labor productivity. The domain of the labor productivity is denoted by $\Omega = \{w^1, w^2, \cdots, w^I\} \in \mathbb{R}^I_{++}$. The idiosyncratic shock evolves over time and let h_t denote a history of the shocks up to period t, in other words, $h_t = (\theta_0, \theta_1, \cdots, \theta_t) \in (\Omega \times \mathbb{R}_+)^{t+1}$. Let $F^h_t : (\Omega \times \mathbb{R}_+)^{t+1} \to \mathbb{R}_+$ denote the cumulative probability distribution function (c.d.f) of h_t. F^θ_t denotes the marginal (conditional or unconditional) c.d.f. of θ_t. The c.d.f.s are public information.

There are two commodities at every period: a divisible consumption good and indivisible labors with various productivities. A household at period t chooses labor supply $l_t \in \{0, 1\}$, after observing the idiosyncratic shock at the period, (w_t, δ_t). If she works ($l_t = 1$), she obtains w_t units of (before tax) labor income and suffers disutility amounts to δ_t unit of consumption. If she does

[2] Blomquist and Micheletto (2008) also show that with the intensive margin, age-dependent income taxation is Pareto improving on an age-independent taxation.

not work, on the other hands, her labor income is zero and she suffers no disutility. Household with productivity w^i cannot mimic other productivity w^j $(i \neq j)$. The opportunity cost to work is private information, while the labor productivity is known to public if and only if she works at the period. The households and government can access outside financial market with constant real interest rate $r \in \mathbb{R}$. There is no market to insure against the idiosyncratic risk.

The government imposes tax on the labor income. Let $y_t(w^i)$ denote the after-tax income, that is, w^i minus labor income tax. $y_t(0)$ represents the subsistence income for not-working household at period t. Thus, $y_t : \{0\} \cup \Omega \to \mathbb{R}$ represents a disposable income schedule at period t. Given the disposable income schedule $y = (y_0, y_1, \cdots, y_T)$, the households choose a plan of labor supply, $l = (l_0, l_1, \cdots, l_T)$, and asset holding, $a = (a_0, a_1, \cdots, a_T)$, where $l_t : (\Omega \times \mathbb{R}_+)^{t+1} \to \{0, 1\}$ and $a_t : (\Omega \times \mathbb{R}_+)^{t+1} \to \mathbb{R}$ respectively represent the labor status and the asset holding at the end of the period, conditional on h_t. All the households have $(1 + r)a_{-1} \in \mathbb{R}$ unit of real asset at the beginning of the initial period.

The consumption net of the pecuniary cost to work at period t conditional on h_t (hereafter called net consumption), denoted by $c_t(h_t)$, is,

$$c_t(h_t) = (1 + r)a_{t-1}(h_{t-1}) + y_t(w_t l_t(h_t)) - \delta_t l_t(h_t) - a_t(h_t), \tag{1}$$

and a plan of net consumption is $c = (c_0, c_1, \ldots, c_T)$. The expected utility is,

$$\sum_t \beta^t \int_{h_t} U(c_t(h_t))\, dF_t^h(h_t), \tag{2}$$

where $\beta \in \mathbb{R}_+$ is time discount factor and U is a twice continuously differentiable utility function satisfying $U' > 0$ and $U'' < 0$.[3] Given a disposable income schedule y, the household chooses a, c and l to maximize (2) subject to (1) and,

$$a_T(h_T) \geq 0, \quad \text{for all } h_T. \tag{3}$$

Given a choice of asset plan a, net consumption at period t is larger at $l_t = 1$ than that at $l_t = 0$, if the opportunity cost to work, δ_t, is smaller than $y_t(w_t) - y_t(0)$. Thus the optimal labor supply plan l_t is 1 (resp. 0) if δ_t is smaller (resp. larger) than $y_t(w_t) - y_t(0)$. Notice that the optimal choice of labor supply solely depends on θ_t and y_t.

The population of households with (before tax) labor income w^i at period t, denoted by n_t^i and the population of not-working households at period t, denoted by n_t^0, are,

$$\begin{aligned} n_t^i &= \int_{\theta_t \in \Theta_t^i} dF_t^\theta(\theta_t), \quad i = 1, 2, \cdots, I, \\ n_t^0 &= 1 - \sum_{i=1}^{I} n_t^i, \end{aligned}$$

where Θ_t^i denotes the region of θ_t defined by,

$$\Theta_t^i \equiv \{(w_t, \delta_t) \mid w_t = w^i \text{ and } 0 < \delta_t < y_t(w^i) - y_t(0)\}.$$

[3]It is natural to restrict the consumption to be non-negative. Consumption net of pecuniary opportunity cost, however, may be negative. We assume the time separable utility function and the common initial asset for notational simplicity. It is not difficult to generalize these assumptions.

The working population of households with w^i at period t, n_t^i for $i \neq 0$, is a non-decreasing function of $y_t(w^i) - y_t(0)$ and, the following symmetric relations hold for all t.

$$\frac{\partial n_t^i}{\partial y_t(w^i)} = -\frac{\partial n_t^i}{\partial y_t(0)} = -\frac{\partial n_t^0}{\partial y_t(w^i)} \geq 0, \quad i = 1, 2, .., I, \tag{4}$$

$$\frac{\partial n_t^0}{\partial y_t(0)} = -\sum_{i=1}^{I} \frac{\partial n_t^i}{\partial y_t(0)} = \sum_{i=1}^{I} \frac{\partial n_t^i}{\partial y_t(w^i)} \geq 0. \tag{5}$$

The first order condition of the households' maximization problem with respect to the asset holding a_t yields,

$$U'(c_t(h_t)) = (1+r)\beta \int_{\theta_{t+1}} U'(c_{t+1}(h_t, \theta_{t+1}))\, dF_{t+1}^{\theta}(\theta_{t+1}|h_t). \tag{6}$$

Given y, (1), (3) with equality, and the Euler conditions (6) uniquely determine the optimal asset plan.[4] Let EU denote the maximized expected utility.

The government must finance an exogenous (presented valued) expenditure G. The government's (presented valued) budget constraint is given by,

$$\sum_{t=0}^{T} \frac{1}{(1+r)^t} \left(-n_t^0 y_t(0) + \sum_{i=1}^{I} n_t^i \left(w^i - y_t(w^i) \right) \right) \geq G. \tag{7}$$

The optimal taxation problem of the government is then to find the disposable income schedule y that maximizes the household's maximized expected utility, EU, subject to the budget constraint (7).

We consider two kinds of optimal taxation problems, namely *optimal age-dependent taxation problem* and *optimal age-independent taxation problem*. The former corresponds to the disposable income schedule which exactly maximizes EU subject to (7). Actual income tax system does not necessary depend on age in most countries, however. Thus we also investigate the latter problem which solves the same maximization problem with the additional constraint that disposable income schedule to be age-independent ($y_0 = y_1 = \cdots = y_T$).

Notice that the optimal age-dependent taxation is not unique due to the Ricardian equivalence. Suppose, for example, $y' = (y_0', y_1', \cdots, y_T')$ is an optimal age-dependent disposable income schedule. For any sequence $(\alpha_0, \alpha_1, \cdots, \alpha_T) \in \mathbb{R}^{T+1}$ satisfying $\sum_t (1+r)^{-t} \alpha_t = 0$, define another disposable income schedule $\tilde{y}$ by,

$$\tilde{y}_t(w^i) = y_t'(w^i) + \alpha_t, \quad \text{for all } i, t.$$

Then $\tilde{y}$ is optimal as well since it does not affect the budget constrains of the households or the government. Thus, we normalize the optimal age-dependent tax to be $y_0(0) = y_1(0) = \cdots = y_T(0)$. This normalization does not affect the conclusions.

[4] The optimal labor supply plan, equation (1) at the last period, and equation (3) with equality uniquely solve c_T as a function of a_{T-1}. The obtained c_T, equation (6), equation (1) and the optimal labor supply rule at $t = T-1$ uniquely solve a_{t-1} as a function of a_{t-2}. Repeating this procedure yields the optimal asset plan.

3 Optimal participation tax

It is well known that the optimal taxation schedule in a static model with extensive margin likely exhibits the property that participation tax rates for some low income households are negative (see Saez (2002)). Our first task here is to examine if this property continue to hold in the both of dynamic settings.

3.1 The case of age-dependent taxation

First, we examine the case of age-dependent taxation. The Lagrangian of the government problem in the case of age-dependent taxation is,

$$\mathcal{L}^D = EU + \lambda^D \left[\sum_{t=0}^{T} \frac{1}{(1+r)^t} \left(-n_t^0 y_t(0) + \sum_{i=1}^{I} n_t^i \left(w^i - y_t(w^i) \right) \right) - G \right],$$

where λ^D is the Lagrange multiplier associated with the government budget constraint in the case of age-dependent taxation. The first order conditions of the Lagrangian with respect to $y_t(w^i)$ and $y_t(0)$ are, using $n_t^0 = 1 - \sum_i n_t^i$,

$$\frac{\partial \mathcal{L}^D}{\partial y_t(w^i)} = \frac{\partial EU}{\partial y_t(w^i)} + \lambda^D \left[\frac{1}{(1+r)^t} \left(-n_t^i + \frac{\partial n_t^i}{\partial y_t(w^i)} (w^i - y_t(w^i) + y_t(0)) \right) \right] = 0, \quad \text{for } i \neq 0,$$
$$\frac{\partial \mathcal{L}^D}{\partial y_t(0)} = \frac{\partial EU}{\partial y_t(0)} + \lambda^D \left[\frac{1}{(1+r)^t} \left(-n_t^0 - \sum_{i=1}^{I} \left(\frac{\partial n_t^i}{\partial y_t(w^i)} \left(w^i - y_t(w^i) + y_t(0) \right) \right) \right) \right] = 0.$$

The expected utility is defined on the net consumption plan c which depends on a, l and y. Suppose the disposable income of working households with productivity w^i at period t increases by marginal 1 unit. By the chain rule, the effect on the expected utility is the sum of (i) the direct effect, that is, the increase of net consumption at period t for the case of $\delta_t \leq y_t(w^i) - y_t(0)$ given a and l, (ii) the effect via the change of l_t for the case of δ_t close to $y_t(w^i) - y_t(0)$, and (iii) the effect via the change of asset plan a. The second effect, however, vanishes because the marginal households switching their labor status are indifferent between work and not-work. The last effect also vanishes because of the envelope theorem. Thus, the partial derivatives of EU with respect to the disposable incomes are,

$$\frac{\partial EU}{\partial y_t(w^i)} = \beta^t \int_{h_{t-1}} \int_{\theta_t \in \Theta_t^i} U'(c_t(h_t))\, dF_t^\theta(\theta_t \mid h_{t-1})\, dF_{t-1}^h(h_{t-1}), \quad \text{for } i \neq 0,$$
$$\frac{\partial EU}{\partial y_t(0)} = \beta^t \int_{h_{t-1}} \sum_{i=1}^{I} \int_{\theta_t \in \bar{\Theta}_t^i} U'(c_t(h_t))\, dF_t^\theta(\theta_t \mid h_{t-1})\, dF_{t-1}^h(h_{t-1}),$$

where $\bar{\Theta}_t^i$ denotes the region of θ_t defined by,

$$\bar{\Theta}_t^i \equiv \{(w_t, \delta_t) \mid w_t = w^i \text{ and } \delta_t \geq y_t(w^i) - y_t(0)\}.$$

We can rewrite the first order conditions in the age-dependent taxation as follows.

$$n_t^i \left(1 - g_t^i\right) = \frac{\partial n_t^i}{\partial y_t(w^i)} \left(w^i - y_t(w^i) + y_t(0)\right), \quad \text{for } i \neq 0, \tag{8}$$

$$n_t^0 \left(1 - g_t^0\right) = -\sum_{i=1}^{I} \frac{\partial n_t^i}{\partial y_t(w^i)} \left(w^i - y_t(w^i) + y_t(0)\right), \tag{9}$$

where g_t^i is the average marginal social weight of period t consumption for the households obtaining disposable income $y_t(w^i)$ at the period, expressed in terms of public funds, that is,

$$g_t^i = \frac{(1+r)^t \beta^t}{n_t^i \lambda^D} \int_{h_{t-1}} \int_{\theta_t \in \Theta_t^i} U'(c_t(h_t))\, dF_t^\theta(\theta_t \mid h_{t-1})\, dF_{t-1}^h(h_{t-1}), \quad \text{for } i \neq 0,$$

$$g_t^0 = \frac{(1+r)^t \beta^t}{n_t^0 \lambda^D} \int_{h_{t-1}} \sum_{i=1}^{I} \int_{\theta_t \in \bar{\Theta}_t^i} U'(c_t(h_t))\, dF_t^\theta(\theta_t \mid h_{t-1})\, dF_{t-1}^h(h_{t-1}).$$

Adding (8) for all $i \neq 0$, and (9), we have,

$$1 = \sum_{i=0}^{I} n_t^i\, g_t^i. \tag{10}$$

The intuition behind (10) is clear. Suppose the tax at period t decreases by $(1+r)^t$ unit for all the labor income. This does not affect the households' labor-leisure choices and requires 1 unit of the government's budget. This tax cut increases all the households' expost utility by their marginal utility of period t consumption. (10) shows the cost must be balanced with the benefit at the optimal taxation schedule.

Households with productivity w^i at period t forgo the subsistence income $y_t(0)$ and pay income tax $w^i - y_t(w^i)$. Thus, $w^i - y_t(w^i) + y_t(0)$ is participation tax for households with productivity w^i at period t. We also define the participation tax rates faced by the households with productivity w^i at age t by,

$$\tau_t(w^i) = \frac{w^i - y_t(w^i) + y_t(0)}{w^i}, \quad \text{for } i \neq 0. \tag{11}$$

Using (11), we can rewrite (8) as an age-specific inverse elasticity rule which is the same shape in static model (see in Saez (2002)),

$$\frac{\tau_t(w^i)}{1 - \tau_t(w^i)} = \frac{1 - g_t^i}{\eta_t^i}, \quad \text{for } i = 1, 2, \cdots, I, \text{and } t = 0, 1, \ldots, T, \tag{12}$$

where η_t^i is the 'age-specific participation elasticity' defined as,

$$\eta_t^i = \frac{y_t(w^i) - y_t(0)}{n_t^i} \frac{dn_t^i}{d(y_t(w^i) - y_t(w^0))} > 0, \quad \text{for } i = 1, 2, \cdots, I, \text{and } t = 0, 1, \ldots, T.$$

Since the weighted average of g_t^i is unity within each period or age group, the right hand side of (12) is likely negative for some working group with lower consumption at that period or age. The participation tax rate for such an income group must be negative.

The participation tax rules both age-dependent case and static case depend on average marginal social weight and participation elasticity, so that it seems to have same interpretation. There are, however, a difference with age-dependent case and static one. In the static model, the average marginal social weight depend on consumptions at only that period. In the age-dependent case, the history of idiosyncratic shocks have an impact on consumption pattern and saving opportunity. Thus, the average marginal social weight depends on consumptions that take account of the idiosyncratic shock with respect to productivity and cost to work until that period as well as savings just prior to that period. In this sense, the age-specific tax rules have more information of household history with productivity. As in the following simulation, age-dependent taxation change their schedule dramatically because elder people's marginal social weight differ from young's one.

3.2 The case of age-independent taxation

Next, we consider the case of age-independent taxation. It appears that this model setting is similar to that of Laroque (2011) and Choné and Laroque (2017). The main difference is, however, that we allow for idiosyncratic shocks about productivities and preferences which is not insured in the outside financial market. In this sense, we incorporate idiosyncratic shocks into the model of Laroque (2011) and Choné and Laroque (2017) without complementary taxes on wealth.

In the case of age-independent taxation, the government's policy instrument is restricted by $y_0 = y_1 = \cdots = y_T$. The Lagrangian of the government's problem is,

$$\mathcal{L}^I = EU + \lambda^I \left[\sum_{t=0}^{T} \frac{1}{(1+r)^t} \left(-n_t^0 y(0) + \sum_{i=1}^{I} n_t^i \left(w^i - y(w^i) \right) \right) - G \right],$$

where λ^I is the Lagrange multiplier associated with government budget constraint in the case of age-independent taxation. The first order conditions of the Lagrangian with respect to $y(w^i)$ and $y(0)$ are, using $n_t^0 = 1 - \sum_i n_t^i$,

$$\frac{\partial \mathcal{L}^I}{\partial y(w^i)} = \frac{\partial EU}{\partial y(w^i)} + \lambda^I \sum_{t=0}^{T} \left[\frac{1}{(1+r)^t} \left(-n_t^i + \frac{\partial n_t^i}{\partial y(w^i)} (w^i - y(w^i) + y(0)) \right) \right] = 0, \quad \text{for } i \neq 0,$$

$$\frac{\partial \mathcal{L}^I}{\partial y(0)} = \frac{\partial EU}{\partial y(0)} + \lambda^I \sum_{t=0}^{T} \left[\frac{1}{(1+r)^t} \left(n_t^0 + \sum_{i=1}^{I} \left(\frac{\partial n_t^i}{\partial y(w^i)} \left(w^i - y(w^i) + y(0) \right) \right) \right) \right] = 0.$$

Similar to the above discussions, the partial derivatives of EU with respect to the disposable incomes are,

$$\frac{\partial EU}{\partial y(w^i)} = \sum_{t=0}^{T} \beta^t \int_{h_{t-1}} \int_{\theta_t \in \Theta^i} U'(c_t(h_t))\, dF_t^{\theta}(\theta_t \mid h_{t-1})\, dF_{t-1}^{h}(h_{t-1}), \qquad \text{for } i \neq 0,$$

$$\frac{\partial EU}{\partial y(0)} = \sum_{t=0}^{T} \beta^t \int_{h_{t-1}} \sum_{i=1}^{I} \int_{\theta_t \in \bar{\Theta}^i} U'(c_t(h_t))\, dF_t^{\theta}(\theta_t \mid h_{t-1})\, dF_{t-1}^{h}(h_{t-1}).$$

The first order conditions in the age-independent taxation problem with respect to $y(w^i)$ and $y(0)$ can be rewritten as follows.

$$\bar{n}^i \left(1 - \bar{g}^i\right) = \frac{\partial \bar{n}^i}{\partial y(w^i)} \left(w^i - y(w^i) + y(0)\right), \quad \text{for } i \neq 0, \tag{13}$$

$$\bar{n}^0 \left(1 - \bar{g}^0\right) = -\sum_{i=1}^{I} \frac{\partial \bar{n}^i}{\partial y(w^i)} \left(w^i - y(w^i) + y(0)\right), \tag{14}$$

where $\bar{n}^i$ and $\bar{g}^i$ are defined as,

$$\bar{n}^i \equiv \sum_{t=0}^{T} \frac{n_t^i}{(1+r)^t}, \quad \text{for all } i = 0, 1, \cdots, I,$$

$$\bar{g}^i \equiv \sum_{t=0}^{T} \frac{(1+r)^t \beta^t}{n_t^i \lambda^I} \int_{h_{t-1}} \int_{\theta_t \in \Theta^i} U'(c_t(h_t))\, dF_t^{\theta}(\theta_t \mid h_{t-1})\, dF_{t-1}^{h}(h_{t-1}), \quad \text{for } i \neq 0,$$

$$\bar{g}^0 \equiv \sum_{t=0}^{T} \frac{(1+r)^t \beta^t}{n_t^0 \lambda^I} \int_{h_{t-1}} \sum_{i=1}^{I} \int_{\theta_t \in \bar{\Theta}^i} U'(c_t(h_t))\, dF_t^{\theta}(\theta_t \mid h_{t-1})\, dF_{t-1}^{h}(h_{t-1}), \quad \text{for } i = 0.$$

Adding (13) for all i and (14), the right hand side vanishes,

$$\sum_{i=0}^{I} \bar{n}^i \left(\bar{g}^i - 1\right) = 0. \tag{15}$$

This means a weighted average of $\bar{g}^i$ is unity.

We also define the age-independent participation tax rates faced by the households with productivity w^i by,

$$\bar{\tau}(w^i) = \frac{w^i - y(w^i) + y(0)}{w^i}, \quad \text{for } i = 1, 2, \cdots, I. \tag{16}$$

Using (16), we can rewrite (13) as an age-independent inverse elasticity rule which is the same form in age-dependent taxation and static model,

$$\frac{\bar{\tau}(w^i)}{1 - \bar{\tau}(w^i)} = \frac{1 - \bar{g}^i}{\bar{\eta}^i}, \quad \text{for } i = 1, 2, \cdots, I, \tag{17}$$

where $\bar{\eta}^i$ is the 'life-cycle participation elasticity' defined as,

$$\bar{\eta}^i \equiv \frac{y(w^i) - y(0)}{\bar{n}^i} \frac{d\bar{n}^i}{d(y(w^i) - y(0))} = \frac{y(w^i) - y(0)}{\bar{n}^i} \sum_t \frac{1}{(1+r)^t} \frac{\partial n_t^i}{\partial y(w^i)} \quad \text{for } i \neq 0.$$

Since the weighted average of $\bar{g}^i$ is unity, the right hand side of (17) is likely negative for some working group with lower consumption. The age-independent participation tax rate for such an income group must be also negative.

Laroque (2011) examines the optimal income taxation in a stationary life-cycle model with an extensive margin and shows that optimal participation tax rate depends on social weight with permanent income rather than current income. In their life-cycle model, they focus exclusively on stationary economy with perfect insurance market, so that at given time there are same characteristics cohort and omit the age. The key factor is the correlation between current and permanent income, which is fixed and determined at birth. In this sense, they need not to consider households history about productivities and costs to work. In our model, social weight depends on realized history of uninsurable idiosyncratic shocks. Thus, the implementation of optimal allocation require more information. Nevertheless, the age-independent participation tax rule with uninsurable idiosyncratic shocks, equation (17), can be described as the same form of Laroque (2011).

4 Numerical simulations in an example economy

The previous section shows that participation tax rates for some working households with small consumptions are likely negative. In a static setting, less productive households tend to obtain lower consumption and negative participation tax. In our dynamic setting, the link between income and consumption is not periodwise. This section numerically solves the optimal tax policies in some example two periods economies and suggests that the negative participation tax should be applied to low-productive young households only.

4.1 Parameter setting

(1) Utility function and interest rate

All the households are two periods lived ($T = 1$). Hereafter, we refer to the households at period 0 as young and the households at period 1 as old. In this economy, one period corresponds to 25 years, the both of annual time discount rate and real interest rate are 2%, and the initial asset holding is zero, that is, $\beta = (1.02)^{-25} \approx 0.61$, $1 + r = 1.02^{25} \approx 1.64$, and $(1 - r)a_{-1} = 0$.

We assume the utility function U is quadratic in this section, and we normalize $U(0) = 0$ and $U'(0) = 1$, that is,

$$U(c) = \begin{cases} -\alpha c^2 + c & \text{for } c \leq \frac{1}{2\alpha} \\ \frac{1}{4\alpha} & \text{otherwise} \end{cases},$$

where $\alpha \in \mathbb{R}_+$ is the parameter determines the curvature of U. Marginal utility of consumption becomes zero when the consumption exceeds the satiation level $1/(2\alpha)$. We normalize the average labor productivity of young households to be unity, as mentioned details below. We consider three possible values for α, $\alpha = 0.1$, 0.05, or 0.025. Corresponding satiation levels are 5, 10, or 20 times large of the average labor productivity for young household.

(2) Labor productivity

There are three possible labor productivity level, ($I = 3$ and $w^1 < w^2 < w^3$). We assume the middle productivity corresponds to the average, $w^2 = 1$ and consider three cases for the domain of labor productivity, namely $\Omega = (w^1, w^2, w^3) = (0.5, 1, 1.5)$, $(0.5, 1, 2)$, and $(0.5, 1, 3)$. Thus, the highest productivity w^3 determines the skewness of the labor productivity distribution.

At period 0, $q_0^i \in [0, 1]$ unit of the young households obtain productivity w^i. Since the total population and the average productivity are unity, we must have,

$$\sum_{i=1}^{3} q_0^i = 1,$$
$$\sum_{i=1}^{3} q_0^i w^i = 1.$$

Let σ denotes the standard deviation of the productivity, in other words,

$$\sigma^2 = \sum_{i=1}^{3} q_0^i (w^i - \bar{w})^2 = q_0^1 (w^1 - 1)^2 + q_0^3 (w^3 - 1).$$

The parameter σ determines the dispersion of the labor productivity distribution. Given (w^1, w^2, w^3)

and σ, the above three conditions determine the initial population distribution. We consider three cases for the standard deviation, namely $\sigma = 1/4$, $1/3$, or $1/2$.

We assume the stochastic process of the labor productivity follows a Markov process independent of the path of the cost to work δ_t. An old household with productivity w^i at period 0 obtain productivity w^j at period 1 with transition probability $\pi^{i,j} \in [0,1]$ ($\sum_j \pi^{i,j} = 1$ for all i). We consider three cases for the transition probabilities, namely,

$$\pi^{i,j} = \begin{cases} 1/3 & \text{if } i = j \\ 1/3 & \text{if } i \neq j \end{cases}, \quad \begin{cases} 0.6 & \text{if } i = j \\ 0.2 & \text{if } i \neq j \end{cases}, \text{ or } \begin{cases} 0.9 & \text{if } i = j \\ 0.05 & \text{if } i \neq j \end{cases}.$$

Let q_1^j denotes the population of old households with productivity w^j at period 1, defined by $q_1^j = \sum_i q_0^i \pi^{i,j}$.

(3) The cost to work

At period 0, the cost to work of a young household with productivity w^i is uniformly distributed over the interval $[\underline{\delta}_0^i, \overline{\delta}_0^i] \subset \mathbb{R}_+$. Thus, working populations and the participation elasticities of the young households are,

$$n_0^i = q_0^i \frac{y_0(w^i) - y_0(0) - \underline{\delta}_0^i}{\overline{\delta}_0^i - \underline{\delta}_0^i}, \quad \text{if } y_0(w^i) - y_0(0) \in [\underline{\delta}_0^i, \overline{\delta}_0^i],$$
$$\eta_0^i = \frac{y_0(w^i) - y_0(0)}{y_0(w^i) - y_0(0) - \underline{\delta}_0^i}.$$

If we set $\underline{\delta}_0^i = 0$, the participation elasticity of the young households is unity.[5]

We assume 90% of young households will work for all the productivity groups, if no tax is imposed,[6] so that,

$$0.1\underline{\delta}_0^i + 0.9\overline{\delta}_0^i = w^i, \text{ for } i = 1, 2, 3. \tag{18}$$

We also assume the coefficient of variation of the cost to work is the same across the age and productivity groups. Let v denote the coefficient of variation of the cost to work. This assumption implies,[7]

$$\frac{\overline{\delta}_0^i - \underline{\delta}_0^i}{\overline{\delta}_0^i + \underline{\delta}_0^i} = \sqrt{3}v, \quad \text{for } i = 1, 2, 3. \tag{19}$$

Given the coefficient of variation, (18) and (19) determine the domains of the cost to work for the young household, $\overline{\delta}_0^i$ and $\underline{\delta}_0^i$. We consider three cases for the coefficient of variation, namely $v = 0.1$, 0.3, or 0.5.[8]

The cost to work of old households with productivity w^i is distributed over the interval $[\underline{\delta}_1^i, \overline{\delta}_1^i]$. Consider an old household at period 1 with a history of productivity ($w_0 = w^j$, $w_1 = w^i$) and

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[5] It seems that this participation elasticity is a bit high. Saez (2002) adopt the participation elasticity for lower income earners equal to 0, 0.5, and 1.

[6] The labor force participation rates of young male in OECD countries average are about 86%, while by age 25 to 64 total are around 75%. See e.g., in OECD.Stat.

[7] Note that δ_0^i is uniformly distributed over $[\underline{\delta}_0^i, \overline{\delta}_0^i]$. The expected value of δ_0^i, $\mathbb{E}(\delta_0^i)$ is $\mathbb{E}(\delta_0^i) = (\overline{\delta}_0^i + \underline{\delta}_0^i)/2$ and the variance of δ_0^i is $(\overline{\delta}_0^i - \underline{\delta}_0^i)^2/12$. Thus, the coefficient of variation v is, by the definition, $v = (\overline{\delta}_0^i - \underline{\delta}_0^i)/(\sqrt{3}(\overline{\delta}_0^i + \underline{\delta}_0^i))$.

[8] $\underline{\delta}_0^i$ become negative if we set v larger than $1/\sqrt{3} \approx 0.58$.

experienced the cost to work δ_0 at period 0. There are two possible cases for the determination of δ_1. The probability p have influence on the stochastic process of the cost to work δ_t. With probability $p \in [0,1]$, the relative position of δ_0 in $[\underline{\delta}_0^j, \overline{\delta}_0^j]$ 'survives' at period 1, that is,

$$\delta_1 = \frac{(\bar{\delta}_1^i - \underline{\delta}_1^i)(\delta_0 - \underline{\delta}_0^j)}{(\overline{\delta}_0^j - \underline{\delta}_0^j)} + \underline{\delta}_1^i.$$

With probability $1-p$, the relative position of δ_0 'expires' and the household draws a new δ from the uniform distribution over the interval $[\underline{\delta}_1^j, \bar{\delta}_1^j]$. This means the distribution of δ_1 of all the old households with productivity w^i is uniform over $[\underline{\delta}_1^i, \bar{\delta}_1^i]$. We consider 3 cases for p, namely $p = 0$, 0.5, or 1.

Working populations and the participation elasticities of the old households are,

$$\begin{aligned} n_1^i &= q_1^i \frac{y_1(w^i) - y_1(0) - \underline{\delta}_1^i}{\bar{\delta}_1^i - \underline{\delta}_1^i}, \\ \eta_1^i &= \frac{y_1(w^i) - y_1(0)}{y_1(w^i) - y_1(0) - \underline{\delta}_1^i}. \end{aligned}$$

Let ρ denotes the work participation ratio of old households. We consider five cases for the work participation ratio of old households, if there is no tax, namely $\rho = 0.1, 0.2, 0.3, 0.4$, or 0.5 for all the productivity groups.[9] The coefficient of variation of δ_1 is equal to v. By the same arguments of $\underline{\delta}_0^i$ and $\overline{\delta}_0^i$, these conditions determine $\underline{\delta}_1^i$ and $\overline{\delta}_1^i$.

Therefore, we consider 3,645 cases for the parameter combinations.[10] Under the specifications of quadratic preference and uniform distribution of δ, the first order conditions of the optimal asset holdings (6) at $t = 0$ are piecewise linear, and we derive a closed form solution of the optimal asset holdings. Substituting these and the labor supply plans, the government's objective function becomes piecewise polynomial. We numerically solved the optimal income tax policy for all the cases.

4.2 Results

(1) The benchmark case

Table 1 shows the optimal age-dependent taxation and the optimal age-independent taxation in the benchmark case where all the parameters are set at the middle choice. The first part, 'no tax', shows the allocation when no labor income tax is imposed, that is, the disposable income y_t^i is equal to the labor productivity w^i. The working population of young and old are 0.9 and 0.3. The resulting GDP (total production of the two generations) is 1.23. The middle part is the age-independent optimal taxation where the disposable income schedule is the same across the different age groups. The redistribution from the more productive workers to the less productive ones improves the welfare (EU increases from 0.33068 to 0.330734) while it has a negative impact on the GDP (y decreases from 1.23 to 1.2182). As shown in the previous section, the participation tax rate for the least productive workers is negative ($\tau_0^1 = \tau_1^1 = -0.0025$). T represents the total amount of redistribution. It is rather small, 0.3% of GDP, in this example. The last part shows the

[9] The labor force participation rates of over age 65 male in OECD countries average are around 20% and over age 65 total are around 14%. The rates by age 55 to 64 male are around 70% and by age 55 to 64 total are around 60%. In our two periods model, the old households is age 55 to 80. The labor force participation rates of old are around 35%. See e.g., in OECD.Stat.

[10] 3 for α, 3 for w, 3 for σ, 3 for π, 3 for v, 3 for p, and 5 for ρ.

Table 1: Optimal taxation of the benchmark case

Parameter setting										
$\alpha = 0.05$, $w = (0.5, 1, 2)$, $\pi^{i,i} = 0.6$, $\pi^{i,j} = 0.2$ $(j \neq i)$, $p = 0.5$, $\sigma = 1/3$, $v = 0.3$, $\rho = 0.3$										
	i	y_0^i	τ_0^i	n_0^i	y_1^i	τ_1^i	n_1^i		
No tax	0	0	—	0.1	0	—	0.7	EU	0.33068
	1	0.50	0	0.133	0.50	0	0.078	GDP	1.23
	2	1.00	0	0.7	1.00	0	0.153	T	0
	3	2.00	0	0.067	2.00	0	0.069	T/GDP	0
Age-independent optimal tax	0	0.003	—	0.104	0.003	—	0.703	EU	0.330734
	1	0.504	-0.0025	0.134	0.504	-0.0025	0.078	GDP	1.2182
	2	1.001	0.0027	0.697	1.001	0.0027	0.152	T	0.0036
	3	1.973	0.0153	0.065	1.973	0.0153	0.066	T/GDP	0.0030
Age-dependent optimal tax	0	0.003	—	0.102	0.003	—	0.706	EU	0.330739
	1	0.506	-0.0056	0.134	0.502	0.0026	0.077	GDP	1.2173
	2	1.001	0.0015	0.698	0.995	0.0074	0.150	T	0.0043
	3	1.973	0.0150	0.065	1.972	0.0155	0.066	T/GDP	0.0035

optimal age-dependent tax, where the participation tax rate for the least productive young worker is negative but the one for the least productive old worker is positive (i.e., $\tau_0^1 < 0$ and $\tau_1^1 \geq 0$). In comparison to the age-independent taxation, the optimal age-dependent tax improves the welfare (EU increases from 0.330734 to 0.330739) since a fine-tuned income tax/transfer is available. It results in larger redistribution (T increases from 0.0036 to 0.0043).

Remember that equation (12) holds within each age group in the age-dependent taxation. Thus, the participation tax rate of the least productive age t household is negative if and only if its marginal social weight is larger than the average of age t households. This negative participation tax, however, is less likely to happen for old households. The reason is as follows. A large part of old households is retiring because of high utility cost to work and the retiree's social weight is likely higher than the working olds who happens to get low utility cost δ. Therefore, it is likely that the marginal social weight of the least productive worker is lower than the average of old households, and negative participation tax for old may not be optimal.

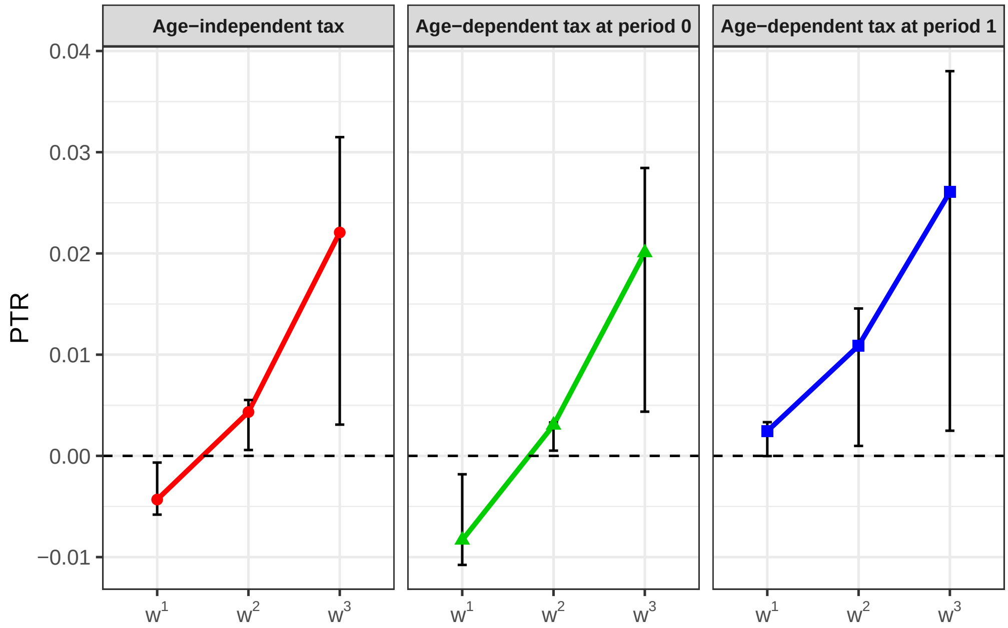
(2) Sensitivity analysis

To see the robustness of this property, we conducted the same calculation for all the 3,645 parameter combinations. Figure 1 shows the average optimal participation tax rates (PTR) with each productivity level in the age-independent taxation, age-dependent taxation at period 0 and period 1 for all cases. The average optimal participation tax rates in figure 1 have same patterns with benchmark case. Furthermore, we calculate the first and third quantile of the optimal participation tax rates in order to see the statical dispersion of the result. One can confirm that in many cases, optimal participation tax rates are similar with the benchmark case. Indeed, these statistics of optimal age-dependent participation tax rates for the least productive working old household are positive, while the corresponding ones for least productive working young household are negative.

Next, we count the frequency of signs of τ^1 in the optimal age-independent taxation, τ_0^1 and τ_1^1 in the optimal age-dependent taxation. The results are shown in Table 2. The first part of Table 2 reports the optimal age-independent participation tax rate for the least productive workers, τ^1 is negative for 3,605 out 3,645 cases of parameter combinations (about 99%). For these cases, 2,652 cases (about 74%) have the same property of the benchmark case (i.e., $\tau_0^1 < 0$ and $\tau_1^1 \geq 0$).

The rest of Table 2 reports the relationships of the sign patterns of τ^1, τ_0^1, and τ_1^1 with various

Figure 1: Optimal average participation tax rate



(Notes) This figure shows the average optimal participation tax rates (PTR) with each productivity level (see color lines). The black lines show the range of first and third quantile of the optimal PTR with each productivity level.

parameters. We first pay attention to the parameters Ω, σ, and π^{ii} affecting the productivity distribution and its stochastic process. If, other things being equal, the sign pattern of the middle case (i.e., $\tau_0^1 < 0$ and $\tau_1^1 \geq 0$) likely maintains when w^3, σ, or π^{ii} is small. The negative participation tax for the worker with the lowest income should be restricted to the young, if the productivity distribution is less skewed, the productivity is less dispersed, or social mobility is high. We especially take notice of the relation between the level of average optimal participation tax rate for the least productive households and the transition probabilities about productivities, π^{ii} (see figure 2). Figure 2 shows that in the case of high social mobility about productivity (i.e., π^{ii} is 0.333 in this case), the optimal participation tax for the least productive working old households would be positive.

Finally, we focus on the parameters v, p, and ρ affecting the labor disutility distribution and its stochastic process. If, other things being equal, the sign pattern of the middle case likely maintains if v is large, or p is middle or large case, or ρ is small. They suggest the negative participation tax should be restricted to the young if the cost of work is dispersed, persistency of the preference shock is middle or large, or work participation ratio of the old is not large. In general, a large part of old is retiring because of high utility cost. Their marginal social weight tends to be larger than those who get low utility cost and participating in the workhorse. Therefore, the marginal social weight of working old of the least income tends to be lower than the average of marginal social weight of the old. Figure 3 plots the relation between the level of average optimal participation tax rate for the least productive households and the work participation ratio of the old household, ρ. It shows that the optimal age-dependent participation tax rate for old would be positive when their work participation ratio is low. The negative participation tax rate for old is inappropriate for the case that the working population of old is small. It is true even when the participation elasticity of old is high since the elasticity does not affect the sign of participation tax rate but

Table 2: The frequency of sign patterns of participation tax by age

	Age-independent taxation $\tau^1 \geq 0$					Age-independent taxation $\tau^1 < 0$				
		Age-dependent taxation					Age-dependent taxation			
		$\tau_0^1 \geq 0$, $\tau_1^1 \geq 0$	$\tau_0^1 \geq 0$, $\tau_1^1 < 0$	$\tau_0^1 < 0$, $\tau_1^1 \geq 0$	$\tau_0^1 < 0$, $\tau_1^1 < 0$		$\tau_0^1 \geq 0$, $\tau_1^1 \geq 0$	$\tau_0^1 \geq 0$, $\tau_1^1 < 0$	$\tau_0^1 < 0$, $\tau_1^1 \geq 0$	$\tau_0^1 < 0$, $\tau_1^1 < 0$
Total Case	40	0	0	40	0	3,605	1	4	2,652	948
parameter settings										
$\alpha = 0.1$	17	0	0	17	0	1,198	0	0	933	265
$\alpha = 0.05$	12	0	0	12	0	1,203	0	0	875	328
$\alpha = 0.025$	11	0	0	11	0	1,204	1	4	844	355
$\Omega = (0.5, 1, 1.5)$	0	0	0	0	0	1,215	0	1	948	266
$\Omega = (0.5, 1, 2)$	9	0	0	9	0	1,206	0	0	918	288
$\Omega = (0.5, 1, 3)$	31	0	0	31	0	1,184	1	3	786	394
$\sigma = 1/4$	40	0	0	40	0	1,175	1	3	913	258
$\sigma = 1/3$	0	0	0	0	0	1,215	0	0	907	308
$\sigma = 1/2$	0	0	0	0	0	1,215	0	1	832	382
$\pi^{ii} = 1/3$	40	0	0	40	0	1,175	1	4	1,007	163
$\pi^{ii} = 0.6$	0	0	0	0	0	1,215	0	0	953	262
$\pi^{ii} = 0.9$	0	0	0	0	0	1,215	0	0	692	523
$v = 0.1$	21	0	0	21	0	1,194	1	4	801	388
$v = 0.3$	12	0	0	12	0	1,203	0	0	877	326
$v = 0.5$	7	0	0	7	0	1,208	0	0	974	234
$p = 0$	0	0	0	0	0	1,215	1	2	672	540
$p = 0.5$	18	0	0	18	0	1,197	0	1	1,008	188
$p = 1$	12	0	0	12	0	1,193	0	1	972	220
$\rho = 0.1$	3	0	0	3	0	726	0	1	635	90
$\rho = 0.2$	20	0	0	20	0	709	1	1	637	70
$\rho = 0.3$	17	0	0	17	0	712	0	0	637	75
$\rho = 0.4$	0	0	0	0	0	729	0	1	532	197
$\rho = 0.5$	0	0	0	0	0	729	0	1	211	516

Figure 2: Optimal average participation tax rates with π^{ii}

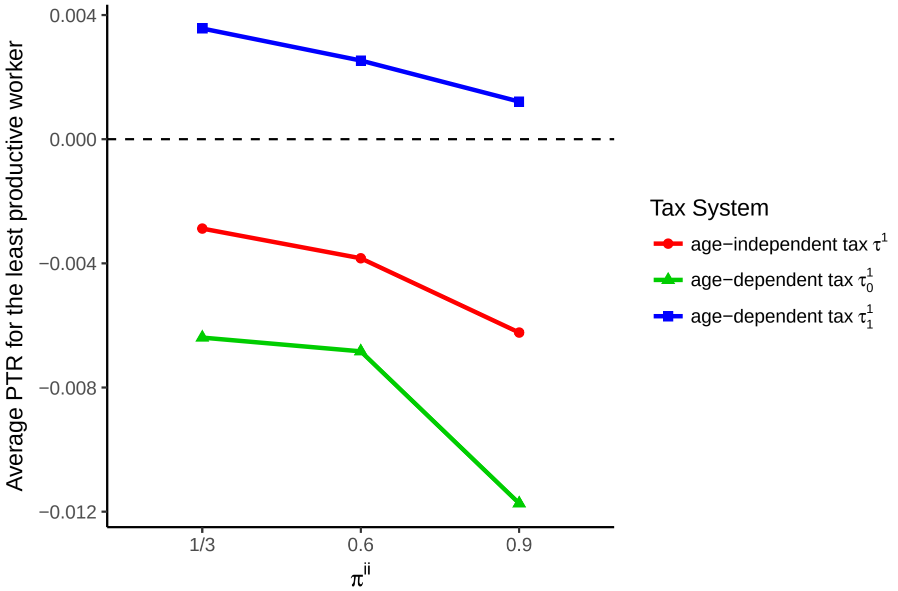


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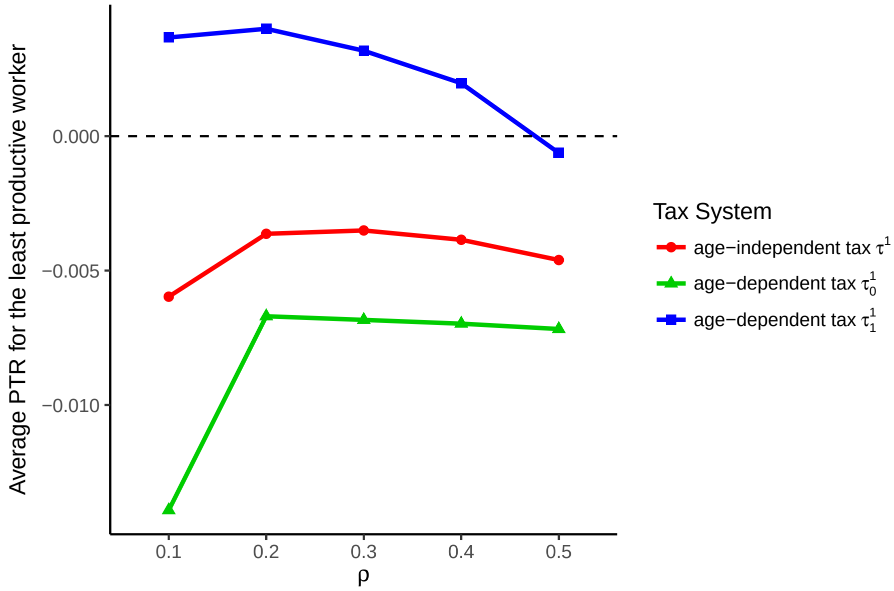
Figure 3: Optimal average participation tax rates with ρ



5 Conclusion

In the present paper, we investigated optimal age-dependent income taxation in a dynamic model with extensive labor margin in each period and uninsurable idiosyncratic shocks of labor productivity and cost to work. When the government can only employ age-independent labor income tax, the optimal participation tax rates are similar to the static model and the stationary life-cycle model. The age-independent participation tax rate for low consumption group must be negative. When the government can employ age-dependent labor income tax, the optimal participation tax rates are age-specific. Negative participation tax rate for low consumption households apply in each age group. The age-specific participation tax rules depend on the average marginal social weight that take account of the idiosyncratic shock with respect to productivity and cost to work until that period. On the contrary, participation tax rates in the static model depend on average marginal social weight at only that period. In this sense, the age-specific tax rules have more information of household history with productivity.

In our example economy, we simulate optimal participation tax rate both age-independent taxation and age-dependent taxation. We calculate the optimal participation tax rate under various parameter specifications and found that well known property of negative participation tax rate for low consumption households are valid in the age-independent case and for only young in the age-dependent case. In our dynamic setting, the link between the income and consumption is not periodwise. Young households tend to work much harder than old households in some reasons (e.g., to raise expense for children or health conditions). Thus, the marginal social weight of the least productive working young households is likely higher than the average of young households. On the contrary, a large part of old households is retiring and their social weight is likely higher than the working olds. Therefore, it is likely that the marginal social weight of the least productive working old is lower than the average of old households, and negative participation tax for old may not be optimal even when the participation elasticity of old is high. In-work policy that targets young people who want to work, have greater benefit. This suggest that in-work credits or wage subsidies for young people are desirable when the extensive labor margin is important.

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