Multi-tier tax competition on Gasoline

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Abstract

This paper analyzes the fiscal interactions arising from gasoline taxation in a federation. We adopt a general theoretical model for studying simultaneous vertical and horizontal tax competition by i) introducing a specific monetary cost of refueling ii) assuming that the price of gasoline is affected by either excise taxes (regional and federal) and the VAT rate, ii) considering elastic demand for gasoline. We show that at the symmetric equilibrium, horizontal taxes are strategic complements but vertical taxes are strategic may be substitutes. Moreover, horizontal excise taxes are strategic substitutes with VAT whereas the result is unclear for the reaction between regional and federal excise taxes. Finally, we show that the tax reaction functions and thus the equilibria crucially differ according to the pattern of decision-making (social planner, Nash or defederalized leadership).

Keywords: Fiscal Federalism, Gasoline Taxation, Horizontal and Vertical Tax Interactions

JEL Codes: E62, H7, Q48

1 Introduction

Subnational governments are in charge of many regional public services (roads, refuse removal) and play a major role in the provision of social services (benefits to disabled, minimum wage etc...). But their available regional fiscal tools do not raise sufficient own revenue and they heavily depend on transfers from the federal government (see Bird (2006)). The increasing role of subnational governments is partly due to the new transfers of competences from the federal government and raises the question of appropriate revenue structure for subnational governments. Apart from property taxes that are the pre-eminent regional taxes in most countries, the idea of a regional gasoline taxation is a rising issue. Such a regional source of funds is justified first by the fact that regional governments are in charged of roads, and second, as an excise tax, gasoline tax

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is one of the simplest and cheapest form of taxation that is recommended for subnational use. However, implementing regional gasoline taxation generates horizontal and vertical tax competition mechanisms that may be inefficient in terms of global tax revenues.
In this paper, we consider a combination of regional, federal excise taxes and VAT on gasoline and analyze the interaction that arise in this gazoline taxation structure. Such a multi-level taxation already exists in federal countries (United States and Canada) but is also partly existing in countries with several tiers of government (France, Japan). In France, taxes represented 61.9% of unleaded gasoline price and 56.2% of gasoline price in 2015 for the French consumer. The federal government levied 14,9 billions of euros from the domestic consumption tax on energy products (the fourth source of federal tax revenue), plus VAT tax revenues. Regional governments levied 5,3 billions of euros from a regional modulation of the domestic consumption tax on energy products and the "départements" received 6,5 billions of euros from the federal government to compensate some transfers of competencies (DGCE (2016)). All in all, tax revenues from the domestic consumption tax on energy products amount to 27,4 billions of euros in 2015 in France. Currently, the differences among prices at the pump essentially depend on the cost of distribution, as all regions except two of them set the upper limit of the regional modulation. However, what would be the consequences of removing the upper limit of the regional modulation, which is an option for ensuring more revenue to newly reinforced regions? This question is of particularly high importance in a context of decreasing transfers granted by the federal government to regional ones together with the devolution of powers from the Central Government to regional authorities which obliges regional governments to raise more revenues. Nowadays, gasoline taxation represents around 30% of the revenue from indirect taxation of regional authorities (regions and départements) and less than 10% of the whole regional revenues. In this context we can wonder if gasoline taxation could be an efficient source of additional regional tax revenue.

To address this question, we assume that two gas stations located on both sides of a regional border could set two very different prices, implied by two different regional tax policies, giving rise to a strategic behaviour of the French driver. The driver, and therefore the tax base (i.e., the number of liters of gasoline purchased), is indeed mobile, but partially as the cost in terms of gasoline consumed of not fueling in the closest gas station must at least be covered by the price difference between two gas stations. This architecture of the taxation of oil products would give rise to horizontal tax interactions among regions –through the regional modulation of the rate of the domestic consumption tax on energy products– and vertical tax interactions –as the domestic consumption tax on energy products is also levied at the national level– as well as tax interdependence between two national instruments, i.e. the domestic consumption tax on energy products and the VAT.

Our paper aims at modelling this two-tier tax competition game, with a specific good which is gasoline, and two tax instruments at the top tier. We thus contribute to the seminal paper of horizontal cross-border shopping by Kanbur and Keen (1993) through i) an endogenous demand for the good whereas it is exogenous in Kanbur and Keen (1993); ii) the specification of two costs, i.e. a psychological cost proportional to the distance traveled to purchase the gasoline and a monetary cost that characterizes the fact that traveling to buy gasoline implies the consumption of gasoline, rather than a cost of crossing the border in Kanbur and Keen (1993), iii) the existence of vertical tax interactions and two tax instruments at the top tier, in addition to horizontal interactions.

Price elasticity of gasoline demand is a key issue in our model. Recent meta-analysis as the one by Brons et al. (2008) have shown that price elasticity demand for gasoline is higher in long run due to the lag of adjustment of the consumer behaviors. Nevertheless, even if the price elasticity of demand is quite small in the short term, we argue that consumers are quite sensitive to a difference in gasoline prices for a quite small distance. The idea that consumers react to a change in gasoline taxes is supported by the paper by Coglianese et al. (2016) who show that the demand for gasoline crucially increases just before a tax increase and is delayed before a tax decrease, rendering the tax instrument endogenous.

Devereux et al. (2007) already considered horizontal and vertical competition in excise taxes, with endogenous demand and cross-border shopping. However, our structure of cost is more sophisticated for gasoline than for their good (e.g. cigarettes). Furthermore, we introduce also the existence of VAT in our model (in addition to excise taxes). A strand of the literature has tried to identify the tax responses of excise taxes in a context of vertical competition (Keen, 1998) or horizontal competition with cross-border shopping (Kanbur and Keen, 1993). In line with the theoretical model by Devereux et al. (2007), we extend the cross boarding shopping models by allowing an individual demand for gasoline to be price-elastic and so tax elastic. Our paper is also related with the literature on tax rate interactions and tax reaction function that has been mainly developed in capital tax competition models (see Vrijburg and de Mooij (2016)). We still adopt a general theoretical model for analyzing simultaneous vertical and horizontal competition that we adapt to the peculiar case of gasoline. Our results show that regional excise taxes are strategic complements, which is currently standard in a model of tax competition with Leviathans. The impact of the regional taxes on the federal ones are more interesting : regional taxes are strategic substitutes with VAT and may be strategic complements or strategic substitutes with the federal excise tax, depending on the curvature of the demand for gasoline. Our simulation show that the excises taxes are more likely to be strategic complements with standard demand function. Finally, we show that the tax reaction functions of a social planner or a federal government with leadership are crucially modified compared with the ones from the Nash game. As a result, we can argue that the type of decision-making crucially matters in the system of gasoline taxation.

The paper is organized as follow: section 2 describes the model and exhibits the fiscal interaction functions. Section 3 compares the federal planner program to the Nash equilibrium and Section 4 exhibits the comparison of the Nash program with a sequential game (defederalized leadership). The last section concludes.

2 The model

We consider a federal country, which is modeled as a segment –in line with Hotelling (1929)– of length 2 with 2 regions of equal size denoted by i ($i = 1, 2$). Region 1 covers the interval $[-1, 0]$ and region 2 the interval $[0, 1]$, the geographical border being 0. We assume that a gasoline station is located in each region, respectively at $S_1 = -1$ in region 1 and $S_2 = 1$ in region 2.

Each region is populated by N identical agents uniformly distributed over the territory. For the sake of simplicity, we assume that $N = 1$. The agent of region i located at point k benefits from the consumption of two goods: a numeraire private good consumed in quantity c_k^i in her location and gasoline purshased in quantity x_k^j in the station S_j of her choice, with $j = 1, 2$.

A special feature of the good "gasoline" is that the act of purchasing itself implies the consumption of part of the purchased amount because driving to the gas station requires gasoline. We denote by α a measure of the gasoline consumption per unit of distance, whose monetary cost depends on the after-tax gasoline price per unit P_j in station S_j [1]. In addition to this monetary cost, the agent bears an opportunity cost δ of not devoting time to another activity, which depends on the time per unit of distance. The distance between the location s_k^i of the agent k and the location S_j of the gas station j is measured by $|s_k^i - S_j|$.

For each unit of gasoline sold by the station S_j at a pre-tax price p_j, the federal government levies a federal gasoline excise tax T and the government of the region of the station levies a regional origine-based gasoline excise tax t_j. In addition, the federal government applies a rate θ of VAT on the purchase of a quantity x_k^j of gasoline at an after-excise-tax price $q_j \equiv p_j + t_j + T$ and on the private good consumption c_k^i. The overall after-tax gasoline price per quantity demanded for gasoline in station j is thus:

$$P_j \equiv q_j \left(1 + \theta\right) \equiv \left(p_j + t_j + T\right) \left(1 + \theta\right).$$

Each agent is endowed with a fixed amount $\overline{y}$. Therefore, we do not consider any income effect and concentrate on a substitution and price effect here. The budget constraint of an agent k located in s_k^i, who chooses the gas station S_j, is

$$\overline{y} = c_k^i(1 + \theta) + x_k^j P_j + (\delta + \alpha P_j) \left| s_k^i - S_j \right|.$$

We assume that the utility function of the agent k located in region i and consuming in station S_j takes a quasi-linear form, i.e., $c_k^i + u(x_k^j)$, where $u\left(.\right)$ is an increasing and strictly concave function.

2.1 The agent's behaviour

Demand for gasoline The demand for gasoline is derived from the maximisation of the agent's utility function under her budget constraint, i.e.,

$$\widehat{x}_k^j = \arg\max\{-x_k^j \frac{P_j}{(1 + \theta)} + u(x_k^j)\}$$

and therefore equalizes the marginal utility to the gasoline price net of VAT

$$u'(x_k^j) = \frac{P_j}{(1 + \theta)} \equiv q_j. \tag{1}$$

The demand for gasoline does not depend on the location of the agent once she has chosen her station. Thereafter, we will denote $\widehat{x}_k^j = x^j$. Using (1) to replace $\frac{P_j}{(1+\theta)}$ by $u'(x^j)$ into the utility

[1] α is normalized to account fo the retrun. In other words, $\frac{\alpha}{2}$ is the gasoline consumption per unit of distance

function, and differentiating with respect to x^j, we show that the indirect utility increases w.r.t. x^j, i.e. $\frac{\partial\left(u(x^j)-x^j q_j\right)}{\partial x^j} - x^j u''(x^j) > 0$, and therefore that:

$$u(x^j) - x^j q_j > u(x^i) - x^i q_i \Longrightarrow q_j < q_i \text{ from (1)} \tag{2}$$

Differenciating the FOC (1) with respect to each tax, we obtain:

$$\frac{dx^j}{dt_j} = \frac{dx^j}{dT} = \frac{1}{u''(x^j)} < 0 \text{ and } \frac{dx^j}{d\theta} = \frac{dx^j}{dt_i} = 0.$$

The demand for gasoline decreases with respect to both excise taxes while it neither reacts to the excise tax of the other region, nor to the VAT because VAT is levied proportionally to c_k^i and x^j and thus does not distort the gasoline demand.

Choice of the station Agent k will purchase gasoline in Station S_1 located in Region 1 if and only if $V_k^1 > V_k^2$ with the indirect utility function $V_k^j \equiv \frac{\overline{y}}{(1+\theta)} - x^j \frac{P_j}{(1+\theta)} - \frac{(\delta+\alpha P_j)}{(1+\theta)} |s_k^i - S_j| + u(x^j)$. Let denote by $\widetilde{s}$ the location of the agent indifferent between purchasing gasoline in Region 1 or in Region 2, i.e., for which $V_k^1 = V_k^2$:

$$\widetilde{s} = \frac{u(x^1) - (x^1+\alpha)\, q_1 - (u(x^2) - (x^2+\alpha)\, q_2)}{\rho}. \tag{3}$$

where $\rho = \frac{2\delta}{(1+\theta)} + \alpha\,(q_1 + q_2) > 0$ is the distance cost. From (2), the numerator of (3) (and therefore $\widetilde{s}$ as the distance cost is positive) is always positive for $q_1 < q_2$, then:

$$\widetilde{s} > 0 \Longleftrightarrow q_1 < q_2. \tag{4}$$

Finally, to ensure that $-1 < \widetilde{s} < 1$, we have to impose the following condition :

$$\frac{2\delta}{1+\theta} \geqslant min[u(x^1) - (x^1+2\alpha)q_1 - u(x^2) + x^2 q_2, u(x^2) - (x^2+2\alpha)q_2 - u(x^1) + x^1 q_1] \tag{5}$$

Only the price of gasoline net of VAT influences the choice of the station. For $q_1 = q_2$, each agent fuels in her region of residence. For $q_1 < q_2$, the threshold $\widetilde{s}$ will be located in region 2, and agents located in $[0, \widetilde{s}]$ will cross the border to fuel in Region 1 (and *vice-versa*).

From comparative statics, we derive the following Lemma:

Lemma 1. *Using the FOC (2) and the expression of $\tilde{s}$, the derivatives of $\tilde{s}$ with respect to taxes are:*

$$\frac{\partial\widetilde{s}}{\partial q_1} = \frac{\partial\widetilde{s}}{\partial p_1} = \frac{\partial\widetilde{s}}{\partial t_1} < 0$$

$$\frac{\partial\widetilde{s}}{\partial q_2} = \frac{\partial\widetilde{s}}{\partial p_2} = \frac{\partial\widetilde{s}}{\partial t_2} > 0$$

$$\frac{\partial \widetilde{s}}{\partial \theta} > 0 \iff q_1 < q_2$$
$$\frac{\partial \widetilde{s}}{\partial T} > 0 \iff q_1 > q_2$$

Proof. The results are derived from the expression of the derivatives:
$$\frac{\partial \widetilde{s}}{\partial q_1} = \frac{\partial \widetilde{s}}{\partial p_1} = \frac{\partial \widetilde{s}}{\partial t_1} = \frac{-\left(x^1+\alpha\right)\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)-\alpha\left(\left(u(x^1)-\left(x^1+\alpha\right)q_1\right)-\left(u(x^2)-\left(x^2+\alpha\right)q_2\right)\right)}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)^2} = \frac{-x^1-\alpha(1+\widetilde{s})}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)}$$
with $\widetilde{s} \in [-1, 1]$
$$\frac{\partial \widetilde{s}}{\partial q_2} = \frac{\partial \widetilde{s}}{\partial p_2} = \frac{\partial \widetilde{s}}{\partial t_2} = \frac{\left(x^2+\alpha\right)\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)-\alpha\left(\left(u(x^1)-\left(x^1+\alpha\right)q_1\right)-\left(u(x^2)-\left(x^2+\alpha\right)q_2\right)\right)}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)^2} = \frac{x^2+\alpha(1-\widetilde{s})}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)}$$
with $\widetilde{s} \in [-1, 1]$
$$\frac{\partial \widetilde{s}}{\partial \theta} = \frac{2\delta}{(1+\theta)^2} \frac{\left(u(x^1)-\left(x^1+\alpha\right)q_1\right)-\left(u(x^2)-\left(x^2+\alpha\right)q_2\right)}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)^2} = \frac{2\delta}{(1+\theta)^2} \frac{\widetilde{s}}{\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)}$$
$$\frac{\partial \widetilde{s}}{\partial T} = \frac{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)\left(x^2-x^1\right)-2\alpha\left(\left(u(x^1)-\left(x^1+\alpha\right)q_1\right)-\left(u(x^2)-\left(x^2+\alpha\right)q_2\right)\right)}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)^2} = \frac{\left(x^2-x^1\right)-2\alpha\tilde{s}}{\left(\frac{2\delta}{(1+\theta)}+\alpha(q_1+q_2)\right)} = \frac{\partial \widetilde{s}}{\partial t_1} + \frac{\partial \widetilde{s}}{\partial t_2}$$
and taking into account the properties (2) and (4). ■

As expected, the threshold consumer $\widetilde{s}$ moves towards Station S_2 (resp. S_1) when the tax rate in region 1(resp. 2) increases. VAT increases the competitive advantage of the less expensive station whereas the federal excise tax tends to reduce it. Indeed, for a lower net of VAT price in region 1, a rise in VAT makes the threshold moving towards region 2 while a rise in the lump sum tax T makes the threshold moving in the opposite way. These opposite effects are due to the fact that the VAT rate does not modify the demand for gasoline contrary to the federal excise tax. Even if the excise tax T affects similarly the price net of VAT in both regions, Expression (3) shows that a rise in VAT, by concerning also the numeraire good, decreases the distance cost which tends to move the threshold towards the less competitive station for a given asymmetry between prices. The effect of the excise tax T is quite different since if affects both the demand for gasoline (negatively) and the net of VAT price (positively). Expression (3) shows that the federal excise tax increases the distance cost which tends to move the threshold towards the federal border (border between regions) for a given asymmetry of p_i and diminishes the utilities net of monetary costs in both regions. Due to the properties of the utility function, this decrease is stronger in the country which applies a smaller p_i. These two effects working in the same way tend to move the threshold agent towards the federal border. Finally, note that when the after excise tax prices are equal in both regions ($q_1 = q_2$), neither the federal excise tax nor the VAT have an impact on the threshold $\widetilde{s}$. Moreover, when the opportunity cost δ is null, VAT does not impact the threshold $\widetilde{s}$ because VAT has no direct effect on gasoline consumption.

2.2 Regional tax reaction functions

In this section, we assume that the federal taxes are given (θ, T) and we look at the tax competition that arises between regions. Local governments act as Leviathan and aim at maximizing their revenue from taxes, i.e. for Region 1:

$$\begin{aligned} r_1(t_1,t_2,T,\theta) &= t_1 X_1 \\ \text{with } X_1(t_1,t_2,T,\theta) &= x^1 s_1(t_1,t_2,T,\theta) \\ \text{where } s_1(t_1,t_2,T,\theta) &= 1+\widetilde{s} \end{aligned}$$

and for Region 2

$$\begin{aligned} r_2(t_1,t_2,T,\theta) &= t_2 X_2 \\ \text{with } X_2(t_1,t_2,T,\theta) &= x^2 s_2(t_1,t_2,T,\theta) \\ \text{where } s_2(t_1,t_2,T,\theta) &= 1-\widetilde{s} \end{aligned}$$

In the above conditions, s_i represents the number of agents that will purchase gasoline in region i and X_i stands for the total demand for gasoline in region i and therefore constitutes the tax base on which the regional tax rate can rely on.

Combining the first order conditions for region i gives

$$\frac{\partial r_i}{\partial t_i} = 0 \Longleftrightarrow X_i(1+\varepsilon_{x^i}+\varepsilon_{s_i}) = 0 \Longrightarrow \Omega^i(t_1,t_2,\theta,T) = 0 \text{ for} X_i > 0 \qquad (6)$$

with $\Omega^i(t_1,t_2,\theta,T) = 1+\varepsilon_{x^i}+\varepsilon_{s_i}$ and $\varepsilon_{x^i} \equiv \frac{t_i}{x^i}\frac{\partial x^i}{\partial t_i}$ and $\varepsilon_{s_i} \equiv \frac{t_i}{s_i}\frac{\partial s_i}{\partial t_i}$.

Here, ε_{x^i} stands for the tax elasticity of the individual demand for gasoline that we call the intensive elasticity and ε_{s_i} stands for the tax elasticity of the number of shoppers or extensive elasticity. The intensive elasticity can also be decomposed as the ratio of the tax over price q_i multiplied by the price elasticity of gasoline $\left(\varepsilon_{x^i} = \frac{t_i}{q_i}\varepsilon_{q_i} \text{ with } \varepsilon_{q_i} = \frac{t_i}{q_i}\frac{\partial x^i}{\partial q_i}\right)$. The extensive elasticity depends on the number of agents that will purchase the gasoline in the region (tax base), and therefore depends on the consumer threshold $\widetilde{s}$.

Lemma 2. *Gasoline expenditures have the following properties:*
i) $\varepsilon_{x^i} < 0 \ \forall i$
ii) $\frac{\partial s_1}{\partial t_1} \leqslant 0$ *and* $\frac{\partial s_2}{\partial t_2} \leqslant 0$
iii) $\varepsilon_{s_i} \leqslant 0 \ \forall i$
iv) $|\varepsilon_{s_i}| < 1 \Longleftrightarrow x^i t_i < \rho s_i - \alpha(1+\tilde{s})t_i$

Proof. The intensive elasticity ε_{x^i} is of the sign of the price elasticity which is negative from (2.1). The second result is derived from the reaction of the threshold agent to a change in the regional taxes ($\frac{\partial s_2}{\partial t_2} = -\frac{\partial \widetilde{s}}{\partial t_2}$ and $\frac{\partial s_1}{\partial t_1} = \frac{\partial \widetilde{s}}{\partial t_1}$) from lemma 1; the third result follows directly from ii). The fourth result is derived from Lemma 1. ■
The fourth result states that the extensive elasticity is lower than 1 in absolute value if the regional government revenue is lower than the transport net of distance cost due to regional tax. At the symmetric equilibrium, it reduces to $|\varepsilon_s| < 1 \Longleftrightarrow xt < \rho - \alpha t$. Moreover, we have $|\varepsilon_{x^i}| < 1$ for any $|\varepsilon_{q_i}| < 1$ which is empirically proved. Indeed, $\left(\varepsilon_{x^i} = \frac{t_i}{q_i}\varepsilon_{q_i}\right)$ and $\frac{t_i}{q_i} < 1$.

The analysis of the comparative statics gives the following results.

Proposition 1. *At the symmetric Nash equilibrium, the slope of horizontal and vertical reaction functions of the regional taxes are:*

$$\begin{aligned}
\frac{\partial t_i}{\partial t_j} &= -\frac{1}{t}\frac{(\varepsilon_s)^2}{\Omega^i_{t_i}} > 0 \ \forall i; \\
\frac{\partial t_i}{\partial \theta} &= -\frac{\frac{\varepsilon_s}{\rho}\frac{2\delta}{(1+\theta)^2}}{\Omega^i_{t_i}} < 0 \ \forall i; \\
\frac{\partial t_i}{\partial T} &= -\frac{-\frac{1}{\rho}\left(2\alpha\varepsilon_s + tx'\right) + \varepsilon_x\left[-\frac{\varepsilon_x}{t} + \frac{x''}{x'}\right]}{\Omega^i_{t_i}} \ \forall i.
\end{aligned}$$

with $\Omega^i_{t_i} = \frac{\partial \Omega^i}{\partial t_i} = \frac{2}{t}\left(-1 + \varepsilon_x\varepsilon_s\right) + \varepsilon_x\frac{x''}{x'} - \frac{1}{\rho}\left(tx' + 2\alpha\varepsilon_s\right) < 0$ *from the concavity condition.*

Proof. See Appendix 1 ∎

Proposition 1 highlights firstly that regional taxes are strategic complements, which is standard in the literature on capital tax competition. Secondly, the VAT rate levied by the federal government decreases the regional excise taxes and this effect goes through the distance cost. The VAT decreases the distance cost so that the extensive elasticity is larger in absolute value when the VAT is large. Since the extensive elasticity is negative, regional governments have to decrease their regional excise tax in order to compensate the overreaction of the extensive margin to an increase in VAT.
In line with Devereux et al. (2007), we are not able to sign the regional tax reaction function to the federal excise tax. The reaction depends on different effects : the first term of the numerator characterizes how the extensive elasticity i.e the elasticity of the number of shoppers with respect to the regional tax reacts to a change in the federal excise tax. This effect is clearly positive and works in the opposite way compared to the VAT effect: an increase in T increases the distance cost and increases the difference in utilities so that a higher T tends to leave the threshold to the center and the extensive elasticity to the regional tax is higher. The second term represents the response of the elasticity of gasoline with respect to the regional tax to an increase in the federal excise tax. This effect is ambiguous and depends on the form of the utility function u.

Proposition 1 applies for symmetric levels of the pre-tax prices $p_1 = p_2$. Figures 1 and 2 illustrate the reaction functions of the regional taxes in an asymmetric case ($p_1 = 0.55$ and $p_2 = 0.5$) for a rise in T (Figure 1) and a rise in θ (Figure 2)[2]. The dashed curves correspond to the reaction functions resulting from a rise in one of the federal taxes. The figures confirm the effects that have been highlighted in the symmetric case: a rise in VAT implies a decrease of the regional taxes.

[2]The model is calibrated for a utility function of the form $u(x_i) = \beta x_i^{1/2}$ with $\overline{y} = 120$, β is calibrated so as to correspond to a refuel of 60 euros, $\alpha = 0.7$ corresponds to a gasoline consumption of 7 liters per 100 kilometers, and $\delta = 2$ is calibrated to correspond to the cost of time evaluated by Pisani-Ferry (2013) for sales. The initial federal taxes are fixed at their effective level, i.e. $T = 0.63$ and $\theta = 0.20$. Prices p_1 and p_2 are also chosen to correspond to the closest effective prices

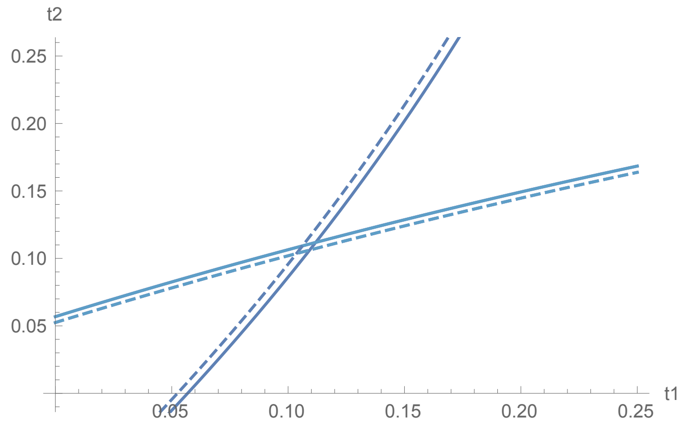

Figure 1: Effect of a rise from $\theta = 0.2$ to $\theta = 0.35$

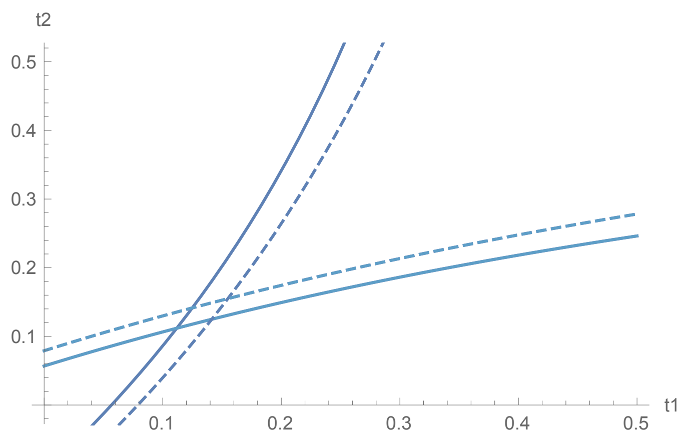

Figure 2: Effect of a rise from $T = 0.63$ to $T = 0.8$

The new equilibrium resulting from the rise in θ implies lower regional taxes. Finally, a rise in the federal excise tax has an opposite effect: it tends to shift upwards the regional tax reaction function and the Nash equilibrium results in higher regional taxes. Note that the utility function that has been chosen to draw the figures implies $x'' < 0$ which reinforces the probability for the excise taxes to be complements.

For particular properties of the demand for gasoline, we are able to state clear-cut effects:

Corollary 1. *i) For an inelastic demand we have* $\frac{\partial t_i}{\partial T} = \frac{\frac{1}{\rho}(2\alpha\varepsilon_s)}{\Omega^1_{t_i}} > 0$ *with* $\Omega^1_{t_i} = -\frac{2}{t} - \alpha\frac{\varepsilon_s}{\rho}$.

ii) For an iso-elastic demand we have $\frac{\partial t_i}{\partial T} = \frac{\frac{1}{\rho}(2\alpha\varepsilon_s + tx') + \frac{\varepsilon_x}{q}}{\Omega^1_{t_i}} > 0$

Proof. An inelastic demand function implies $x' = 0$ and then $\varepsilon_x = 0$. For an iso elastic demand we have $q\frac{x''}{x'} = (\varepsilon_x \frac{q}{t} - 1)$. ■
Corollary 1 identifies peculiar demand functions for which the regional and federal excise taxes

are strategic complements. For an inelastic demand, only the extensive elasticity matters. For an iso-elastic demand, the response of the elasticity is identical along the demand curve and both the extensive margin and the intensive margin play in the same way.

2.3 Federal tax reaction functions

We assume that the federal government also acts as a Leviathan and maximizes his revenue with respect to the federal taxes (θ, T) with

$$R(t_1, t_2, T, \theta) = \theta C + \sum_{i=1}^{2} (\theta q_i + T) X_i$$

and

$$C(t_1, t_2, T, \theta) = \int_{-1}^{\widetilde{s}} c^1 ds + \int_{\widetilde{s}}^{1} c^2 ds$$

with $c^1 = \frac{\overline{y} - x^1 P_1 - (\delta + \alpha P_1)(s+1)}{(1+\theta)}$ and $c^2 = \frac{\overline{y} - x^2 P_2 - (\delta + \alpha P_2)(1-s)}{(1+\theta)}$
c^1 and c^2 are respectively the consumptions of the numeraire good in Regions 1 and 2. They depend on the threshold consumer and therefore on the taxes on both regions.

The first-order conditions with respect to the two federal fiscal tools give

$$\begin{aligned}
\frac{\partial R}{\partial T} &= \theta \frac{\partial C}{\partial T} + \sum_{i=1}^{2} \left((\theta + 1) X_i + \left(\frac{\theta}{1+\theta} P_i + T \right) \frac{\partial X_i}{\partial T} \right) = 0 \Longleftrightarrow \Theta^T (\theta, T; t_1, t_2) = 0 \\
\frac{\partial R}{\partial \theta} &= C + \theta \frac{\partial C}{\partial \theta} + \sum_{i=1}^{2} \left(q_i X_i + \left(\frac{\theta}{1+\theta} P_i + T \right) \frac{\partial X_i}{\partial \theta} \right) = 0 \Longleftrightarrow \Theta^\theta (\theta, T; t_1, t_2) = 0
\end{aligned}$$

From the first-order conditions, we are able to derive the tax reaction functions:

Proposition 2. *At the symmetric Nash equilibrium, the slope of horizontal and vertical tax reaction functions of the federal taxes are:*

$$\begin{aligned}
\frac{\partial T}{\partial t_i} &= -\frac{2x' + Tx^{''} + \theta x'}{\Theta_T^T} < 0 \\
\frac{\partial \theta}{\partial t_i} &= \frac{\frac{\alpha}{2}}{\Theta_\theta^\theta} < 0; \\
\frac{\partial \theta}{\partial T} &= \frac{\alpha}{\Theta_\theta^\theta} = 2\frac{\partial \theta}{\partial t_i} < 0 \\
\frac{\partial T}{\partial \theta} &= \frac{\alpha}{\Theta_T^T} < 0
\end{aligned}$$

with $\Theta_\theta^\theta = 2\left[\left(\frac{\delta - 2\overline{y}}{(1+\theta)^3}\right)\right] < 0$ *and* $\Theta_T^T = 2\left[2x' + Tx^{''}\right] < 0$ *from the concavity condition.*

Proof. see Appendix 2 ■

From the federal tax reaction functions we can deduce that the VAT rate is a decreasing function of the regional taxes. Most of the effects collapse at the federal level mainly because VAT does not affect the demand for gasoline and because we consider the symmetric equilibrium. The only effect of the regional taxes that remains effective goes through the monetary cost α. An increase in the regional tax t_i increases the monetary costs of traveling to refuel (even for agents that buy gasoline in their region) that diminishes the consumption of the numeraire good. In order to compensate this diminution of the demand for the numeraire good, the federal government is inclined to decrease the VAT.

The impact of the regional tax on the federal excise tax is also negative. Contrary to the VAT rate, the federal and regional excise taxes affect the demand for gasoline. On one hand, the increase of the regional tax modifies the reaction of the gasoline demand to the federal excise tax (Tx'') in the numerator. The sign of this effect depends on the form of the utility function u. On the other hand, the regional tax affects directly the regional demand for gasoline. This effect (first term) is clearly negative and pushes the federal government to diminish its excise tax to limit the decrease in gasoline consumption. Finally, the increase in gazoline price due to a rise in t_i affects negatively the VAT revenue so that the federal excise tax is inclined to diminish in order to raise gasoline demand (third term).

Finally, the vertical interactions between the two federal taxes are also negative. At the symmetric equilibrium, the effect of the federal excise tax on the number of shoppers collapses and the only effect goes through the monetary traveling cost that diminishes the gasoline demand. Following an increase in the federal excise tax, the federal government is then inclined to reduce the VAT in order to compensate the diminution of consumption and vice-versa. To sump up, all taxes are strategic substitutes.

2.4 Nash equilibrium

The Nash equilibrium is the result of the intersection of both the horizontal and vertical tax reaction functions i.e. $\left(t_1^N, t_2^N; \theta^N, T^N\right)$ is obtained by combining:

$$\begin{aligned}\Omega^1\left(t_1, t_2; \theta, T\right) &= 0\\ \Omega^2\left(t_1, t_2; \theta, T\right) &= 0\\ \Theta^\theta\left(t_1, t_2, \theta, T\right) &= 0\\ \Theta^T\left(t_1, t_2, \theta, T\right) &= 0\end{aligned}$$

Even in the symmetric case, we are not able to derive expressions that could give us useful information about the level of the taxes. As an illustration, we simulate the Nash equilibrium with the parameters already used to graph the horizontal reaction functions of the asymmetric cases. As a benchmark, we also simulate the regional taxes for given federal taxes (the rates that are applied in France):

	t_i	t_j	T	θ
T=0.63 and $\theta = 0.20$	0.112	0.112	fixed	fixed
Nash equilibrium	0.149	0.149	0.91	0.816

The interesting result from these simulations are not so far the levels of the taxes but the low level of the regional taxes compared to the national ones that could explain the existence of a ceiling for the regional taxes. However, for the effective values of the Federal taxes ($T = 0.63$ and $\theta = 0.20$), the efficient regional taxes are far from the effective one ($t = 0.025$).

3 Nash versus Social Planner

In this section we determine how the program of a social planner who cares about the whole revenue of the governments (regional and federal) may modify the tax reaction functions compared with the Nash game. The whole revenue of the social planner (SP) writes:

$$\begin{aligned} SP &= R\left(t_1, t_2, T, \theta\right) + r_1\left(t_1, t_2, T, \theta\right) + r_2\left(t_1, t_2, T, \theta\right) \\ &= t_1 X_1 + t_2 X_2 + \theta C + \sum_{i=1}^{2}\left(\theta q_i + T\right) X_i \end{aligned} \tag{7}$$

To compare the tax reaction functions, we first have to focus on the tax externalities which are presented in the following lemma:

Lemma 3. *In the symmetric case:*
i) regional taxes exhibit positive horizontal externalities and negative vertical externalities: $\frac{\partial r_j}{\partial t_i} > 0$ *and* $\frac{\partial R}{\partial t_i} < 0$
ii) the federal excise tax exhibits negative vertical externalities: $\frac{\partial r_1}{\partial T} + \frac{\partial r_2}{\partial T} < 0$.
iii) the federal sales tax exhibits has no impact on the regional tax revenues: $\frac{\partial r_1}{\partial \theta} + \frac{\partial r_2}{\partial \theta} = 0$.

Proof. With $r_1 = t_1 (1 + \widetilde{s}) x_1$ and $r_2 = t_2 (1 - \widetilde{s}) x_2$

$$\frac{\partial r_1}{\partial t_2} = t_1 x_1 \frac{\partial \widetilde{s}}{\partial t_2} > 0 \text{ and } \frac{\partial r_2}{\partial t_1} = -t_2 x_2 \frac{\partial \widetilde{s}}{\partial t_1} > 0$$

$$\frac{\partial r_1}{\partial T} + \frac{\partial r_2}{\partial T} = t_1 x_1 \frac{\partial \widetilde{s}}{\partial T} + (1 + \widetilde{s}) t_1 \frac{\partial x_1}{\partial T} + (1 - \widetilde{s}) t_2 \frac{\partial x_2}{\partial T} - t_2 x_2 \frac{\partial \widetilde{s}}{\partial T} = t_1 \frac{\partial x_1}{\partial T} + t_2 \frac{\partial x_2}{\partial T} < 0 \text{ in the symmetric case}$$

$$\frac{\partial r_1}{\partial \theta} + \frac{\partial r_2}{\partial \theta} = t_1 x_1 \frac{\partial \widetilde{s}}{\partial \theta} - t_2 x_2 \frac{\partial \widetilde{s}}{\partial \theta} = 0 \text{ in the symmetric case}$$

With $R = \theta C + \sum_{i=1}^{2} (\theta q_i + T) X_i$

$$\begin{aligned} \frac{\partial R}{\partial t_1} &= \theta \frac{\partial C}{\partial t_1} + \theta X_1 + (\theta q_1 + T) \left(\frac{\partial x^1}{\partial t_1} (1 + \widetilde{s}) + \frac{\partial \widetilde{s}}{\partial t_1} x^1 \right) + (\theta q_2 + T) \left(-\frac{\partial \widetilde{s}}{\partial t_1} x_2 \right) \\ &= \theta \frac{\partial C}{\partial t_1} + \theta x^1 + (\theta q + T) \frac{\partial x^1}{\partial t_1} \end{aligned}$$

From (10) we know that for the symmetric case we have $\frac{\partial C}{\partial t_1} = -\left(\frac{\partial x^1}{\partial t_1} q_1 + x^1 \right) - \frac{\alpha}{2}$ so that

$$\frac{\partial R}{\partial t_i} = -\frac{\alpha}{2}\theta + T x' < 0.$$

■

The explanation of the spillover effects is the following: a rise in one of the regional tax (let us say t_1) makes the threshold agent, initially in $\tilde{s} = 0$, moving on the left side of the line i.e. towards -1. This implies a lower number of agents who choose to refuel in region 1. This benefits to the other region whose tax revenue increases. A rise in t_1 also diminishes the demand for gasoline which affects the federal government revenue through both the excise tax revenue and the sale tax revenue. A rise in the federal excise tax does not modify the threshold agent (remains in $\tilde{s} = 0$) but affects the demand for gasoline that diminishes the regional government revenue. Finally, an increase in θ does not affect the sum of the regional revenues because the VAT rate does not alter the gasoline demand. The threshold agent is not affected either at the symmetric equilibrium because both the regions are symmetrically affected.

From the previous lemma, we can compare the tax reaction functions derived from the symmetric Social Planner program and the symmetric Nash game:

Proposition 3. *In the symmetric case, compared to the Nash game, the reaction functions from the Social Planner program are:*
i) driven downwards for the federal excise tax
ii) unchanged for the sales tax.
iii) driven downwards for the regional excise taxes if vertical externalities of the regional taxes are higher than horizontal externalities.

Proof. Evaluated at the symmetric Nash equilibrium, the first order conditions rewrite:

$$\frac{\partial SP}{\partial T} = \sum \frac{\partial r_i}{\partial T} < 0$$
$$\frac{\partial SP}{\partial \theta} = \sum \frac{\partial r_i}{\partial \theta} = 0$$
$$\frac{\partial SP}{\partial t_i} = \frac{\partial R}{\partial t_i} + \frac{\partial r_j}{\partial t_i}$$

from the lemma above.

■

The only undetermined case is for regional tax reaction functions. Indeed, a rise in a regional tax raises the revenue of the other regional government through a standard horizontal tax competition that leads some of the consumers to refuel in the other region. The vertical externalities are negative through two different effects: a decrease in demand for gasoline and a rise in the transport cost. Both effects diminish the federal revenue.

Since we have four different taxes in our social planner program it is quite difficult to compare the equilibria resulting from the Nash game and the Social planner solution. We are only able to compare the equilibria, fixing two of the instruments.

Let us first assume that t_1 and t_2 are fixed. Due to the fact that the reaction function resulting from CPO with respect to the federal excise tax is shifted downward, we can deduce that the federal excise tax is lower when decided by a Social planner but the VAT rate is higher: $T^N > T^{SP}$ and $\theta^N < \theta^{SP}$. These results are directly explained by the negative externalities that follow an increase in excise taxes through the decrease of gasoline demand. They are valid under the assumption that the federal Nash equilibrium i.e Nash equilibrium with (t_1, t_2) given is stable. [3]

With fixed federal taxes (θ and T) we can conclude that regional taxes are lower when decided by a federal planner if vertical tax externalities tax dominate horizontal ones. In this case, regional revenues are lower compared to the Nash equilibrium levels: $r_i^N > r_i^{SP}$. Conversely, horizontal externalities that dominate the vertical one will drive the regional reaction functions upward (for given federal taxes) and lead to higher regional taxes under the social planner choice. It directly results that regional revenues are higher: $r_i^N < r_i^{SP}$. Here again, we assume that the Nash equilibrium is stable i.e. $\left|-\frac{t(\epsilon_s^2)}{\Omega_{t\,i}^1}\right| < 1$.

4 Sequential vertical interactions

Now if we quite realistically assume that the setting of the federal taxes is more rigid than the decision about the regional excise taxes, we can consider the choice of the taxes as a sequential

[3] The stability condition requires $|\frac{\partial \theta}{\partial T}| < \left|\frac{1}{\frac{\partial T}{\partial \theta}}\right| \iff \left|\frac{\alpha}{\Theta_\theta^\theta}\right| < \left|\frac{\theta_T^T}{\alpha}\right|$.

game in which the federal taxes are decided in a first stage and the regional governments adjust their choice in a second stage. Once again, our aim is to determine how the tax reaction functions are modified in this setting compared to a symmetric Nash game.
Solving this program backward, the first stage of the game corresponds to the regional government first order condition (6). Compiling the two first order conditions (for Region 1 and 2) we are able to express t_1 and t_2 as functions of the federal instruments θ and T and derive the impact of both federal instruments on the regional taxes as follows [4]:

$$\left.\frac{\partial t_i}{\partial \theta}\right|_{SG} = -\frac{\frac{\varepsilon_s}{\rho}\frac{2\delta}{(1+\theta)^2}}{\Omega^1_{t_i}}\frac{1}{1+t\frac{(\varepsilon_s)^2}{\Omega^1_{t_i}}} < 0\ \forall i;$$

$$\left.\frac{\partial t_i}{\partial T}\right|_{SG} = -\frac{-\frac{1}{\rho}\left(2\alpha\varepsilon_s + tx'\right) + \varepsilon_x\left[-\frac{\varepsilon_x}{t} + \frac{x''}{x'}\right]}{\Omega^1_{t_i}}\frac{1}{1+t\frac{(\varepsilon_s)^2}{\Omega^1_{t_i}}}\ \forall i.$$

Now, the first order conditions with respect to the two fiscal tools reduce to:

$$\frac{\partial R}{\partial T} + \sum \frac{\partial R}{\partial t_i}\left.\frac{\partial t_i}{\partial T}\right|_{SG} = 0$$

$$\frac{\partial R}{\partial \theta} + \sum \frac{\partial R}{\partial t_i}\left.\frac{\partial t_i}{\partial \theta}\right|_{SG} = 0$$

Proposition 4. *In the symmetric case, compared to the Nash game, the federal reaction functions with the sequential game are:*
i) driven downward (resp. upward) for the excise tax T if excise taxes (t_i and T) are strategic complements (resp. strategic substitutes)
ii) driven upward for the sales tax.

Proof. Evaluated at the Nash equilibrium, the first order conditions with respect to federal taxes rewrite:

$$\frac{\partial R}{\partial t_i}\left.\frac{\partial t_i}{\partial \theta}\right|_{SG} > 0$$

$$\frac{\partial R}{\partial t_i}\left.\frac{\partial t_i}{\partial T}\right|_{SG} > 0 \Longleftrightarrow \left.\frac{\partial t_i}{\partial T}\right|_{SG} < 0.$$

Since at the symmetric Nash equilibrium we have:

$$\frac{\partial R}{\partial t_i} = -\frac{\alpha\theta}{2} + Tx' < 0$$

■

Let first assume that the federal excise tax is fixed. Then the sequential game will exhibit a higher

[4] We are considering the case of a stable Nash equilibrium in the first stage which implies $1 + t\frac{(\varepsilon_s)^2}{\Omega^1_{t_i}} > 0$

VAT rate than at the Nash game but lower regional excise taxes due to the negative interactions between the VAT rate and the regional excise taxes. This implies that $r_i^{SG} < r_i^N$.
Now, if we fix the VAT rate and analyze the choice of the excise taxes (game which is more realistic), the sequential game exhibits a higher federal excise tax if excise taxes are strategic complements and a lower federal excise tax if excise taxes are strategic substitutes. In both cases, the regional taxes are higher at the defederalized leadership equilibrium than at the Nash equilibrium. Indeed, the defederalized leadership equilibrium gives a leadership power to the federal government that is able to internalize the tax reaction functions and the regional externalities on the federal revenue. As a result, when the federal and regional taxes are strategic substitutes, the regional revenue is higher with a sequential game: $r_i^{SG} > r_i^N$. The lower federal excise tax implies a higher demand and more revenue whereas $|\varepsilon_x| < 1$ guarantees that a higher t_i implies a higher regional revenue.

Now, let us assumed that none of the federal tax is fixed.

Proposition 5. *For $\varepsilon_x > -1$, if federal and regional taxes are strategic complements ($\frac{\partial t_i}{\partial T} > 0$), compared to the Nash equilibrium, the sequential game equilibrium implies*
- a higher VAT rate
- a lower federal excise tax
- lower regional excise taxes
- lower revenues for the regional governments.

Proof. $\varepsilon_x > -1$ implies $\frac{\partial \theta}{\partial T} < 0$ and strategic complementarities imply $\frac{\partial t_i}{\partial T} > 0$.
We have

$$\theta^N(T^N) < \theta^{SG}(T^N) < \theta^{SG}(T^{SG}(\theta^N))$$
$$T^N(\theta^N) > T^{SG}(\theta^N) > T^{SG}(\theta^{SG}(T^N))$$

Combining both we obtain $\theta^{SG} > \theta^N$ and $T^{SG} > T^N$
Finally, since $\frac{\partial t_i}{\partial \theta} < 0$ and $\frac{\partial t_i}{\partial T} > 0$ we obtain $t_i^{SG} < t_i^N \forall i$. ■

A sequential game tends to diminish the level of the excise taxes and increase the VAT rate when the regional excise tax is a complement to the federal one $\left(\frac{\partial t_i}{\partial T}\right) > 0$. This is due to the vertical tax competition mechanism of the excise taxes: regional excise taxes create negative externalities on the federal revenue that are reinforced by strategic complementaries between regional and federal excise taxes. To reach the highest federal revenue, the federal government is inclined to limit the level of the federal excise taxes and raise the VAT rate. This induces a loss of revenue for the regional governments compared to the Nash equilibrium.

5 Conclusion

This paper aims to analyze the multiple strategic interactions operating in a complex system of gasoline taxation where both regional and federal governments set their own taxes. This corresponds to the American and Canadian systems, and to a lesser extend, to the French and Japaneses ones. The complexity of the system leads its evaluation in terms of fiscal efficiency difficult. By disentangling the different mechanisms, we are able to present some key results that drive the main forces of the gasoline taxation.

Even if our theoretical results are presented in a symmetric case, important elements can be extracted for this peculiar case. One of the first important result is that VAT and excise taxes (both at the regional and federal levels) appear to be strategic substitutes. This results is mainly explained by the fact that the VAT rate affects also the numeraire good. Moreover, while the regional excise taxes are strategic complements, the interactions between the regional and federal excise taxes are not clear-cut. They definitely depend on the slope of the demand function. However, the demand for gasoline is quite complicated to grasp as shown by the empirical literature on the price elasticity that has not reached yet a clear consensus on the size of the price elasticity, a key feature of our analysis. Finally, we show that the way of determining the regional taxes (by a federal planner, a Nash competition of a federalized leadership game) impact the level of the taxes and the regional revenues. A federalized leadership decision may diminish the regional revenues when excise taxes are complements.

6 Appendices

6.1 Appendix 1

For all i the first order condition writes

$$\begin{aligned}\frac{\partial r_i}{\partial t_i} = 0 \qquad &\Longleftrightarrow X_i \left(1 + \frac{t_i}{x^i}\frac{\partial x^i}{\partial t_i} + \frac{t_i}{s_i}\frac{\partial s_i}{\partial t_i}\right) = 0 \\ &\Longleftrightarrow \Omega^i\left(t_1, t_2, \theta, T\right) = \left(1 + \frac{t_i}{x^i}\frac{\partial x^i}{\partial t_i} + \frac{t_i}{s_i}\frac{\partial s_i}{\partial t_i}\right) = 1 + \varepsilon_{x^i} + \varepsilon_{s_i} = 0 \end{aligned} \tag{8}$$

At the equilibrium, the concavity condition from the regional government program yields $\Omega^1_{t_1} < 0$ and $\Omega^2_{t_2} < 0$ with

$$\Omega^i_{t_i} = \underbrace{\left(\frac{1}{x^i}\frac{\partial x^i}{\partial t_i} + \frac{1}{s_i}\frac{\partial s_i}{\partial t_i}\right)}_{=\frac{1}{t_i}(\varepsilon^i_x + \varepsilon_{si})<0 \text{ at the equilibrium}} + t_i\left(\underbrace{-\frac{1}{\left(x^i\right)^2}\left(\frac{\partial x^i}{\partial t_i}\right)^2}_{<0} + \frac{1}{x^i}\frac{\partial^2 x^i}{\partial t_i^2} \underbrace{- \frac{1}{s_i^2}\left(\frac{\partial s_i}{\partial t_i}\right)^2}_{<0} + \frac{1}{s_i}\frac{\partial^2 s_i}{\partial t_i^2}\right) < 0$$

with

$$\frac{\partial^2 s_1}{\partial t_1^2} = \frac{\partial^2 \widetilde{s}}{\partial t_1^2} = \frac{-\frac{\partial x^1}{\partial t_1} - 2\alpha\frac{\partial \tilde{s}}{\partial t_1}}{\rho}$$

and

$$\frac{\partial^2 s_2}{\partial t_2^2} = -\frac{\partial^2 \widetilde{s}}{\partial t_2^2} = \frac{-\frac{\partial x^2}{\partial t_2} + 2\alpha\frac{\partial \tilde{s}}{\partial t_2}}{\rho}$$

Thus we obtain:

$$\Omega^1_{t_1} = \frac{1}{t-2}(\varepsilon^1_x + \varepsilon_{s_1}) - \frac{1}{t_1}(\varepsilon^2_{x^1} + \varepsilon^2_{s_1}) + \frac{x^{1\prime\prime}}{x^1}t_1 + \frac{t_1}{s_1}\left(\frac{-\frac{\partial x^1}{\partial t_1} - 2\alpha\frac{\partial \tilde{s}}{\partial t_1}}{\rho}\right)$$

and

$$\Omega^2_{t_2} = \frac{1}{t-2}(\varepsilon^2_x + \varepsilon_{s_2}) - \frac{1}{t_2}(\varepsilon^2_{x^2} + \varepsilon^2_{s_2}) + \frac{x^{2\prime\prime}}{x^2}t_2 + \frac{t_2}{s_2}\left(\frac{-\frac{\partial x^2}{\partial t_2} + 2\alpha\frac{\partial \tilde{s}}{\partial t_2}}{\rho}\right)$$

At the symmetric equilibrium, we have $s_i = 1 \quad \forall i$ and $\frac{\partial \tilde{s}}{\partial t_1} = -\frac{\partial \tilde{s}}{\partial t_2}$ so that $\varepsilon_{s_1} = \varepsilon_{s_2}$. Using the condition $\Omega^i = 0$ we obtain

$$\Omega^i_{t_i} = \frac{2}{t}\left(-1 + \varepsilon_x\varepsilon_s\right) + \varepsilon_x\frac{x^{''}}{x'} - \frac{1}{\rho}\left(tx' + 2\alpha\varepsilon_s\right)$$

with $x' = \frac{\partial x}{\partial q} = \frac{\partial x}{\partial t_i} = \frac{1}{u"} < 0$ and $x'' = \frac{\partial^2 x}{\partial t_i^2}$

We proceed to the comparative statics :

We first start analyzing the vertical tax competition
1- with respect to θ
By differentiating (8), we obtain:

$$\frac{\partial t_1}{\partial \theta} = -\frac{\Omega^1_\theta}{\Omega^1_{t_1}} \text{ and } \frac{\partial t_2}{\partial \theta} = -\frac{\Omega^2_\theta}{\Omega^2_{t_2}}$$

The concavity condition from the regional government program yields $\Omega^1_{t_1} < 0$ and $\Omega^2_{t_2} < 0$. The sign of Ω^i_θ gives the sign of $\frac{\partial t_i}{\partial \theta}$

$$\Omega^i_\theta = \frac{\partial \varepsilon_{s_i}}{\partial \theta} \text{ since } \frac{\partial \varepsilon_{x^i}}{\partial \theta} = 0 \text{ from } \frac{\partial x^i}{\partial \theta} = 0$$

we have

$$\frac{\partial \varepsilon_{s_i}}{\partial \theta} = t_i \left[-\frac{1}{s_i^2} \frac{\partial s_i}{\partial t_i} \frac{\partial s_i}{\partial \theta} + \frac{1}{s_i} \frac{\partial^2 s_i}{\partial t_i \partial \theta} \right]$$

with

$$s_1 = (1 + \widetilde{s}) \text{ and } s_2 = (1 - \widetilde{s})$$

then

$$\begin{aligned} \frac{\partial s_1}{\partial t_1} &= \frac{\partial \widetilde{s}}{\partial t_1} \text{ and } \frac{\partial s_2}{\partial t_2} = -\frac{\partial \widetilde{s}}{\partial t_2} \\ \frac{\partial s_1}{\partial \theta} &= -\frac{\partial s_2}{\partial \theta} = \frac{\partial \widetilde{s}}{\partial \theta} \text{ and } \frac{\partial^2 s_1}{\partial t_1 \partial \theta} = \frac{\partial^2 \widetilde{s}}{\partial t_1 \partial \theta} \text{ and } \frac{\partial^2 s_2}{\partial t_2 \partial \theta} = -\frac{\partial^2 \widetilde{s}}{\partial t_2 \partial \theta} \end{aligned}$$

with

$$\begin{aligned} \frac{\partial^2 \widetilde{s}}{\partial t_1 \partial \theta} &= \frac{-\alpha \frac{\partial \widetilde{s}}{\partial \theta} + \frac{2\delta}{(1+\theta)^2} \frac{\partial \widetilde{s}}{\partial t_1}}{\rho} \\ &\text{and} \\ \frac{\partial^2 \widetilde{s}}{\partial t_2 \partial \theta} &= -\frac{-\alpha \frac{\partial \widetilde{s}}{\partial \theta} + \frac{2\delta}{(1+\theta)^2} \frac{\partial \widetilde{s}}{\partial t_2}}{\rho} \end{aligned}$$

At the symmetric equilibrium ($p_1 = p_2$ so that $t_1 = t_2$) we have $\widetilde{s} = 0$, $\frac{\partial \widetilde{s}}{\partial \theta} = 0$ and $s_i = 1 > 0 \quad \forall i$ so that

$$\begin{aligned} \frac{\partial^2 s_1}{\partial t_1 \partial \theta} &= \frac{\frac{2\delta}{(1+\theta)^2} \frac{\partial \widetilde{s}}{\partial t_1}}{\rho} < 0 \\ &\text{and} \\ \frac{\partial^2 s_2}{\partial t_2 \partial \theta} &= -\frac{\frac{2\delta}{(1+\theta)^2} \frac{\partial \widetilde{s}}{\partial t_2}}{\rho} < 0 \end{aligned}$$

and

$$\frac{\partial \varepsilon_{s_i}}{\partial \theta} = t_i \frac{\partial^2 s_i}{\partial t_i \partial \theta} = \frac{\frac{2\delta}{(1+\theta)^2} \frac{\varepsilon_s}{t}}{\rho} < 0 \text{ so that } \frac{\partial t_i}{\partial \theta} < 0$$

2- with respect to T

$$\frac{\partial t_1}{\partial T} = -\frac{\Omega_T^1}{\Omega_{t_1}^1} \text{ and } \frac{\partial t_2}{\partial T} = -\frac{\Omega_T^2}{\Omega_{t_2}^2}$$

the sign of Ω_T^i gives the sign of $\frac{\partial t_i}{\partial T}$

$$\begin{aligned} \Omega_T^i &= \left(\frac{\partial \varepsilon_{x^i}}{\partial T} + \frac{\partial \varepsilon_{s_i}}{\partial T}\right) \\ &\text{with} \\ \frac{\partial \varepsilon_{x^i}}{\partial T} &= t_i \left[\underbrace{-\frac{1}{(x^i)^2}\frac{\partial x^i}{\partial t_i}\frac{\partial x^i}{\partial T}}_{<0} + \frac{1}{x^i}\frac{\partial^2 x^i}{\partial t_i \partial T}\right] \\ \frac{\partial \varepsilon_{s_i}}{\partial T} &= t_i \left[-\frac{1}{s_i^2}\frac{\partial s_i}{\partial t_i}\frac{\partial s_i}{\partial T} + \frac{1}{s_i}\frac{\partial^2 s_i}{\partial t_i \partial T}\right] \end{aligned}$$

Let us start by analysing $\frac{\partial \varepsilon_{s_i}}{\partial T}$ with $s_1 = (1+\widetilde{s})$ and $s_2 = (1-\widetilde{s})$

$$\frac{\partial s_1}{\partial T} = -\frac{\partial s_2}{\partial T} = \frac{\partial \widetilde{s}}{\partial T} \text{ and } \frac{\partial^2 s_1}{\partial t_1 \partial T} = \frac{\partial^2 \widetilde{s}}{\partial t_1 \partial T} \text{ and } \frac{\partial^2 s_2}{\partial t_2 \partial T} = -\frac{\partial^2 \widetilde{s}}{\partial t_2 \partial T}$$

$$\begin{aligned} \frac{\partial^2 \widetilde{s}}{\partial t_1 \partial T} &= \frac{-\alpha \frac{\partial \widetilde{s}}{\partial T} - \frac{\partial x_k^1}{\partial T} - 2\alpha \frac{\partial \widetilde{s}}{\partial t_1}}{\rho} \\ &\text{and} \\ \frac{\partial^2 \widetilde{s}}{\partial t_2 \partial T} &= \frac{-\alpha \frac{\partial \widetilde{s}}{\partial T} + \frac{\partial x_k^2}{\partial T} - 2\alpha \frac{\partial \widetilde{s}}{\partial t_2}}{\rho} \end{aligned}$$

At the symmetric equilibrium ($p_1 = p_2$ so that $t_1 = t_2$) we have $\widetilde{s} = 0$, $\frac{\partial \widetilde{s}}{\partial T} = 0$, $\frac{\partial \widetilde{s}}{\partial t_1} < 0$, $\frac{\partial \widetilde{s}}{\partial t_2} > 0$ and $s_i = 1 > 0$ so that

$$\begin{aligned} \frac{\partial^2 \widetilde{s}}{\partial t_1 \partial T} &= \frac{-\frac{\partial x^1}{\partial T} - 2\alpha \frac{\partial \widetilde{s}}{\partial t_1}}{\frac{2\delta}{(1+\theta)} + \alpha(q_1 + q_2)} = \frac{\partial^2 \widetilde{s}}{\partial t_1^2} = -\frac{\partial^2 s_1}{\partial t_1 \partial T} > 0 \\ &\text{and} \\ \frac{\partial^2 \widetilde{s}}{\partial t_2 \partial T} &= \frac{\frac{\partial x^2}{\partial T} - 2\alpha \frac{\partial \widetilde{s}}{\partial t_2}}{\frac{2\delta}{(1+\theta)} + \alpha(q_1 + q_2)} = \frac{\partial^2 \widetilde{s}}{\partial t_2^2} = -\frac{\partial^2 s_2}{\partial t_2 \partial T} < 0 \end{aligned}$$

so that

$$\frac{\partial \varepsilon_{s_i}}{\partial T} = \frac{t_i}{s_i}\frac{\partial^2 s_i}{\partial t_i \partial T} = -\frac{1}{\rho}\left(2\alpha \varepsilon_s + t x'\right) > 0$$

Let us now analyse $\frac{\partial \varepsilon_{x^i}}{\partial T}$

$$\frac{\partial \varepsilon_{x^i}}{\partial T} = t_i \left[-\frac{1}{(x^i)^2} \frac{\partial x^i}{\partial t_i} \frac{\partial x^i}{\partial T} + \frac{1}{x^i} \frac{\partial^2 x^i}{\partial t_i \partial T} \right] = \varepsilon_x \left[-\frac{\varepsilon_x}{t} + \frac{x''}{x'} \right]$$

Then

$$\Omega^i_T = \left(\frac{\partial \varepsilon_{x^i}}{\partial T} + \frac{\partial \varepsilon_{s_i}}{\partial T} \right) = \frac{1}{\rho} \left(-2\alpha \varepsilon_s - t x' \right) + \varepsilon_x \left[-\frac{\varepsilon_x}{t} + \frac{x''}{x'} \right]$$

3- Horizontal tax competition $\frac{\partial t_i}{\partial t_j}$

$$\frac{\partial t_i}{\partial t_j} = -\frac{\Omega^i_{t_j}}{\Omega^i_{t_i}}$$

As $\Omega^i_{t_i} < 0$,

$$sign \frac{\partial t_i}{\partial t_j} = sign \Omega^i_{t_j}$$

$$\Omega^i_{t_j} = \left(\frac{\partial \varepsilon_{x^i}}{\partial t_j} + \frac{\partial \varepsilon_{s_i}}{\partial t_j} \right) = t_i \left[-\frac{1}{s_i^2} \frac{\partial s_i}{\partial t_i} \frac{\partial s_i}{\partial t_j} + \frac{1}{s_i} \frac{\partial^2 s_i}{\partial t_i \partial t_j} \right] \text{ since } \frac{\partial \varepsilon_{x^i}}{\partial t_j} = 0$$

$$\frac{\partial s_1}{\partial t_i} = \frac{\partial \widetilde{s}}{\partial t_i};\ \frac{\partial s_2}{\partial t_i} = -\frac{\partial \widetilde{s}}{\partial t_i} \text{ and } \frac{\partial^2 s_1}{\partial t_1 \partial t_2} = \frac{\partial^2 \widetilde{s}}{\partial t_1 \partial t_2} \text{ and } \frac{\partial^2 s_2}{\partial t_2 \partial t_1} = -\frac{\partial^2 \widetilde{s}}{\partial t_2 \partial t_1}$$

with

$$\frac{\partial^2 \widetilde{s}}{\partial t_1 \partial t_2} = \frac{-\alpha \frac{\partial \widetilde{s}}{\partial t_2} - \alpha \frac{\partial \widetilde{s}}{\partial t_1}}{\rho} = \frac{-\alpha \frac{\partial \widetilde{s}}{\partial T}}{\rho} = \frac{\partial^2 \widetilde{s}}{\partial t_2 \partial t_1}$$

At the symmetric equilibrium ($p_1 = p_2$ so that $t_1 = t_2$) we have $\widetilde{s} = 0$, $\frac{\partial \widetilde{s}}{\partial T} = 0$, $\frac{\partial \widetilde{s}}{\partial t_1} < 0$, $\frac{\partial \widetilde{s}}{\partial t_2} > 0$ and $s_i = 1 > 0$ so that

$$\Omega^i_{t_j} = t_i \left[-\frac{1}{s_i^2} \frac{\partial s_i}{\partial t_i} \frac{\partial s_i}{\partial t_j} \right] = -\frac{1}{t} \left(\varepsilon_s \right)^2 < 0$$

From Lemma 1.

6.2 Appendix 2

1/ Lump sum tax T

$$\frac{\partial R}{\partial T} = \theta \frac{\partial C}{\partial T} + \sum_{i=1}^{2} (1+\theta) s_i x^i + \theta \sum_{i=1}^{2} q_i \frac{\partial (s_i x^i)}{\partial T} + T \sum_{i=1}^{2} \frac{\partial (s_i x^i)}{\partial T} = 0 \Longleftrightarrow \Theta^T (\theta, T; t_1, t_2) = 0$$

The existence of an equilibrium requires

$$\Theta^T_T = \left[\theta \frac{\partial^2 C}{\partial T^2} + \sum_{i=1}^{2} (1+\theta) \frac{\partial s_i x^i}{\partial T} + \theta \sum_{i=1}^{2} \left(\frac{\partial q_i}{\partial T} \frac{\partial (s_i x^i)}{\partial T} + q_i \frac{\partial^2 (s_i x^i)}{\partial T^2} \right) + \sum_{i=1}^{2} \frac{\partial (s_i x^i)}{\partial T} + T \sum_{i=1}^{2} \frac{\partial^2 (s_i x^i)}{\partial T^2} \right] < 0$$

We have

$$\frac{\partial T}{\partial t_i} = -\frac{\Theta^T_{t_i}}{\Theta^T_T}$$

Then the sign of $\Theta^T_{t_i}$ gives the sign of $\frac{\partial T}{\partial t_i}$

$$\Theta^T_{t_k} = \theta\frac{\partial C}{\partial T\partial t_k} + \sum_{i=1}^{2}(1+\theta)\frac{\partial\left(s_i x^i\right)}{\partial t_k} + \theta\sum_{i=1}^{2} q_i\frac{\partial^2\left(s_i x^i\right)}{\partial T\partial t_k} + T\sum_{i=1}^{2}\frac{\partial^2\left(s_i x^i\right)}{\partial T\partial t_k} + \theta\frac{\partial\left(s_k x^k\right)}{\partial T}$$

Since $s_1 = 1+\widetilde{s}$ and $s_2 = 1-\widetilde{s}$ we have (symmetric cases are in brackets)

$$\begin{aligned}
\frac{\partial\left(s_1x^1\right)}{\partial t_1} &= \frac{\partial\widetilde{s}}{\partial t_1}x^1 + \frac{\partial x^1}{\partial t_1}\left(\widetilde{s}+1\right)\left(=\frac{\partial\tilde{s}}{\partial t}x + x'\right)\\
\frac{\partial\left(s_2x^2\right)}{\partial t_1} &= -\frac{\partial\widetilde{s}}{\partial t_1}x^2\\
\frac{\partial\left(s_1x^1\right)}{\partial T} &= \frac{\partial\widetilde{s}}{\partial T}x^1 + \frac{\partial x^1}{\partial T}\left(\widetilde{s}+1\right)(=x')\\
\frac{\partial\left(s_2x^2\right)}{\partial T} &= -\frac{\partial\widetilde{s}}{\partial T}x^2 + \frac{\partial x^2}{\partial T}\left(1-\widetilde{s}\right)(=x')\\
\frac{\partial^2\left(s_1x^1\right)}{\partial T\partial t_1} &= \frac{\partial^2\widetilde{s}}{\partial T\partial t_1}x^1 + \frac{\partial\widetilde{s}}{\partial T}\frac{\partial x^1}{\partial t_1} + \frac{\partial^2 x^1}{\partial T\partial t_1}\left(\widetilde{s}+1\right) + \frac{\partial x^1}{\partial T}\frac{\partial\widetilde{s}}{\partial t_1}\\
\frac{\partial^2\left(s_2x^2\right)}{\partial T\partial t_1} &= -\frac{\partial^2\widetilde{s}}{\partial T\partial t_1}x^2 - \frac{\partial\widetilde{s}}{\partial t_1}\frac{\partial x^2}{\partial T}\\
\frac{\partial^2\left(s_1x^1\right)}{\partial T^2} &= \frac{\partial^2\widetilde{s}}{\partial T^2}x^1 + 2\frac{\partial\widetilde{s}}{\partial T}\frac{\partial x^1}{\partial T} + \frac{\partial^2 x^1}{\partial T^2}\left(\widetilde{s}+1\right)(=x'')\\
\frac{\partial^2\left(s_2x^2\right)}{\partial T^2} &= -\frac{\partial^2\widetilde{s}}{\partial T^2}x^2 - 2\frac{\partial\widetilde{s}}{\partial T}\frac{\partial x^2}{\partial T} + \frac{\partial^2 x^2}{\partial T^2}\left(1-\widetilde{s}\right)(=x'')
\end{aligned}$$

$$C\left(t_1,t_2,T,\theta\right) = \int_{-1}^{\widetilde{s}} c^1 ds + \int_{\widetilde{s}}^{1} c^2 ds$$

Since $c^1 = \frac{\overline{y}-x^1P_1-(\delta+\alpha P_1)(s+1)}{(1+\theta)}$ we have $\int_{-1}^{\widetilde{s}} c^1 ds = \int_{-1}^{\widetilde{s}}\frac{\overline{y}-x^1P_1-(\delta+\alpha P_1)}{(1+\theta)}ds - \int_{-1}^{\widetilde{s}}\frac{(\delta+\alpha P_1)s}{(1+\theta)}ds = \frac{\overline{y}-x^1P_1-(\delta+\alpha P_1)}{(1+\theta)}\left(\widetilde{s}+1\right) - \frac{(\delta+\alpha P_1)}{2(1+\theta)}\left(\widetilde{s}^2-1\right)$

and similarly for c^2 we have $c^2 = \frac{\overline{y}-x^2P_2-(\delta+\alpha P_2)(1-s)}{(1+\theta)}$ so that $\int_{\widetilde{s}}^{1} c^2 ds = \int_{\widetilde{s}}^{1}\frac{\overline{y}-x^2P_2-(\delta+\alpha P_2)}{(1+\theta)}ds + \int_{\widetilde{s}}^{1}\frac{(\delta+\alpha P_2)s}{(1+\theta)}ds = \frac{\overline{y}-x^2P_2-(\delta+\alpha P_2)}{(1+\theta)}\left(1-\widetilde{s}\right) + \frac{(\delta+\alpha P_2)}{2(1+\theta)}\left(1-\widetilde{s}^2\right)$

Then we can rewrite C as

$$C = \frac{\overline{y}-x^1P_1-(\delta+\alpha P_1)}{(1+\theta)}\left(\widetilde{s}+1\right) - \frac{(\delta+\alpha P_1)}{2(1+\theta)}\left(\widetilde{s}^2-1\right) + \frac{\overline{y}-x^2P_2-(\delta+\alpha P_2)}{(1+\theta)}\left(1-\widetilde{s}\right) + \frac{(\delta+\alpha P_2)}{2(1+\theta)}\left(1-\widetilde{s}^2\right)$$

with simplifications we obtain

$$C = \frac{\overline{y} - x^1 P_1}{(1+\theta)} (\widetilde{s}+1) - \frac{(\delta + \alpha P_1)}{2(1+\theta)} (1+\widetilde{s})^2 + \frac{\overline{y} - x^2 P_2}{(1+\theta)} (1-\widetilde{s}) - \frac{(\delta + \alpha P_2)}{2(1+\theta)} (1-\widetilde{s})^2 \tag{9}$$

Then

$$\begin{aligned}
\frac{\partial C}{\partial T} &= \frac{\partial \widetilde{s}}{\partial T} \left(\frac{\overline{y} - x^1 P_1}{(1+\theta)} \right) - \left(\frac{\partial x^1}{\partial T} q_1 + x^1 \right) (\widetilde{s}+1) - 2 \frac{\partial \widetilde{s}}{\partial T} \frac{(\delta + \alpha P_1)}{2(1+\theta)} (1+\widetilde{s}) - \frac{\alpha}{2} (1+\widetilde{s})^2 \\
&\quad - \frac{\partial \widetilde{s}}{\partial T} \left(\frac{\overline{y} - x^2 P_2}{(1+\theta)} \right) - \left(\frac{\partial x^2}{\partial T} q_2 + x^2 \right) (1-\widetilde{s}) + 2 \frac{\partial \widetilde{s}}{\partial T} \frac{(\delta + \alpha P_2)}{2(1+\theta)} (1-\widetilde{s}) - \frac{\alpha}{2} (1-\widetilde{s})^2 \\
&= \frac{\partial \widetilde{s}}{\partial T} \left(\frac{x^2 P_2 - x^1 P_1}{(1+\theta)} \right) - \left(\frac{\partial x^1}{\partial T} q_1 + x^1 \right) (\widetilde{s}+1) - \frac{\partial \widetilde{s}}{\partial T} \frac{(\delta + \alpha P_1)}{(1+\theta)} (1+\widetilde{s}) - \alpha \left(1 + \widetilde{s}^2 \right) - \\
&\quad \left(\frac{\partial x^2}{\partial T} q_2 + x^2 \right) (1-\widetilde{s}) + \frac{\partial \widetilde{s}}{\partial T} \frac{(\delta + \alpha P_2)}{(1+\theta)} (1-\widetilde{s})
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 C}{\partial T^2} &= \frac{\partial^2 \widetilde{s}}{\partial T^2} \left(\frac{x^2 P_2 - x^1 P_1}{(1+\theta)} \right) + \frac{\partial \widetilde{s}}{\partial T} \left(x^2 - x^1 \right) + \frac{\partial \widetilde{s}}{\partial T} \left(\frac{\frac{\partial x^2}{\partial T} P_2 - \frac{\partial x^1}{\partial T} P_1}{(1+\theta)} \right) - \left(2 \frac{\partial x^1}{\partial T} + \frac{\partial^2 x^1}{\partial T^2} q_1 \right) (\widetilde{s}+1) - \\
&\quad \left(\frac{\partial x^1}{\partial T} q_1 + x^1 \right) \frac{\partial \widetilde{s}}{\partial T} - \frac{\partial^2 \widetilde{s}}{\partial T^2} \frac{(\delta + \alpha P_1)}{(1+\theta)} (1+\widetilde{s}) - \left(\frac{\partial \widetilde{s}}{\partial T} \right)^2 \frac{(\delta + \alpha P_1)}{(1+\theta)} - \frac{\partial \widetilde{s}}{\partial T} \alpha (1+\widetilde{s}) - 2\alpha \left(\frac{\partial \widetilde{s}}{\partial T} \right) \widetilde{s} \\
&\quad - \left(2 \frac{\partial x^2}{\partial T} + \frac{\partial^2 x^2}{\partial T^2} q_2 \right) (1-\widetilde{s}) + \left(\frac{\partial x^2}{\partial T} q_2 + x^2 \right) \frac{\partial \widetilde{s}}{\partial T} + \frac{\partial^2 \widetilde{s}}{\partial T^2} \frac{(\delta + \alpha P_2)}{(1+\theta)} (1-\widetilde{s}) + \frac{\partial \widetilde{s}}{\partial T} \alpha (1-\widetilde{s}) \\
&\quad - \left(\frac{\partial \widetilde{s}}{\partial T} \right)^2 \frac{(\delta + \alpha P_2)}{(1+\theta)}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2 C}{\partial T \partial t_1} &= \frac{\partial^2 \widetilde{s}}{\partial T \partial t_1} \left(\frac{x^2 P_2 - x^1 P_1}{(1+\theta)} \right) + \frac{\partial \widetilde{s}}{\partial T} \left(-\frac{\partial x^1}{\partial t_1} q_1 - x^1 \right) \\
&- \left(\frac{\partial x^1}{\partial T} q_1 + x^1 \right) \frac{\partial \widetilde{s}}{\partial t_1} - \left(\frac{\partial x^1}{\partial T \partial t_1} q_1 + \frac{\partial x^1}{\partial T} + \frac{\partial x^1}{\partial t_1} \right) (\widetilde{s}+1) \\
&- \frac{\partial^2 \widetilde{s}}{\partial T \partial t_1} \frac{(\delta + \alpha P_1)}{(1+\theta)} (1+\widetilde{s}) - \frac{\partial \widetilde{s}}{\partial T} \frac{\partial \widetilde{s}}{\partial t_1} \frac{(\delta + \alpha P_1)}{(1+\theta)} \\
&- \alpha \frac{\partial \widetilde{s}}{\partial T} (1+\widetilde{s}) - 2\alpha \widetilde{s} \frac{\partial \widetilde{s}}{\partial t_1} + \frac{\partial \widetilde{s}}{\partial t_1} \left(\frac{\partial x^2}{\partial T} q_2 + x^2 \right) + \frac{\partial^2 \widetilde{s}}{\partial T \partial t_1} \frac{(\delta + \alpha P_2)}{(1+\theta)} (1-\widetilde{s}) - \frac{\partial \widetilde{s}}{\partial t_1} \frac{\partial \widetilde{s}}{\partial T} \frac{(\delta + \alpha P_2)}{(1+\theta)}
\end{aligned}$$

At the symmetric equilibrium we have $p_1 = p_2$ so that $t_1 = t_2$ and $\widetilde{s} = 0$, $\frac{\partial \widetilde{s}}{\partial T} = 0$,

$$\frac{\partial C}{\partial T} = -2 \left(x' q + x \right) - \alpha$$

$$\begin{aligned}\frac{\partial^2 C}{\partial T \partial t_1} &= -\left(\frac{\partial^2 x^1}{\partial T \partial t_1} q_1 + \frac{\partial x^1}{\partial T} + \frac{\partial x^1}{\partial t_1}\right) \\ &= -\left(x^{''} q + 2x'\right)\end{aligned}$$

and

$$\frac{\partial^2 C}{\partial T^2} = -\left(2\frac{\partial x^1}{\partial T} + \frac{\partial^2 x^1}{\partial T^2} q_1\right) - \left(2\frac{\partial x^2}{\partial T} + \frac{\partial^2 x^2}{\partial T^2} q_2\right) = -2\left(2x' + qx^{''}\right)$$

Combining all the effects in $\Theta^T_{t_k}$, we obtain at the symmetric equilibrium

$$\begin{aligned}\Theta^T_{t_k} &= -\theta\left(x^{''} q + 2x'\right) + 2(1+\theta)x' + \theta\left(x^{''} q\right) + Tx^{''} + \theta x' \\ &= 2x' + Tx^{''} + \theta x'\end{aligned}$$

At the symmetric equilibrium we have

$$\Theta^T_T = 2\left[2x' + Tx^{''}\right] < 0$$

so that $\Theta^T_{t_k} < 0$ since $2x' + Tx^{''} + \theta x' < 2x' + Tx^{''} < 0$

2/ VAT θ

$$\frac{\partial R}{\partial \theta} = C + \theta \frac{\partial C}{\partial \theta} + \sum_{i=1}^{2} q_i s_i x^i + \theta \sum_{i=1}^{2} q_i x^i \frac{\partial s_i}{\partial \theta} + T \sum_{i=1}^{2} x^i \frac{\partial s_i}{\partial \theta} \Longleftrightarrow \Theta^\theta(\theta, T; t_1, t_2) = 0$$

The existence of an equilibrium requires

$$\Theta^\theta_\theta = \left[2\frac{\partial C}{\partial \theta} + \theta \frac{\partial^2 C}{\partial \theta^2} + 2\sum_{i=1}^{2} q_i \frac{\partial s_i}{\partial \theta} x^i + \theta \sum_{i=1}^{2} q_i x^i \frac{\partial^2 s_i}{\partial \theta^2} + T\sum_{i=1}^{2} x^i \frac{\partial^2 s_i}{\partial \theta^2}\right] \leq 0$$

From (9) we obtain

$$\begin{aligned}\frac{\partial C}{\partial \theta} &= \frac{\partial \widetilde{s}}{\partial \theta}\left(\frac{\overline{y} - x^1 P_1}{(1+\theta)}\right) - \left(\frac{\overline{y}}{(1+\theta)^2}\right)(\widetilde{s}+1) - 2\frac{\partial \widetilde{s}}{\partial \theta}\frac{(\delta + \alpha P_1)}{2(1+\theta)}(1+\widetilde{s}) + \frac{\delta}{2(1+\theta)^2}(1+\widetilde{s})^2 - \\ &\quad \frac{\partial \widetilde{s}}{\partial \theta}\left(\frac{\overline{y} - x^2 P_2}{(1+\theta)}\right) - \left(\frac{\overline{y}}{(1+\theta)^2}\right)(1-\widetilde{s}) + 2\frac{\partial \widetilde{s}}{\partial \theta}\frac{(\delta + \alpha P_2)}{2(1+\theta)}(1-\widetilde{s}) + \frac{\delta}{2(1+\theta)^2}(1-\widetilde{s})^2 \\ &= \frac{\partial \widetilde{s}}{\partial \theta}\left(\frac{x^2 P_2 - x^1 P_1}{(1+\theta)}\right) - 2\left(\frac{\overline{y}}{(1+\theta)^2}\right) - \frac{\partial \widetilde{s}}{\partial \theta}\frac{(\delta + \alpha P_1)}{(1+\theta)}(1+\widetilde{s}) + \frac{\delta}{(1+\theta)^2}\left(1+\widetilde{s}^2\right) + \\ &\quad \frac{\partial \widetilde{s}}{\partial \theta}\frac{(\delta + \alpha P_2)}{(1+\theta)}(1-\widetilde{s})\end{aligned}$$

which gives at the symmetric equilibrium ($p_1 = p_2$ so that $t_1 = t_2$, $\widetilde{s} = 0$ and $\frac{\partial \widetilde{s}}{\partial \theta} = 0$)

$$\frac{\partial C}{\partial \theta} = \frac{\delta - 2\overline{y}}{(1+\theta)^2}$$

and

$$\frac{\partial^2 C}{\partial \theta^2} = -2\frac{\delta - 2\overline{y}}{(1+\theta)^3}$$

so that

$$\Theta_\theta^\theta = 2\left[\left(\delta - \frac{2\overline{y}}{(1+\theta)^3}\right)\right] \leq 0$$

We have

$$\frac{\partial \theta}{\partial t_i} = -\frac{\Theta_{t_i}^\theta}{\Theta_\theta^\theta}$$

Then the sign of $\Theta_{t_i}^\theta$ gives the sign of $\frac{\partial \theta}{\partial t_i}$

$$\Theta_{t_1}^\theta = \frac{\partial C}{\partial t_1} + \theta\frac{\partial^2 C}{\partial \theta \partial t_1} + \sum_{i=1}^{2}\left(\frac{\partial\left(q_i x^i s_i\right)}{\partial t_1} + \theta\frac{\partial\left(q_i x^i\right)}{\partial t_1}\frac{\partial s_i}{\partial \theta} + \theta q_i x^i\frac{\partial^2 s_i}{\partial \theta \partial t_1} + T\frac{\partial^2 s_i}{\partial \theta \partial t_1}x^i + T\frac{\partial s_i}{\partial \theta}\frac{\partial x^i}{\partial t_1}\right)$$

$$\begin{aligned}
\frac{\partial\left(q_1 s_1 x^1\right)}{\partial t_1} &= q_1\frac{\partial \widetilde{s}}{\partial t_1}x^1 + q_1\frac{\partial x^1}{\partial t_1}\left(\widetilde{s}+1\right) + x^1\left(1+\widetilde{s}\right)\\
\frac{\partial\left(q_2 s_2 x^2\right)}{\partial t_1} &= -q_2\frac{\partial \widetilde{s}}{\partial t_1}x^2\\
\frac{\partial\left(q_1 x^1\right)}{\partial t_1} &= x^1 + q_1\frac{\partial x^1}{\partial t_1}\\
\frac{\partial\left(q_2 x^2\right)}{\partial t_1} &= 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 C}{\partial \theta \partial t_1} = &\frac{\partial^2 \widetilde{s}}{\partial \theta \partial t_1}\left(\frac{x^2 P_2 - x^1 P_1}{(1+\theta)}\right) + \frac{\partial \widetilde{s}}{\partial \theta}\left(-\frac{\partial x^1}{\partial t_1}q_1 - x^1\right) - \frac{\partial^2 \widetilde{s}}{\partial \theta \partial t_1}\frac{(\delta + \alpha P_1)}{(1+\theta)}\left(1+\widetilde{s}\right) - \frac{\partial \widetilde{s}}{\partial \theta}\frac{\partial \widetilde{s}}{\partial t_1}\frac{(\delta + \alpha P_1)}{(1+\theta)}\\
&-\alpha\frac{\partial \widetilde{s}}{\partial \theta}\left(1+\widetilde{s}\right) + 2\widetilde{s}\frac{\delta}{(1+\theta)^2}\frac{\partial \widetilde{s}}{\partial t_1} + \frac{\partial^2 \widetilde{s}}{\partial \theta \partial t_1}\frac{(\delta + \alpha P_2)}{(1+\theta)}\left(1-\widetilde{s}\right) - \frac{\partial \widetilde{s}}{\partial t_1}\frac{\partial \widetilde{s}}{\partial \theta}\frac{(\delta + \alpha P_2)}{(1+\theta)}
\end{aligned}$$

Finally

$$\begin{aligned}
\frac{\partial C}{\partial t_1} = &\frac{\partial \widetilde{s}}{\partial t_1}\left(\frac{\overline{y} - x^1 P_1}{(1+\theta)}\right) - \left(\frac{\partial x^1}{\partial t_1}q_1 + x^1\right)\left(\widetilde{s}+1\right) - 2\frac{\partial \widetilde{s}}{\partial t_1}\frac{(\delta + \alpha P_1)}{2(1+\theta)}\left(1+\widetilde{s}\right) - \frac{\alpha}{2}\left(1+\widetilde{s}\right)^2\\
&-\frac{\partial \widetilde{s}}{\partial t_1}\left(\frac{\overline{y} - x^2 P_2}{(1+\theta)}\right) + 2\frac{\partial \widetilde{s}}{\partial t_1}\frac{(\delta + \alpha P_2)}{2(1+\theta)}\left(1-\widetilde{s}\right)
\end{aligned}$$

At the symmetric equilibrium we have $p_1 = p_2$ so that $t_1 = t_2$ and $\widetilde{s} = 0$, $\frac{\partial \widetilde{s}}{\partial T} = 0$,

$$\frac{\partial^2 C}{\partial \theta \partial t_1} = 0$$

and

$$\frac{\partial C}{\partial t_1} = -\left(x'q + x\right) - \frac{\alpha}{2} \tag{10}$$

Combining all the effects in $\Theta^T_{t_k}$, we obtain at the symmetric equilibrium

$$\begin{aligned}
\Theta^\theta_{t_i} &= \frac{\partial C}{\partial t_1} + \theta\frac{\partial^2 C}{\partial\theta\partial t_1} + \sum_{i=1}^{2}\left(\frac{\partial\left(q_i x^i s_i\right)}{\partial t_1} + \theta\frac{\partial\left(q_i x^i\right)}{\partial t_1}\frac{\partial s_i}{\partial\theta} + \theta q_i x^i\frac{\partial^2 s_i}{\partial\theta\partial t_1} + T\frac{\partial^2 s_i}{\partial\theta\partial t_1}x^i + T\frac{\partial s_i}{\partial\theta}\frac{\partial x^i}{\partial t_1}\right) \\
&= -\left(\frac{\partial x^1}{\partial t_1}q_1 + x^1\right) - \frac{\alpha}{2} + q_1\frac{\partial x^1}{\partial t_1} + x^1 \\
&= -\frac{\alpha}{2} \leq 0
\end{aligned}$$

3/ Federal tax interactions

$$\frac{\partial R}{\partial\theta} = C + \theta\frac{\partial C}{\partial\theta} + \sum_{i=1}^{2}q_i s_i x^i + \theta\sum_{i=1}^{2}q_i x^i\frac{\partial s_i}{\partial\theta} + T\sum_{i=1}^{2}x^i\frac{\partial s_i}{\partial\theta} \Longleftrightarrow \Theta^\theta\left(\theta, T; t_1, t_2\right) = 0$$

We have

$$\frac{\partial\theta}{\partial T} = -\frac{\Theta^\theta_T}{\Theta^\theta_\theta}$$

Then the sign of Θ^θ_T gives the sign of $\frac{\partial\theta}{\partial T}$

$$\Theta^\theta_T = \frac{\partial C}{\partial T} + \theta\frac{\partial^2 C}{\partial\theta\partial T} + \sum_{i=1}^{2}\left(\frac{\partial\left(q_i x^i s_i\right)}{\partial T} + \theta\frac{\partial\left(q_i x^i\right)}{\partial T}\frac{\partial s_i}{\partial\theta} + \theta q_i x^i\frac{\partial^2 s_i}{\partial\theta\partial T} + T\frac{\partial^2 s_i}{\partial\theta\partial T}x^i + T\frac{\partial s_i}{\partial\theta}\frac{\partial x^i}{\partial T}\right)$$

$$\begin{aligned}
\frac{\partial C}{\partial T} &= \frac{\partial\widetilde{s}}{\partial T}\left(\frac{x^2P_2 - x^1P_1}{(1+\theta)}\right) - \left(\frac{\partial x^1}{\partial T}q_1 + x^1\right)\left(\widetilde{s}+1\right) - \frac{\partial\widetilde{s}}{\partial T}\frac{\left(\delta + \alpha P_1\right)}{(1+\theta)}\left(1+\widetilde{s}\right) - \alpha\left(1+\widetilde{s}^2\right) \\
&\quad - \left(\frac{\partial x^2}{\partial T}q_2 + x^2\right)\left(1-\widetilde{s}\right) + \frac{\partial\widetilde{s}}{\partial T}\frac{\left(\delta + \alpha P_2\right)}{(1+\theta)}\left(1-\widetilde{s}\right)
\end{aligned}$$

$$\frac{\partial C}{\partial\theta} = \frac{\partial\widetilde{s}}{\partial\theta}\left(\frac{x^2P_2 - x^1P_1}{(1+\theta)}\right) - 2\left(\frac{\overline{y}}{(1+\theta)^2}\right) - \frac{\partial\widetilde{s}}{\partial\theta}\frac{\left(\delta+\alpha P_1\right)}{(1+\theta)}\left(1+\widetilde{s}\right) + \frac{\delta}{(1+\theta)^2}\left(1+\widetilde{s}^2\right) + \frac{\partial\widetilde{s}}{\partial\theta}\frac{\left(\delta+\alpha P_2\right)}{(1+\theta)}\left(1-\widetilde{s}\right)$$

$$\begin{aligned}
\frac{\partial^2 C}{\partial\theta\partial T} &= \frac{\partial^2\widetilde{s}}{\partial\theta\partial T}\left(\frac{x^2P_2 - x^1P_1}{(1+\theta)}\right) + \frac{\partial\widetilde{s}}{\partial\theta}\left(-\frac{\partial x^1}{\partial T}q_1 - x^1 + \frac{\partial x^2}{\partial T}q_2 + x^2\right) - \frac{\partial^2\widetilde{s}}{\partial\theta\partial T}\frac{\left(\delta+\alpha P_1\right)}{(1+\theta)}\left(1+\widetilde{s}\right) \\
&\quad - \frac{\partial\widetilde{s}}{\partial\theta}\frac{\partial\widetilde{s}}{\partial T}\frac{\left(\delta+\alpha P_1\right)}{(1+\theta)} - \alpha\frac{\partial\widetilde{s}}{\partial\theta}\left(1+\widetilde{s}\right) + 2\widetilde{s}\frac{\delta}{(1+\theta)^2}\frac{\partial\widetilde{s}}{\partial T} + \frac{\partial^2\widetilde{s}}{\partial\theta\partial T}\frac{\left(\delta+\alpha P_2\right)}{(1+\theta)}\left(1-\widetilde{s}\right) - \\
&\quad \frac{\partial\widetilde{s}}{\partial T}\frac{\partial\widetilde{s}}{\partial\theta}\frac{\left(\delta+\alpha P_2\right)}{(1+\theta)} + \alpha\frac{\partial\widetilde{s}}{\partial\theta}\left(1-\widetilde{s}\right)
\end{aligned}$$

At the symmetric equilibrium we have

$$\frac{\partial C}{\partial T} = -2\left(x'q + x\right) - \alpha$$

$$\frac{\partial^2 C}{\partial \theta \partial T} = 0$$

$$\begin{aligned}\Theta_T^\theta &= \frac{\partial C}{\partial T} + \theta \frac{\partial^2 C}{\partial \theta \partial T} + \sum_{i=1}^{2}\left(\frac{\partial\left(q_i x^i s_i\right)}{\partial T} + \theta \frac{\partial\left(q_i x^i\right)}{\partial T}\frac{\partial s_i}{\partial \theta} + \theta q_i x^i \frac{\partial^2 s_i}{\partial \theta \partial T} + T\frac{\partial^2 s_i}{\partial \theta \partial T} x^i + T\frac{\partial s_i}{\partial \theta}\frac{\partial x^i}{\partial T}\right) \\ &= -2\left(x'q + x\right) - \alpha + 2\left(x'q + x\right) = -\alpha \leq 0\end{aligned}$$

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