Demographics and competition for capital in political economy*

Tadashi Morita† Yasuhiro Sato‡ Kazuhiro Yamamoto§

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Abstract

We examine possible impacts of demographics on outcomes of competition for capital in political economy. For this purpose, we develop a multi-region overlapping generations model wherein public good provision financed by capital tax is determined by majority voting. When a population is growing, younger people represent the majority, whereas when a population is decreasing, older people represent the majority. We show that the race to the bottom is likely to emerge as a result of tax competition in an economy with a growing population whereas the race to the top might emerge in an economy with a decreasing population.

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1 Introduction

This paper investigates the possible impacts of demographics on the results of competition for capital in political economy. Given the drastic increases in capital flows across countries

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†Faculty of Economics, Kindai University, *e-mail*: t-morita@kindai.ac.jp

‡*Corresponding author*, Faculty of Economics, University of Tokyo, *e-mail*: ysato@e.u-tokyo.ac.jp

§Graduate School of Economics, Osaka University, *e-mail*: yamamoto@econ.osaka-u.ac.jp

and regions, many scholars have analyzed the effects of globalization in the capital market over the past few decades. One of the most important strands in this field is the theory of capital tax competition, which has a long history dating back at least to Zodrow and Mieszkowski (1986) and Wilson (1986).[1] Researchers in this strand investigated the role of governments in attracting capital to their jurisdictions. In standard tax competition models, governments are benevolent and maximize the representative resident's welfare. Nonetheless, they set inefficiently low capital tax rates because capital taxation causes capital flight, which increases the tax bases in other countries and causes positive fiscal externalities. This result is known as the *race to the bottom* and has attracted much attention (see e.g., OECD, 1998).

Around the same time, we observe large differences in demographic structure among countries and drastic demographic changes in many of them. In fact, when we consider the old-age dependency ratio, which is the ratio of people older than 65 years of age to the working-age population, we find large differences among countries. For example, the 2014 ratios were 9.6 in Mexico, 11.1 in Turkey, 21.6 in the United States, 27.7 in Spain, 34.4 in Italy, and 41.9 in Japan.[2] Similarly, the median ages in 2010 were 26.6 in Mexico, 28.3 in Turkey, 36.9 in the United States, 40.1 in Spain, 46.4 in Italy, and 47.8 in Japan.[3] Moreover, we also observe drastic changes in these figures: the old-age dependency ratios and median ages of OECD countries rose from 13.7 to 24.2 between 1960 and 2014 and from 28.9 to 45.4 between 1950 and 2010, respectively. These facts imply that in political economy, decisive voters are younger generations in countries such as Mexico and Turkey, whereas they are older generations in countries such as Italy and Japan. Moreover, they are getting older in OECD countries. Put plainly, decisive voters and hence, the objectives of governments might change over time and place. This would, in turn, affect the outcomes of inter-governmental competition for capital.

In this paper, we ask how demographics are related to outcomes of governments' policy competition for capital and firms. To be more specific, we ask whether the race to the bottom, which is believed to prevail in the world, emerges under any demographic

[1] For surveys on this strand, see Wilson (1999), Wilson and Wildasin (2004) and Zodrow (2003) among others.

[2] World Bank, http://data.worldbank.org/indicator/SP.POP.DPND.OL

[3] OECD, http://www.oecd-ilibrary.org/education/trends-shaping-education-2013/median-age-going-up-into-the-next-century_trends_edu-2013-graph43-en

structure.

To analyze this issue, we develop a capital tax competition model involving the overlapping generations structure wherein policies are determined by majority voting. In our model, all individuals live for two periods, young and old, and the population grows with an exogenous constant growth rate. If the population growth rate is positive, then the young individuals represent the majority because their population size is larger than that of old individuals. In contrast, if the population growth rate is negative, then the old individuals represent the majority. We consider multiple countries and each country's government supplies public goods and finances them with capital tax. A government chooses the level of public good provision and capital tax rate to maximize the utility of the individuals representing the majority.

In our model, young individuals supply labor to firms, which produce private consumption goods with labor and capital. The wage income of young individuals increases with the capital inputs in the country, whereas the savings income of old individuals increases with the rate of return on savings. Young individuals consume private and public goods and save the wage income for old-age consumption. Old individuals consume private and public goods. Governments at a certain period ignore the effects of taxes and public goods on instantaneous utility of minority, because the subjective of government is to maximize the majority's utility only. When the decisive voter is in the young (old) generation, the government disregards the utility of old (young) individuals. Hence, capital taxation causes inter-generational externalities. The government disregards the effects of capital taxation on the benefits of public good provision to the minority generation irrespective of the majority's generation, which we call the *Public-good externality.* The sign of the Public-good externality is ambiguous because a rise in the capital tax rate increases the tax revenues from one unit of capital but causes capital outflows to decrease the tax base. In addition, when old individuals represent the majority, the government ignores the negative effect of capital taxation on wages of young individuals, which is caused by capital outflows. We call this the *Gerontocracy externality*, which makes a government have a higher incentive to tax capital in an economy with a decreasing population than in an economy with a growing population.

Moreover, our model has three types of inter-country externalities caused by competition for capital. First, because we assume free mobility of capital between countries, capital taxation in one country causes capital flight to other countries and increases their tax bases, which is the *Fiscal externality*, as is standard in models of capital tax competition. Second, the capital flight also raises the wage rates in the other countries, which we call the *Wage externality.* Finally, capital taxation decreases the rate of return on savings, which reduces the savings income of old individuals in the other countries. We call this effect the *Interest-rate externality.* The Fiscal and Wage externalities are positive whereas the Interest-rate externality is negative.

The relative significance of positive and negative externalities characterizes the (in)efficiency of the equilibrium capital tax rate. When individuals strongly prefer public good consumption, sum of positive externalities overwhelms the sum of negative externalities, which yields an inefficiently low capital tax rate, i.e., we observe the race to the bottom. When individuals do not strongly prefer public good consumption, the capital tax rate is inefficiently high, which we call the *race to the top.* Furthermore, because an economy with a decreasing population has the Interest-rate and Gerontocracy externalities as negative externalities but an economy with a growing population has only the Interest-rate externality a negative externality, the race to the top is more likely to emerge in the former than in the latter.

We also consider asymmetric countries. When the population is growing or decreasing in all countries, we can obtain qualitatively similar results with the case of symmetric countries. When the population is growing in some countries and decreasing in other countries, we show a possibility that capital tax rates are inefficiently low in countries with a growing population and inefficiently high in countries with a decreasing population.

Our results has the following strong policy implications: Demographic structure matters for the outcomes of tax competition. Although the race to the bottom has prevailed as a popular outcome of tax competition in the past few decades during which population has increased in many countries, we might observe the race to the top as aging would proceed in many countries. In fact, Keen and Konrad (2013) mentioned the minimum excise duty rate adopted by the European Union to limit downward spiralling of rate by tax competition. However, if aging proceeds, policy debates should refer to the possibility of the race to the top and coordinated tax decreases.

Several existing papers have investigated capital tax competition in political economy models. Persson and Tabellini (1992), Borck (2003), Lockwood and Makris (2006), Grazzini and van Ypersele (2003), Fuest and Huber (2001), and Ihori and Yang (2009) assumed that individuals have different endowments of labor and capital, and that capital tax rates are determined by the political economy process.[4] These papers commonly showed that if the decisive voter's capital endowment is smaller than the average, then the equilibrium capital tax rate tends to be inefficiently high. Our paper also studies the tax competition in a political economy model. However, in our model, individuals are not heterogeneous with respect to endowments. Alternatively, we consider a difference between generations by using an overlapping generations model. Put differently, we focus on inter-generational political conflicts, whereas existing studies focus on political conflicts among individuals with heterogeneous endowments.

We also refer to existing studies that used overlapping generations models in political economy to investigate a macro economy. Alesina and Rodrik (1994) and Persson and Tabellini (1994a, b) constructed overlapping generation models wherein individuals have heterogeneous endowments of labor and capital, and a median voter chooses the capital income tax rate. In these models, when the median voter has more capital endowments, the equilibrium capital tax rate becomes lower, which raises the equilibrium growth rate. These papers analyzed the closed economy models and focused on the effects of income distribution on growth rates. Mateos-Planas (2010) analyzed the effects of demographics on the mix of tax rates on households' labor and capital income by using a median voter model. He focused on the quantitative effects of increases in young population on the capital income tax rate in a close economy. Our paper also constructs an overlapping generation model in political economy. However, we consider an open economy wherein capital is mobile among countries and focus on the effects of demographics on the results of policy competition.

Moreover, our analysis regarding asymmetric countries relates to studies of asymmetric tax competition, which have considered differences in many aspects between regions and countries. In particular, whereas previous studies focused on regional characteristics and disparities in population size, technology, preferences, labor market, and initial endowment, we add a new and significant view to asymmetric tax competition by considering

[4] Montén and Thum (2010) considered the effects of demographics on the inter-generational conflict under tax competition. Their focus is on to derive conditions under which gerontocracy improves the young generation's welfare. Hence, efficiency issues of tax competition are out of the scope of their paper.

international differences in demographic structure.[5]

This paper is structured as follows. Section 2 presents the baseline framework. Section 3 provides the various efficiency properties of our model. Section 4 extends the baseline framework by considering asymmetric countries. Section 5 concludes.

2 Baseline framework

Consider an overlapping generations model wherein time is discrete and each individual lives for two (young and old) periods. At the first (young) period, an individual works to earn wage income, consumes, and saves, whereas at the second (old) period, she/he does not work and spends her/his savings to consume. At the end of the old period, she/he exits the economy. We call a cohort of individuals who are young at time t as generation t. This economy has M countries, and each country i $(i = 1, ..M)$ has a population of size $\overline{L}_{it} + \overline{L}_{it-1}$, where $\overline{L}_{it}$ represents the population size of generation t. We assume that the population growth rate, n, is exogenous, and in the baseline model, we assume symmetric countries so that n is common to all countries. Hence, we have $\overline{L}_{it+1} = (1+n)\overline{L}_{it}$. In order to keep population positive, we assume that $n > -1$.

2.1 Individuals

Individuals obtain utility from private good consumption, c, and public good consumption, g. We specify the utility function as follows:

$$U_{it} = u_{iyt} + \beta u_{iot+1}, \tag{1}$$

where β $(\in (0,1))$ is the time discount rate. The subscripts y and o represent the young and old periods, respectively. u_{iy} is the utility from consumption at the young period in country i and u_{io} is that from consumption at the old period in country i. We assume that u_{ij} $(j = y, o)$ is given by

$$u_{ijt} = \ln c_{ijt} + \alpha \ln g_{it}, \tag{2}$$

where α is a positive constant that represents the preference for public good consumption. Budget constraints are given by

$$w_{it} = c_{iyt} + s_{it}, \qquad (1 + r_{it+1})s_{it} = c_{iot+1},$$

[5] See Ogawa et al (2016) for a brief survey of the existing studies on asymmetric tax competition.

where w, s, and r are the wage income, savings, and rate of return on savings, respectively. We assume that individuals are price-takers. At period t, an individual in country i and generation t inelastically supplies her/his labor endowments, of which amounts are normalized to one, to earn w_{it}, and chooses c_{iyt} and s_{it} given all prices. At period $t+1$, she/he receives $(1+r_{it+1})s_{it}$ and chooses c_{iot+1} given all prices. We assume perfect foresight regarding individual's expectation on r_{it+1}. Standard life-time utility maximization yields

$$c_{iyt} = \frac{w_{it}}{1+\beta}, \qquad s_{it} = \frac{\beta w_{it}}{1+\beta}, \qquad c_{iot+1} = \frac{\beta}{1+\beta}(1+r_{it+1})w_{it}. \tag{3}$$

2.2 Firms

Firms produce the numéraire using labor and capital under constant returns to scale. The numéraire can be traded with no cost between countries. We assume perfectly competitive goods, labor, and capital markets. We employ a Cobb-Douglas production function:

$$y_{it} = L_{it}^{\gamma} K_{it}^{1-\gamma},$$

where y is the output level, γ ($\in (0,1)$) is a positive constant representing the labor share in production, and L and K are labor and capital inputs, respectively. Letting k denote the capital per capita (capital-labor ratio, $= K/L$), profit maximization yields

$$w_{it} = \gamma k_{it}^{1-\gamma}, \qquad k_{it} = \left(\frac{1-\gamma}{r_{it}+\tau_{it}} \right)^{1/\gamma}, \tag{4}$$

where τ represents the capital tax rate. As is standard in capital tax competition models, capital taxation decreases the capital per capita ($\partial k_{it}/\partial \tau_{it} < 0$).

2.3 Market clearing conditions

In this paper, we assume that individuals are immobile between countries, implying that the labor market is local, whereas capital is freely mobile, implying that the capital market is global. Hence, the labor market clearing condition in country i is given by

$$L_{it} = \overline{L}_{it}.$$

The global capital market clearing condition is given by

$$\sum_{i=1}^{M} K_{it} = \sum_{i=1}^{M} s_{it-1} \overline{L}_{it-1}.$$

Because capital is assumed to be freely mobile among countries, the rate of return on savings becomes common to all countries ($r_{it} = r_t$, $\forall i$). Then, the capital market clearing condition can be written as

$$\sum_{i=1}^{M} \left(\frac{1-\gamma}{r_t + \tau_{it}} \right)^{1/\gamma} \overline{L}_{it} = \frac{\beta\gamma}{1+\beta} \sum_{i=1}^{M} \left(\frac{1-\gamma}{r_{t-1} + \tau_{it-1}} \right)^{(1-\gamma)/\gamma} \overline{L}_{it-1}. \tag{5}$$

2.4 Governments

In each country, a government uses capital tax revenues to finance public good provision. We assume that policies are determined by majority voting: the capital tax rate, τ_{it}, and the level of public good provision, g_{it}, are determined so that they maximize the utility of the majority at period t.[6] Hence, when the population size of generation t is larger than that of generation $t-1$ ($\overline{L}_{it} > \overline{L}_{it-1}$), the government at period t chooses τ_{it} and g_{it} that maximize U_{it}. When the opposite holds true ($\overline{L}_{it} < \overline{L}_{it-1}$), it chooses τ_{it} and g_{it} that maximize u_{iot}.[7] When deciding on τ_{it} and g_{it}, governments regard past variables (w_{it-1}), other countries' policies (τ_{jt} and g_{jt}), prices determined in the global market (r_t and r_{t+1}), and own future policy (g_{it+1}) as given.[8] We make the last assumption because governments make decisions at each period, which implies that they cannot commit to future decisions. We assume perfect foresight regarding government's expectations on g_{it+1}, and r_{t+1}. The following figure summarizes the structure of the model.

[Figure 1 around here]

[6] In our setting wherein only two types of individuals exist, this corresponds to maximize the utility of the median voter.

[7] When $\overline{L}_{it} = \overline{L}_{it-1}$, we assume that the government chooses to maximize U_{it} or u_{iot} with equal probability. If it chooses to maximize U_{it}, then the results are the same as those in the case of $\overline{L}_{it} > \overline{L}_{it-1}$; if it chooses to maximize u_{iot}, then the results are the same as those in the case of $\overline{L}_{it} < \overline{L}_{it-1}$. For the sake of expositional simplicity, we omit the case of $\overline{L}_{it} = \overline{L}_{it-1}$.

[8] We assume that a government regards global prices as given for analytical simplicity. Such an assumption would be appropriate when many countries exist. Even if we do not assume this, the government's incentive to tax capital is different between an economy with a growing population and an economy with a decreasing population. Hence, the economy would have equilibrium inefficiencies that depend on demographics as will be shown in this paper.

2.4.1 Economy with a growing population

We start from the case of $n > 0$, which implies the population is increasing. In this case, because $\overline{L}_{it} > \overline{L}_{it-1}$, the young individuals represent the majority of the population and the government maximizes U_{it}. Plugging (2), (3) and the government budget constraint $g_{it} = \tau_{it} K_{it}$ into (1), we obtain

$$U_{it} = \alpha \ln \tau_{it} - \frac{\alpha}{\gamma} \ln(r_t + \tau_{it}) - \frac{(1+\beta)(1-\gamma)}{\gamma} \ln(r_t + \tau_{it}) \tag{6}$$
$$+ \beta \ln(1 + r_{t+1}) + \alpha\beta \ln g_{it+1} + \alpha \ln \overline{L}_{it} + \Upsilon,$$

where Υ is defined as $\Upsilon \equiv \beta \ln \beta - (1+\beta) \ln(1+\beta) + \alpha \ln(1-\gamma)^{1/\gamma} + (1+\beta) \ln \gamma (1-\gamma)^{(1-\gamma)/\gamma}$.

For the later use, we define MW_{iyt} as the differentiation of (6) with respect to τ_{it}:

$$MW_{iyt} \equiv \alpha \left[\frac{1}{\tau_{it}} - \frac{1}{\gamma (r_t + \tau_{it})} \right] - \frac{(1+\beta)(1-\gamma)}{\gamma (r_t + \tau_{it})}.$$

The first term of MW_{iyt} represents the effect of τ_{it} on the supply of public goods and the second term of MW_{yt} denotes the effect of τ_{it} on the wage rate. The first order condition of the government's maximization is given by $MW_{iyt} = 0$, yielding

$$\tau_{it} = \frac{\alpha\gamma}{(1+\alpha+\beta)(1-\gamma)} r_t. \tag{7}$$

2.4.2 Economy with a decreasing population

Next, we consider the case of $n < 0$, which implies that the population is decreasing. In this case, because $\overline{L}_{it} < \overline{L}_{it+1}$, old individuals represent the majority and the government maximizes u_{iot}. Plugging (3) and $g_{it} = \tau_{it} K_{it}$ into (2), we obtain

$$u_{iot} = \alpha \ln \tau_{it} - \frac{\alpha}{\gamma} \ln(r_t + \tau_{it}) + \ln(1 + r_t) + \alpha \ln \overline{L}_{it} + \ln w_{it-1} + \Psi, \tag{8}$$

where Ψ is defined as

$$\Psi \equiv \ln \left(\frac{\beta}{1+\beta} \right) + \alpha \ln(1-\gamma)^{1/\gamma}.$$

Again, for the later use, we define MW_{iot} as the differentiation of (8) with respect to τ_{it}:

$$MW_{iot} \equiv \alpha \left[\frac{1}{\tau_{it}} - \frac{1}{\gamma (r_t + \tau_{it})} \right].$$

The first order condition of the government's maximization is given by $MW_{iot} = 0$, yielding

$$\tau_{it} = \frac{\gamma}{1-\gamma} r_t. \tag{9}$$

Equations (7) and (9) imply that for a given rate of return on savings, an economy with a decreasing population has a higher capital tax rate than an economy with a growing population. Because the government regards the global price, r_t, as given, capital taxation affects the utility through changes in capital tax revenues, $\tau_{it}K_{it}$, and changes in wage rate, w_{it}. Although the government recognizes the former effect in both economies, it disregards the latter effect in an economy with a decreasing population. Moreover, as shown in (4), capital taxation, by decreasing capital per capita, k_{it}, lowers w_{it} and U_{it} because capital and labor are complementary in production. Hence, the government has a stronger incentive to tax capital in an economy with a decreasing population than in an economy with a growing population.[9]

2.5 Transitional dynamics

In the baseline model, we assume symmetric countries, which implies that all countries have the same capital holdings at period 0, the same population size, and the same population growth rate, implying that $\overline{L}_{it} = \overline{L}_{jt}$ $(i \neq j)$ for all t. From the capital demand (4), and the capital market clearing condition (5), the sequence of capital per capita, k, can be written as

$$k_t = \frac{\beta\gamma}{(1+\beta)(1+n)} k_{t-1}^{1-\gamma}, \tag{10}$$

regardless of population growth rate. Note here that k_t is common to all countries. Moreover, (3) and (4) result in common consumption levels, i.e., $c_{iyt} = c_{yt}$ and $c_{iot} = c_{ot}$, $\forall i$.

2.6 Steady-state

We focus on steady-state equilibrium, wherein the level of individual's consumption, c, and capital per capita, k, are constant over time ($c_{yt} = c_{yt+1} = c_y^*$, $c_{ot} = c_{ot+1} = c_o^*$, and

[9] This result is similar to the results shown in the existing studies of tax competition with the median voter principle cited in the Introduction, wherein the incentive of governments to impose tax on capital increases with the decrease in capital endowments of the median voter. In our framework, the median voter is young individuals in an economy with a growing population and old individuals in an economy with a decreasing population. Because young individuals have no capital, the government has a weaker incentive to tax capital in an economy with a growing population than in an economy with a decreasing population.

$k_t = k_{t+1} = k^*$).[10] Then, from (4), we readily know that $r_t + \tau_t = r_{t+1} + \tau_{t+1}$. Using this and (4), (10) can be rewritten as

$$r_t + \tau_t = \frac{(1+n)(1+\beta)(1-\gamma)}{\beta\gamma}. \tag{11}$$

The higher the population growth rate, the smaller the capital per capita becomes, which results in higher marginal productivity of capital (i.e., higher gross rate of return on capital). We can solve (7) and (11) to derive equilibrium τ and r in an economy with a growing population whereas we can use (9) and (11) to obtain equilibrium τ and r in an economy with a decreasing population.

3 Equilibrium and its efficiency

Because our model is an overlapping generations model, the equilibrium savings rate is not optimal because of the dynamic inefficiency (Blanchard and Fischer, 1989: and Romer, 2011).[11] Moreover, capital taxation in the presence of capital mobility and political process also cause externalities. Hence, the equilibrium capital tax rate departs from the optimal one. In this section, we characterize the (in)efficiency of capital tax rate. Following Blanchard and Fischer (1989), we employ the Benthamite social welfare function as the welfare criterion:

$$W = \sum_{i=1}^{M} \left(u_{io0} L_{o0} + \sum_{t=0}^{\infty} \beta^t U_{it} \overline{L}_t \right). \tag{12}$$

In order to ensure the existence of optimal tax rate, we assume that $\sum_{i=1}^{\infty} \beta^t \overline{L}_t < \infty$, which implies that $\beta(1+n) < 1$. Using (3), (4), and $g_{it} = \tau_{it} K_{it}$, U_{it} becomes

$$U_{it} = \alpha \log \tau_{it} + \alpha\beta \log \tau_{it+1} - \frac{\alpha\beta}{\gamma} \ln(r_{t+1} + \tau_{it+1}) - \frac{\alpha + (1+\beta)(1-\gamma)}{\gamma} \ln(r_t + \tau_{it})$$
$$+ \beta \ln(1 + r_{t+1}) + \alpha \ln \overline{L}_{it} + \alpha\beta \ln \overline{L}_{it+1} + \Upsilon + \alpha\beta \ln(1-\gamma)^{1/\gamma}.$$

[10] Note here that (10) has a unique steady state. Combined with (3) and (4), this implies that there exists a unique consumption level.

[11] In Appendix A, we examine the first best case, and show that our model has dynamic inefficiency. Note further that from (3), (4) and (11), the savings rate is independent of the capital tax rate in the steady state. Thus, the dynamic inefficiency cannot be internalized by capital taxation in our model. Hence, when we discuss the effciency of equilibrium tax rate, the efficient allocation is the second-best optimum.

Then, by differentiating W with respect to τ_{it}, we obtain the effects of capital taxation on the welfare as follows.

$$\frac{1}{\overline{L}_{t-1}R^{t-1}}\frac{\partial W}{\partial \tau_{it}} = \beta(1+n)MW_{iyt} + \beta MW_{iot} - \sum_{i=1}^{M}\left(\Gamma_{i1t} + \Gamma_{i2t} - \Gamma_{i3t}\right), \tag{13}$$

where Γ_{ilt} $(l = 1, 2, 3)$ is defined as

$$\Gamma_{i1t} \equiv \frac{\alpha\left[\beta + \beta(1+n)\right]}{\gamma\left(r_t + \tau_{it}\right)}\frac{\partial r_t}{\partial \tau_{it}},$$

$$\Gamma_{i2t} \equiv \frac{\beta(1+n)(1+\beta)(1-\gamma)}{\gamma(r_t + \tau_{it})}\frac{\partial r_t}{\partial \tau_{it}},$$

$$\Gamma_{i3t} \equiv \frac{\beta}{1+r_t}\frac{\partial r_t}{\partial \tau_{it}}.$$

The first and second terms of (13) represent the effect of an increase in τ_{it} on the young agent's welfare and that on the old agent's welfare in country i at time t. The third term is the effect of an increase in τ_{it} on the social welfare via changes in the world rate of return on savings, which includes the effects on the foreign (country j $(j \neq i)$) agents' welfare as well as those on the domestic (country i) agents' welfare. We discuss the interpretation of Γ_{i1t}, Γ_{i2t}, and Γ_{i3t} in the next subsection.

3.1 Possible externalities

In equilibrium, the government sets its capital tax rate so that $MW_{iy_t} = 0$ when the population is increasing and $MW_{iot} = 0$ when the population is decreasing. However, the optimal tax rate needs to satisfy $\partial W/\partial \tau_{it} = 0$ where $\partial W/\partial \tau_{it}$ is given by (13). Hence, when the population is increasing, the government ignores the effects represented by the two terms, βMW_{iot} and $-\sum_{i=1}^{M}\left(\Gamma_{i1t} + \Gamma_{i2t} - \Gamma_{i3t}\right)$. When the population is decreasing, it ignores the effects represented by the two terms, $\beta(1+n)MW_{iyt}$ and $-\sum_{i=1}^{M}\left(\Gamma_{i1t} + \Gamma_{i2t} - \Gamma_{i3t}\right)$. Such ignorance causes externalities in our framework.

In maximizing the social welfare, one needs to consider the effects of capital taxation, τ_{it}, on the global price, r_t. However, in the equilibrium, the government treats r_t as given because there exist a large mass, M, of countries in the world. Such a difference causes distortions.[12] More specifically, capital flight caused by capital taxation in a country

[12] Even if a government does not regard r as given, it considers the effects of τ on its own country's welfare. In the optimal, one needs to consider the effects of τ, through changes in r, on the other countries' welfare and policies as well. Hence, even in this case, we observe a distortion caused by capital taxation.

enlarges the amount of capital in other countries and this capital flight induces three types of externalities, which we call as the *Fiscal externality*, the *Wage externality*, and the *Interest-rate externality.* The Fiscal externality is a positive externality that the capital flight increases the tax bases in other countries, which is standard in tax competition models. In (13), it is represented by Γ_{i1t}. The Wage externality is a positive externality that the capital flight raises the wages of young individuals in other countries, which is represented by Γ_{i2t} in (13). The Interest-rate externality is a negative externality that the capital flight implies a lower capital demand, which results in a lower rate of return on savings and decreases the income level of old individuals. It is represented by Γ_{i3t} in (13).

This is not the end of the story. Our framework yields additional externalities because the equilibrium policies are chosen though political process (majority voting). A government chooses its current tax rate to maximize the utility level of majority and disregards the minority's utility. This implies that the government in an economy with a growing population disregards the effects of capital taxation on old individuals utility, i.e., MW_{iot} in (13). Put differently, when the young individuals represent the majority, the government disregards the effect on the benefits of public good provision to old individuals. We call it as the *Public-good externality.* On the other hand, when the old individuals represent the majority, the government takes no thought of the following two effects of capital taxation on young individuals' utility: One is that it disregards the negative effect on young individuals through changes in the wage rate, which is represented by the second term of MW_{iyt} in (13), which we call as the *Gerontocracy externality.* The other is the *Public-good externality, i.e.,* it ignores the effect on the benefits of public good provision to young individuals, which is represented by the first term of MW_{iyt} in (13).

Summarizing above arguments, our framework has five externalities associated to capital taxation. The Fiscal and Wage externalities are positive, whereas the Interest-rate externality is negative. The sign of the Pubic-good externality is ambiguous because it is not clear whether an increase in the capital tax rate might increases the tax revenues. However, the sum of the Public-good and Fiscal externalities always becomes positive and is increasing in α.[13] Finally, the Gerontocracy externality works as a negative externality in an economy with a decreasing population, whereas it does not exists in the economy

[13] In an economy with a growing population, the sum of Public-good and Fiscal externalities is $R(1+n)MW_{iot} + \Gamma_{i1t} = (\alpha/\tau_{it})\{1+\beta\tau_{it}/[\gamma(r_t+\tau_{it})]\} > 0$. In an economy with a decreasing population, it is $MW_{iot} + \Gamma_{i1t} = \alpha\beta/[\gamma(r_t+\tau_{it})] > 0$. Therefore, it is positive in both economies.

with a growing population. Hence, the overall negative external effects are stronger in an economy with a decreasing population than in an economy with a growing population.

3.2 Preliminary analysis: closed economy

Before proceeding to the analysis of capital tax competition, we present the political economy outcomes in the absence of capital mobility. For this purpose, temporarily suppose that capital is immobile among countries. Then, all variables become local variables and the government takes taxation effects on the rate of return on savings in country i, r_{it}, into consideration in addition to other local variables. Moreover, because $r_{it}+\tau_{it}$ is determined by past variables, the government now considers that any increases in τ_{it} are exactly offset by decreases in r_{it} ($\partial r_{it}/\partial\tau_{it} = -1$).

In an economy with a growing population, U_{it} given by (6) is an increasing function of τ_{it} (i.e., $MW_{iyt} \equiv \partial U_{it}/\partial\tau_{it} = \alpha/\tau_{it} > 0$) under consideration of $\partial r_{it}/\partial\tau_{it} = -1$. Therefore, the government sets its capital tax rate as high as possible (as long as $r_{it} \geq 0$), which, combined with (11), results in[14]

$$\tau_{iy}^{im} = \frac{(1+n)(1+\beta)(1-\gamma)}{\beta\gamma}. \tag{14}$$

The superscript im represents the case of capital immobility.

In an economy with a decreasing population, the government maximizes u_{iot} with respect to τ_{it}, and the equilibrium tax rate satisfies $MW_{iot} \equiv \partial u_{iot}/\partial\tau_{it} = \alpha/\tau_{it} - 1/(1+r_{it}) = 0$ and is given by

$$\tau_{io}^{im} = \alpha(1+r_{io}^{im}). \tag{15}$$

Comparing MW_{iyt} with MW_{iot}, we know that the government in an economy with a decreasing population sets a lower tax rate than in an economy with a growing population, if capital is immobile between countries. Because changes in rate of return on savings absorb the capital tax effects ($\partial r_{it}/\partial\tau_{it} = -1$), capital taxation does not affect capital per capita, k_{it}, and hence, the wage rate, w_{it}. The government considers such a relationship, implying that a government in an economy with a growing population cares only about the level of public good provision in maximizing U_{it} whereas a government in an economy with a decreasing population considers decreases in returns from savings when maximizing u_{iot} as well. Thus, a government in an economy with a decreasing population is more tentative

[14] We obtain (11) from (5) by setting $M = 1$.

in taxing capital than a government in an economy with a growing population if capital is immobile.[15]

With tax rates in hand, we can examine the efficiency properties of these tax rates by looking at the effects of an increase in τ on the social welfare (12). Such effects can be derived by differentiating (12) with respect to τ_{it}, and evaluating it at τ_{iy}^{im} or at τ_{io}^{im}. The social welfare function (12) in the closed economy is given by

$$W = u_{io0}L_{o0} + \sum_{t=0}^{\infty} \beta^t U_{it} \overline{L}_t.$$

By using $\partial r_{it}/\partial \tau_{it} = -1$, we can write its derivative as

$$\begin{aligned} \frac{1}{\overline{L}_{t-1}\beta^{t-1}} \frac{\partial W}{\partial \tau_{it}} &= \beta(1+n)MW_{iyt} + \beta MW_{iot} \\ &= \frac{\alpha\beta(1+n)}{\tau_{it}} + \frac{\alpha\beta}{\tau_{it}} - \frac{\beta}{1+r_{it}}. \end{aligned} \tag{16}$$

Evaluating (16) at $\tau_{it} = \tau_{iy}^{im}$ yields

$$\mathsf{sgn}\left[\left.\frac{\partial W}{\partial \tau_{it}}\right|_{\tau=\tau_{iy}^{im}}\right] = \mathsf{sgn}\left[\frac{\alpha\beta\gamma(2+n)}{(1+\beta)(1-\gamma)(1+n)} - 1\right].$$

Because τ_{it} cannot be higher than τ_{iy}^{im} because of the non-negative constraint of r_{it}, we know that in an economy with a growing population, the equilibrium tax rate is (second-best) optimal if the preference for public good consumption is sufficiently large (i.e., $\alpha \geq (1+\beta)(1-\gamma)(1+n)/\left[\beta\gamma(2+n)\right]$), and inefficiently high otherwise (i.e., $\alpha < (1+\beta)(1-\gamma)(1+n)/\left[\beta\gamma(2+n)\right]$). Similarly, evaluating (16) at $\tau_{it} = \tau_{io}^{im}$ yields

$$\mathsf{sgn}\left[\left.\frac{\partial W}{\partial \tau_{it}}\right|_{\tau=\tau_{o}^{im}}\right] = \mathsf{sgn}\left[\frac{1+n}{1+r_{io}^{im}}\right] > 0.$$

Hence, we observe an inefficiently low capital tax rate in an economy with a decreasing population.

[15] Mateos-Planas (2010) analyzed the effects of demographics on the mix of tax rates on households' labor and capital income by using a median voter model, and showed that when the decisive voter changes from old individuals to young individuals, the capital tax rate increases. He confirmed the quantitative relevance of this result by calibrating his model to United States data. Our result on capital tax rate in the capital immobile case is consistent with this. However, he showed that when the proportion of old individuals decreases while keeping the decisive voter type unaltered, the capital tax decreases, which is not consistent with our result wherein a higher n and hence, a lower proportion of old individuals implies a higher capital tax rate. Such a departure would come from the fact we endogenize governments' expenditure and ignore labor income tax whereas Mateos-Planas (2010) fixed governments' expenditure and introduced labor income tax.

Proposition 1 *Suppose immobility of capital between countries. Then, the capital tax rate is optimal or inefficiently high in an economy with a growing population whereas it is inefficiently low in an economy with a decreasing population.*

In the absence of capital mobility between countries, there exists no externality induced by tax competition and therefore only the externalities induced by the political process are relevant.

When the population is decreasing, we observe the Public-good externality, which is represented by $\alpha\beta(1+n)/\tau_{it}$ of the right hand side of (16) and is always positive in the closed economy.[16] As a result, the capital tax rate becomes inefficiently low.

In a similar vein, when the population is growing, we observe the positive Public-good externality, which is represented by $\alpha\beta/\tau_{it}$ of the right hand side of (16). In addition, we observe the negative Interest-rate externality, which is represented by $-\beta/(1+r_t)$ of the right hand side of (16). The Public-good externality is stronger than the Interest-rate externality when the preference for public good consumption, α, is sufficiently large and weaker when α sufficiently is small. However, when α is sufficiently large, individuals require the capital tax rate to be the maximum possible rate, implying that the resulting equilibrium tax rate becomes identical to the optimal one. When α is sufficiently small, the Interest-rate externality overwhelms the Public-good externality, yielding an inefficiently high capital tax rate in equilibrium.

Note here that the interpretations of the Interest-rate externality are different between the open and closed economies. In the open economy, because each country is sufficiently small, each government regards the rate of return on savings as constant. This misunderstanding of government is the source of the Interest-rate externality. Hence, the Interest-rate externality is related to tax competition in the open economy. In contrast, in the closed economy, a government considers the effect of capital taxation on the rate of return on savings. Still, when the population is growing, it ignores the effect of capital taxation on the capital income of old individuals, which induces the Interest-rate externality. Hence, the Interest-rate externality is related to the political process in the closed economy.[17]

[16] Because capital taxation does not affect the wage rate in the closed economy, there exists no Gerontocracy externality.

[17] If each government considers the effect of capital taxation on the rate of return from savings in an open economy, the interpretations of the Interest-rate externality become the same across the two economies.

3.3 Economy with a growing population

Now we return to the baseline model wherein capital is mobile between countries, and move to the efficiency analysis of capital tax competition in political economy. Start from an economy with a growing population. From (7) and (11), we obtain the equilibrium capital tax rate as follows:

$$\tau_y^* = \frac{\alpha(1+n)(1+\beta)(1-\gamma)}{\beta\left[\alpha+(1+\beta)(1-\gamma)\right]}. \tag{17}$$

The rate of return on savings becomes

$$r_y^* = \frac{(1+n)(1+\beta)(1-\gamma)^2(1+\alpha+\beta)}{\beta\gamma\left[\alpha+(1+\beta)(1-\gamma)\right]}. \tag{18}$$

Similarly to the previous section, we examine the equilibrium efficiency properties by looking at the effects of an increase in τ_{it}. By evaluating (13) at $\tau_{it} = \tau_y^*$ (see also (11)), we know that

$$\frac{1}{\overline{L}_{t-1}\beta^{t-1}}\frac{\partial W}{\partial \tau_{it}}\bigg|_{\tau_{it}=\tau_y^*} = \frac{\alpha}{\tau_y^*(1+r_y^*)}\frac{\alpha(n+2)\Omega-(\beta+1)(1-\gamma)\Phi}{\gamma\left[\alpha+(1+\beta)(1-\gamma)\right]}, \tag{19}$$

where Φ and Ω are defined as

$$\Phi \equiv \gamma\left\{n\left[\beta(n+2)+n+4\right]+3\right\}-(\beta+1)(n+1)(n+2),$$
$$\Omega \equiv \beta\left[1-(1-\gamma)\gamma\right]+(1-\gamma)^2+n(\beta+1)(1-\gamma)^2>0.$$

Hence, we can see that

$$\mathsf{sgn}\left[\frac{\partial W}{\partial \tau_t}\bigg|_{\tau=\tau_y^*}\right] = \mathsf{sgn}\left[\alpha(n+2)\Omega-(\beta+1)(1-\gamma)\Phi\right].$$

Therefore, (19) is positive if and only if

$$\alpha > \widetilde{\alpha},$$

where $\widetilde{\alpha}$ is defined as

$$\widetilde{\alpha} \equiv \frac{(\beta+1)(1-\gamma)\Phi}{(n+2)\Omega}.$$

Proposition 2 *Capital tax competition in an economy with a growing population results in an inefficiently low (resp. high) capital tax rate if and only if the preference for public good consumption, α, is larger than $\widetilde{\alpha}$ (resp. smaller than $\widetilde{\alpha}$).*

In a closed economy with a growing population, the capital tax rate is never inefficiently low. In an open economy with a growing population, we have the positive Fiscal and Wage externalities, and the negative Interest-rate externality. Although the sign of the Public-good externality is ambiguous, the sum of the Fiscal and Public-good externalities is always positive and is increasing in α. Hence, if α is sufficiently large to satisfy $\alpha > \widetilde{\alpha}$, then the overall externality becomes positive, which results in an inefficiently low capital tax rate, i.e., capital tax competition in political economy results in the *race to the bottom.* In contrast, when the opposite holds true ($\alpha < \widetilde{\alpha}$), the negative Interest-rate externality dominates positive externalities, resulting in an inefficiently high capital tax rate, i.e., we observe the *race to the top.*

3.4 Economy with a decreasing population

Next, consider an economy with a decreasing population. From (9) and (11), we obtain the equilibrium capital tax rate and rate of return on savings as follows:[18]

$$\begin{aligned} \tau_o^* &= \frac{(1+n)(1+\beta)(1-\gamma)}{\beta}, \\ r_o^* &= \frac{(1+n)(1+\beta)(1-\gamma)^2}{\beta\gamma}. \end{aligned} \tag{20}$$

To examine the efficiency of equilibrium, we evaluate (13) at $\tau_{it} = \tau_o^*$ (see also (11)), resulting in

$$\frac{1}{\overline{L}_{t-1}\beta^{t-1}} \frac{\partial W}{\partial \tau_{it}}\bigg|_{\tau_{it}=\tau_o^*} = \frac{1}{\tau_y^*(1+r_y^*)} \frac{[\alpha(n+2)\Omega - (\beta+1)(1-\gamma)\gamma(n+1)]}{\gamma}.$$

This is positive if and only if

$$\alpha > \widehat{\alpha},$$

where $\widehat{\alpha}$ is defined as

$$\widehat{\alpha} \equiv \frac{(\beta+1)(1-\gamma)\gamma(n+1)}{(n+2)\Omega}. \tag{21}$$

Proposition 3 *Capital tax competition in an economy with a decreasing population results in an inefficiently low (resp. high) capital tax rate if and only if the preference for public good consumption, α, is larger than $\widehat{\alpha}$ (resp. smaller than $\widehat{\alpha}$).*

[18] Note that from the assumption of positive population size ($n > -1$), τ_o^* and r_o^* are positive.

In a closed economy with a decreasing population, the race to the bottom always emerges. However, when capital becomes mobile, we have the negative Gerontocracy and Interest-rate externalities, yielding the possibility of the race to the top. When α is sufficiently large to satisfy $\alpha > \widehat{\alpha}$, the sum of Fiscal and Public good externalities becomes large, and we observe the race to the bottom. When α is sufficiently small, the Gerontocracy and Interest-rate externalities become prominent, resulting in the race to the top.

3.5 Possibility of the race to the top

As we saw, both of the race to the bottom and the race to the top are possible regardless of the population growth rate when capital is mobile. It would then be interesting to ask which economy is more likely to face the race to the top. From (21), we can show that the race to the top never emerge in an economy with a growing population (i.e., $\widetilde{\alpha} < 0$ holds true for $n \geq 0$) when $2\beta - 3\gamma + 2 > 0$ and $3\gamma^2 - \gamma - 1 < 0$ hold true.[19] However, we can readily know that $\widehat{\alpha}$ is always positive because of the assumption that $n > -1$, implying that there always exists a possibility of the race to the top in an economy with a decreasing population.

Proposition 4 *When $2\beta - 3\gamma + 2 > 0$ and $3\gamma^2 - \gamma - 1 < 0$, tax competition always results in the race to the bottom in an economy with a growing population (i.e., $\widetilde{\alpha} < 0$), and it might result in the race to the top in an economy with a decreasing population (i.e., $\widehat{\alpha} > 0$).*

In this sense, we can say that the race to the top is more likely to emerge when a population is decreasing than when it is increasing. As shown in Section 2.4.2, when capital is mobile, the government's incentive to tax capital is stronger in an economy with a decreasing population than in an economy with a growing population because the government in an economy with a decreasing population ignores the effect of the increase in tax rate on the current wage rate of young generation. This generates a higher possibility of the race to the top in an economy with a decreasing population than in an economy with a growing population.

In fact, if we follow Romer (2011) and Acemoglu (2009) to set the labor share, γ to 2/3, we obtain $2\beta - 3\gamma + 2 > 0$ and $3\gamma^2 - \gamma - 1 < 0$. Moreover, Karabarbounis

[19] Proof is given in Appendix.

and Neiman (2014) estimated the global labor share and showed that γ has exhibited a relatively steady downward trend from 0.64 to 0.59 during the past several decades. If $0.59 \leq \gamma \leq 0.64$, $3\gamma^2 - \gamma - 1 < 0$ holds true.[20] In this case, $2\beta - 3\gamma + 2 > 0$ requires that the time discount rate, β, is larger than 0.115, which is likely to be satisfied in the real world. Because most countries have experienced population growth during the past several decades, we can think that the case of $\widetilde{\alpha} < 0$ is relevant to the real world. However, even when $2\beta - 3\gamma + 2 > 0$ and $3\gamma^2 - \gamma - 1 < 0$ hold true, $\widehat{\alpha} > 0$ and there always exists a possibility of the race to the top if a population becomes decreasing.

Figure 2 illustrates the above arguments.

[Figure 2 around here]

In Figure 2, we set $n = 0.1$ for an economy with a growing population and $n = -0.1$ for an economy with a decreasing population. As to γ, we try two alternative values, 0.75 and 0.6. The former is higher than the values observed in Karabarbounis and Neiman (2014) and the latter is close to the observed values. We take β in the horizontal axis and α in the vertical axis. The shaded areas represent combinations of α and β that result in the race to the top. Figures 2-(a-1) and 2-(a-2) describe an economy with a growing population and that with a decreasing population when $\gamma = 0.75$, respectively. In both of these figures, we observe the possibility of the race to the top for lower values of α although the shaded area is larger in an economy with a decreasing population than in an economy with a growing population. When $\gamma = 0.6$, which is represented by Figures 2-(b-1) and 2-(b-2), we observe no possibility of the race to the top in an economy with a growing population (Figure 2-(b-1)) whereas we do observe it in an economy with a decreasing population (Figure 2-(b-2)).

When $2\beta - 3\gamma + 2 > 0$ and $3\gamma^2 - \gamma - 1 < 0$, and hence, $\widetilde{\alpha} < 0$ hold true, declines in the population growth rate can shift the economy from the race to the bottom to the race to the top. Because we know that $\widehat{\alpha} > 0 > \widetilde{\alpha}$ in this case, an economy with $\alpha \in (0, \widehat{\alpha}|_{n=0})$ experiences the shift from the race to the bottom to the race to the top as the population growth rate, n, changes from positive to negative. This yields a strong policy implication given the downward trends in the population growth rate as observed in many developed countries. Such trends imply that many countries, which have experienced population

[20] When $0 < \gamma < \left(1 + \sqrt{13}\right)/6 \fallingdotseq 0.77$, $3\gamma^2 - \gamma - 1 < 0$.

growth in the past, will experience population declines as Japan has already come to face.[21] Then, we might observe the race to the top, under which coordinated decreases in the capital tax rate among countries can improve the welfare. This is in contrast to the standard result in the tax competition literature wherein we observe the race to the bottom and coordinated increases in the capital tax rate can improve the welfare.

4 Asymmetric countries

In this section, we extend the baseline framework by considering asymmetric countries. Consider two groups of countries (groups h and l), where group h has M_h countries and group l has M_l countries. We assume that countries in each group have the same population growth rate. Let n_k denote the population growth rate of group k countries ($k = h, l$). Without loss of generality, we assume that $n_h > n_l$. Noticing that $r_{kt} + \tau_{kt} = r_{kt-1} + \tau_{kt-1}$ holds true in the steady state and that the assumption of global capital market yields $r_{ht} = r_{lt} = r_t$, we obtain the capital market clearing condition as

$$\begin{aligned}&\left(\frac{1-\gamma}{r_t+\tau_{ht}}\right)^{1/\gamma}\overline{L}_{ht} + \left(\frac{1-\gamma}{r_t+\tau_{lt}}\right)^{1/\gamma}\overline{L}_{lt} \\ &= \frac{\beta\gamma}{1+\beta}\left[\left(\frac{1-\gamma}{r_t+\tau_{ht}}\right)^{(1-\gamma)/\gamma}\frac{\overline{L}_{ht}}{1+n_h} + \left(\frac{1-\gamma}{r_t+\tau_{lt}}\right)^{(1-\gamma)/\gamma}\frac{\overline{L}_{lt}}{1+n_l}\right],\end{aligned} \tag{22}$$

where $\overline{L}_{ht}$ and $\overline{L}_{lt}$ are total population sizes in each group of countries and defined as $\overline{L}_{ht} \equiv \sum_{i=1}^{M_h} \overline{L}_{it}$ and $\overline{L}_{lt} \equiv \sum_{j=1}^{M_l} \overline{L}_{jt}$.

In this extended setting, we have the following three cases: (i) all countries have a growing population (i.e., $n_h > n_l > 0$), (ii) all countries experience population decreases (i.e., $0 > n_h > n_l$), and (iii) group h countries have a growing population whereas group l countries have a decreasing population (i.e., $n_h > 0 > n_l$). In case (i) (resp. case (ii)), young (resp. old) individuals represent the majority in both groups of countries. In case (iii), young individuals represents the majority in group h countries whereas old individuals do so in group l countries.

[21] The total population in Japan started to decrease in 2016, and will contiously decrease with time.

4.1 Equilibrium and its welfare properties

Case (i): economy with a growing population

In this case, each government maximizes the young individual's utility, U_{it}, and the first-order condition yields (7) for all countries, implying that $\tau_{ht} = \tau_{lt} = \tau_t$. We substitute this into (22) to obtain

$$r_t + \tau_t = \frac{(1+n_w)(1+\beta)(1-\gamma)}{\beta\gamma}, \tag{23}$$

where n_w describes the world population growth rate and is defined as

$$n_w \equiv \frac{\overline{L}_{ht} + \overline{L}_{lt}}{\overline{L}_{ht}/(1+n_h) + \overline{L}_{lt}/(1+n_l)} - 1.$$

Note that we obtain (23) if we replace n with n_w in (11). Therefore, we obtain the equilibrium tax rate and rate of return on savings very similar to those shown in the baseline framework. More specifically, (7) and (23) yield

$$\begin{aligned} \tau^{(i)*} &= \frac{\alpha(1+n_w)(1+\beta)(1-\gamma)}{\beta\left[\alpha + (1+\beta)(1-\gamma)\right]}, \\ r^{(i)*} &= \frac{(1+n_w)(1+\beta)(1-\gamma)^2(1+\alpha+\beta)}{\beta\gamma\left[\alpha + (1+\beta)(1-\gamma)\right]}. \end{aligned} \tag{24}$$

The superscript (i) represents case (i). We can obtain (24) by replacing n with n_w in τ_y^* and r_y^* (given by (17) and (18)) in the baseline framework. Hence, welfare properties in this case are similar to those in the symmetric case. In fact, replace n in $\widetilde{\alpha}$ with n_w and denote it by $\widetilde{\alpha}^{(i)}$. Then, we can see that (13) evaluated at $\tau = \tau^{(i)*}$ is positive if and only if $\alpha > \widetilde{\alpha}^{(i)}$.

Proposition 5 *Suppose all countries have a growing population but with different rates. Then, capital tax competition results in an inefficiently low (resp. high) capital tax rate if and only if the preference for public good consumption, α, is larger than $\widetilde{\alpha}^{(i)}$ (resp. smaller than $\widetilde{\alpha}^{(i)}$).*

Case (ii): economy with a decreasing population

When all countries have a decreasing population, each government maximizes the old individual's utility, u_{iot}, and the first-order condition yields (9), implying that $\tau_{ht} = \tau_{lt} =$

τ_t. From (9) and (22), we obtain the equilibrium tax rate and rate of return on savings as

$$\tau^{(ii)*} = \frac{(1+n_w)(1+\beta)(1-\gamma)}{\beta}, \qquad (25)$$
$$r^{(ii)*} = \frac{(1+n_w)(1+\beta)(1-\gamma)^2}{\beta\gamma}.$$

The superscript (ii) represents case (ii). Again, we can obtain (25) by replacing n with n_w in τ_o^* and r_o^* (given by (20)) in the baseline framework. Hence, welfare properties again become similar to those obtained in the baseline framework: (13) evaluated at $\tau = \tau^{(ii)*}$ is positive if and only if $\alpha > \widehat{\alpha}^{(ii)}$, where we replace n with n_w in $\widehat{\alpha}$ and denote it by $\widehat{\alpha}^{(ii)}$.

Proposition 6 *Suppose all countries have a decreasing populations but with different rates. Then, capital tax competition results in an inefficiently low (resp. high) capital tax rate if and only if the preference for public good consumption, α, is larger than $\widehat{\alpha}^{(ii)}$ (resp. smaller than $\widehat{\alpha}^{(ii)}$).*

Case (iii): countries with a growing population v.s. countries with a decreasing population

In this case, young individuals represent the majority in countries with a growing population (group h countries) whereas old individuals do so in countries with a decreasing population (group l countries). Then, the equilibrium tax rates become

$$\tau_{ht} = \frac{\alpha\gamma}{(1+\alpha+\beta)(1-\gamma)} r_t, \qquad (26)$$
$$\tau_{lt} = \frac{\gamma}{1-\gamma} r_t.$$

Comparing the two tax rates, we know that the equilibrium tax rate is lower in group h countries than in group l countries. From (26), we can derive

$$r_t + \tau_{ht} = \frac{\alpha + (1+\beta)(1-\gamma)}{(1+\alpha+\beta)(1-\gamma)} r_t, \qquad (27)$$
$$r_t + \tau_{lt} = \frac{1}{1-\gamma} r_t,$$

implying that

$$r_t + \tau_{ht} = \frac{\alpha + (1+\beta)(1-\gamma)}{1+\alpha+\beta} (r_t + \tau_{lt}).$$

We substitute this into (22) to rewrite the capital market equilibrium condition as

$$\left[\frac{(1-\gamma)\,\Lambda}{r_t+\tau_{lt}}\right]^{1/\gamma}\overline{L}_{ht}+\left(\frac{1-\gamma}{r_t+\tau_{lt}}\right)^{1/\gamma}\overline{L}_{lt}$$
$$=\frac{\beta\gamma}{1+\beta}\left\{\left[\frac{(1-\gamma)\,\Lambda}{r_{t-1}+\tau_{lt-1}}\right]^{(1-\gamma)/\gamma}\frac{\overline{L}_{ht}}{1+n_h}+\left(\frac{1-\gamma}{r_{t-1}+\tau_{lt-1}}\right)^{(1-\gamma)/\gamma}\frac{\overline{L}_{lt}}{1+n_l}\right\},$$

where Λ is defined as $\Lambda\equiv(1+\alpha+\beta)\,/\,[\alpha+(1+\beta)(1-\gamma)]$. Furthermore, if we define z_t and H_t as

$$z_t\equiv r_t+\tau_{lt},$$
$$H_t\equiv\frac{\beta\gamma}{(1+\beta)\,(1-\gamma)}\frac{\Lambda^{(1-\gamma)/\gamma}+\overline{L}_{lt}/\overline{L}_{ht}}{\Lambda^{1/\gamma}(1+n_{h)}+(1+n_l)\overline{L}_{lt}/\overline{L}_{ht}},$$

this becomes

$$z_{t+1}=H_t^{1-\gamma}z_t^{1-\gamma}. \tag{28}$$

In the long run, the total population size of group h countries expands whereas that of group l countries shrinks, implying that $\lim_{t\to\infty}\overline{L}_{lt}/\overline{L}_{ht}=0$. Hence, the steady state value of H_t becomes

$$H^*=\lim_{t\to\infty}H_t=\frac{\beta\gamma}{\Lambda\,(1+\beta)\,(1-\gamma)\,(1+n_h)}.$$

Combined with (28), this yields the steady state value of z^* as

$$z^*=H^{*(1-\gamma)/\gamma}.$$

Plugging this into (27), we obtain the steady state values of the rate of return on savings and tax rates as follows:

$$r_t^*=(1-\gamma)H^{*(1-\gamma)/\gamma}, \tag{29}$$
$$\tau_h^{(iii)*}=\frac{\alpha\gamma}{1+\alpha+\beta}H^{*(1-\gamma)/\gamma},$$
$$\tau_l^{(iii)*}=\gamma H^{*(1-\gamma)/\gamma}.$$

We substitute (29) into (13) to obtain

$$\left.\frac{\partial W}{\partial\tau_{ht}}\right|_{\tau_{ht}=\tau_h^{(iii)*}}=\beta^t(1+n_h)^{t-1}\left[\frac{1}{1+\gamma H^{*(1-\gamma)/\gamma}}+\frac{(2+n)(1+\alpha+\beta)}{\gamma H^{*\frac{1-\gamma}{\gamma}}}\right],$$
$$\left.\frac{\partial W}{\partial\tau_{lt}}\right|_{\tau_{lt}=\tau_l^{(iii)*}}=\beta^t(1+n_l)^{t-1}\left[\frac{1}{1+\gamma H^{*(1-\gamma)/\gamma}}+\frac{(2+n)\alpha}{\gamma H^{*\frac{1-\gamma}{\gamma}}}\right].$$

From this, we know that $\partial W/\partial \tau_{ht}|_{\tau_{ht}=\tau_h^{(iii)*}} > \partial W/\partial \tau_{lt}|_{\tau_{lt}=\tau_l^{(iii)*}}$, resulting in the following proposition.[22]

Proposition 7 *If group l countries set an inefficiently low capital tax rate, then so do group h countries ($\partial W/\partial \tau_{ht}|_{\tau_{ht}=\tau_h^{(iii)*}} > \partial W/\partial \tau_{lt}|_{\tau_{lt}=\tau_l^{(iii)*}} > 0$). If group h countries set an inefficiently high capital tax rate, then so do group l countries ($0 > \partial W/\partial \tau_{ht}|_{\tau_{ht}=\tau_h^{(iii)*}} > \partial W/\partial \tau_{lt}|_{\tau_{lt}=\tau_l^{(iii)*}}$). Additionally, there is a possibility that group h countries set an inefficiently low capital tax rate whereas group l countries set an inefficiently high capital tax rate ($\partial W/\partial \tau_{ht}|_{\tau_{ht}=\tau_h^{(iii)*}} > 0 > \partial W/\partial \tau_{lt}|_{\tau_{lt}=\tau_l^{(iii)*}}$).*

This result is consistent with that shown in Proposition 4. Put differently, we again find that countries with a decreasing population are more likely to exhibit the race to the top.

Equation (29) shows that if countries with a decreasing population compete for capital with countries with a growing population, the former countries set higher capital tax rates than the latter countries. This prediction is consistent with differences in the corporate income taxes across countries. In fact, if we compare the five oldest OECD countries with the five youngest OECD countries in terms of the median age, the average corporate income tax in 2010 is around 30 percent for the former and 26 percent for the latter.[23][24] Such a difference is robust even if we take the oldest and youngest ten countries. Of course, differences in capital tax rates can arise from other sources. The most prominent example would be the country size differences, and many papers have shown that large countries

[22] In fact, we have the second case when γ is large. When γ is sufficiently small, we have the first and the last cases. In fact, if we set $\gamma = 1/2$, the first case holds true when α is sufficiently large and the last case holds true when α is sufficiently small.

[23] Data sources are as follows:

median age: OECD, http://www.oecd-ilibrary.org/education/trends-shaping-education-2013/median-age-going-up-into-the-next-century_trends_edu-2013-graph43-en,

population: OECD, http://www.oecd-ilibrary.org/economics/national-accounts-at-a-glance-2014/population_na_glance-2014-table105-en,

corporate income tax: OECD, http://www.oecd-ilibrary.org/taxation/data/corporate-income-tax/corporate-income-tax-rates_7cde787f-en,

GDP: OECD, https://data.oecd.org/gdp/gross-domestic-product-gdp.htm.

[24] Here, we use the combined corporate income tax, which is the sum of the central government's corporate income tax and the sub-central government's corporate income tax. In 2010, we have 34 OECD countries, and Japan, Germany, Italy, and Finland are the five oldest countries in terms of the median age whereas Mexico, Turkey, Israel, Chile, and Ireland are the five youngest countries.

set higher capital tax rates than smaller countries.[25] If we control for size differences in terms of population or GDP, we can still find higher tax rates for older countries than for younger countries.

4.2 Utility difference between asymmetric countries

We finally discuss which group of countries gain from tax competition in political economy. In so doing, we assume that countries in each group have the same population size (i.e., $\overline{L}_{it} = \widehat{L}_{kt} \equiv \overline{L}_{kt}/M_k$). The indirect utility of an individual in a group k country is written as

$$\begin{aligned} U_{kt} = \alpha(1+\beta)\ln\tau_{kt} - \frac{(1+\beta)(1+\alpha-\gamma)}{\gamma}\ln(r_t+\tau_{kt}) + \beta\ln(1+r_t) \\ + \alpha(1+\beta)\ln\widehat{L}_{kt} + \alpha\beta\ln(1+n_k) + \alpha(1+\beta)\ln(1-\gamma)^{1/\gamma} \\ + (1+\beta)\ln\gamma(1-\gamma)^{(1-\gamma)/\gamma} - \ln(1+\beta) + \beta\ln\left(\frac{\beta}{1+\beta}\right). \end{aligned}$$

In cases (i) and (ii), we know that $\tau_{ht} = \tau_{lt}$, implying that

$$U_{ht} - U_{lt} = \alpha(1+\beta)\left(\ln\widehat{L}_{ht} - \ln\widehat{L}_{lt}\right) + \alpha\beta\left(\ln(1+n_h) - \ln(1+n_l)\right). \qquad (30)$$

Thus, in these cases, we can decompose the welfare difference into two terms: the first term represents the population size effect and the second term represents the population growth effect. The larger the population size, the more the country attracts capital because of the complementarity between capital and labor in production, resulting in a larger tax base. Hence, individuals in a larger country can consume a larger amount of public goods than those in a smaller country, which makes the welfare of the larger country higher than that of the smaller country. Moreover, in a similar vein, a higher population growth rate increases the individual's public good consumption at the old period because it implies a larger population size of the next generation. This makes the welfare of group h countries higher than that of group l countries.

In case (iii), the welfare difference between countries becomes

$$\begin{aligned} U_{ht} - U_{lt} = \alpha(1+\beta)\left(\ln\tau_{ht} - \ln\tau_{lt}\right) + \frac{(1+\beta)(1+\alpha-\gamma)}{\gamma}\left[\ln(r_t+\tau_{lt}) - \ln(r_t+\tau_{ht})\right] \\ + \alpha(1+\beta)\left(\ln\widehat{L}_{ht} - \ln\widehat{L}_{lt}\right) + \alpha\beta\left(\ln(1+n_h) - \ln(1+n_l)\right). \end{aligned}$$

[25] See e.g., Baldwin and Krugman (2004), Borck and Pflüger (2006), Bucovetsky (1991), Haufler and Wooton (1999), and Ottaviano and van Ypersele (2005).

Thus, in addition to the population size and growth effects, we have two other effects. The first-term of the right-hand side of the above equation represents the tax effect. A higher tax rate increases the tax revenues and individuals' public good consumption, resulting in a higher welfare. As we know from (26) that $\tau_{ht} < \tau_{lt}$, the first term is negative. The second term represents the capital cost effect. A higher capital tax rate implies a higher capital cost, reducing capital input per capita. This results in a lower wage rate and lower welfare. As we know that $\tau_{ht} < \tau_{lt}$, the second term becomes positive.

5 Summary and discussions

In this paper, we developed an overlapping generations model wherein public good provision financed by capital taxation is determined by majority voting. When population is growing (resp. decreasing), young (resp. old) individuals represent the majority, implying that the government's decision depends on the demographic structure. We showed that young individuals suffer more from capital flight than old individuals, and that the race to the bottom is more likely to emerge when the population is growing than when it is decreasing. It is even possible to observe the race to the bottom when the population is growing whereas the race to the top might emerge when the population is decreasing. Such dependence on the outcomes of capital tax competition on demographics provides us a new viewpoint in policy debates regarding competition for capital and firms. Particularly, because we observe drastic aging in many developed countries, our results indicate an increasing relevance of the race to the top.

We briefly discuss the robustness of our results against two alternative extensions.[26] First, suppose that in addition to capital taxation, the government has another instrument to finance its expenditure. As an example, we consider labor income tax on households. Then, the government in an economy with a growing population would finance its expenditure solely by labor income tax to prevent capital flight, whereas the government in an economy with a decreasing population would impose a positive tax on capital while trying to set income tax as high as possible because it cares only about tax revenues. Moreover, we can show that under reasonable parameter values, the equilibrium capital tax rate is inefficiently low in an economy with a growing population whereas it is so in an economy with a decreasing population if and only if the preference for public good consumption

[26] Formal analyses on these extensions are available in online appendices.

is sufficiently large, implying the possibility of the race to the top. In this sense, the introduction of income tax does not alter our main results qualitatively.

Second, in our framework, governments provide public goods. Alternatively, we can assume that governments provides public inputs that affect productivity of firms. In such a case, the positive externality associated to capital taxation is further strengthened because public inputs and capital are complementary in production. Consequently, we observe the race to the bottom when the public input elasticity of output is sufficiently large. Hence, the introduction of public inputs would make the race to the bottom more likely to emerge as the public inputs can contribute to production.

We here refer to two possible extensions. First, it would be worth endogenizing the demographic structure by introducing decisions on the number of children to have. Governments' behavior would impact, via changes in economic conditions, such decisions. Then, by endogenizing the fertility rate, we might be able to analyze the long-run mutual interdependences between policies and demographics. Second, it would also be significant to consider households' mobility between countries. If we introduce households' mobility, countries need to compete for people as well as capital, which would result in additional policy externality. These are important topics for future research.

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Appendix

Derivation of the possibility of $\widetilde{\alpha} < 0$ when $n \geq 0$

When $n = 0$, $\widetilde{\alpha}$ is

$$\widetilde{\alpha}|_{n=0} = -\frac{(\beta+1)(1-\gamma)(2\beta-3\gamma+2)}{2\left(1-\beta((1-\gamma)\gamma)+(1-\gamma)^2\right)}. \tag{31}$$

From this equation, when $2\beta - 3\gamma + 2 > 0$, $\widetilde{\alpha}|_{n=0} < 0$ holds true. By differentiating $\widetilde{\alpha}$ with respect to n, we obtain

$$\frac{\partial \widetilde{\alpha}}{\partial n} = -\frac{(\beta+1)(\gamma-1)\gamma\Delta}{(n+2)^2\left\{\beta\left[(\gamma-1)\gamma+1\right]+(\gamma-1)^2+(\beta+1)(\gamma-1)^2 n\right\}^2}, \tag{32}$$

where Δ is defined as

$$\Delta \equiv -\gamma\left[\beta-\beta n^2+\beta^2(n+2)^2-2(n+1)^2\right]+(\beta+1)\gamma^2\left[\beta(n+2)^2-(n+1)^2\right]-(\beta+1)(n+1)^2$$

We differentiate Δ with respect to n to obtain

$$\frac{\partial \Delta}{\partial n} = -2(\beta+1)(1-\gamma)(2\beta\gamma+\beta\gamma n+n(1-\gamma)+1-\gamma) < 0.$$

Substituting $n = 0$ into Δ, we can observe

$$\Delta|_{n=0} = -(1-\gamma)^2 - 4\beta^2(1-\gamma)\gamma + \beta\left(3\gamma^2-\gamma-1\right).$$

If $3\gamma^2 - \gamma - 1 < 0$ hold true, we obtain $\Delta|_{n=0} < 0$. In such a case, $\Delta < 0$ holds true for $n > 0$ because $\partial\Delta/\partial n < 0$ and $\Delta|_{n=0} < 0$. $\Delta < 0$, in turn, implies that $\partial\widetilde{\alpha}/\partial n < 0$. When $2\beta - 3\gamma + 2 > 0$, $\widetilde{\alpha}|_{n=0} < 0$ and hence we know that $\widetilde{\alpha} < 0$ for $n > 0$.

Online appendices (not for publication)

Appendix A: First best allocation

In this Appendix, we analyze the first best outcome, in which the social planner can directly choose the amount of young consumption, old consumption, savings, and public goods, and capital. The social welfare function is

$$W = \sum_{i=1}^{M} \left[u_{io0} L_{o0} + \sum_{t=0}^{\infty} \beta^t U_{it} \overline{L}_t \right],$$

where we assume that the social planner has the same discount rate as individuals. Since the planner perfectly internalize international externalities, the first best optimum with M symmetric countries is the same as that with M autarkic countries. Then, the objective function of the planner can be written as

$$W = M \left[u_{io0} L_{o0} + \sum_{t=0}^{\infty} \beta^t U_{it} \overline{L}_t \right].$$

In autarky, the resource constraint is

$$k_t^{1-\gamma} = (1+n)k_{t+1} + c_{yt} + \frac{c_{ot}}{1+n} + b_t,$$

where b_t is the per capita opportunity costs to provide public goods. The first order conditions of the planner's maximization are

$$c_{ot} : \frac{\beta}{c_{ot}} - \frac{\beta(1+n)}{(1+n)\left[k_t^{1-\gamma} - (1+n)k_{t+1} - c_{ot}/(1+n) - b_t\right]} = 0,$$

$$k_t : -\frac{(1+n)}{k_{t-1}^{1-\gamma} - (1+n)k_t - c_{ot-1}/(1+n) - b_{t-1}} - \frac{\beta(1-\gamma)k_t^{-\gamma}(1+n)}{k_t^{1-\gamma} - (1+n)k_{t+1} - c_{ot}/(1+n) - b_t} = 0,$$

$$b_t : -\frac{\beta(1+n)}{k_t^{1-\gamma} - (1+n)k_{t+1} - c_{ot}/(1+n) - b_t} + \frac{\alpha\beta(2+n)}{b_t} = 0.$$

Here, we focus our attention on the steady state ($k = k_t = k_{t+1}, c = c_{ot} = c_{ot+1}$ and $b = b_t = b_{t+1}$). The first order condition can be rewritten as follows:

$$\frac{\beta}{c} - \frac{\beta(1+n)}{[k^{1-\gamma} - (1+n)k - c/(1+n) - \tau k]} = 0,$$

$$-\frac{(1+n)\left[\beta(1-\gamma)k^{-\gamma} - 1\right]}{k^{1-\gamma} - (1+n)k - c/(1+n) - \tau k} = 0, \tag{A1}$$

$$-\frac{R(1+n)k}{k^{1-\gamma} - (1+n)k - c/(1+n) - b} + \frac{\alpha\beta(2+n)}{\tau} = 0.$$

Equation (A1) states that the first best amount of capital per capita is

$$k^f = [\beta(1-\gamma)]^{1/\gamma}. \tag{A2}$$

In contrast, the equilibrium and the second best amount of capital per capita can be obtained from (4) and (11) and is given by[27]

$$k^* = \left[\frac{\beta\gamma}{(1+n)(1+\beta)}\right]^{1/\gamma}. \tag{A3}$$

Equations (A2) and (A3) show that $k^f \neq k^*$ .Thus, the equilibrium and the second best savings level can not coincide with the first best level, from which we can confirm that the economy in this model brings about dynamic inefficiency as that in Acemoglu (2009), Blanchard and Fischer (1989) and Romer (2011).

Appendix B: Income tax

Suppose now that governments can impose tax on households in addition to capital tax. As an example, we introduce labor income tax, $\sigma_{it} \in [0, 1]$, into the baseline model developed in Section 2. Such tax modifies the individual's demand (3) as

$$c_{iyt} = \frac{(1-\sigma_{it})w_{it}}{1+\beta}, \qquad s_{it} = \frac{\beta(1-\sigma_{it})w_{it}}{1+\beta}, \qquad c_{iot+1} = \frac{\beta}{1+\beta}(1+r_{it+1})(1-\sigma_{it})w_{it}. \tag{B1}$$

The budget constraint of the government becomes $g_{it} = \sigma_{it}w_{it}\overline{L}_{it} + \tau_{it}K_{it} = [\sigma_{it}w_{it} + \tau_{it}k_{it}]\,\overline{L}_{it}$, which, from (4), can be written as

$$\begin{aligned} g_{it} &= \left[\sigma_{it}\gamma\left(\frac{1-\gamma}{r_{it}+\tau_{it}}\right)^{(1-\gamma)/\gamma} + \tau_{it}\left(\frac{1-\gamma}{r_{it}+\tau_{it}}\right)^{1/\gamma}\right]\overline{L}_{it} \\ &= \frac{(1-\gamma)^{(1-\gamma)/\gamma}\overline{L}_{it}\left[\sigma_{it}\gamma(r_{it}+\tau_{it}) + (1-\gamma)\tau_{it}\right]}{(r_t+\tau_{it})^{1/\gamma}}. \end{aligned} \tag{B2}$$

In the steady state, the per capita capital becomes

$$k = \left[\frac{\beta\gamma(1-\sigma)}{(1+\beta)(1+n)}\right]^{1/\gamma}.$$

This equation implies that

$$r + \tau = \frac{(1+\beta)(1+n)(1-\gamma)}{\beta\gamma(1-\sigma)}.$$

[27] The second best tax rate is obtained by setting (13)= 0.

Substituting (4), (B1), and (B2) into (6), we obtain

$$U_{it} = \alpha \ln\left[\sigma_{it}\gamma\left(r_t + \tau_{it}\right) + (1-\gamma)\tau_{it}\right] - \frac{\alpha + (1+\beta)(1-\gamma)}{\gamma}\ln(r_t + \tau_{it}) + (1+\beta)\ln(1-\sigma_{it}) \tag{B3}$$

$$+ \alpha \ln(1-\gamma)^{(1-\gamma)/\gamma}\overline{L}_{it} + (1+\beta)\ln\gamma(1-\gamma)^{(1-\gamma)/\gamma} - \ln(1+\beta)$$
$$+ \beta \ln\left(\frac{\beta}{1+\beta}\right)(1+r_{t+1}) + \alpha\beta\ln g_{it+1}.$$

The social welfare function is

$$W^{IT} = \sum_{i=1}^{M}\left[u_{io0}L_{o0} + \sum_{t=0}^{\infty}\beta^t U_{it}\overline{L}_t\right], \tag{B4}$$

which yields

$$\frac{1}{\overline{L}_{t-1}\beta^{t-1}}\frac{\partial W^{IT}}{\partial \tau_{it}} = \frac{-\beta}{1+r_t} + \frac{\alpha\beta(1-\gamma)(2+n)}{\sigma_{it}\gamma(r_{it}+\tau_{it}) + (1-\gamma)\tau_{it}}.$$

Economy with a growing population

In the economy with a growing population, the country i's government at period t chooses τ_{it} and σ_{it} to maximize (B3) while regarding r_t and future variables as given. The first-order conditions yield

$$\tau'_{yit} = 0 \qquad \text{and} \qquad \sigma'_{yit} = \frac{\alpha}{1+\alpha+\beta} > 0.$$

Hence, governments have no incentive to tax on mobile capital. Still, we can show that such zero-tax rate on capital is excessively low and an increase in capital tax rate can improve welfare. By keeping σ_{it} as fixed, an increase in τ_{it} affects (B4) if evaluated at $\tau_{it} = 0$ and $\sigma_{it} = \sigma'_{yit}$ as follows:

$$\left.\frac{\partial W^{IT}}{\partial \tau_{it}}\right|_{\tau_{it}=0 \text{ and } \sigma_{it}=\sigma'_{yit}} = \beta^{t+1}\overline{L}_{it-1}\left[\frac{2+n}{1+n} - \frac{\gamma}{\beta\gamma + (1+\alpha+\beta)(1-\gamma)(1+n)}\right].$$

If we follow Romer (2011) and Acemoglu (2009) in assuming that $\gamma = 2/3$ and Eckstein et al. (1999) in assuming that $\beta = 2/3$, we observe

$$\left.\frac{\partial W^{IT}}{\partial \tau_{it}}\right|_{\tau=0 \text{ and } \sigma=\sigma'_{yit}} > 0.$$

Under the realistic parameter values, tax competition results in the race to the bottom in an economy with a growing population.

Economy with a decreasing population

In the economy with a decreasing population, a government chooses τ_{it} and σ_{it} to maximize u_{iot} while regarding r_t and past variables as given. Substituting (4), (B1), and (B2) into u_{iot}, we obtain

$$
\begin{aligned}
u_{iot} &= \ln\left(\frac{\beta}{1+\beta}\right)(1+r_{it})(1-\sigma_{it-1})w_{it-1} + \alpha\ln\frac{(1-\gamma)^{(1-\gamma)/\gamma}\overline{L}_{it}\left[\sigma_{it}\gamma(r_{it}+\tau_{it})+(1-\gamma)\tau_{it}\right]}{(r_t+\tau_{it})^{1/\gamma}} \\
&= \ln\left(\frac{\beta}{1+\beta}\right)(1+r_{it})(1-\sigma_{it-1})w_{it-1} + \alpha\ln\left[\sigma_{it}\gamma(r_{it}+\tau_{it})+(1-\gamma)\tau_{it}\right] \\
&- \alpha\ln(r_t+\tau_{it})^{1/\gamma} + \alpha\ln(1-\gamma)^{(1-\gamma)/\gamma}\overline{L}_{it}.
\end{aligned}
$$

Here, we know that u_{iot} is monotonously increasing in σ_{it}. To avoid the non-existence of equilibrium rate of return on savings, we assume the upper-bound of labor income tax $\overline{\sigma} \in (0,1)$.[28] The government determines tax rates as

$$
\tau'_{oit} = \frac{r\gamma(1-\sigma)}{1-\gamma(1-\sigma)} \qquad \text{and} \qquad \sigma'_{oit} = \overline{\sigma}.
$$

By keeping σ as fixed, an increase in τ_{it}, evaluated at $\tau_{it} = \tau'_{oit}$ and $\sigma_{it} = \overline{\sigma}$, results in[29]

$$
\begin{aligned}
&\frac{1}{\overline{L}_{t-1}\beta^{t-1}}\frac{\partial W^{IT}}{\partial \tau_{it}}\bigg|_{\tau_{it}=\tau'_{oit} \text{ and } \sigma_{it}=\overline{\sigma}} \\
&= \left[\frac{\alpha\beta(1-\gamma)(2+n)}{\sigma\gamma(r_t+\tau'_{oit})+(1-\gamma)\tau'_{oit}}\right] - \frac{\beta}{1+r_r}
\end{aligned}
$$

Hence, we readily know this is positive if and only if

$$
\alpha > \widetilde{\alpha}',
$$

where $\widetilde{\alpha}'$ is defined as

$$
\widetilde{\alpha}' \equiv \frac{\overline{\sigma}(1+n)(1+\beta)+\beta\tau'_{oit}}{\alpha\beta^2\gamma(1+r)(2+n)} > 0.
$$

Therefore, if governments can impose tax on households, capital tax competition under decreasing population results in an inefficiently high (resp. low) capital tax rate if the household's preference for public good consumption is sufficiently small i.e., $\alpha < \widetilde{\alpha}'$ (resp. large, i.e., $\alpha > \widetilde{\alpha}$).

[28] When $\sigma = 1$, the equilibrium rate of return on savings diverges to infinity.

[29] Note again that (11) implies that $\partial r/\partial\tau = -1$.

Appendix C: Public inputs

Suppose that individuals obtain utility only from private good consumption, c. We specify the utility function as follows:

$$U_{it} = \ln c_{iyt} + \beta \ln c_{iot+1}, \tag{C1}$$

where β ($\in (0,1)$) is the time discount rate. From utility maximization, we can obtain the following equations:

$$c_{iyt} = \frac{w_{it}}{1+\beta}, \qquad s_{it} = \frac{\beta w_{it}}{1+\beta}, \qquad c_{iot+1} = \frac{\beta}{1+\beta}(1+r_{it+1})w_{it}.$$

Firms produce the numéraire by using labor and capital under constant returns to scale. Here, we assume that public inputs raise productivity of firms. We employ a Cobb-Douglas production function:

$$y_{it} = g_{it}^{\epsilon} L_{it}^{\gamma} K_{it}^{1-\gamma},$$

where ϵ is a positive constant that represents the public input elasticity of output. We assume that $\gamma > \epsilon$. Profit maximization yields

$$w_{it} = \gamma g_{it}^{\epsilon} k_{it}^{1-\gamma}, \qquad k_{it} = \left[\frac{g_t^{\epsilon}(1-\gamma)}{r_{it}+\tau_{it}} \right]^{1/\gamma}, \tag{C2}$$

where τ represents the capital tax rate. We substitute $g_{it} = \tau_{it} K_{it}$ into (C2) to get

$$g_{it} = \tau_{it}^{\gamma/(\gamma-\epsilon)} \left(\frac{1-\gamma}{r_{it}+\tau_{it}} \right)^{1/(\gamma-\epsilon)} \overline{L}_{it}^{\gamma/(\gamma-\epsilon)}. \tag{C3}$$

The social welfare function is

$$W^P = \sum_{i=1}^{M} \left[u_{io0} L_{o0} + \sum_{t=0}^{\infty} \beta^t U_{it} \overline{L}_t \right],$$

which yields

$$\frac{1}{\overline{L}_{t-1}\beta^{t-1}} \frac{\partial W^P}{\partial \tau_{it}} = -\frac{\beta}{1+r_{it}} + \frac{\beta(1+n)(1+\beta)\epsilon}{\gamma-\epsilon}\frac{1}{\tau_{it}}$$
$$+ \beta \ln(1+r_{it}) + \beta(1+n)\left[(1+\beta)\frac{\epsilon}{\gamma-\epsilon} \ln \tau_{it} - (1+\beta)\frac{1-\gamma+\epsilon}{\gamma-\epsilon} \ln(r_{it}+\tau_{it}) \right].$$

Economy with a growing population

We start with the case of $n > 0$, which implies that the population is increasing. In this case, because $\overline{L}_{it} > \overline{L}_{it+1}$, young individuals represent the majority and the government maximizes U_{it}. Plugging (3), (C2), and the government budget constraint (C3) into (C1), we obtain

$$U_{it} = (1+\beta)\frac{\epsilon}{\gamma-\epsilon}\ln\tau_{it} - (1+\beta)\frac{1-\gamma+\epsilon}{\gamma-\epsilon}\ln(r_{it}+\tau_{it}) + \beta\ln(1+r_{it+1})$$
$$+\beta\ln\left(\frac{\beta}{1+\beta}\right) + \ln\left(\frac{1}{1+\beta}\right) + (1+\beta)\ln\left[(1-\gamma)^{(1-\gamma)/\gamma}\,\overline{L}_{it}^{\varepsilon/(\gamma-\varepsilon)}\right].$$

The first-order condition regarding τ yields

$$\tau_{it} = \frac{\epsilon}{1-\gamma}r_t. \tag{C4}$$

Economy with a decreasing population

Next, we consider the case of $n < 0$, which implies that the population is decreasing. In this case, because $\overline{L}_{it} < \overline{L}_{it+1}$, old individuals represent the majority and the government maximizes u_{iot}. Plugging (3) and $g_{it} = \tau_{it}K_{it}$ into (2), we obtain

$$u_{iot} = \ln\left(\frac{\beta}{1+\beta}\right) + \ln(1+r_{it}) + \ln w_{it-1}.$$

In this case, the utility of the old individual does not depend on τ_{it}, implying that old agents are indifferent to any tax rate. Therefore, policies that maximize U_{it} are supported by the majority, and the first-order condition of the maximization again yields (C4).

Steady-state

We assumed symmetric countries, which implies that all countries have the same capital holdings at period 0, the same population size, and the same population growth rate, implying that $\overline{L}_{it} = \overline{L}_{jt}$ $(i \neq j)$ for all t. From $\overline{L}_{it} = \overline{L}_{jt}$ $(i \neq j)$, we obtain $c_{it} = c_{jt} = c_t$ and $k_{it} = k_{jt} = k_t$. We focus on the steady-state equilibrium, wherein the level of individual's consumption, c_t, and capital per capita, k_t, are constant over time ($c_t = c_{t+1} = c^*$ and $k_t = k_{t+1} = k^*$). Then, from (4), we readily know that $r_t + \tau_t = r_{t+1} + \tau_{t+1}$. Using this, the capital market clearing condition (5) can be rewritten as

$$\left[\frac{g^\epsilon(1-\gamma)}{r_t+\tau_t}\right]^{1/\gamma}\sum_{j=1}^{M}\overline{L}_{it} = \frac{\beta\gamma}{1+\beta}g^\epsilon\left[\frac{g^\epsilon(1-\gamma)}{r_t+\tau_t}\right]^{(1-\gamma)/\gamma}\sum_{j=1}^{M}\overline{L}_{it-1}.$$

From $\overline{L}_{it+1} = (1+n)\overline{L}_{it}$ for all countries, we obtain

$$r_t + \tau_t = \frac{(1+n)(1+\beta)(1-\gamma)}{\beta\gamma}. \tag{C5}$$

Equilibrium and its efficiency

From (C4) and (C5), we obtain the equilibrium capital tax rate in both economies as follows:

$$\tau_{y2}^{*} = \frac{\epsilon(1+n)(1+\beta)(1-\gamma)}{\beta\gamma(1-\gamma+\epsilon)}.$$

An increase in τ_{it}, evaluated at $\tau_{it} = \tau_{y2}^{*}$ results in

$$\frac{1}{\overline{L}_{t-1}\beta^{t-1}} \frac{\partial W^{P}}{\partial \tau_{it}}\bigg|_{\tau_{it}=\tau_{y2}^{*}} = \frac{1}{\tau_{y2}^{*}(1+r)} \frac{(\beta+1)\varepsilon(n+1)\chi}{\gamma(1-\gamma+\varepsilon)}, \tag{C6}$$

where

$$\chi \equiv \varepsilon\left[1-(1-\beta)\gamma\right] + n(\beta+1)(1-\gamma)^{2} - (1-\gamma)(\beta-2\gamma+1).$$

The sign of χ is positive if and only if

$$\varepsilon > \widetilde{\varepsilon},$$

where

$$\widetilde{\varepsilon} \equiv \frac{(1-\gamma)(\beta-2\gamma+1) - n(\beta+1)(1-\gamma)^{2}}{1-(1-\beta)\gamma}.$$

Thus, when the public input elasticity of output is higher than $\widetilde{\varepsilon}$, (C6) becomes positive and we observe the race to the bottom.

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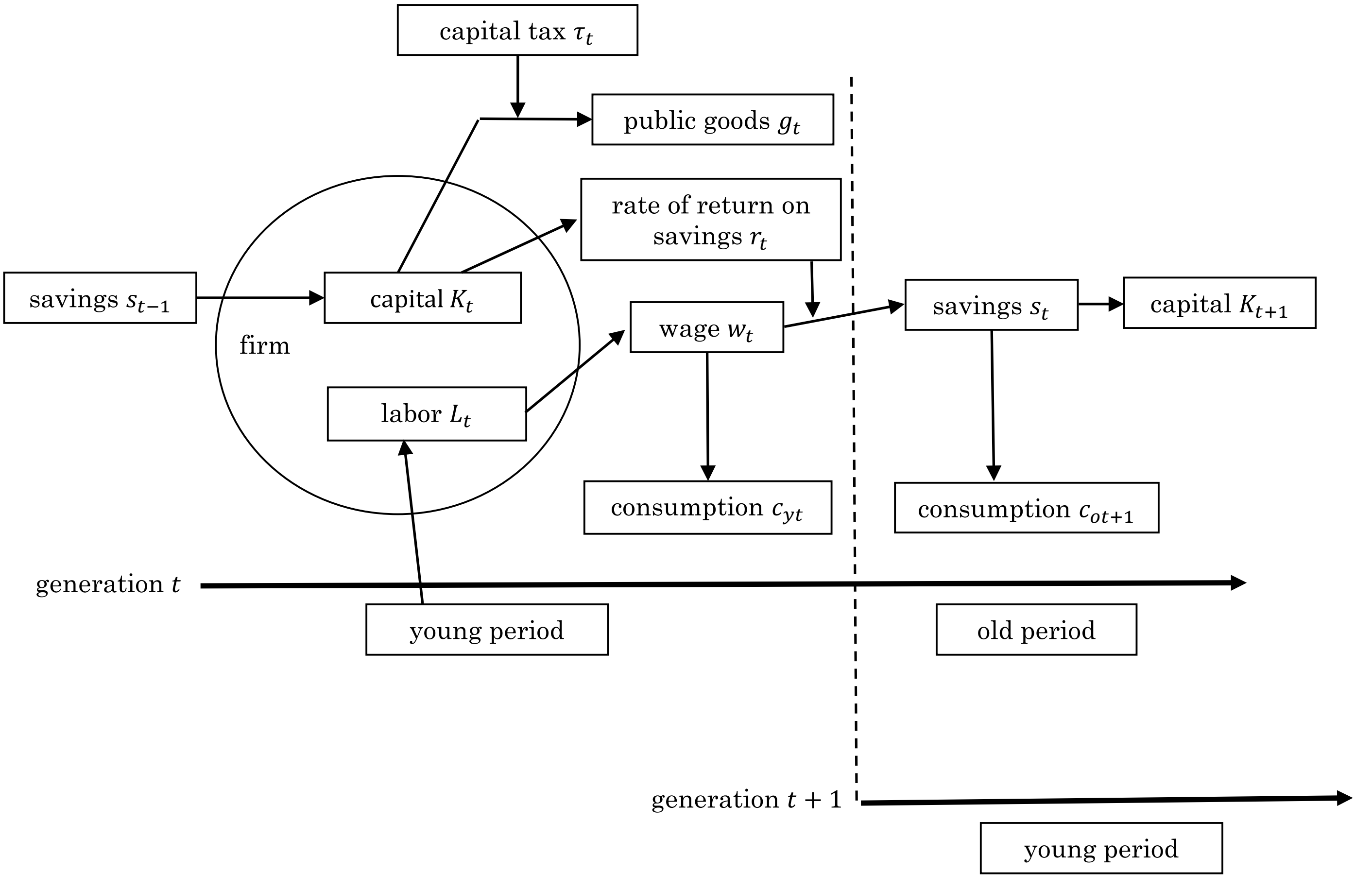

Figure 1： OLG structure of the model

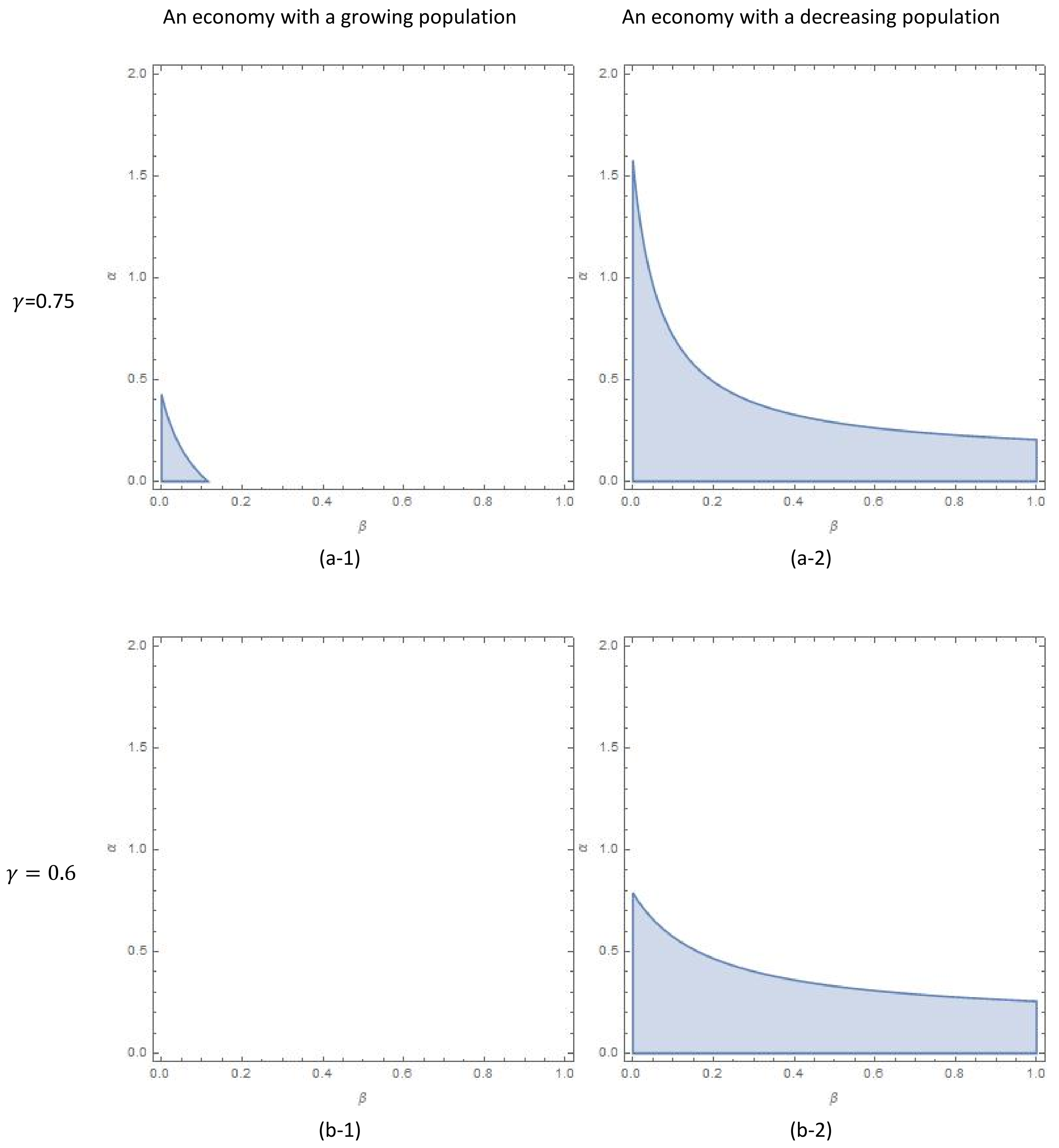

Figure 2: Possibility of the race to the top

Notes: We set $n = 0.1$ for an economy with a growing population and $n = -0.1$ for an economy with a decreasing population. For each case, we consider two alternative values of γ ($\gamma = 0.75$ and $\gamma = 0.6$). The shaded areas represent combinations of α and β that result in the race to the top.