Cash-Flow Business Taxation Revisited: Bankruptcy and Asymmetric Information

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Abstract

It is well-known that cash-flow business taxes with full loss-offset, and their present-value equivalents, are neutral with respect to firms' investment decisions when firms are risk-neutral and there are no distortions. We study the effects of cash-flow business taxation when there is bankruptcy risk, and financial intermediaries face asymmetric information problems in financing heterogeneous firms. In these circumstances, investment decisions are distorted, with investment being less than in the full-information case. Cash-flow taxation applying to both real and financial cash flows corrects the distortion by inducing more investment in rent-generating projects and increases social welfare.

Key Words: cash-flow tax, bankruptcy, asymmetric information
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1 Introduction

A classic result in the design of business taxes due to Brown (1948) concerns the neutrality of cash-flow taxation. Investment decisions undertaken in a world of full certainty will be unaffected by a tax imposed on firms' cash flows, assuming there is full loss-offsetting (and the tax rate is constant, as shown by Sandmo, 1979). In effect, a cash-flow tax will divert a share of the pure profits or rents from the firm's owners to the government. This is of obvious policy interest since it represents a non-distorting source of tax revenue.

In this paper, we revisit the use of cash-flow taxation as a rent-collecting device when firms face asymmetries of information in capital markets. We do so in a simple partial equilibrium model of risk-averse entrepreneurs who vary in their productivity, so that returns to inframarginal entrepreneurs generate rents. We assume that banks can observe entrepreneurs' types so there is no adverse selection. Investment outcomes are uncertain and entrepreneurs face the possibility of bankruptcy, which banks can verify only by engaging in costly monitoring. In these circumstances, investment is inefficiently low compared with the full-information case.

We study the effects of cash-flow taxation on both the entry decision of potential entrepreneurs (the extensive margin) and on the decision as to how much to borrow and invest (the intensive margin). We assume the cash flow tax is levied on both real and financial cash-flows, so applies to both entrepreneurs and banks. The effect of the tax on intensive-margin decisions turns out to be especially important. Unlike in the full-information case, cash-flow taxation is not neutral. On the contrary, it tends to encourage rent-generating investment and improve efficiency thereby acting as a corrective device in a distorted environment.

The Brown neutrality result inspired a sizable literature on neutral business tax design, much of which generalized his result to taxes that are equivalent to cash-flow taxes in present value terms. Boadway and Bruce (1984) show that the cash-flow tax is a special case of a more general class of neutral business taxes that have the property that the present value of deductions for future capital costs (interest plus depreciation) arising from any investment just equals initial investment expenditures. A special case of this is the Capital Account Allowance (CAA) tax in which investment expenditures are added to a capital account each year, and each year the capital account is depreciated at a rate specified for tax purposes. Annual deductions for capital costs then consist of the sum of the cost of capital plus the depreciation rate multiplied by the book value of the capital account.

More generally, neutrality can be achieved by a business tax in which the present value of future tax bases just equals the present value of cash flows. An example of a cash-flow equivalent tax system of this sort is the Resource Rent Tax (RRT) proposed by Garnaut and

Clunies-Ross (1975) for the taxation of non-renewable natural resources. In their version, firms starting out are allowed to accumulate negative cash flows in an account that rises each year with the cost of capital. Once the account becomes positive, cash flows are taxed as they occur. Negative cash flows are carried forward at the interest rate to achieve the equivalent of cash-flow taxation.

These basic results continue to apply if returns to investment are uncertain, provided firms are risk-neutral. Fane (1987) shows that neutrality holds under uncertainty as long as tax credits and liabilities are carried forward at the risk-free nominal interest rate, and tax credits and liabilities are eventually redeemed. Bond and Devereux (1995) show that the CAA tax remains neutral in the presence of uncertainty and the possibility of bankruptcy provided that a risk-free interest rate applied to the value of the capital account is used for the cost of capital deduction, that any unused negative tax credits are refunded in the event of bankruptcy, and that the valuation of risky assets satisfy the value additivity principle.[1] The use of a risk-free discount rate reflects the assumption that there is no risk to the firm associated with postponing capital deductions into the future (i.e., no political risk). Boadway and Keen (2015) show that the same neutrality result applies to the RRT in the presence of uncertainty.

The above results focussed on taxes applied to real cash flows or their equivalent, what Meade (1978) called R-base cash-flow taxation. Bond and Devereux (2003) show that neutrality can be achieved using the more general case of cash-flow taxation proposed by Meade, referred to as (R+F)-base cash-flow taxation, in which both real and financial cash flows are included in the base. They also study the neutrality of the Allowance for Corporate Equity (ACE) tax, which is a version of the CAA tax that allows actual interest deductions alongside a cost of capital deduction for equity-financed investment. (They assume that there are no rents earned by bond-holders.) Notably, Bond and Devereux assume full information in the sense that banks can observe incomes of firms that claim to be bankrupt. As well, they do not allow firms to choose the quantity of investment.

These neutrality results have inspired various well-known policy proposals, some of which have been implemented. A cash-flow business tax was recommended by the US Treasury (1977), Meade (1978) in the UK, and the President's Panel (2005) in the USA. The latter two both recommended additional cash-flow taxation to apply to financial institutions. The Australian Treasury (2010) (the Henry Report) recommended an RRT for the mining industries in Australia. Several bodies have recommended an ACE corporate tax system,

[1] The value additivity principle implies that the present value of the sum of stochastic future payoffs is equal to the sum of the present values of these payoffs, and is consistent with a no-arbitrage principle in the valuation of assets.

including the Institute for Fiscal Studies (1991), the Mirrlees Review (2011) and Institut d'Economia de Barcelona (2013). ACE taxes have been deployed in a few countries, including Brazil, Italy, Croatia and Belgium. Reviews of their use may be found in Klemm (2007), de Mooij (2011), Panteghini, Parisi, and Pighetti (2012), and Princen (2012). Cash-flow-type taxes with full loss-offset are used in the Norwegian offshore petroleum industry (Lund, 2014), and the RRT was applied temporarily in the Australian mining industry.

The neutrality of cash-flow taxation no longer applies when firms' owners are risk-averse so that part of the return to investment is compensation for risk. A cash-flow tax applies to both rents and returns to risk-taking, and these two streams cannot be distinguished. As Domar and Musgrave (1944) famously show, risk-averse savers faced with a proportional tax on capital income with full loss-offset would be expected to increase the proportion of their portfolio held as risky assets, although the results become murkier when the proceeds from the tax are returned to savers by the government, as discussed in Atkinson and Stiglitz (1980) and Buchholz and Konrad (2014). These results readily extend to an entrepreneurial firm, as shown in Mintz (1981). This non-neutrality is not necessarily a bad thing if the government is better able to diversify risk that private savers.

In our analysis, investment is undertaken by entrepreneurs of differing productivity. We begin with the case where entrepreneurs are risk-neutral, and then extend the analysis to allow for risk-averse entrepreneurs. Our main results are as follows. With risk-neutral entrepreneurs and the possibility of bankruptcy, the (R+F)-base cash-flow tax is neutral in the absence of asymmetric information as in Bond and Devereux (1995, 2003), but distorts investment decisions if banks must incur monitoring costs when firms declare bankruptcy. Remarkably, the cash-flow tax increases investment while leaving bankruptcy risk and firms' expected profits unchanged. Expected rents, government expected revenues and social surplus all increase with the tax rate. In effect, as well as raising revenue the cash-flow tax serves as a corrective device for the market failure arising from asymmetric information. With risk-averse entrepreneurs, the cash-flow tax results in some risk-sharing between firms and the government. The tax induces higher investment but is neutral with respect to bankruptcy risk and expected utility as in Domar and Musgrave (1944).

We begin by outlining the main elements of the model under risk-neutrality. We show that under-investment occurs in the absence of taxes. We then study the effects of cash-flow taxation on the entry and investment decisions of entrepreneurs in the basic model. Finally, we extend the analysis to the case where entrepreneurs are risk-averse.

2 The Base Case with Risk-Neutral Entrepreneurs

2.1 Outline of the model

Our model is designed to capture the following key features. Firms' investments are heterogeneous such that inframarginal investments generate pure profits or rents. The purpose of profit taxation is to tax these rents, and in a first-best world cash-flow profits taxation would do so in a non-distorting way. Our model departs from the first best by assuming that firms, who rely on the banks to finance their investments, face the possibility of bankruptcy, but that banks cannot observe without cost the profits of firms that declare bankruptcy. Banks can learn these profits at a cost by ex post monitoring or verification. The consequence is that relative to the full-information setting, there are too few loans. In this context, cash-flow taxation can actually encourage lending and improve social efficiency. The manner in which cash-flow taxation affects efficiency depends on whether the owners of firms, which we refer to as entrepreneurs, are risk-neutral or risk-averse.

There is a population of potential entrepreneurs with an identical endowment of wealth who can undertake an investment project. They differ in the productivity of their projects. We simplify our analysis by assuming there is a single period so we can suppress the entrepreneurs' consumption-savings decision and focus on production decisions. At the beginning of the period, potential entrepreneurs decide whether to enter a risky industry and invest their wealth in a risky project. Those who do not enter invest their wealth in a risk-free asset and consume the proceeds at the end of the period. Entrepreneurs who enter the risky industry choose how much to borrow to leverage their own equity investment, which determines their capital stock. After investment has been undertaken, risk is resolved. Those with good outcomes earn profits for the entrepreneur. Those with bad outcomes go bankrupt. Their production goes to their creditors, which are risk-neutral competitive banks.

There are thus two decisions made by potential entrepreneurs. First, they decide whether to enter, which is an extensive-margin decision; and second, they decide how much to borrow to expand their capital, which is an intensive-margin decision. For simplicity, we suppress their labor income: all income comes from profits they earn if they enter the risky sector, or from their initial wealth if they do not. Adding labor income (as in Kanniainen and Panteghini (2012)) would make no substantial difference for our result on business taxation. Our purpose is to study how cash-flow corporate taxation affects the extensive- and intensive-margin decisions of firms, and as a result the efficiency of firms' behavior.

We assume that banks know the productivity of entrepreneurs, so can offer type-specific interest rates. The interest rate offered to a given type of entrepreneur depends on the borrowing the entrepreneur chooses. More borrowing increases the risk of bankruptcy, which in turn affects the expected profit of the lending bank. Since banks are competitive, their expected profits from loans to each type of entrepreneur will be zero in equilibrium, and this zero-profit condition determines the interest rate. Entrepreneurs know how their borrowing affects their interest rate, and that influences how much they borrow.

The details of the model with risk-neutral entrepreneurs are as follows. Later we consider the consequences of entrepreneurs being risk-averse and unable to insure against the risk of their uncertain incomes.

2.2 Details of the model with risk-neutral entrepreneurs

A continuum of potential entrepreneurs are all endowed with initial wealth E. For simplicity, we assume that the production function is linear in capital K. The average product of capital, denoted R, is constant, but differs across entrepreneurs, and is distributed over $[0, R_{\max}]$ by the distribution function $H(R)$. The value of output is subject to idiosyncratic risk, and the stochastic value of a type$-R$ entrepreneur's output is $\tilde{\varepsilon}RK$, where $\tilde{\varepsilon}$ is distributed uniformly over $[0, \varepsilon_{\max}]$, with density $g = 1/\varepsilon_{\max}$. The expected value of ε is:[2]

$$\overline{\varepsilon} \equiv \mathbb{E}[\tilde{\varepsilon}] = \frac{\varepsilon_{\max}}{2} = \frac{1}{2g} \tag{1}$$

We assume that the distribution of $\tilde{\varepsilon}$ is the same for all entrepreneurs, so they differ only by their productivity R. Capital is financed by the entrepreneur's own equity and debt, and depreciates at the proportional rate δ per period. Entrepreneurs who do not enter invest all their wealth in a risk-free asset with rate of return ρ, so consume $(1+\rho)E$. Since all potential entrepreneurs have the same alternative income, those with the highest productivity as entrepreneurs will enter the entrepreneurial sector. Let $\widehat{R}$ denote the productivity of the marginal entrepreneur.

Entrepreneurs who enter invest all their wealth in the risky firm. Then, E will be the common value of own-equity of all entrepreneurs. The type$-R$ entrepreneur who has entered borrows an amount $B(R)$ so his aggregate capital stock is $K(R) = E + B(R)$. Denote the leverage rate by $\phi(R)$, where $\phi(R) = B(R)/K(R)$. Then $K(R)$ can be written:

$$K(R) = \frac{E}{1 - \phi(R)} \tag{2}$$

We assume that there is a maximum value of the capital stock, such that $K(R) \leqslant K_{\max}$, and moreover that $E < K_{\max}$ so the entrepreneur's wealth is less than the maximum size of the capital stock. By (2), this implies that $0 \leqslant \phi(R) \leqslant 1 - E/K_{\max} < 1$. Since we

[2] A stochastic variable is denoted $\tilde{x}$; its expected value is $\overline{x}$; and its cutoff value where relevant is $\hat{x}$.

assume that all the entrepreneur's wealth is invested, the minimum level of capital for entrepreneurs who enter is E. Allowing entrepreneurs to invest only part of their wealth would complicate the analysis slightly without adding any insight.[3] The entrepreneur's capital stock is therefore in the range $K(R) \in [E, K_{\max}]$. The assumption of a maximal capital stock reflects the notion that after some point additional capital is non-productive. It is like a strong concavity assumption on the production function, which precludes extreme outcomes that would otherwise occur with linear production. In most of our analysis, entrepreneurs choose an interior solution so $K(R) \leqslant K_{\max}$ is not binding.

Since we assume that banks can identify entrepreneurs by type and set a type-specific interest rate, equilibrium analysis applies separately to entrepreneurs of each type.[4] Accordingly, consider a representative type$-R$ entrepreneur and drop the identifier R from all functions for simplicity. After the shock $\tilde{\varepsilon}$ is revealed, an entrepreneur's ex post after-tax profits (or return to own-equity) evaluated at the end of the period is given by:

$$\widetilde{\Pi} = \tilde{\varepsilon}RK + (1-\delta)K - (1+r)B - \widetilde{T} \tag{3}$$

where $\widetilde{T}$ is the tax paid and r is the interest rate, so $(1+r)B$ is the repayment of interest and principal on the borrowing B. The term $(1-\delta)K$ is the value of capital remaining after production, given the depreciation rate δ. The type-specific interest rate r will depend upon the leverage ϕ chosen by the entrepreneur since this affects bankruptcy risk. The manner in which ϕ affects r depends upon the behavior of the lending banks as discussed below.

In our base case, we assume the government deploys an (R+F)-base cash-flow tax on both the firms and the banks. As we show below, this is equivalent to ACE taxation. In the absence of market failures, these taxes are non-distorting. However, in our asymmetric information setting, they affect firms' investment choices and serve partly to correct the market distortions. Tax liability under cash-flow taxation, again evaluated at the end of the period after $\tilde{\varepsilon}$ is revealed, is given by:

$$\widetilde{T} = \tau\Big(\underbrace{\tilde{\varepsilon}RK - (1+\rho)K + (1-\delta)K}_{R-\text{base}} + \underbrace{(1+\rho)B - (1+r)B}_{F-\text{base}}\Big) \tag{4}$$

[3] If entrepreneurs were to invest part of their wealth in the safe asset, leverage would increase for any given level of investment. That would increase bankruptcy risk and the interest rate faced by the entrepreneur. If entrepreneurs face unlimited liability in the case of bankruptcy, there would be no incentive to invest less than total wealth in the risky project since the interest rate on borrowing will be higher than the rate of return on the safe asset. If there is limited liability in the case of bankruptcy, entrepreneurs may choose to hold wealth in the safe asset although that would result in higher interest costs on borrowing.

[4] If wealth differed among entrepreneurs, leverage ϕ and therefore the interest rate could vary with both R and E. This would not affect the qualitative results of our analysis.

where τ is the tax rate and, as noted above, ρ is the risk-free interest rate. The real component of the cash-flow tax base (R-base) in (4) consists of three terms. The first is the revenue of the firm, $\tilde{\varepsilon}RK$. The second, $(1+\rho)K$, is the end-of-period value of the deduction for investment. Since investment K occurs at the beginning of the period, we assume that the tax savings from deducting investment are either refunded immediately or are carried over to the end of the period with interest at rate ρ. Third, the cash-flow tax is levied on selling or winding-up the depreciated value of business assets, $(1-\delta)K$, at the end of the period. Financial cash-flows (F-base) include the end-of-period value of the borrowing B obtained by the firm less the principal and interest repaid, $(1+r)B$. Eq. (4) applies whether $\widetilde{T}$ is positive or negative, so implicitly assumes that the tax system allows full loss-offsetting. The tax liability $\widetilde{T}$ is incurred by the firm as long as it is not bankrupt. If the firm goes bankrupt, the bank pays taxes on the bankrupt cash flows.

Using $B = E - K$, (4) may be rewritten as:

$$\widetilde{T} = \tau\Big(\tilde{\varepsilon}RK - \delta K - rB - \rho E\Big) = \widetilde{T}^{\text{ACE}} \tag{5}$$

where the term in brackets is the ACE tax base. It includes revenues $\tilde{\varepsilon}RK$ less three deductions for capital at the end of the period: depreciation δK, interest rB, and the cost of equity finance ρE. This verifies that the R+F and ACE bases are equivalent.

Using (3) and (4), ex post after-tax profits under cash-flow taxation can be written:

$$\widetilde{\Pi} = (1-\tau)\Big(\tilde{\varepsilon}RK + (1-\delta)K - (1+r)B\Big) + \tau(1+\rho)E \tag{6}$$

Entrepreneurs are confronted with bankruptcy when $\tilde{\varepsilon}$ is too low to meet debt repayment obligations, that is, when $\widetilde{\Pi} < 0$. This occurs for entrepreneurs with $\tilde{\varepsilon} < \hat{\varepsilon}$, where $\hat{\varepsilon}$ (which is specific to type R) satisfies:

$$0 = (1-\tau)\Big(\hat{\varepsilon}RK + (1-\delta)K - (1+r)B\Big) + \tau(1+\rho)E \tag{7}$$

In what follows, we refer to $\hat{\varepsilon}$ as *bankruptcy risk*. The higher the value of $\hat{\varepsilon}$, the greater the chances of the entrepreneur going bankrupt. Combining (6) and (7), we obtain:

$$\widetilde{\Pi} = (1-\tau)(\tilde{\varepsilon} - \hat{\varepsilon})RK \quad \text{for} \quad \tilde{\varepsilon} \geqslant \hat{\varepsilon} \tag{8}$$

Eq. (7) determining $\hat{\varepsilon}$ can be rewritten, using $B = \phi K$ and $E = (1-\phi)K$, as:

$$(1-\tau)\Big(\hat{\varepsilon}R + (1-\delta) - (1+r)\phi\Big) + \tau(1+\rho)(1-\phi) = 0 \tag{9}$$

As this expression indicates, bankruptcy risk $\hat{\varepsilon}$ depends on both the leverage chosen by the entrepreneur, ϕ, and the interest rate, r. The latter is determined by a competitive banking sector as follows. Assume that banks are risk-neutral and can observe R and ϕ for each entrepreneur, but cannot observe $\tilde{\varepsilon}$ or ex post profits. Thus, there is no adverse selection since banks know entrepreneurs' types, but there is moral hazard since entrepreneurs may have an incentive to declare bankruptcy to avoid repaying the loan. Imperfection of the financial market due to asymmetric information is addressed by an ex post verification or monitoring cost in the event a firm declares bankruptcy. Following the financial accelerator model of Bernanke *et al* (1999), we assume that the verification cost is proportional to ex post output so takes the form $c\tilde{\varepsilon}RK$, for $\tilde{\varepsilon} \leqslant \hat{\varepsilon}$. This might reflect the fact that the verification cost includes the costs of seizing the firm's output in a default.[5] We assume for simplicity that there are no errors of monitoring. Then, only entrepreneurs with $\tilde{\varepsilon} < \hat{\varepsilon}$ will declare bankruptcy in equilibrium. The expected total monitoring cost incurred by a bank for a given type of entrepreneur will be:

$$\int_0^{\hat{\varepsilon}} c\tilde{\varepsilon}RKg d\tilde{\varepsilon} = cRKg\frac{\hat{\varepsilon}^2}{2} \tag{10}$$

so the expected monitoring cost increases with bankruptcy risk, $\hat{\varepsilon}$. This specific form of the monitoring cost is chosen for analytical convenience and is not critical for our results.

In the event of bankruptcy, the firm no longer repays it debt and interest, $(1+r)B$, and its after-tax profits go to the bank. Using (6), these after-tax profits become:

$$\widetilde{\Pi} = (1-\tau)\Big(\tilde{\varepsilon}RK + (1-\delta)K\Big) + \tau(1+\rho)E \quad \text{for} \quad \tilde{\varepsilon} < \hat{\varepsilon} \tag{11}$$

As this expression indicates, we assume that negative tax liabilities owing to bankrupt firms are refundable to the banks. Competition among banks ensures that expected profits earned from lending to the representative entrepreneur of each type are zero. We assume that banks will not go bankrupt, so they pay the risk-free interest rate ρ on their deposits. We also assume that banks incur no operating costs for simplicity.

The banks pay the (R+F)-base tax on their financial income less monitoring costs plus any revenues they obtain from bankrupt firms. The expected tax liability of a bank from a loan B to a given type of entrepreneur is:

$$\overline{T}_B = \tau\Bigg(\int_{\hat{\varepsilon}}^{\varepsilon_{max}} \Big((1+r)B - (1+\rho)B\Big)gd\tilde{\varepsilon}$$

[5] Bernanke and Gertler (1989) introduced a fixed verification cost in a business cycle model where there is asymmetric information between lenders and borrowers about the realized return on risky projects, while Townsend (1979) explored the design of debt contracts with verification costs that could either be fixed or functions of realized project output. See also Bernanke *et al* (1996) for an analysis of the implications of agency costs in lending contracts arising from asymmetric information about project outcome.

$$+\int_0^{\hat{\varepsilon}}\Big(\hat{\varepsilon}RK+(1-\delta)K-(1+\rho)E-(1+\rho)B-c\hat{\varepsilon}RK\Big)gd\hat{\varepsilon}\Big) \tag{12}$$

The first term of the tax base is net financial cash flow when the loan is repaid. The second includes the net revenues from the bankrupt firms less the unclaimed tax credit owing to those firms, the deduction for the cost of repaying deposits and the cost of monitoring. Using (12), the bank's zero-expected profit condition can be written:

$$(1-\tau)(1+\rho)B=(1-\tau)\Bigg(\int_{\hat{\varepsilon}}^{\varepsilon_{\max}}(1+r)Bgd\tilde{\varepsilon}$$

$$+\int_0^{\hat{\varepsilon}}\Big(\tilde{\varepsilon}RK+(1-\delta)K+\frac{\tau(1+\rho)}{1-\tau}E\Big)gd\tilde{\varepsilon}-cRKg\frac{\hat{\varepsilon}^2}{2}\Bigg) \tag{13}$$

This zero-profit condition, which applies for each type of entrepreneur, determines the interest rate the entrepreneur of a given type pays, given their bankruptcy risk, or equivalently, their leverage. Using (13) in (12), expected tax liabilities of the bank simplify to:

$$\overline{T}_B=-\int_0^{\hat{\varepsilon}}\frac{\tau}{1-\tau}(1+\rho)Egd\hat{\varepsilon} \tag{14}$$

Though this is negative in expected terms, the bank's tax liabilities will be positive if the firm does not go bankrupt.

2.3 Leverage, bankruptcy risk and the interest rate

The bankruptcy condition (9) and the bank's zero-profit condition (13) jointly determine the relations among r, ϕ and $\hat{\varepsilon}$. By combining these equations, we can eliminate $(1+r)B$ and obtain a relationship between ϕ and $\hat{\varepsilon}$ as shown in the following lemma.

Lemma 1 *The leverage rate $\phi=B/K$, for $0<\phi<1-E/K_{\max}$, is given by:*

$$\phi(\hat{\varepsilon},R,\tau,c)=\frac{1-\tau}{1+\rho}\Bigg(\Big(1-\frac{g\hat{\varepsilon}}{2}-\frac{cg\hat{\varepsilon}}{2}\Big)\hat{\varepsilon}R+(1-\delta)\Bigg)+\tau \tag{15}$$

The proofs of all lemmas are given in the Appendix. Routine differentiation of (15) gives properties of $\phi(\hat{\varepsilon},R,\tau,c)$ that are useful in what follows:

$$\phi_{\hat{\varepsilon}}=\frac{1-\tau}{1+\rho}(1-g\hat{\varepsilon}-cg\hat{\varepsilon})R;\quad \phi_c=-\frac{1-\tau}{1+\rho}\frac{Rg\hat{\varepsilon}^2}{2};\quad \phi_R=\frac{1-\tau}{1+\rho}\Big(1-\frac{g\hat{\varepsilon}}{2}-\frac{cg\hat{\varepsilon}}{2}\Big)\hat{\varepsilon};$$

$$\phi_\tau=\frac{1-\phi}{1-\tau};\quad \phi_{\hat{\varepsilon}\hat{\varepsilon}}=-\frac{1-\tau}{1+\rho}(1-c)Rg\hat{\varepsilon};\quad \phi_{\hat{\varepsilon}\tau}=-\frac{1-g\hat{\varepsilon}-cg\hat{\varepsilon}}{1+\rho}R \tag{16}$$

Eqs. (9) and (15) represent two equations in three unknowns: ϕ, $\hat{\varepsilon}$ and r. In practice, an entrepreneur chooses leverage ϕ, and this determines both bankruptcy risk $\hat{\varepsilon}$ and the interest rate r through (9) and (15). Bankruptcy risk is increasing in leverage by (16), and, as the following lemma states, so is the interest rate. That is, an increase in leverage increases the probability of the entrepreneur going bankrupt, and that increases the interest rate that banks must charge if their zero-expected-profit condition is to be satisfied.

Lemma 2 *The interest rate r facing an entrepreneur is increasing in leverage ϕ.*

To interpret the role of (15) in our analysis, we assume that entrepreneurs understand how the leverage they choose affects the probability of bankruptcy and the interest rate they face through the bankruptcy condition (9) and the bank's zero-profit condition (13). Therefore, they know the relationship between ϕ and $\hat{\varepsilon}$ in (15), and how it implicitly takes account of the interest rate they face. In what follows, we take advantage of Lemma 1 to suppress the interest rate r from our analysis. While in practice, entrepreneurs choose leverage ϕ, it is convenient for us to assume that they choose bankruptcy risk $\hat{\varepsilon}$, which is related to leverage via (15). The choice of $\hat{\varepsilon}$ is equivalent to choosing leverage ϕ because, even though ϕ is not necessarily monotonic in $\hat{\varepsilon}$, $\phi_{\hat{\varepsilon}\hat{\varepsilon}} < 0$ by (16). We proceed by deriving an expression for expected profits as a function of $\hat{\varepsilon}$.[6]

2.4 Entrepreneurs' expected after-tax profits

Prior to $\tilde{\varepsilon}$ being revealed, the expected after-tax profits of a representative entrepreneur of a given type are $\overline{\Pi} \equiv \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi} g d\tilde{\varepsilon}$. (Recall that for $\tilde{\varepsilon} < \hat{\varepsilon}$, profits are claimed by the bank.) Given the expression for $\widetilde{\Pi}$ in (6), this becomes:

$$\overline{\Pi} = \int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big((1-\tau)\Big(\tilde{\varepsilon} R K + (1-\delta)K - (1+r)B\Big) + \tau(1+\rho)E \Big) g d\tilde{\varepsilon} \tag{17}$$

The entrepreneur takes into account the fact that the interest rate r depends on the zero-profit condition of the bank, which in turn depends upon the bankruptcy risk or leverage he chooses. Using the bank's zero-profit condition (13) to eliminate $(1-t)\int_{\hat{\varepsilon}}^{\varepsilon_{\max}}(1+r)Bgd\tilde{\varepsilon}$ from (17), and using (1) and (2) along with $B = K - E$, (17) may be written after integration as:

$$\overline{\Pi} = \left(\frac{1-\tau}{1-\phi(\hat{\varepsilon}, R, \tau, c)} \Big(\bar{\varepsilon} R - \delta - \rho - cgR\frac{\hat{\varepsilon}^2}{2} \Big) + 1 + \rho \right) E \equiv \overline{\pi}(\hat{\varepsilon}, R, \tau, c) E \tag{18}$$

[6] We could instead have used (15) to determine $\hat{\varepsilon}$ as a function of ϕ, and obtained derivatives of $\hat{\varepsilon}$ with respect to ϕ and the other variables. We could then use ϕ as the choice variable of entrepreneurs. While this would more accurately reflect entrepreneurial choices, it would make the analysis more complicated and would not change the results.

where $\overline{\pi}(\hat{\varepsilon}, R, \tau, c)$ is expected profit per unit of own equity. For future use, differentiate $\overline{\pi}(\cdot)$ in (18) with respect to $\hat{\varepsilon}$ to obtain:

$$\overline{\pi}_{\hat{\varepsilon}} = \frac{1-\tau}{1-\phi(\cdot)}\left(\Delta(\hat{\varepsilon}, R, \tau, c)\left(\overline{\varepsilon}R - \delta - \rho - \frac{cRg\hat{\varepsilon}^2}{2}\right) - c\hat{\varepsilon}gR\right) \tag{19}$$

where

$$\Delta(\hat{\varepsilon}, R, \tau, c) \equiv \frac{\phi_{\hat{\varepsilon}}(\hat{\varepsilon}, R, \tau, c)}{1-\phi(\hat{\varepsilon}, R, \tau, c)} \tag{20}$$

Using the relationship between leverage and bankruptcy risk in (15), $\overline{\Pi} = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E$ satisfies the following lemma.

Lemma 3

$$\overline{\Pi} = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E = \frac{1-\tau}{1-\phi(\cdot)}\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2 E \tag{21}$$

The expression for expected profits in (21) takes into account both the endogeneity of the interest rate r facing the entrepreneur through the bank's zero-profit condition (13) and the relationship between leverage and bankruptcy risk through (15). Since we assume risk-neutrality in this basic model, the expected utility of entrepreneurs, and thus their objective function, is given by $\overline{\Pi} = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E$ in (18) or (21). We make use of both of these in what follows.

3 Behavior of Risk-Neutral Entrepreneurs

Recall that entrepreneurs make two decisions in sequence. First, they decide whether to undertake risky investments, given their productivity R. This is the extensive-margin decision. Then, if they enter, they decide how much to borrow to acquire more capital over and above their own equity, E. This is the intensive-margin decision. Once their shock $\tilde{\varepsilon}$ is revealed, their after-tax profits and therefore ex post utility are determined. We consider the intensive and extensive decisions in reverse order for an entrepreneur of a given type, and continue to suppress the type identifier R for simplicity.

3.1 Choice of leverage: intensive margin

Consider a type$-R$ entrepreneur who decides to enter. As mentioned, given (15) the choice of $\hat{\varepsilon}$ is equivalent to the choice of leverage ϕ, and we use the former as the entrepreneur's choice variable. Differentiating (21) with respect to $\hat{\varepsilon}$, we obtain:

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}}E = \left(\Delta(\hat{\varepsilon}, R, \tau, c) - \frac{2}{\varepsilon_{\max} - \hat{\varepsilon}}\right)\frac{(1-\tau)Rg}{1-\phi}(\varepsilon_{\max} - \hat{\varepsilon})^2 E \tag{22}$$

where $\Delta(\hat{\varepsilon}, R, \tau, c)$ was defined in (20).

Let $\hat{\varepsilon}^*$ be the optimal choice of $\hat{\varepsilon}$. It could be in the interior or it could take on corner solutions at the top or bottom. From (15), $\hat{\varepsilon}^*$ takes on a minimum value of $\hat{\varepsilon}^* = 0$ when $\phi \leqslant \phi(0, R, \tau, c) = (1-\tau)(1-\delta)/(1+\rho) + \tau$. The maximum value of $\hat{\varepsilon}^*$ satisfies $\phi(\hat{\varepsilon}, R, \tau, c) = 1 - E/K_{\max}$, which is assumed to be smaller than $\varepsilon_{\max}$ for any entrepreneur type.

If it is in the interior, $d\overline{\Pi}/d\hat{\varepsilon} = 0$, so by (22) the first-order condition on $\hat{\varepsilon}$ can be written:

$$\Delta(\hat{\varepsilon}^*, R, \tau, c) = \frac{2}{\varepsilon_{\max} - \hat{\varepsilon}^*} > 0 \tag{23}$$

Using this, we obtain the following lemma.

Lemma 4 *For $\hat{\varepsilon}^*$ in the interior, $\phi_{\hat{\varepsilon}} > 0$, $\phi_R > 0$, $\phi_{\hat{\varepsilon}R} > 0$ and, if the second-order condition is satisfied, $d\hat{\varepsilon}^*/dR > 0$.*

Thus, the probability of bankruptcy increases with the productivity R of the entrepreneur. This occurs because entrepreneurs with higher productivity choose higher leverage. Although a higher value of R tends to reduce bankruptcy risk directly through the bankruptcy condition (7), this direct impact on bankruptcy risk is more than offset by the increase in leverage, $\phi_R > 0$.

3.2 Decision to undertake risky investment: extensive margin

Ex ante, entrepreneurs decide whether to undertake the risky investment or to opt for the risk-free option. In the risk-free option, they invest their wealth E at a risk-free return ρ, leading to consumption of $(1+\rho)E$. They enter if their expected after-tax income as an entrepreneur, given by $\overline{\Pi}$ in (18) or (21), is at least as great as their certain income if they invest their wealth in a safe asset and obtain consumption of $(1+\rho)E$, that is,

$$\overline{\Pi} = \overline{\pi}(\hat{\varepsilon}, R, \tau, c)E \geqslant (1+\rho)E \quad \text{or} \quad \overline{\pi}(\hat{\varepsilon}, R, \tau, c) \geqslant 1+\rho \tag{24}$$

Differentiating $\overline{\pi}(\cdot)$ in (21) by R and using $\phi_R > 0$ by Lemma 4, we obtain that $\overline{\pi}(\cdot)$ is increasing in R. Given that $\hat{\varepsilon}$ is being optimized, the cutoff value of R, denoted $\widehat{R}$, will be uniquely determined by $\overline{\pi}(\hat{\varepsilon}, \widehat{R}, \tau, c) = 1+\rho$. Using the expression for $\overline{\pi}$ in (18), the following lemma is apparent.

Lemma 5 *The cutoff value of R is determined by:*

$$\overline{\varepsilon}\widehat{R} - \delta - \rho - \frac{c\widehat{R}g\hat{\varepsilon}^2}{2} = 0 \tag{25}$$

Entrepreneurs with $R > \widehat{R}$ enter the risky sector and earn a rent. Those with $R < \widehat{R}$ invest their wealth in a risk-free asset, so earn no rent. Moreover, as the following lemma shows, bankruptcy risk is zero for marginal entrepreneurs, and positive for all inframarginal ones.

Lemma 6 *The marginal entrepreneur $\widehat{R}$ chooses $\hat{\varepsilon}^* = 0$, while all entrepreneurs $R > \widehat{R}$ choose $\hat{\varepsilon}^* > 0$.*

This has implications for the effect of the cash-flow tax in what follows. To study this, consider first the social optimum as a benchmark.

4 The Social Optimum

To study the efficiency properties of cash-flow business taxation, it is useful to characterize the full-information social optimum, that is, the outcome where $c = 0$ so banks can observe without cost the output of the bankrupt firms. Social surplus includes only the surplus of projects of entrepreneurs who invest in the risky sector since no surplus is generated either by the banks, which earn zero expected profits, or by potential entrepreneurs who invest in the safe outcome and earn $(1+\rho)E$. Expected social surplus can be defined as the expected value of production by entrepreneurs less the opportunity cost of financing their capital. Financing costs include the cost of both debt and equity finance, so are given by $(1+\rho)B + (1+\rho)E = (1+\rho)K$.

For the representative entrepreneur of type$-R$, end-of-period expected social surplus $S(R)$ can be written as follows, using $K = E/(1-\phi)$:

$$S(R) = \int_0^{\varepsilon_{\max}} \Big(\tilde{\varepsilon}RK + (1-\delta)K\Big) g d\tilde{\varepsilon} - (1+\rho)K = \Big(\overline{\varepsilon}R - \delta - \rho\Big)\frac{E}{1-\phi(\cdot)} \qquad (26)$$

where $\phi(\cdot)$ satisfies (15) with $c = 0$ and $\tau = 0$, or:

$$\phi(\hat{\varepsilon}, R, 0, 0) = \frac{1}{1+\rho}\left(\Big(1 - \frac{g\hat{\varepsilon}}{2}\Big)\hat{\varepsilon}R + (1-\delta)\right) \qquad (27)$$

and $\hat{\varepsilon}$ satisfies the bankruptcy condition, (7). Note that $S(R)$ includes the surplus earned by the investments that go bankrupt since this accrues to the banks. This expression for $S(R)$ applies whether taxes are in place or not.

In a social optimum, both the extensive and intensive margins are optimized. Entry is optimized if $S(R) = 0$ for the marginal entrepreneur, or by (26),

$$\overline{\varepsilon}\widehat{R}^o - \delta - \rho = 0 \qquad (28)$$

where $\widehat{R}^o$ is marginal entrepreneur in the social optimum. All entrepreneurs $R \geq \widehat{R}^o$ enter in the social optimum. Changes in leverage, or equivalently in bankruptcy risk, affect $S(R)$ in (26) as follows:

$$\frac{dS(R)}{d\hat{\varepsilon}} = \frac{\phi_{\hat{\varepsilon}}}{(1-\phi)^2}\big(\overline{\varepsilon}R - \delta - \rho\big)E \tag{29}$$

This implies by (28) that $\widehat{R}^o$, $dS(\widehat{R}^o)/d\hat{\varepsilon} = 0$ for the marginal entrepreneur so social surplus is independent of leverage and K. For $R > \widehat{R}^o$, $dS(R)/d\hat{\varepsilon} > 0$ for all $\hat{\varepsilon}$ since $\phi_{\hat{\varepsilon}} > 0$ by Lemma 4. Inframarginal entrepreneurs will therefore maximize leverage and choose $K = K_{\max}$.

As expected, when $c = 0$ so the full-information social optimum is achieved, the cash-flow tax has no effect on market outcomes. It simply diverts rents from inframarginal entrepreneurs to the government. To see this, consider first the extensive-margin decision. When $c = 0$, (25) implies that $\overline{\varepsilon}\widehat{R} - \delta - \rho = 0$ so $\widehat{R}$ is independent of τ. Thus, $\widehat{R} = \widehat{R}^o$ by (28) so entry is socially optimal. Next, consider the effect of the cash-flow tax on leverage. With $c = 0$, (19) implies:

$$\frac{d\overline{\Pi}}{d\hat{\varepsilon}} = \overline{\pi}_{\hat{\varepsilon}}E = \frac{1-\tau}{1-\phi}\Delta(\hat{\varepsilon}, R, \tau, c)(\overline{\varepsilon}R - \delta - \rho)E$$

For the marginal entrepreneur, (28) implies that $\overline{\pi}_{\hat{\varepsilon}} = 0$, so $d\overline{\Pi}/d\hat{\varepsilon}|_{R=\widehat{R}} = 0$. Therefore, leverage ϕ and thus K are indeterminate for the marginal entrepreneur and independent of τ. For inframarginal entrepreneurs, $\overline{\varepsilon}R - \delta - \rho > 0$ since $R > \widehat{R}$, so $\overline{\pi}_{\hat{\varepsilon}}$ has the same sign as $\Delta(\cdot) = \phi_{\hat{\varepsilon}}/(1-\phi)$, which is positive by Lemma 4. Therefore, $\hat{\varepsilon}$ takes its maximum value with $\phi(\hat{\varepsilon}^*, R, \tau, c) = 1 - E/K_{\max}$. Since $\phi_{\hat{\varepsilon}} > 0$ and $\phi_\tau > 0$ by (16), we have that $d\hat{\varepsilon}^*/d\tau < 0$ to keep ϕ constant. While $\hat{\varepsilon}^*$ changes, τ does not distort ϕ or the capital stock $K = K_{\max} = E/(1-\phi)$.

When banks must incur a monitoring cost c to observe the profits of bankrupt firms, the social optimum will not be achieved. We saw above in Lemma 6 that $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur. Therefore by (25), the productivity of the marginal entrepreneur satisfies $\overline{\varepsilon}\widehat{R} - \delta - \rho = 0$, which implies that $\widehat{R} = \widehat{R}^o$ so entry is optimal. At the same time, since $\hat{\varepsilon}^* > 0$ for inframarginal entrepreneurs by Lemma 6, $\hat{\varepsilon}^*$ will be in the interior for large enough values of R, so leverage and therefore investment will be below the maximum level obtained in the social optimum. Thus, while entry is socially optimal in the presence of imperfect information, there is too little investment for entrepreneurs that incur bankruptcy risk.

Before turning to the implications of cash-flow taxation in the imperfect information setting, it is useful to define *constrained social surplus* as social surplus less the costs of monitoring incurred by the banks since the government confronts the same information problem that the banks do. For a type$-R$ entrepreneur, constrained social surplus can be expressed as follows, analogous to (26):

$$\overline{S}(R) = \Big(\overline{\varepsilon}R - \delta - \rho - cRg\frac{\hat{\varepsilon}^2}{2}\Big)\frac{E}{1-\phi(\cdot)} \tag{30}$$

Combining the entrepreneur's expected profits in (17) with the bank's zero profits expression (13), we obtain:

$$\begin{aligned}\overline{\Pi} - (1+\rho)E &= \int_0^{\varepsilon_{\max}} \Big((1-\tau)\big(\tilde{\varepsilon}RK + (1-\delta)K\big) - (1+\rho)(B+E)\Big)gd\tilde{\varepsilon} - (1-\tau)cRKg\frac{\hat{\varepsilon}^2}{2} \\ &= (1-\tau)\overline{S}(R) \end{aligned} \tag{31}$$

In the absence of taxation, maximizing private surplus $\overline{\Pi} - (1+\rho)E$ maximizes constrained social surplus, but that will no longer be the case with $\tau > 0$. We use (31) below to interpret the efficiency consequences of cash-flow taxation in an information-constrained setting.

To summarize, in the full-information social optimum, inframarginal entrepreneurs maximize leverage and choose $K = K_{\max}$, while marginal entrepreneurs are indifferent to the level of K. The cash-flow tax has no effect on entry or leverage, but diverts to the government the rents of inframarginal entrepreneurs. If there are monitoring costs, entry remains optimal and marginal entrepreneurs assume no bankruptcy risk, while some inframarginal entrepreneurs underinvest.

5 Cash-Flow Taxation with Risk-Neutral Entrepreneurs

The model discussed in the previous sections includes both bankruptcy, when entrepreneurs are unable to repay their loans fully, and asymmetric information, in the sense that banks can only verify bankruptcy with costly ex post monitoring. In this section, we consider the effect of (R+F)-base cash-flow taxation on leverage and entry as well as on after-tax profits, tax revenue and social surplus.

5.1 Cash-flow taxation and leverage

The leverage decision for an inframarginal type$-R$ entrepreneur is governed by (22), where $d\overline{\Pi}/d\hat{\varepsilon} = 0$ if $\hat{\varepsilon}^*$ is in the interior. To determine the effect of taxes on leverage, differentiate $\phi\big(\hat{\varepsilon}(\cdot), R, \tau, c\big)$ to obtain:

$$\frac{d\phi}{d\tau} = \phi_{\hat{\varepsilon}}\frac{d\hat{\varepsilon}^*}{d\tau} + \phi_\tau \tag{32}$$

where $\phi_\tau = (1-\phi)/(1-\tau)$ by (16). To evaluate (32), we can use the first-order condition on $\hat{\varepsilon}$, (23), to obtain the following lemma.

Lemma 7 *Assume $\hat{\varepsilon}^*$ is in the interior. Then,*

$$\frac{d\hat{\varepsilon}^*}{d\tau} = 0$$

Thus, the cash-flow tax does not affect bankruptcy risk for firms with $\hat{\varepsilon}^*$ in the interior.

Using Lemma 7 and (16), (32) reduces to

$$\frac{d\phi}{d\tau} = \phi_\tau = \frac{1-\phi}{1-\tau} > 0 \tag{33}$$

While the tax does not affect bankruptcy risk, it does increase leverage and therefore investment. Some explanation for this comes from the following lemma.

Lemma 8 *For $\hat{\varepsilon}$ in the interior,*

$$\frac{dr}{d\tau} < 0 \tag{34}$$

The intuition is that the cash-flow tax allows the banks to claim a refund of the opportunity cost of investment, $\tau(1+\rho)K$, on bankrupt projects. An increase in τ increases the gain that the bank can collect from the bankrupt entrepreneurs, which improves its expected profits and thus leads to a reduction in r. By reducing r, the increase in τ induces entrepreneurs to borrow and therefore invest more.

5.2 Cash-flow taxation and entry

Consider now the extensive-margin decision. The productivity of the marginal entrepreneur $\widehat{R}$ is determined by (25). Since $\hat{\varepsilon}^*$ is independent of τ by Lemma 7, so is $\widehat{R}$ and therefore entry. Therefore, the cash-flow tax is neutral with respect to entry.

5.3 Cash-flow taxation and expected profits

Next, consider the effect of the cash-flow tax on expected profits of a type$-R$ firm. Lemma 3 applies, so $\overline{\pi}$ is given by:

$$\overline{\pi} = \frac{1-\tau}{1-\phi}\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2 \equiv D\frac{Rg}{2}(\varepsilon_{\max} - \hat{\varepsilon})^2 \tag{35}$$

Using (33), $D \equiv (1-\tau)/(1-\phi)$ is independent of τ,[7] so expected profits $\overline{\pi}E$ are as well. Therefore, while the tax increases leverage and therefore investment, it leaves expected

[7] Proof:

$$D_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\phi_\tau = -\frac{1}{1-\phi} + \frac{1-\tau}{(1-\phi)^2}\frac{1-\phi}{1-\tau} = 0 \qquad \blacksquare$$

after-tax profits unchanged. This is analogous to the Domar and Musgrave (1944) result albeit for a different reason in this context since entrepreneurs are risk-neutral. We obtain a similar result below for risk-averse entrepreneurs.

5.4 Cash-flow taxation and expected tax revenue

Expected government revenue from the cash-flow tax can be written, using (4), as:

$$\overline{T} = \tau \int_{\widehat{R}}^{R_{\max}} \left(\bar{\varepsilon}R - \rho - \delta - cRg\frac{\hat{\varepsilon}^2}{2}\right)\frac{E}{1-\phi(\cdot)}dH(R) \equiv \tau\overline{Y} \tag{36}$$

where $\overline{Y}$ is the aggregate expected tax base and $H(R)$ has been defined as the distribution of entrepreneur types. Given from above that neither $\widehat{R}$ nor $\hat{\varepsilon}$ are affected by the tax, differentiating $\overline{T}$ yields:

$$\frac{d\overline{T}}{d\tau} = \overline{Y} + \tau \int_{\widehat{R}}^{R_{\max}} \left(\bar{\varepsilon}R - \rho - \delta - cRg\frac{\hat{\varepsilon}^2}{2}\right)\frac{E}{(1-\phi)^2}\frac{d\phi}{d\tau}dH(R) > 0 \tag{37}$$

where the inequality follows from (33). The first term is the mechanical effect of an increase in the tax rate on revenues which is positive. The second term is also positive given that leverage increases with the tax rate as shown above. Since leverage and therefore investment increase with τ, more rents are created and this induces an increase in the tax base $\overline{Y}$.

5.5 Cash-flow taxation and expected social surplus

Finally, consider the effect of the tax on constrained expected social surplus $\overline{S}$. Using $\overline{\Pi} - (1+\rho)E = (1-\tau)\overline{S}$ from (31), we have:

$$\overline{S} = \frac{\overline{\Pi} - (1+\rho)E}{1-\tau}$$

Since a tax increase leaves $\overline{\Pi} = \overline{\pi}E$ and therefore the numerator unchanged, it will increase $\overline{S}$. In effect, the tax induces the firms to increase leverage while keeping $\hat{\varepsilon}$ constant. Since investment satisfies $K = E/(1-\phi)$ by (2), introducing the tax increases K, and as can be seen from (30), $\overline{S}$ increases. Equivalently, the increase in K holding $\hat{\varepsilon}$ constant generates more pre-tax profits, or rents. The government taxes away those profits, leaving after-tax expected profits unchanged and improving constrained expected social surplus. Thus, while the no-tax outcome replicates the constrained social optimum, implementing a cash-flow tax improves social outcomes without changing firms' expected profits. It does so by breaking the connection between leverage and bankruptcy risk. This reflects the fact that levels of K in the absence of the tax are less than in the unconstrained social optimum for some entrepreneurs as discussed above.

The optimal tax rate for a given entrepreneur-type would be that which just induced the entrepreneur to choose maximum leverage. Since the government cannot observe R, it cannot implement optimal type-specific tax rates.

The main results of the analysis in the base-case model are summarized as follows.

Proposition 1 *With risk-neutral entrepreneurs, equilibrium has the following properties:*

i. Entrepreneurs with productivity R above some threshold level $\widehat{R}$ enter the risky industry and earn a rent. For those with K in the interior, leverage ϕ and bankruptcy risk $\hat{\varepsilon}^*$ are increasing with R.

ii. In the absence of taxation, entry is socially efficient in equilibrium, but leverage and therefore investment are below the full information socially optimal levels.

iii. $(R+F)$-base cash-flow taxation increases leverage, while bankruptcy risk and expected profits remain unchanged, and expected tax revenues increases. Expected rents and expected social surplus both increase, so the cash-flow tax act as a corrective device.

We assumed in this section that the government deployed an (R+F)-base cash-flow tax, or equivalently an ACE corporate tax. In a previous version of this paper (Boadway, Sato and Tremblay, 2016), we considered the case of an R-base cash-flow tax where financial cash-flows are not taxed. If banks are exempt from the tax, marginal entrepreneurs still assume no bankruptcy risk and entry is not affected by the tax. However, bankruptcy risk falls with the tax and the change in leverage is smaller than in the (R+F)-base case. Leverage may even fall in which case the tax would move the equilibrium away from the social optimum. On the other hand, if the real cash-flows of banks were taxable, the same results as in the (R+F)-base cash-flow tax are obtained.

6 Risk-Averse Entrepreneurs

Assume now that entrepreneurs are risk-averse and unable to insure their uncertain project outcomes. Assume also that the government deploys an (R+F)-base cash-flow tax to both entrepreneurs and banks as in our basic model. The key new element in this setting is that because the tax system cannot distinguish returns to risk from rents, the cash-flow tax necessarily applies to both.

Recall that all consumption takes place at the end of the period. Entrepreneurs who enter the risky industry invest all their wealth in their firm at the beginning of the period and consume the after-tax profits $\widetilde{\Pi}$ at the end, where the latter are given by (6) or (8). As in the base case with risk neutrality, entrepreneurs get no income in the event of bankruptcy since that goes to the bank. Let their end-of-period expected utility V be:

$$V = \frac{1}{1-\gamma}\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \widetilde{\Pi}^{1-\gamma} g d\tilde{\varepsilon} \tag{38}$$

where $0 < \gamma < 1$, so the entrepreneur's utility function exhibits constant relative risk aversion. Using (8), (2) and $D = (1-\tau)/(1-\phi)$ in (38), expected utility becomes:

$$V = \frac{1}{1-\gamma}\int_{\hat{\varepsilon}}^{\varepsilon_{\max}} \Big(D(\tilde{\varepsilon}-\hat{\varepsilon})RE\Big)^{1-\gamma} g d\tilde{\varepsilon} = \frac{1}{1-\gamma}\Big(DRE\Big)^{1-\gamma}(\varepsilon_{\max}-\hat{\varepsilon})^{2-\gamma}\frac{g}{2-\gamma} \tag{39}$$

As before, the entrepreneur decides whether to enter, and if so, how much to borrow and therefore how much risk to take on. Consider these decisions in reverse order.

6.1 Intensive-margin decision

The choice of leverage ϕ is again equivalent to the choice of bankruptcy risk $\hat{\varepsilon}$ through (15). Differentiating (39) with respect to $\hat{\varepsilon}$ and using the definition of $\Delta(\cdot)$ in (20), we obtain after straightforward simplification:

$$\frac{dV}{d\hat{\varepsilon}} \ \propto\ \Delta(\hat{\varepsilon}, R, \tau, c) - \frac{2-\gamma}{1-\gamma}\frac{1}{\varepsilon_{\max}-\hat{\varepsilon}} \tag{40}$$

If the optimal choice of $\hat{\varepsilon}$, $\hat{\varepsilon}^*$, is in the interior, $dV/d\hat{\varepsilon} = 0$, and (40) gives:

$$\Delta(\hat{\varepsilon}^*, R, \tau, c) = \frac{2-\gamma}{1-\gamma}\frac{1}{\varepsilon_{\max}-\hat{\varepsilon}^*} > 0 \tag{41}$$

From (41) we obtain the following analogue to Lemma 7.[8]

Lemma 9 *If $\hat{\varepsilon}^*$ is in the interior, then*

$$\frac{d\hat{\varepsilon}^*}{d\tau} = 0 \qquad \text{and} \qquad \frac{d\hat{\varepsilon}^*}{d\gamma} < 0$$

Note that Lemma 4 applies here as well, so bankruptcy risk and therefore leverage are increasing in entrepreneurial productivity: $d\hat{\varepsilon}^*/dR > 0$.

[8] We have assumed that entrepreneurs who enter invest all of their initial wealth E in the firm. Risk-averse entrepreneurs may choose to invest only a portion of their wealth, say, α, in the firm to diversify their risk. Suppose entrepreneurs choose both $\hat{\varepsilon}$ and α. It is straightforward to show that Lemma 9 applies to both α and $\hat{\varepsilon}$ if they are in the interior. We omit α as a choice variable for simplicity.

6.2 Extensive-margin decision

The entry decision involves comparing expected utility as an entrepreneur with that obtained from the safe alternative yielding end-of-period consumption of $(1+\rho)E$. Entrepreneurs will enter if $V \geqslant \big((1+\rho)E\big)^{1-\gamma}/(1-\gamma)$. From (39), V is increasing in R, so the cutoff value $\widehat{R}$ will be uniquely determined.

To characterize the extensive-margin decision, we show first that in equilibrium $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur, $\widehat{R}$. The following lemma indicates when $\hat{\varepsilon}^* = 0$.

Lemma 10

$$\hat{\varepsilon}^* = 0 \qquad \textit{if} \qquad \frac{\overline{\varepsilon}R}{\delta+\rho} \leq \frac{1}{2}\frac{2-\gamma}{1-\gamma} \tag{42}$$

Let R_0 be the value of R such that $\hat{\varepsilon}^*$ just becomes zero. Then, by (42), R_0 satisfies the following:

$$\frac{\overline{\varepsilon}R_0}{\delta+\rho} = \frac{1}{2}\frac{2-\gamma}{1-\gamma} \tag{43}$$

so is increasing in γ. For any R, higher risk-aversion will lead an entrepreneur to choose lower leverage and bankruptcy risk. Therefore, the value of R at which bankruptcy risk just becomes zero will increase with the degree of risk aversion.

Next, we can establish the following relationship between R_0 and the productivity of the marginal entrepreneur, $\widehat{R}$.

Lemma 11 $\widehat{R} \leqslant R_0 \quad \textit{as} \quad \gamma \geqslant 0$

The implication of Lemma 11 is that $\hat{\varepsilon}^* = 0$ at $R = \widehat{R}$ as long as risk-aversion γ is non-negative, and this applies regardless of the values of c or τ. This result nests what we found earlier in the risk-neutral model with $\gamma = 0$ where the marginal entrepreneur choose $\hat{\varepsilon}^* = 0$ as well. However, in the risk-neutral case, $\widehat{R} = R_0$, so $\hat{\varepsilon}^* > 0$ for all inframarginal entrepreneurs.

Lemma 11 implies that with $\gamma > 0$, $\hat{\varepsilon}^* = 0$ for all entrepreneurs with $R \in [\widehat{R}, R_0]$. In that range, there is no risk of bankruptcy, so $r = \rho$ by the banks' zero-profit condition (13) and ex post after-tax profits in (6) can be written, using $B = K - E = E/(1-\phi) - E$:

$$\widetilde{\Pi} = \Big(\frac{1-\tau}{1-\phi}\big(\tilde{\varepsilon}R - \delta - \rho\big) + 1 + \rho\Big)E = \big(D(\tilde{\varepsilon}R - \delta - \rho) + 1 + \rho\big)E$$

where, recall, $D \equiv (1-\tau)/(1-\phi)$. Therefore, expected utility can be written:

$$V = \int_0^{\varepsilon_{\max}} \frac{\widetilde{\Pi}^{1-\gamma}}{1-\gamma} g d\tilde{\varepsilon} = \frac{E^{1-\gamma}}{1-\gamma}\int_0^{\varepsilon_{\max}} \big(D(\tilde{\varepsilon}R - \delta - \rho) + 1 + \rho\big)^{1-\gamma} g d\tilde{\varepsilon} \tag{44}$$

The expression for V in (44) applies for all $R \in [\widehat{R}, R_0]$. In this range, an entrepreneur of given R will choose leverage, or equivalently $D \equiv (1-\tau)/(1-\phi)$, to maximize V. The first-order condition $\partial V/\partial D = 0$ reduces to:

$$\int_0^{\varepsilon_{\max}} (\tilde{\varepsilon}R - \delta - \rho)\big(D^*(\tilde{\varepsilon}R - \delta - \rho) + 1 + \rho\big)^{-\gamma} g d\tilde{\varepsilon} = 0 \tag{45}$$

where D^* is the optimal choice of D. This value of D^*, and therefore ϕ will be increasing in R. As $R \to R_0$, $\phi \to (1-\tau)(1-\delta)/(1+\rho) + \tau$, which is the value of ϕ that just satisfies (15) when $\hat{\varepsilon} = 0$.

The extensive margin is determined by value of $\widehat{R}$ where $V = (1+\rho)^{1-\gamma} E^{1-\gamma}/(1-\gamma)$ or using (44) with $D = D^*$,

$$\int_0^{\varepsilon_{\max}} \big(D^*(\tilde{\varepsilon}\widehat{R} - \delta - \rho) + 1 + \rho\big)^{1-\gamma} g d\tilde{\varepsilon} = (1+\rho)^{1-\gamma} \tag{46}$$

Entrepreneurs will enter the risky sector only if $R \geqslant \widehat{R}$. Those with $R < \widehat{R}$ will invest their wealth in a safe asset.

6.3 The effects of cash-flow taxation

We next turn to the effects of (R+F)-base cash-flow taxation on entrepreneurs' decisions and equilibrium outcomes when entrepreneurs are risk-averse. We begin with the intensive- and extensive-margin decisions, and then consider the effects on expected profit and on government revenues.

6.3.1 Intensive-margin decision

By Lemma 11, $\widehat{R} < R_0$ since $\gamma > 0$. Consider first entrepreneurs in the range $R \in [\widehat{R}, R_0]$. As we have seen, for these entrepreneurs, $\hat{\varepsilon}^* = 0$ and the first-order condition (45) applies. This yields a unique value of D^* for each R which is independent of the tax rate. Since $D^* = (1-\tau)/(1-\phi)$, leverage will rise by the same amount as an increase in τ. Given that D^* is invariant with the tax rate, V will be as well as seen by (44). Thus, for entrepreneurs in this range, a result analogous to Domar-Musgrave (1944) applies. Entrepreneurs offset an increase in τ by increasing private risk-taking, while achieving the same level of expected utility V. The increase in leverage ϕ entails more investment and therefore more rent generation, which accrues to the government in increased tax revenues as discussed below.

For entrepreneurs of type $R > R_0$, $\hat{\varepsilon}^* > 0$ so they incur some bankruptcy risk. Their intensive-margin decision is similar to the base case with risk-neutrality above. Eq. (32) again applies, and since $d\hat{\varepsilon}^*/d\tau = 0$ by Lemma 9 (32) simplifies to $d\phi/d\tau = \phi_\tau$. By (16),

$\phi_\tau = (1-\tau)/(1-\phi)$, so τ encourages leverage. However, the increase in leverage does not translate into an increase in bankruptcy risk $\hat{\varepsilon}$, since $d\hat{\varepsilon}^*/d\tau = 0$. As well, as we have shown in footnote 7, $D = (1-\tau)/(1-\phi)$ is constant when $\phi_\tau = (1-\tau)/(1-\phi)$. Therefore, with D and $\hat{\varepsilon}$ constant, so is expected utility V in (39) as long as leverage ϕ and K are in the interior. This again is analogous to the Domar and Musgrave (1944) result on portfolio investment: a tax on capital income with full loss-offset encourages risk-taking by risk-averse individuals because the government is sharing the risk with them on actuarially fair terms. In our case of an (R+F)-base cash-flow tax on firms, the government is sharing the risk of the entrepreneur, and as a result the tax does not affect private risk $\hat{\varepsilon}$.

6.3.2 Extensive-margin decision

Since $\widehat{R} < R_0$ we have that $\hat{\varepsilon}^* = 0$ for the marginal entrepreneur. For a given tax rate τ, the marginal entrepreneur $\widehat{R}$ chooses leverage such that the first-order condition (45) is satisfied at $R = \widehat{R}$. This leads to a unique value of the optimal $D^* = (1-\tau)/(1-\phi)$ regardless of the tax rate. Recall that $\widehat{R}$ satisfies (46). Since the choice of D^* by the marginal entrepreneur is independent of the tax rate, the value of $\widehat{R}$, and therefore the extensive-margin decision, that satisfies (46) is independent of τ.

6.3.3 Expected profits and tax revenues

As well as V being unaffected by the cash-flow tax, so are expected profits $\overline{\Pi}$. To see this, consider the expression for the expected rate of return on equity, $\overline{\pi}$, in (21). Since both $\hat{\varepsilon}^*$ and $D^* = (1-\tau)/(1-\phi)$ are independent of τ, $\overline{\pi}$ will also be invariant with τ. Therefore, entrepreneurs are able to offset the effect of the tax on their expected after-tax profits by increasing their leverage. The increase in leverage will correspond to an increase in borrowing and investment, which in turn will increase expected before-tax profits and therefore expected tax revenue to the government since after-tax profits are unchanged.

To see that expected tax revenues will rise, note that (36) still applies, and the change in expected tax revenues will again be given by (37), which is positive since $d\phi/d\tau > 0$. Expected tax revenues rise due both to a mechanical effect and to an increase in the tax base because of increased leverage and investment. Social welfare will increase if the increase in expected tax revenues is valuable to the government, and that depends on how the government evaluates the increase in risk that might accompany the tax revenues.

Our findings with risk-averse entrepreneurs are summarized as follows.

Proposition 2 *With risk-averse entrepreneurs subject to (R+F)-base cash-flow taxation, equilibrium has the following properties:*

i. *There is a range of entrepreneurs with* $R \in [\widehat{R}, R_0]$ *for whom there is no risk of bankruptcy, so* $\hat{\varepsilon}^* = 0$ *and* $r = \rho$.

ii. *For all entrepreneurs with* $\hat{\varepsilon}^*$ *below the maximum, leverage increases with the cash-flow tax rate, while bankruptcy risk, expected profits and expected utility are unchanged.*

iii. *The (R+F)-base cash-flow tax is neutral with respect to entry decisions.*

iv. *Expected tax revenues increase with the tax rate.*

7 Concluding Remarks

In this paper, we analyzed the impact of cash-flow business taxation on entrepreneurs' decision of whether to enter a risky industry and, if so, how much to borrow when entrepreneurs face bankruptcy risk, and when there is asymmetric information between entrepreneurs and financial intermediaries. The neutrality of cash-flow taxation found by Bond and Devereux (1995, 2003) in the absence of asymmetric information and risk-aversion no longer applies under these features. Moreover, there is too little investment in the information-constrained social optimum compared with the full-information case. The issue is then whether cash-flow taxation improves social efficiency. We assume that cash-flow taxation is of the (R+F)-base sort so it applies both to entrepreneurs and banks.

With risk-neutral entrepreneurs, cash-flow taxation taxes rents only. When banks must undertake costly monitoring of firms that declare bankruptcy, the tax does not affect entry decisions, given that the marginal entrepreneur earns no rent, but it does affect leverage and therefore investment. In particular, the tax will actually increase social welfare. By inducing firms to increase leverage, and therefore investment, the cash-flow tax leads to higher pre-tax profits, or rents, without affecting bankruptcy risk. These additional pre-tax profits are taxed away by the government leading to a higher constrained social surplus. By inducing more investment, the cash-flow tax is implicitly correcting for the inefficiencies of the information-constrained outcome.

When entrepreneurs are risk-averse, cash-flow taxation taxes both rents and return to risk. The cash-flow tax does not affect the entry decision and increases leverage, while leaving bankruptcy risk unchanged as in the risk-neutral case. In addition, neither expected profits nor expected utility are affected. Expected tax revenues of the government increase, and the government may incur greater risk if it is unable to pool risk better then the private sector. This is analogous to the results of Domar and Musgrave (1944) and Atkinson and Stiglitz (1980).

We have assumed that asymmetric information involved moral hazard rather than adverse selection, so banks can observe firm types but not their profits. If banks cannot observe the productivity of entrepreneurs ex ante, there will be an adverse selection problem as in Stiglitz and Weiss (1981) and de Meza and Webb (1987), among others. In this case, if banks cannot offer separating contracts, all entrepreneurs face the same interest rate. In contrast to the case considered here, the equilibrium without taxation will be inefficient along the extensive margin and cash-flow taxation will generally not be neutral. In particular, there will be excessive entry by the least-productive entrepreneurs to take advantage of the favorable interest rate. A cash-flow tax will discourage entry, thereby improving efficiency. It would be interesting to consider the case where banks are able to offer contracts in which the interest rate varies with the size of loan, so firms can be separated by type. There will be informational rents that might influence the effect of cash-flow taxes.

Appendix

Proof of Lemma 1

Using (7) and integrating, (13) can be written:

$$(1-\tau)(1+\rho)B = \big((1-\tau)(\varepsilon R+(1-\delta))K+\tau(1+\rho)E\big)(1-g\hat{\varepsilon})+(1-\tau)RK\frac{g\hat{\varepsilon}^2}{2}$$

$$+\big((1-\tau)(1-\delta)+\tau(1+\rho)\big)Eg\hat{\varepsilon}-(1-\tau)\frac{cRKg\hat{\varepsilon}^2}{2}$$

$$=(1-\tau)\Big(\hat{\varepsilon}(1-g\hat{\varepsilon})+\frac{g\hat{\varepsilon}^2}{2}\Big)RK+(1-\tau)(1-\delta)K+\tau(1+\rho)E-(1-\tau)\frac{cRKg\hat{\varepsilon}^2}{2}$$

Using $E = K - B$, this becomes

$$(1+\rho)B = (1-\tau)\Big(1-\frac{g\hat{\varepsilon}}{2}-\frac{cg\hat{\varepsilon}}{2}\Big)\hat{\varepsilon}RK+(1-\tau)(1-\delta)K+\tau(1+\rho)K$$

Using $\phi = B/E$,

$$(1+\rho)\phi = (1-\tau)\Big(1-\frac{g\hat{\varepsilon}}{2}-\frac{cg\hat{\varepsilon}}{2}\Big)\hat{\varepsilon}R+(1-\tau)(1-\delta)+\tau(1+\rho) \qquad \blacksquare$$

Proof of Lemma 2

Differentiate (9) with respect to $\hat{\varepsilon}$ and r, and use $\phi_r = 0$ to obtain:

$$0 = Rd\hat{\varepsilon}-\phi dr-\frac{1}{\phi}\Big[\hat{\varepsilon}R+(1-\delta)+\frac{\tau(1+\rho)}{1-\tau}\Big]\phi_{\hat{\varepsilon}}$$

Using $\phi_{\hat{\varepsilon}}$ from (16) gives:

$$0 = Rd\hat{\varepsilon}-\phi dr-\frac{1}{\phi}\Big[\hat{\varepsilon}R+(1-\delta)+\frac{\tau(1+\rho)}{1-\tau}\Big]\frac{1-\tau}{1+\rho}\big(1-(1+c)g\hat{\varepsilon}\big)R$$

This can be rewritten using (15) as:

$$\frac{\phi^2}{R}\frac{dr}{d\hat{\varepsilon}} = \frac{1-\tau}{1+\rho}\left(\left(1-\frac{1+c}{2}g\hat{\varepsilon}\right)\hat{\varepsilon}R+(1-\delta)\right)+\tau-\left[\hat{\varepsilon}R+(1-\delta)+\frac{\tau(1+\rho)}{1-\tau}\right]\frac{1-\tau}{1+\rho}\big(1-(1+c)g\hat{\varepsilon}\big)$$

which simplifies to:

$$\frac{\phi^2}{R}\frac{dr}{d\hat{\varepsilon}} = (1+c)g\hat{\varepsilon}\left(\tau+\frac{1-\tau}{1+\rho}\left((1-\delta)+\frac{\hat{\varepsilon}}{2}R\right)\right) \ > \ 0 \qquad \blacksquare$$

Proof of Lemma 3

Rewrite $\overline{\pi}$ in (18) as

$$\overline{\pi}(\hat{\varepsilon},R,\tau,c) = \frac{1}{1-\phi}\left((1-\tau)\left(R\overline{\varepsilon}-\delta-\rho-\frac{cRg\hat{\varepsilon}^2}{2}\right)+(1+\rho)(1-\phi)\right)$$

From (15), we obtain

$$(1+\rho)(1-\phi) = -(1-\tau)\left(1-\frac{g\hat{\varepsilon}}{2}-\frac{cg\hat{\varepsilon}}{2}\right)\hat{\varepsilon}R+(1-\tau)(\rho+\delta)$$

Substituting this in the expression for $\overline{\pi}$ gives, using $\overline{\varepsilon} = \varepsilon_{\max}/2$ and $\varepsilon_{\max} = 1/g$:

$$\overline{\pi}(\hat{\varepsilon},R,\tau,c) = \frac{1-\tau}{1-\phi}R\left(\overline{\varepsilon}-\hat{\varepsilon}\left(1-\frac{g}{2}\hat{\varepsilon}\right)\right) = \frac{1-\tau}{1-\phi}\frac{Rg}{2}(\hat{\varepsilon}-\varepsilon_{\max})^2 \qquad \blacksquare$$

Proof of Lemma 4

Since $\Delta(\hat{\varepsilon}^*,R,\tau,c) > 0$ for $\hat{\varepsilon}^*$ in the interior by (23), $\phi_{\hat{\varepsilon}} > 0$ by (20). Therefore, $\phi_R > 0$ and $\phi_{\hat{\varepsilon}R} > 0$ by (16). The second-order condition on $\hat{\varepsilon}$ is:

$$\Delta_{\hat{\varepsilon}}(\hat{\varepsilon},R,\tau,c) - \frac{2}{(\varepsilon_{\max}-\hat{\varepsilon})^2} \ < \ 0$$

Differentiate the first-order condition (23), and use $\Delta_R = \phi_{\hat{\varepsilon}R}/(1-\phi)+\phi_{\hat{\varepsilon}}\phi_R/(1-\phi)^2 > 0$ and the second order-conditions on $\hat{\varepsilon}$ to obtain $d\hat{\varepsilon}^*/dR > 0$. $\blacksquare$

Proof of Lemma 6

For the marginal entrepreneur, Lemma 5 implies by (19) that $\overline{\pi}_{\hat{\varepsilon}} < 0$ for $\hat{\varepsilon} > 0$. Therefore, the marginal entrepreneur chooses $\hat{\varepsilon}^* = 0$ and incurs no bankruptcy risk. Then, with $\hat{\varepsilon} = 0$, $\overline{\pi}_{\hat{\varepsilon}}$ in (19) becomes zero. Increasing R above $\widehat{R}$ than causes $\overline{\pi}_{\hat{\varepsilon}}$ to become positive, so $\hat{\varepsilon}^*$ increases above zero. $\blacksquare$

Proof of Lemma 7

Differentiating $\Delta(\hat{\varepsilon}, \tau, c) = \phi_{\hat{\varepsilon}}/(1-\phi)$, we have $\Delta_\tau = \phi_{\hat{\varepsilon}\tau}/(1-\phi) + \phi_\tau \phi_{\hat{\varepsilon}}/(1-\phi)^2$. Using (16) for ϕ_τ, $\phi_{\hat{\varepsilon}}$ and $\phi_{\hat{\varepsilon}\tau}$,

$$\Delta_\tau = -\frac{1 - g\hat{\varepsilon} - cg\hat{\varepsilon}}{(1+\rho)(1-\phi)}R + \frac{1-\phi}{1-\tau}\frac{1-\tau}{1+\rho}(1 - g\hat{\varepsilon} - cg\hat{\varepsilon})R\frac{1}{(1-\phi)^2} = 0 \tag{47}$$

Since $\Delta_\tau = 0$, the solution of first-order condition (23) for $\hat{\varepsilon}^*$ is independent of τ. ■

Proof of Lemma 8

Differentiate (9) with respect to τ and use $d\hat{\varepsilon}/d\tau = 0$ by Lemma 7 to obtain:

$$-\Big(\hat{\varepsilon}R + (1-\delta) - (1+\rho) + (1+r)\phi_\tau\Big)d\tau - \phi dr = 0$$

Since $\phi_\tau = (1-\phi)/(1-\tau)$ by (16), this becomes:

$$\frac{dr}{d\tau} = -\frac{1}{\phi}\Big(\hat{\varepsilon}R - (\delta+\rho) + (1+r)\frac{1-\phi}{1-\tau}\Big)$$

Using (9), this can be written:

$$\frac{dr}{d\tau} = -\frac{1}{\phi}\frac{r-\rho}{1-\tau}$$

which is negative since $r > \rho$ by the bank's zero-profit condition. ■

Proof of Lemma 9

Note that τ enters into (41) through $\Delta(\cdot) = \phi_{\hat{\varepsilon}}/(1-\phi)$. Differentiating $\Delta(\cdot)$ with respect to τ and using (16), we obtain:

$$\frac{\partial\Delta}{\partial\tau} = \frac{\phi_{\hat{\varepsilon}\tau}}{1-\phi} + \frac{\phi_{\hat{\varepsilon}}}{(1-\phi)^2}\phi_\tau = -\frac{1 - g\hat{\varepsilon} - cg\hat{\varepsilon}}{(1+\rho)(1-\phi)}R + \frac{1-\tau}{1+\rho}\frac{1 - g\hat{\varepsilon} - cg\hat{\varepsilon}}{(1-\phi)^2}R\phi_\tau = 0$$

Therefore, Δ remain unchanged when τ changes as long as the solution is in the interior, so $\hat{\varepsilon}^*$ not affected by changes in τ. Then, differentiation of (41) gives $\partial\hat{\varepsilon}^*/\partial\gamma > 0$. ■

Proof of Lemma 10

We have that $\hat{\varepsilon}^* = 0$ if $dV/d\hat{\varepsilon}|_{\hat{\varepsilon}=0} \leq 0$, or $\Delta(0, R, \tau, c) \leq (2-\gamma)/((1-\gamma)2\overline{\varepsilon})$ by (40), using (1). By (20) and using (15) and (16), we obtain

$$\Delta(0, R, \tau, c) = \frac{\phi_{\hat{\varepsilon}}(0, R, \tau, c)}{1 - \phi(0, R, \tau, c)} = \frac{(1-\tau)R/(1+\rho)}{1 - (1-\tau)(1-\delta)/(1+\rho) - \tau} = \frac{R}{\rho+\delta}$$

Eq. (42) follows immediately. ■

Proof of Lemma 11

Using (21) for $\overline{\pi}(\hat{\varepsilon}, R, \tau, c)$, expected utility in (39) may be written:

$$V = \frac{E^{1-\gamma}}{1-\gamma}\overline{\pi}(\hat{\varepsilon}, R, \tau, c)^{1-\gamma}(\varepsilon_{\max} - \hat{\varepsilon})^{\gamma}\frac{2^{1-\gamma}g^{\gamma}}{2-\gamma}$$

At $\hat{\varepsilon} = 0$ and using $\varepsilon_{\max} = 1/g$, this becomes:

$$V_0 = \frac{E^{1-\gamma}}{1-\gamma}\overline{\pi}(0, R_0, \tau, c)^{1-\gamma}\frac{2^{1-\gamma}}{2-\gamma}$$

Using (21) for $\overline{\pi}$, (15) for ϕ, and (43), this may be written after manipulation:

$$V_0 = \frac{(1+\rho)^{1-\gamma}E^{1-\gamma}}{1-\gamma}\frac{1}{(1-\gamma)^{1-\gamma}(2-\gamma)^{\gamma}} \tag{48}$$

Recall that for the marginal entrepreneur, $\widehat{R}$,

$$V(\widehat{R}) = \frac{(1+\rho)^{1-\gamma}E^{1-\gamma}}{1-\gamma}$$

We have that $\widehat{R} \leqslant R_0$ if $V(\widehat{R}) \leqslant V_0$, or from (48) if $(1-\gamma)^{1-\gamma}(2-\gamma)^{\gamma} \leqslant 1$, or equivalently $\gamma \geqslant 0$, where the equality applies for $\gamma = 0$. ∎

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