Optimal income taxation and unemployment benefit with involuntary unemployment*

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Abstract

We consider optimal income taxation and search-contingent unemployment benefits in a model where there are both voluntary and involuntary unemployed households, where households' labor productivity and labor disutilities are heterogeneous, and households choose whether to participate and seek jobs in the labor market. We derive an optimal employment tax rule. We show that employment tax rates depend on the size of the search-contingent unemployment benefit. Our numerical simulations suggest that negative employment tax rates for some income groups with low consumption are less likely to be negative.

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1 Introduction

Recent literature has emphasized the role of an extensive margin of labor. With an extensive margin of labor, households choose whether to participate in the labor force with fixed hours of work. In this setting, it can be optimal to adopt wage subsidies or in-work credits with a negative participation tax for low-income households (see, e.g., Diamond (1980), Saez (2002), Choné and Laroque (2005, 2011), Jacquet et al. (2013), and Christiansen (2015)).

The most simple setting for the extensive margin does not consider the demand side of the labor market. This means that all households who choose to participate in the labor force can find employment, and there is no involuntary employment. Some studies develop optimal income taxation with involuntary employment. First, Boone and Bovenberg (2004, 2006, 2013) consider that households bear a search cost when they enter the labor market

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and seek jobs, and households choose work effort as well as search effort with quasi-linear preferences for leisure. Boone and Bovenberg (2004) show that with exogenous welfare benefit, the negative marginal tax rate and welfare benefit have a U-shaped relationship. Additionally, search subsidies (negative employment taxes) are likely to be optimal if the welfare benefit and the government's required revenue is low while search costs are high. Second, Hungerbuhler et al. (2006), Lehmann et al. (2011), and Jacquet et al. (2014) consider a search-matching framework in which the wage is determined by a bargaining process. Jacquet et al. (2014) consider wages are determined by Kalai bargaining and show that this matching environment induces significantly lower employment tax rates than the usual competitive model with endogenous participation only.

Although these studies have provided influential findings with both theoretical and policy implications, a large component of the literature does not focus on the role of wage subsidies (or in-work credits) and unemployment benefits. In the real world, governments employ wage subsidies for low-income workers as well as unemployment benefits for both voluntary and involuntary unemployed households. Governments often restrict the qualifications for receiving benefits to those who engage in a job search. With involuntary employment, the search-contingent unemployment benefit could be a more efficient policy to encourage labor market participation than wage subsidies or in-work credits. A recent paper by Boadway and Cuff (2018) only investigates the joint design of a piecewise linear income tax/transfer and unemployment benefits with a search-matching framework. The authors' model adopts the extensive margin under which households choose whether to participate in job search and the intensity of that search. The authors find analytically that the optimal employment tax depends on the relative size of the transfer for the involuntarily unemployed and the voluntarily unemployed. Instead, we adopt a simple search cost model in which households are heterogeneous about labor productivities, labor disutilities, and search costs, and the search-contingent unemployment benefit is endogenous. This model is a simple extension of the original extensive margin model used in Diamond (1980) and Saez (2002) (hereafter referred to as the full employment model) in which households are heterogeneous about labor productivities and disutilities. We use this model to consider involuntary unemployment and to compare both a full employment model and an extensive model with search and involuntary unemployment.

Thus, we examine the relation and the role of wage subsidies and unemployment benefit in the extensive model with search and involuntary unemployment both analytically and numerically. We derive the following results. First, in the case of involuntary unemployment, we identify the optimal employment tax rule, which depends on the optimal search-contingent unemployment benefit. In contrast to the full employment model, negative employment tax rates for households with lower consumption are less likely to occur. Second, the optimal search-contingent unemployment benefit should be higher when the effect of the benefit on the population of working households is greater. Finally, we numerically solve the optimal tax policies for example economies. We confirm that employment tax rates and search-contingent unemployment benefit have a positive correlation. Negative employment tax rates (in-work credits or wage subsidies) are valid only when the employment probability is close to unity or the search cost is close to zero. This suggests that the search-contingent unemployment benefit is a substitute for in-work credits.

The remainder of the paper is organized as follows. In Section 2, we present a model with an extensive margin of labor and involuntary unemployment. In Section 3, we investigate the optimal income tax problem with involuntary unemployment and derive optimal employment tax rates in the case of partial optimal taxation and full optimal taxation. In the former case, the government only employs labor income tax/transfers. In the latter case, the government employs labor income tax/transfers as well as unemployment benefit. In Section 4, we consider an example economy with various parameters and numerically determine optimal income tax policies and optimal search-contingent unemployment benefits. In Section 5, we present our conclusions.

2 The model

The economy is populated by a government and one unit of continuous households with an indivisible labor supply. Each household is heterogeneous about the two-dimensional characteristic $\theta = (w, \delta)$ where $w \in \mathbb{R}_+$ and $\delta \in \mathbb{R}$, respectively, represent labor productivity and labor disutility. There are $I \in \mathbb{N}$ possible levels of labor productivity. The domain of labor productivity is denoted by $\Omega = \{w_1, w_2, \cdots, w_I\} \in \mathbb{R}^I_{++}$ where $(w_1 < w_2 < \cdots < w_I)$. Let $F : \Omega \times \mathbb{R} \to \mathbb{R}_+$ denote the cumulative probability distribution function (c.d.f) of the population. The c.d.f. is public information.

There are I labor markets for each labor productivity. A household with productivity w_i cannot mimic another productivity w_j $(i \neq j)$. A household chooses labor participation status $s \in \{0, 1\}$ given its θ. If a household with productivity w_i decides to participate $(s = 1)$ in the labor market of productivity w_i, that household suffers the utility cost of job search, Ψ_i, and has the probability of job matches, $p_i \in [0, 1]$. If the household pays the search cost and receives a job offer, then the household works fixed hours and earns w_i units of (before-tax) labor income and suffers the utility cost of labor, δ. If the household pays the search cost but does not find a job, then the household does not get w_i and does not pay δ. If the household does not participate $(s = 0)$, their labor income is zero, and the household suffers no search cost and disutility of labor. The labor productivity is known to the public if and only if the household works. Note that nonworking households in this economy are composed of both the involuntary unemployed and the voluntary unemployed.

There is a divisible consumption good. Let $U(c)$ denote the household's utility from consumption, c. U is a twice continuously differentiable utility function satisfying $U' > 0$ and $U'' < 0$. All households have no real assets at the beginning of the economy.

The government's policy is a tax on labor income and unemployment benefits conditional on the job search. Let $T(w_i)$ and $-T(0)$ denote the labor income tax of working households with productivity w_i and the uniform subsistence income for a nonworking household, respectively. $T : \{0\} \cup \Omega \to \mathbb{R}$ represents a labor income tax schedule. In addition, the government provides unemployment benefits conditional on a job search, b. This unemployment benefit is paid only if a household searched for a job in the labor market and was not employed. We assume that the government can monitor households' job search activities, but a job offer is not observable. If $U(-T(0) + b) - \Psi_i > U(-T(0))$, some low-productivity workers pretend to be involuntary unemployed. Thus, we assume the following incentive compatible conditions hold for all i:

$$U(-T(0) + b) - \Psi_i \leq U(-T(0)). \tag{1}$$

Given $T(0)$, (1) determines an upper bound of b. The condition (1) also implies no rejection of a job offer.

Given labor income tax schedule T, search-contingent unemployment benefit b, and individual characteristic θ, the household chooses labor participation status $s \in \{0, 1\}$ to maximize the expected utility,

$$s[p_i[U(w_i - T(w_i)) - \Psi_i - \delta] + [1 - p_i][U(-T(0) + b) - \Psi_i)] + [1 - s]U(-T(0)).$$

Let V_i denote the difference of expected utility between a household that participates in labor market i and a household that does not participate divided by p_i as follows:

$$V_i(p_i, T, b) = U(w_i - T(w_i)) + \frac{[1 - p_i]}{p_i} U(-T(0) + b) - \frac{\Psi_i + U(-T(0))}{p_i}. \tag{2}$$

Suppose a household has a δ smaller than V_i. The household's expected utility is greater at $s = 1$. Thus, the optimal labor participation plan s is 1 (resp. 0) if δ is smaller (resp. larger) than V_i.

The population of households who participate in labor market i, denoted by k_i, is,

$$k_i = \int_{\theta \in \Theta_i} dF(\theta), \quad i = 1, 2, \cdots, I, \tag{3}$$

where Θ_i denotes the region of θ defined by

$$\Theta_i \equiv \{(w, \delta) \mid w = w_i \text{ and } \delta < V_i\}.$$

Thus, the population of households with earning (before-tax) labor income w_i, denoted by n_i, and the population of nonworking households, denoted by n_0, are,

$$n_i = p_i \times k_i, \quad i = 1, 2, \cdots, I,$$

$$n_0 = 1 - \sum_{i=1}^{I} n_i.$$

The following symmetric relations hold:

$$\frac{\partial n_i}{\partial T(w_i)} = -\frac{\partial n_0}{\partial T(w_i)} \leq 0, \quad i = 1, 2, .., I,$$
$$\frac{\partial n_0}{\partial T(0)} = -\sum_{i=1}^{I} \frac{\partial n_i}{\partial T(0)} \geq 0.$$

Let EU denote the unconditional expected utility as follows:

$$\begin{aligned} EU &= \sum_{i=1}^{I} [1 - k_i] U(-T(0)) \\ &\quad + \sum_{i=1}^{I} k_i \left[p_i [U(w_i - T(w_i)) - \mathbb{E}_i(\delta)] + [1 - p_i] U(-T(0) + b) - \Psi_i \right] \\ &= \sum_{i=1}^{I} \left[U(-T(0)) + p_i k_i \left[V_i - \mathbb{E}_i(\delta) \right] \right], \end{aligned} \tag{4}$$

where $\mathbb{E}_i(\delta)$ denotes $\mathbb{E}(\delta | w = w_i$ and $\delta \leq V_i)$. We have $\frac{\partial EU}{\partial T(w_i)} = -p_i k_i U'$ for $i = 1, \ldots, I$ where U' represents the average marginal utility of consumption over working households with w_i. The government must finance an exogenous expenditure G. The government's budget constraint is given by,

$$\sum_{i=1}^{I} k_i \left[p_i T(w_i) - [1 - p_i] b \right] + n_0 T(0) \geq G. \tag{5}$$

The optimal taxation problem for the government is to find the labor income tax schedule T and the job search-contingent unemployment benefit b that maximizes the household's unconditional expected utility, EU, subject to the budget constraint (5) and IC condition (1).

We consider two types of optimal taxation problems: *the full optimal taxation problem* and *the partial optimal taxation problem*. The former corresponds to the labor income tax schedule T and the job search-contingent unemployment benefit pair b that exactly maximize EU subject to (5) and IC condition (1). However, the actual unemployment benefit system does not necessarily depend on households' job search activity. Thus, we also investigate the latter problem, which solves the same maximization problem but with the additional constraint that the government does not employ the unemployment benefit ($b = 0$).

3 Employment tax rate

We first consider the case of partial optimal taxation. Then, we examine the full optimal taxation problem.

3.1 Partial optimal taxation

For the partial optimal taxation problem, the government does not employ the unemployment benefit ($b = 0$). The Lagrangian of the government problem in the case of full optimal taxation is

$$\mathcal{L}^P = EU^P + \lambda \left[\sum_{i=1}^{I} p_i k_i T(w_i) + n_0 T(0) - G \right],$$

where EU^P is the unconditional expected utility in the case of partial optimal taxation, and λ^P is the Lagrange multiplier associated with the government budget constraint in the case of partial optimal taxation. The first-order conditions of the Lagrangian with respect to $T(w_i)$ and $T(0)$ are,

$$\frac{\partial \mathcal{L}^P}{\partial T(w_i)} = \frac{\partial EU^P}{\partial T(w_i)} + \lambda^P \left[\frac{\partial k_i}{\partial T(w_i)} p_i T(w_i) + p_i k_i + \frac{\partial n_0}{\partial T(w_i)} T(0) \right] = 0, \quad \text{for } i \neq 0,$$
$$\frac{\partial \mathcal{L}^P}{\partial T(0)} = \frac{\partial EU^P}{\partial T(0)} + \lambda^P \left[\sum_{i=1}^{I} \frac{\partial k_i}{\partial T(0)} p_i T(w_i) + n_0 + \frac{\partial n_0}{\partial T(0)} T(0) \right] = 0.$$

With involuntary unemployment, households with productivity w_i that participate in the labor market and find a job forgo subsistence income $-T(0)$ and pay income tax $T(w_i)$. Thus, $T(w_i) - T(0)$ is the employment tax for households with productivity w_i. We also define the employment tax rates faced by households with productivity w_i in the case of partial optimal taxation by

$$\tau_i^P = \frac{T(w_i) - T(0)}{w_i}, \quad \text{for } i \neq 0. \tag{6}$$

Using (10), we rewrite the first-order conditions and obtain the optimal employment tax rule for partial optimal taxation as follows (see Appendix A.1):

$$\frac{\tau_i^P}{1 - \tau_i^P} = \frac{1 - g_i^P}{\eta_i^P}, \quad \text{for } i = 1, 2, \ldots, I, \tag{7}$$

where g_i^P is the average marginal social weight of consumption for the working households with productivity w_i in the case of partial optimal taxation and expressed in terms of public funds as follows:

$$g_i^P = -\frac{1}{\lambda^P p_i k_i} \frac{\partial EU^P}{\partial T(w_i)}, \quad \text{for } i = 1, 2, \ldots, I,$$

and η_i^P is the participation elasticity in the case of partial optimal taxation defined as

$$\eta_i^P = -\frac{\partial k_i}{k_i} \frac{w_i - T(w_i) + T(0)}{\partial T(w_i)}, \quad \text{for } i = 1, 2, \ldots, I.$$

Equation (7) is similar to the standard optimal participation tax rule in the extensive margin with full employment (see Diamond (1980), Saez (2002), Choné and Laroque (2005, 2011), Jacquet et al. (2013), and Christiansen (2015)). We consider the full employment case as a special case of partial optimal taxation. In the full employment case, there is no involuntary unemployment and no utility costs for a job search, so $p_i = 1$ and $\Psi_i = 0$ for all i. Thus, the government need not employ the unemployment benefit ($b = 0$).

In the full employment case, households with productivity w_i that participate in labor markets forgo the subsistence income $-T(0)$ and pay income tax $T(w_i)$. Thus, $T(w_i) - T(0)$ is the participation tax for households with productivity w_i. We also define the participation tax rates faced by households with productivity w_i by

$$\tau_i^E = \frac{T(w_i) - T(0)}{w_i}, \quad \text{for } i \neq 0. \tag{8}$$

Using (8), we rewrite the first-order conditions and derive the optimal participation tax rule in the case of full employment as follows (See Appendix A.2):

$$\frac{\tau_i^E}{1 - \tau_i^E} = \frac{1 - g_i^E}{\eta_i^E}, \quad \text{for } i = 1, 2, \ldots, I, \tag{9}$$

where η_i^E is the participation elasticity in the case of full employment defined as

$$\eta_i^E = -\frac{\partial n_i}{n_i} \frac{w_i - T(w_i) + T(0)}{\partial T(w_i)}, \quad \text{for } i = 1, 2, \ldots, I,$$

and g_i^E is the average marginal social weight of consumption for the working households with productivity w_i expressed in terms of public funds in the case of full employment as follows:

$$g_i^E = -\frac{1}{\lambda^E n_i} \frac{\partial EU^E}{\partial T(w_i)}, \quad \text{for } i = 1, 2, \ldots, I$$

where EU^E is the unconditional expected utility in the case of full employment, and λ^E is the Lagrange multiplier associated with the government budget constraint in the case of full employment. Note that the average marginal social weight of consumption in the case of full employment, g_i^E, is the same shape as the average marginal social weight of consumption in the case of partial optimal taxation, g_i^P. However, the unconditional expected utility in the case of full employment, EU^E, does not depend on the search cost, Ψ. The arguments of the unconditional expected utility are different between the partial optimal taxation and full employment case. Nevertheless, equations (7) and (9) have the same pattern. We can apply the same interpretation with both rules. If the right-hand side of (7) (resp. (9)) is negative for some working group with lower consumption, then the employment (resp. participation) tax rate for such an income group must be negative.

3.2 Full optimal taxation

In the full optimal taxation case, there is involuntary unemployment, and the government employs labor income tax schedule T as well as the job search-contingent unemployment benefit b. The Lagrangian of the government problem in the case of full optimal taxation is

$$\mathcal{L}^F = EU^F + \lambda^F \left[\sum_{i=1}^{I} k_i \left[p_i T(w_i) - [1-p_i]b \right] + n_0 T(0) - G \right]$$

where EU^F is the unconditional expected utility in the case of full optimal taxation, and λ^F is the Lagrange multiplier associated with the government budget constraint in the case of full optimal taxation. Assuming the interior solution, the first-order conditions of the Lagrangian with respect to $T(w_i)$, $T(0)$ and b are

$$\frac{\partial \mathcal{L}^F}{\partial T(w_i)} = \frac{\partial EU^F}{\partial T(w_i)} + \lambda^F \left[\frac{\partial k_i}{\partial T(w_i)} \left[p_i T(w_i) - [1-p_i]b \right] + p_i k_i + \frac{\partial n_0}{\partial T(w_i)} T(0) \right] = 0, \quad \text{for } i \neq 0,$$

$$\frac{\partial \mathcal{L}^F}{\partial T(0)} = \frac{\partial EU^F}{\partial T(0)} + \lambda^F \left[\sum_{i=1}^{I} \frac{\partial k_i}{\partial T(0)} \left[p_i T(w_i) - [1-p_i]b \right] + n_0 + \frac{\partial n_0}{\partial T(0)} T(0) \right] = 0,$$

$$\frac{\partial \mathcal{L}^F}{\partial b} = \frac{\partial EU^F}{\partial b} + \lambda^F \left[\sum_{i=1}^{I} \left[\frac{\partial k_i}{\partial b} \left[p_i T(w_i) - [1-p_i]b \right] - k_i [1-p_i] \right] + \frac{\partial n_0}{\partial b} T(0) \right] = 0.$$

Using the same method as the previous subsection, we define the employment tax rates faced by households with productivity w_i in the case of full optimal taxation by

$$\tau_i^F = \frac{T(w_i) - T(0)}{w_i}, \quad \text{for } i \neq 0. \tag{10}$$

Using (10), we rewrite the first-order conditions and the optimal employment tax rate rule in the case of full optimal taxation as follows (See Appendix A.3):

$$\frac{\tau_i^F}{1-\tau_i^F} = \frac{1}{\eta_i^F} \left[1 - g_i^F - \frac{1}{p_i k_i} \frac{\partial k_i}{\partial T(w_i)} [1-p_i] b \right], \quad \text{for } i = 1, 2, \ldots, I, \tag{11}$$

where g_i^F is the average marginal social weight of consumption for working households with productivity w_i in the case of full optimal taxation expressed in terms of public funds as follows:

$$g_i^F = -\frac{1}{\lambda^F p_i k_i} \frac{\partial EU^F}{\partial T(w_i)}, \quad \text{for } i = 1, 2, \ldots, I,$$

and η_i^F is the participation elasticity in the case of full optimal taxation defined as

$$\eta_i^F = -\frac{\partial k_i}{k_i} \frac{w_i - T(w_i) + T(0)}{\partial T(w_i)}, \quad \text{for } i = 1, 2, \ldots, I.$$

Equation (11) is the optimal employment tax rule with involuntary unemployment and search-contingent unemployment benefit. Equation (11) shows that the optimal employment tax rate depends on the search-contingent unemployment benefit (the third term of the right-hand side of (11)). Since $\frac{\partial k_i}{\partial T(w_i)} \leq 0$ and $b \geq 0$, this term would be non-negative in total. Thus, even when $g_i^F > 1$ and $1 - g_i^F < 0$, the third term cancels it out, and the negative employment tax rate is less likely to occur.

The search-contingent unemployment benefit is endogenous in full optimal taxation. We examine the effect of the search-contingent unemployment benefit on optimal employment tax rates. To do so, we rewrite the first-order condition with respect to b (see Appendix A.4), and we obtain the optimal search-contingent unemployment benefit as follows:

$$b^* = \frac{g_b^F - 1 + \frac{\sum_{i=1}^{I} \frac{\partial k_i}{\partial b} p_i [T(w_i) - T(0)]}{\sum_{i=1}^{I} k_i [1 - p_i]}}{h^F}, \tag{12}$$

where g_b^F is the average marginal social weight of unemployment benefits for the involuntary unemployed expressed in terms of public funds, and h^F is the rate of the increase in the involuntary unemployed due to a marginal increase in unemployment benefit as follows:

$$\begin{aligned} g_b^F &= \frac{1}{\lambda^F \sum_{i=1}^{I} k_i [1 - p_i]} \frac{\partial EU^F}{\partial b}, \\ h^F &= \frac{\sum_{i=1}^{I} \frac{\partial k_i}{\partial b} [1 - p_i]}{\sum_{i=1}^{I} k_i [1 - p_i]}. \end{aligned}$$

The denominator on the right-hand side of (12) shows that when the unemployment benefit results in a higher number of involuntary unemployed, the benefit should be decreased. The third term of the numerator on the right-hand side of (12) is positive and shows the marginal increase in tax revenue from an increase in the number of working households due to the unemployment benefit divided by the number of involuntary unemployed. If this value is higher, then a larger optimal unemployment benefit is desirable. However, the total effect of the right-hand side of (12) is ambiguous since the numerator and the denominator on the right-hand side of (12) could have positive correlations. Numerical simulations are required to investigate the effect of the search-contingent unemployment benefit on the optimal employment tax rates.

4 Numerical simulations

The previous section shows that employment tax rates for some working households with low consumption are less likely to be negative when there is involuntary unemployment and search-contingent unemployment benefits. This section numerically solves for optimal tax policies using an example economy. We suggest that there is a positive correlation between employment tax rates for the least productive workers and search-contingent unemployment benefits, which implies that unemployment benefits in full optimal taxation are a substitute for in-work credits.

4.1 Parameter setting

(1) Utility function

We assume the utility function U is

$$U(c) = c^{\alpha},$$

where $\alpha \in \mathbb{R}_+$ determines the curvature of U. We assume that the range of α is $\alpha \in [0.75, 0.99]$.

(2) Labor productivity

There are three possible labor productivity levels, ($I = 3$ and $w_1 < w_2 < w_3$). We assume the domain of labor productivity $(w_1, w_2, w_3) = (1, 2, 4)$. This means that the labor productivity distribution is skewed.

Let $q = (q_1, q_2, q_3)$ denote the vector of the population where q_i denotes that the household population obtains productivity w_i. We assume that the range q_1 and q_3 is $q_1 \in [0.2, 0.45]$ and $q_3 \in [0.2, 0.45]$. Since the total population is unity, $\sum_i^3 q_i = 1$, the range q_2 is $q_2 \in [0.1, 0.6]$.

(3) Participation ratio at the laissez-faire equilibrium

Let $\rho = (\rho_1, \rho_2, \rho_3)$ denote the vector of the participation ratio where ρ_i is the participation ratio of households who have labor productivity w_i when there is no income tax and no unemployment benefits. We assume $\rho_i \in [0.8, 0.99]$ for all the productivity groups.

(4) Employment probability

Let $p = (p_1, p_2, p_3)$ denote the vector of the employment probability where p_i is the employment probability in labor market i. We assume $p_i \in [0.7, 1]$ for all the productivity groups.

(5) Search cost

Let $\Psi = (\Psi_1, \Psi_2, \Psi_3)$ denote the vector of the utility cost of job search. We assume the utility cost of job search in labor market i is specified by $\Psi_i = (1-p_i)\gamma p_i w_i^{\alpha}$ where $\gamma \in [0, 1]$ is a common parameter, and $p_i w_i^{\alpha}$ are the expected utility from earnings when there is no income tax and no unemployment benefits in market i. Our specification assumes that the magnitude of the search costs in market i are some ratio of the expected utility from earnings in the laissez-faire equilibrium. The ratio is the same for all markets. This means that highly productive jobs tend to be more difficult to find. The search cost is higher when the employment probability in the market is smaller.

(6) Labor disutility

The labor disutility δ_i is uniformly distributed over the interval $[\underline{\delta}_i, \bar{\delta}_i] \subset \mathbb{R}$. Thus, the density of the labor disutility δ_i is $\frac{q_i}{\bar{\delta}_i - \underline{\delta}_i}$. Let δ_i^* denote the labor disutility, which is equal to the difference of expected utility between participation in the labor market and nonparticipation divided by p_i such that $\delta = V_i$ when there is no income tax and no unemployment benefits. With uniform distributions, the population of households that participate in labor market i, k_i becomes $k_i = \frac{q_i[\delta_i^* - \underline{\delta}_i]}{\bar{\delta}_i - \underline{\delta}_i}$, and the participation ratio, ρ_i, is $\rho_i = \frac{k_i}{q_i} = \frac{\delta_i^* - \underline{\delta}_i}{\bar{\delta}_i - \underline{\delta}_i}$.

Let e denote the labor participation elasticity with respect to labor productivity when there is no income tax and no unemployment benefits, $e = \frac{\partial k_i / k_i}{\partial w_i / w_i}$ for all i. Using the labor participation elasticity, e, we obtain the following equations:[1]

$$\underline{\delta}_i = \delta_i^* - \frac{\alpha (w_i)^\alpha}{e},$$
$$\bar{\delta}_i = \delta_i^* + \frac{\alpha (w_i)^\alpha}{e} \left[\frac{1}{\rho_i} - 1 \right]$$

where $\delta_i^* = w_i^\alpha - \frac{\Psi_i}{p_i}$. Given α, e and w_i, ρ_i, p_i, and Ψ_i determine the domains of the labor disutility for households with productivity w_i, $\bar{\delta}_i$, and $\underline{\delta}_i$, respectively. We consider $e \in [0.01, 1]$.

4.2 Benchmark case

Table 1 shows optimal income taxation for full employment, optimal partial taxation, and the optimal full taxation in the benchmark case where all parameter settings are listed at the top of the table.

We first focus on full employment (see Table 1 (A)). In this case, the population of households who participate in labor market i corresponds to the population of households who work in labor market i, (i.e., $p_i = 1$). The first part, "No tax," shows the allocation when no labor income tax is imposed; that is, disposable income y_i is equal to labor productivity w_i. The resulting GDP (total production) is 2.07. The last part, "Optimal income taxation," is the optimal income taxation for full employment. The redistribution from more productive workers to less productive workers improves the welfare (EU increases from 1.609 to 1.622) but has a negative impact on GDP (GDP decreases from 2.07 to 1.906). As shown in the previous section, the participation tax rate for some working

[1] We rewrite e as follows:

$$e = \frac{\partial k_i}{\partial V_i} \frac{\partial V_i}{\partial w_i} \frac{w_i}{k_i} = \frac{q_i}{\bar{\delta}_i - \underline{\delta}_i} \alpha (w_i)^{\alpha - 1} \frac{w_i}{k_i} = \frac{\alpha (w_i)^\alpha}{\rho_i [\bar{\delta}_i - \underline{\delta}_i]}.$$

Thus, using the participation ratio, $\delta_i^* - \underline{\delta}_i = \frac{\alpha (w_i)^\alpha}{e}$ and $\bar{\delta}_i - \delta_i^* = [1 - \rho_i][\bar{\delta}_i - \underline{\delta}_i] = \frac{\alpha (w_i)^\alpha}{e} \left[\frac{1}{\rho_i} - 1 \right]$.

Table 1: Optimal taxation for the benchmark case

A. Full employment case

Parameter setting

$\alpha = 0.875$, $q = (0.3, 0.4, 0.3)$, $\rho = (0.9, 0.9, 0.9)$, $p = (1, 1, 1)$, $e = 0.5$

$w = (1, 2, 4)$, $\Psi = (0, 0, 0)$, $G = 0$

	i	y_i	$T(w_i)$	τ_i	k_i	n_i		
No tax	0	0	0	—	0.1	0.1	EU	1.609
	1	1.00	0	0	0.27	0.27	GDP	2.07
	2	2.00	0	0	0.36	0.36		
	3	4.00	0	0	0.27	0.27		
Optimal income taxation	0	0.207	-0.207	—	0.156	0.156	EU	1.622
	1	1.253	-0.253	-0.047	0.265	0.265	GDP	1.906
	2	2.062	-0.062	0.072	0.337	0.337		
	3	3.502	0.498	0.176	0.242	0.242		

B. Involuntary unemployment case

Parameter setting

$\alpha = 0.875$, $q = (0.3, 0.4, 0.3)$, $\rho = (0.9, 0.9, 0.9)$, $p = (0.85, 0.85, 0.85)$, $\gamma = 0.5$, $e = 0.5$

$w = (1, 2, 4)$, $\Psi = (0.064, 0.117, 0.214)$, $G = 0$

	i	y_i	$T(w_i)$	τ_i	k_i	n_i		
No tax	0	0	0	—	0.1	0.235	b	0
	1	1.00	0	0	0.27	0.23	EU	1.367
	2	2.00	0	0	0.36	0.306	GDP	1.759
	3	4.00	0	0	0.27	0.229		
Partial optimal taxation	0	0.249	-0.249	—	0.184	0.306	b	0
	1	1.228	-0.228	0.021	0.255	0.216	EU	1.391
	2	1.998	0.002	0.126	0.327	0.278	GDP	1.571
	3	3.374	0.626	0.219	0.235	0.2		
Full optimal taxation	0	0.232	-0.232	—	0.174	0.298	b	0.214
	1	1.193	-0.193	0.039	0.259	0.22	EU	1.397
	2	1.97	0.03	0.131	0.33	0.281	GDP	1.587
	3	3.357	0.643	0.219	0.237	0.201		

group with lower consumption could be negative. This example shows that the participation tax rate for the least productive workers is negative ($\tau_1^E = -0.047$), and in-work credits are desirable.

Next, we focus on involuntary unemployment (see Table 1 (B)). Even with no tax, the population of households that participate in labor market i is different from the population of households that work in labor market i due to involuntary unemployment, (i.e., $p_i < 1$). The second part, "Partial optimal taxation," shows the case where the government does not employ the unemployment benefit ($b = 0$). In this case, the employment tax rate for the least productive worker is positive (i.e., $\tau_1^P = 0.021$). The third part, "Full optimal taxation," shows the case where the government employs labor income tax as well as the search-contingent unemployment benefit. In comparison to partial optimal taxation, the employment tax rate for the least productive workers is higher (i.e., $\tau_1^F = 0.039$), and the search-contingent unemployment benefit is useful ($b = 0.214$). Full optimal taxation improves GDP (GDP increases from 1.571 to 1.587) as well as welfare (EU increases from 1.391 to 1.397). The search-contingent unemployment benefit reduces the uniform income transfer for a nonworking household ($-T(0)$ decreases from 0.249 to 0.232) so that a fine-tuned income tax/transfer is available.[2]

4.3 Sensitivity analysis: Monte Carlo simulations

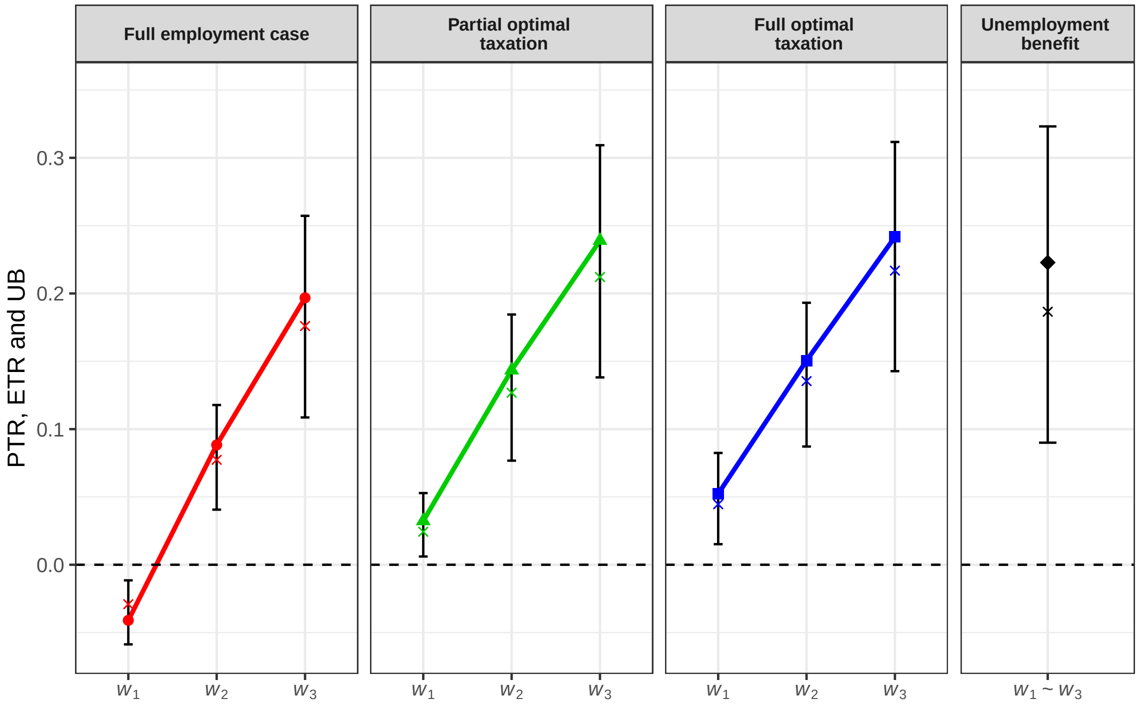
To investigate the robustness of this property, we conducted a Monte Carlo simulation from 10,000 samples where all the parameters (i.e., α, q, ρ, p, γ, and e) are uniformly distributed over the specified range. Figure 1 shows the average optimal participation tax rate for each productivity level in the full employment case, the average optimal employment tax rate for each productivity level for partial optimal taxation and full optimal taxation, and the average optimal unemployment benefit. Average optimal employment (participation) tax rates and optimal unemployment benefits in Figure 1 have almost the same patterns as the benchmark case. Additionally, we calculate the first and third quartiles of these cases to determine the statistical dispersion of the result. The results show that, in many cases, the optimal employment (participation) tax rates are similar to the benchmark case. These statistics for optimal participation tax rates for the least productive working households are negative while those of optimal employment tax rates for the least productive working households are positive, and the statistics are higher for full optimal taxation. We also examine the distribution of optimal employment tax rates for the least productive working households for both partial optimal taxation and full optimal taxation, τ_1^P and τ_1^F (see Figure 2). The distribution of employment tax rates for full optimal taxation is more dispersed and has a thicker right tail than that of partial optimal taxation.[3]

Next, we examine the relationships between the optimal employment tax rate of the least productive working households and the optimal unemployment benefit with various employment probabilities and search costs. To do so, we derive several fitted curves from 10,000 samples. We first focus on the employment probabilities (See Figure 3 (A)). When the employment probability is equal to unity, all households that participate in the labor market find a job, and there is no involuntary unemployment. From equation (11) above, the higher the p_1, the closer to zero is the third term on the right-hand side of (11), and the optimal employment tax rate rule becomes similar in form to the optimal participation tax rule. Figure 3 (A) indicates that this intuition is valid. If the employment probability p_1 is higher, then the optimal employment tax rate for full optimal taxation is close to zero or negative. On the other hand, parameter p_1 does not seem to affect the level of unemployment benefit. Second, we examine the effect of search costs (see Figure 3 (B)). If the utility cost of job search Ψ_1 is smaller, then the unemployment benefit is rapidly decreasing and close to zero. Following this move in unemployment benefits, the optimal employment tax rate for full optimal taxation is also decreasing. This implies a positive correlation between unemployment benefits and the optimal employment tax rate for full optimal taxation.

[2] In this example, the difference in welfare between full optimal taxation and partial optimal taxation, expressed in terms of public funds, amounts to 0.467% of GDP.

[3] We count the sign pattern about τ_1^P and τ_1^F. In our Monte Carlo simulation, 78.5 % of the samples show that τ_1^F is greater than τ_1^P with the same parameter settings.

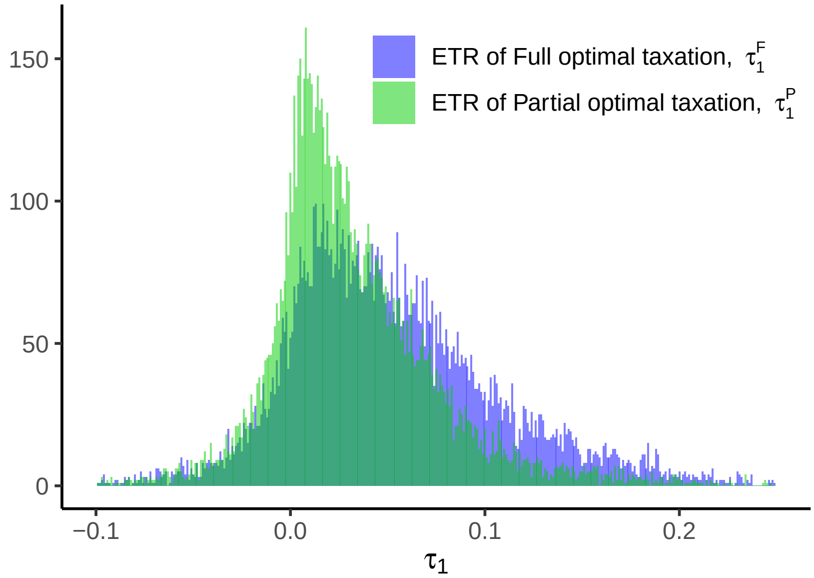
Figure 1: Optimal average PTR, ETR, and UB



(Notes) This figure shows the average optimal participation tax rate (PTR) for each productivity level (red line), the average optimal employment tax rate (ETR) for each productivity level (green and blue lines), and the average optimal unemployment benefit (UB) for all productivity levels (see black diamond point). The black lines show the range for the first and third quartiles of the optimal PTR, ETR for each productivity level, and UB for all productivity levels. The cross points show the medians.

Finally, we examine the relationships between the optimal employment tax rates of various types of working households, uniform income transfer for nonworking households, and the optimal unemployment benefit (see Figure 4). Note again that these policy instruments are all endogenous. Figure 4 shows that the optimal employment tax rates for various type of working households and the optimal search-contingent unemployment benefit have positive correlations, and the optimal uniform income transfer for nonworking households and the optimal search-contingent unemployment benefit have negative correlations. Particularly, the slope of the fitted curve of the employment tax rate is steepest for the least productive working households so that when the optimal unemployment benefit is small and close to zero, the optimal employment tax rates for the least productive working households should also be small. Thus, in-work credits (negative employment tax rates) for the least productive working households and the search-contingent unemployment benefit are substitutes. This suggests that when there are fewer search frictions in the labor market or the employment probability is high, it is more desirable to employ in-work credits than the search-contingent unemployment benefit. In contrast, when there are many search frictions and households cannot easily obtain a job offer, the government should employ the search-contingent unemployment benefit rather than in-work credits as

Figure 2: Histogram of τ_1^P and τ_1^F



(Notes) This figure shows the histogram of the optimal employment tax rate for the lowest productivity level.

a redistribution policy. This is because there is a need to raise incentives among households to participate and search in the labor market.

5 Conclusion

In this study, we investigated optimal income taxation and optimal unemployment benefits in a model with an extensive margin of labor and involuntary unemployment. In the case of involuntary unemployment, the government can employ labor income tax/transfers as well as a search-contingent unemployment benefit. We derive optimal employment tax rules, which depend on the average marginal social weight, the participation elasticity, the tax effect on the participant population in labor markets, and the optimal search-contingent unemployment benefit. Due to the existence of the search-contingent unemployment benefit, a negative employment tax rate for some working households with low consumption is less likely to occur. In addition, the optimal search-contingent unemployment benefit should be higher when the effect of the benefit on the population of working households is greater.

For our example economy, we simulate an optimal participation tax rate in the case of full employment and an optimal employment tax rate in the case of involuntary unemployment with both partial optimal taxation and full optimal taxation. We calculate the optimal participation and employment tax rate under various parameter specifications and find that the participation tax rates for low consumption households are likely to be negative in the case of full employment. However, this is not the case for involuntary unemployment. We confirm that employment tax rates and the search-contingent un-

Figure 3: Fitted curve plot of τ_1 and b with search related parameters

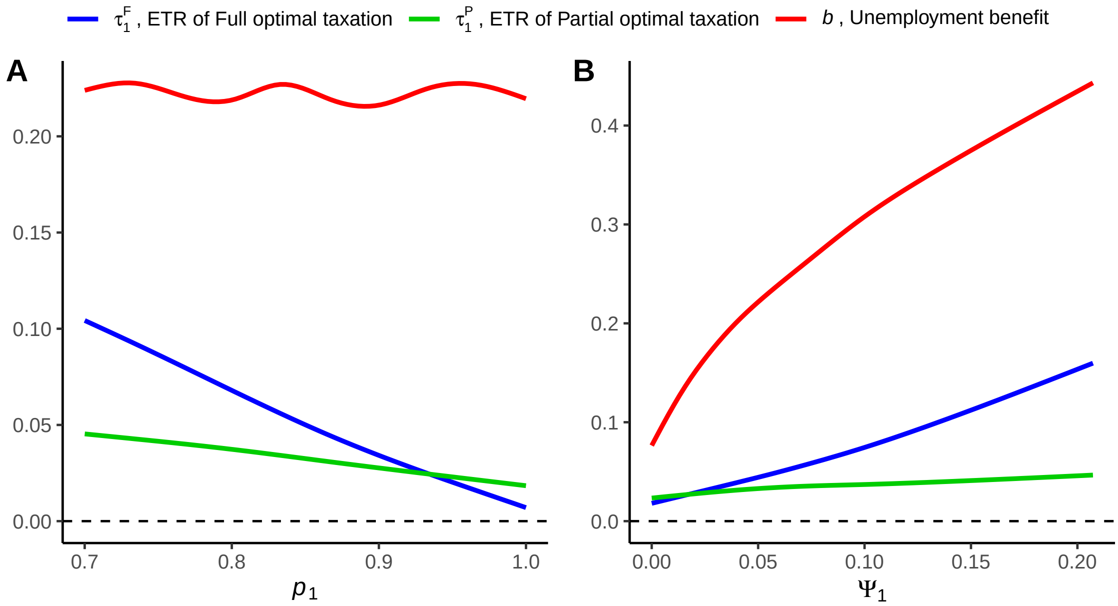
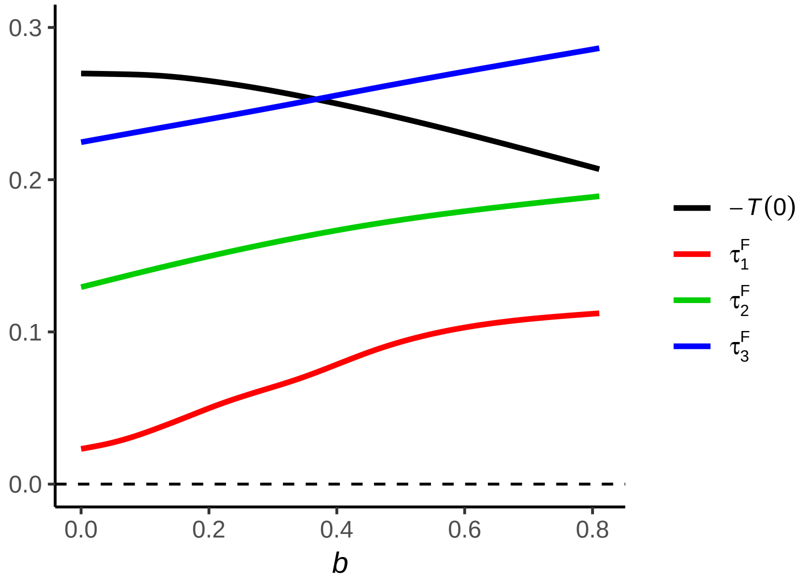


Figure 4: Fitted curve plot between $-T(0)$, τ_i^F, and b



employment benefit have a positive correlation. Only when the employment probability is close to unity or the search cost is close to zero can negative employment tax rates be valid. When the search cost is high for households, the role of the search-contingent unemployment benefit is significant. The unemployment benefit that targets households who want to work and participate in the labor market have a greater benefit than in-work credits or wage subsidies when there are serious search frictions. These findings suggest that the search-contingent unemployment benefit is a substitute for in-work credits.

Although we show that the search-contingent unemployment benefit improves welfare with involuntary unemployment, the underlying assumption is that the government can completely verify whether households search for a job in labor markets. Previous studies on unemployment benefit shed light on the problem of monitoring beneficiaries (see, e.g., Boadway and Cuff (1999) , Boadway et al. (2003), and Boone and Bovenberg (2013)). Thus, it is important that future studies clarify the role of in-work credits and unemployment benefits in the case of incomplete monitoring of job searches.

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A Appendix

A.1 Derivation of equation (7)

The first-order conditions of the Lagrangian with respect to $T(w_i)$ are

$$\frac{\partial EU^P}{\partial T(w_i)} + \lambda^P \left[\frac{\partial k_i}{\partial T(w_i)} p_i T(w_i) + p_i k_i + \frac{\partial n_0}{\partial T(w_i)} T(0) \right] = 0.$$

Note that $n_0 = 1 - \sum_{i=1}^n n_i = 1 - \sum_{i=1}^n p_i k_i$. We rewrite the first-order conditions using $\frac{\partial n_0}{\partial T(w_i)} = -p_i \frac{\partial k_i}{\partial T(w_i)}$ as follows:

$$\frac{\partial EU^P}{\partial T(w_i)} + \lambda^P \left[\frac{\partial k_i}{\partial T(w_i)} p_i T(w_i) + p_i k_i - p_i \frac{\partial k_i}{\partial T(w_i)} T(0) \right] = 0.$$

Dividing both sides by $\lambda^P p_i k_i$ and using $g_i^P = -\frac{1}{\lambda^P p_i k_i} \frac{\partial EU^P}{\partial T(w_i)}$ and $\eta_i^P = -\frac{\partial k_i}{k_i} \frac{w_i - T(w_i) + T(0)}{\partial T(w_i)}$, we rearrange the above equation as follows:

$$1 - g_i^P = -\eta_i^P \frac{T(w_i) - T(0)}{w_i - T(w_i) + T(0)}.$$

Finally, using $\tau_i^P = \frac{T(w_i) - T(0)}{w_i}$, we obtain the following equation,

$$\frac{\tau_i^P}{1 - \tau_i^P} = \frac{1 - g_i^P}{\eta_i^P}.$$

This equation corresponds to equation (7).

A.2 Derivation of equation (9)

The Lagrangian of the government problem in the case of full employment is

$$\mathcal{L}^E = EU^E + \lambda^E \left[\sum_{i=1}^{I} n_i T(w_i) + n_0 T(0) - G \right],$$

where λ^E is the Lagrange multiplier associated with the government budget constraint in the case of full employment. The first-order conditions of the Lagrangian with respect to $T(w_i)$ and $T(0)$ are

$$\frac{\partial \mathcal{L}^E}{\partial T(w_i)} = \frac{\partial EU^E}{\partial T(w_i)} + \lambda^E \left[\frac{\partial n_i}{\partial T(w_i)} T(w_i) + n_i + \frac{\partial n_0}{\partial T(w_i)} T(0) \right] = 0, \quad \text{for } i \neq 0,$$
$$\frac{\partial \mathcal{L}^E}{\partial T(0)} = \frac{\partial EU^E}{\partial T(0)} + \lambda^E \left[\sum_{i=1}^{I} \frac{\partial n_i}{\partial T(0)} T(w_i) + n_0 + \frac{\partial n_0}{\partial T(0)} T(0) \right] = 0.$$

We rewrite the first-order conditions with respect to $T(w_i)$ using $\frac{\partial n_0}{\partial T(w_i)} = -\frac{\partial n_i}{\partial T(w_i)}$ as follows:

$$\frac{\partial EU^E}{\partial T(w_i)} + \lambda^E \left[\frac{\partial n_i}{\partial T(w_i)} T(w_i) + n_i - \frac{\partial n_i}{\partial T(w_i)} T(0) \right] = 0.$$

Dividing both sides by $\lambda^E n_i$ and using $g_i^E = -\frac{1}{\lambda^E n_i} \frac{\partial EU^E}{\partial T(w_i)}$ and $\eta_i^E = -\frac{\partial n_i}{n_i} \frac{w_i - T(w_i) + T(0)}{\partial T(w_i)}$, we rearrange the above equation as follows:

$$1 - g_i^E = -\eta_i^E \frac{T(w_i) - T(0)}{w_i - T(w_i) + T(0)}.$$

Finally, using $\tau_i^E = \frac{T(w_i) - T(0)}{w_i}$, we obtain equation (9).

A.3 Derivation of equation (11)

The first-order conditions of the Lagrangian with respect to $T(w_i)$ in the full optimal taxation are

$$\frac{\partial EU^F}{\partial T(w_i)} + \lambda^F \left[\frac{\partial k_i}{\partial T(w_i)} [p_i T(w_i) - [1 - p_i] b] + p_i k_i + \frac{\partial n_0}{\partial T(w_i)} T(0) \right] = 0.$$

We rewrite the first-order conditions with respect to $T(w_i)$ using $\frac{\partial n_0}{\partial T(w_i)} = -p_i \frac{\partial k_i}{\partial T(w_i)}$ as follows:

$$\frac{\partial EU^F}{\partial T(w_i)} + \lambda^F \left[\frac{\partial k_i}{\partial T(w_i)} [p_i T(w_i) - [1 - p_i] b] + p_i k_i - p_i \frac{\partial k_i}{\partial T(w_i)} T(0) \right] = 0.$$

Dividing both sides by $\lambda^F p_i k_i$ and using $g_i^F = -\frac{1}{\lambda^F p_i k_i}\frac{\partial EU^F}{\partial T(w_i)}$ and $\eta_i^F = -\frac{\partial k_i}{k_i}\frac{w_i - T(w_i) + T(0)}{\partial T(w_i)}$, we rearrange the above equation as follows:

$$1 - g_i^F - \frac{1}{p_i k_i}\frac{\partial k_i}{\partial T(w_i)}[1 - p_i]b = -\eta_i^F \frac{T(w_i) - T(0)}{w_i - T(w_i) + T(0)}.$$

Finally, using $\tau_i^F = \frac{T(w_i) - T(0)}{w_i}$, we obtain the following equation,

$$\frac{\tau_i^F}{1 - \tau_i^F} = \frac{1}{\eta_i^F}\left[1 - g_i^F - \frac{1}{p_i k_i}\frac{\partial k_i}{\partial T(w_i)}[1 - p_i]b\right].$$

This equation corresponds to equation (11).

A.4 Derivation of equation (12)

The first-order condition of the Lagrangian with respect to b for full optimal taxation is

$$\frac{\partial EU^F}{\partial b} + \lambda^F \left[\sum_{i=1}^{I}\left[\frac{\partial k_i}{\partial b}\left[p_i T(w_i) - [1 - p_i]b\right] - k_i[1 - p_i]\right] + \frac{\partial n_0}{\partial b}T(0)\right] = 0.$$

Noting that $\frac{\partial n_0}{\partial b} = -\sum_{i=1}^{I} p_i \frac{\partial k_i}{\partial b}$, we rewrite the first-order condition as follows:

$$\frac{\partial EU^F}{\partial b} + \lambda^F \left[\sum_{i=1}^{I}\frac{\partial k_i}{\partial b}\left[p_i T(w_i) - [1 - p_i]b\right] - \sum_{i=1}^{I} k_i[1 - p_i] - \sum_{i=1}^{I} p_i \frac{\partial k_i}{\partial b}T(0)\right] = 0.$$

Dividing both sides by $\lambda^F \sum_{i=1}^{I} k_i[1 - p_i]$, we rearrange the above equation as follows:

$$\frac{1}{\lambda^F \sum_{i=1}^{I} k_i[1 - p_i]}\frac{\partial EU^F}{\partial b} - 1 = -\frac{\sum_{i=1}^{I}\frac{\partial k_i}{\partial b} p_i[T(w_i) - T(0)]}{\sum_{i=1}^{I} k_i[1 - p_i]} + \frac{b\sum_{i=1}^{I}\frac{\partial k_i}{\partial b}[1 - p_i]}{\sum_{i=1}^{I} k_i[1 - p_i]}.$$

Finally, using $g_b^F = \frac{1}{\lambda^F \sum_{i=1}^{I} k_i[1 - p_i]}\frac{\partial EU^F}{\partial b}$ and $h^F = \frac{\sum_{i=1}^{I}\frac{\partial k_i}{\partial b}[1 - p_i]}{\sum_{i=1}^{I} k_i[1 - p_i]}$, we obtain the following equation,

$$b^* = \frac{g_b^F - 1 + \frac{\sum_{i=1}^{I}\frac{\partial k_i}{\partial b} p_i[T(w_i) - T(0)]}{\sum_{i=1}^{I} k_i[1 - p_i]}}{h^F}.$$

This equation corresponds to equation (12).