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Discussion Paper 02-12

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ABSTRACT

A new benchmark of volatility implicit in Nikkei 225 stock option prices is constructed and its stochastic dynamics are examined in comparison with the S&P implied volatility index. There is a tendency for the levels of implied volatility to move in tandem during the late 1990s despite the idiosyncrasies of the booming U.S. economy and Japanese recession. The long-term convergence dynamics of implied volatility are found to be similar but the convergence of the transitory component towards zero is inherently different across markets. There is evidence from the estimation of asymmetric GARCH models that useful information on leverage effects is embedded in implied volatility. Albeit upward biased, implied volatility is found to constitute a good approximate of future levels of realized volatility in the Japanese market. Based on a two-factor model of risk components consistent with the intertemporal CAPM theory, there is also evidence that implied volatility is function of stochastic changes in the investment opportunity set proxied by measures of uncertainty about the domestic real economy and the international financial economy.

JEL Classification: C51, G13

1. Introduction

For plausible explanations of dramatic changes in stock market volatility, it is essential to understand how and why market volatility does ebb and flow. Given the widespread perception that option trading is akin to volatility trading, the empirical questions of whether the volatility implied in option prices is redundant, or whether it constitutes a substitute or complement to alternative measures of volatility, have gained in importance. In understanding the complexity of risk and the dynamics of risk hedging, there is no doubt that the contribution of theoretical and methodological advances in option pricing has been significant. But the limited empirical evidence on the information content of implied volatility leaves large rooms for further analysis that may shed lights on the properties of financial market volatility and the economics and efficiency of options markets as well.

The present study is an attempt to enrich the existing literature with new perspectives from alternative options markets and the construction of a new implied volatility index. In gathering information contained in the term structure of implied volatility and volatility smiles, this benchmark allows for the aggregation of short-term volatility expectations across option investors, hedgers and arbitrageurs. In contrast to the numerous indices of financial asset prices, volatility benchmarks are rather rare. The Chicago Board of Options Exchange uses the real-time OEX bid-ask quotes on S&P 100 American options to publish since 1993 the CBOE-VIX index of implied volatility. We construct a similar index of implied volatility on the Nikkei 225 stock index, which is the average price of representative stocks listed on the first section of Tokyo Stock Exchange. Despite the fact that the Nikkei 225 stock average is the underlying asset of financial derivatives traded on three major Asia-Pacific derivatives markets, albeit under different contract specifications, there is to the knowledge of the authors, no volatility benchmark for Japanese equity markets.[1]

In constructing the implied volatility index from Nikkei 225 stock index options traded on the Osaka Stock Exchange, there is an attempt to provide a useful addition to the list of financial benchmarks, which can facilitate comparative analysis of volatility across international markets. The replication exercise provides also the opportunity to examine four core empirical issues addressed in sequence in the remainder of the paper, which is structured as follows. The next section presents a review of literature on option pricing as far as theoretical and empirical issues in implied volatility are concerned. Section 3 outlines the methodology underlying the construction of the implied volatility index and the measurement problems encountered in its implementation with respect to Japanese stock index options. The distributional and time-series properties of the implied volatility index are discussed in section 4. Section 5 examines its relationship with conditional volatility in asymmetric Generalized Autoregressive Conditional Heteroskedasticity (GARCH) modeling of returns. It analyzes the information embedded on the asymmetric impacts of bad and good news, or leverage effects, which derive from the increased risk of holding stocks when prices are sharply decreasing, thereby discounting the value of equity relative to corporate debt and increasing corporate leverage. The analysis in Section 6 focuses on the ability of implied volatility to forecast the realized levels of market volatility over options expirations. A structural examination of its relation with risk factors related to the domestic real economy and the international financial economy is made in Section 7. Section 8 concludes the paper.

2. Option pricing theory and implied volatility

The underlying asset price S is generally assumed to follow the diffusion process described in equation (1) where the drift $\mu(S,t)$ and volatility $\sigma(S,t)$ terms is function of S and time t and where W is a standard Brownian motion.

$$dS(t) = \mu(S,t)dt + \sigma(S,t)S(t)dW(t) \tag{1}$$

The theoretical value of an European call according to Black and Scholes (1973) option pricing model is function of the exercise price K, constant volatility $\sigma(S,t) = \sigma$ over the option's life, and risk-free interest rate r in addition to the state variables, time to expiration τ and the underlying asset price S. The option valuation model extended by Merton (1973a) to account for stochastic dividends at the anticipated continuous yield q can be expressed as

$$\begin{aligned} &C = C(S,\tau;K,r,q,\sigma) = Se^{-q\tau}N(d_1) - Ke^{-r\tau}N(d_2) \\ &\text{where} \quad N(d) = \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx \\ &\qquad d_1 = \{\ln(S/K) + (r - q + \sigma^2/2)\tau\} \big/ (\sigma\sqrt{\tau}) \\ &\qquad d_2 = d_1 - \sigma\sqrt{\tau} \end{aligned} \tag{2}$$

Whereas the mathematical advances in option valuation by Black and Scholes (1973) spurred an extant theoretical literature on option pricing, the idea first introduced by Latane and Rendleman (1976) of extracting market volatility from option premium added another dimension to the empirical research on derivatives markets. An estimate of the unobservable state variable market volatility σ can be derived as a function of the option price and remaining parameters using a numerical iterative process that reduces the error between the observed option price and its theoretical value. Convergence problems arise when equating the observed and model values fails. The numerical difficulties stem in part from the fact that option pricing models do not typically provide closed-form solutions and that the volatility function cannot be reduced to a scalar.[2]

The rich literature on volatility implicit in option premium seems to evolve around the numerical and estimation problems, distributional properties, informational content of implied volatility and the characterization of its relationship with conditional and realized volatilities. The empirical evidence, indisputable so far, is that daily estimates of implied volatility vary across exercise prices and exhibit a 'volatility smile' that is inconsistent with the theoretical foundations of the parsimonious Black-Scholes valuation model, which assumes volatility to be constant, independent of exercise prices and time-invariant. Volatility smiles are suggestive of Black-Scholes mispricing of deep in- and out-of-the-money options or options market inefficiency. They may represent *de facto* empirical violations of the underlying assumptions including the normal distribution of returns. Pena, Rubio and Serno (1999) attribute volatility smiles to transaction costs and find a significant inverse relationship between the degree of curvature and time to expiration. Even in the presence of 'flat' smile when Black-Scholes implied volatility is constant across exercise prices, the square of implied volatility is shown by Britten-Jones and Neuberger (2000) to be equal to the risk-neutral expectation of squared volatility and shown to be positively biased estimate of realized volatility. Unless volatility is constant, it can be shown using Jensen's inequality,

$$E\left[\int_t^{t+\tau} \sigma(t)^2\, dt\right] \le 2\int_0^{\infty} \frac{C(t+\tau, K) - C(t, K)}{K^2}\, dK \qquad (3)$$

that under risk-neutral probability distribution and continuous time, it is the square of volatility rather than volatility *per se* that is an unbiased estimate of realized volatility.

The examination of the properties of implied volatility constitutes a joint test of options pricing model and market efficiency. To mitigate inherent inconsistencies in Black-Scholes modeling, a wide array of competing approaches to recover unbiased estimates of implied volatility and alternative option pricing models has been advocated.[3] At one end of the implied volatility spectrum, a "model-free" approach is advocated by Britten-Jones and Neuberger (2000) who adjust arbitrary volatility processes to option prices, drawing upon the standard practice of fitting interest rate processes to bond prices. Modeling the implied volatility function to depend on the exercise price includes studies using polynomials, interpolation or splines smoothing of the pricing function by Shimko (1993), Longstaff (1995), Malz (1997), and Ait-Sahalia and Lo (1998), inter alia. GARCH option pricing including Duan, Gauthier and Simonato (1999) and Ritchken and Trevor (1999) attempt to capture the path-dependence dynamics of volatility and the negative correlation between volatility and returns.

The stochastic properties of volatility are also addressed in earlier studies by Hull and White (1987), Stein and Stein (1991) and Heston (1993) among others, who develop models where the constant volatility parameter is substituted with the entire joint probability distribution of returns and changes in volatility. Although Ball and Roma (1994) find the modeling of stochastic volatility to be consistent with implied volatility smiles, the approach is complicated with the estimation of the market price of volatility risk. Assuming volatility to be a deterministic function of the underlying asset price and time, as in models developed by Rubinstein (1994), Derman and Kani (1994) and Dupire (1997), is also inconsistent with the findings by Buraschi and Jackwerth (2001). The examination by Dumas, Fleming and Whaley (1998) of the predictive and hedging performances of the *ad hoc* Black-Scholes model and quadratic-form modeling of the deterministic volatility function $\sigma(K,t)$ suggests that the latter outperforms the former on both accounts. There is evidence that the estimated deterministic function is itself unstable over time and Black-Scholes hedge-ratios are more reliable. As far as at-the-money options are concerned, there is also earlier evidence by Ledoit and Santa-Clara (1998) that as expiration draws near, Black-Scholes implied volatility converges to the asset's instantaneous or stochastic volatility.

The final issue addressed in the literature concerns the informational content of implied volatility, where early empirical evidence by Day and Lewis (1992) suggests that volatility implied by S&P 100 index options is almost unbiased and constitutes an informative forecast but the conditional volatility based on GARCH modeling of returns does contain incremental information. The significance of the informational content for forecasting purposes is also supported by evidence from Lamoureux and Lastrapes (1993) using volatilities implicit in the price of individual stock options. Albeit upwardly biased, implied volatility is also found by Fleming (1998) to constitute a reliable forecast of ex post volatility, with forecast errors being orthogonal to parameters embedded in ARCH models. Further evidence provided by Christensen and Prabhala (1998) explains away part of the bias in implied volatility with regime shifts in the pricing of index options around the October 1987 stock market crashes. More recent evidence from Blair, Poon and Taylor (2001) indicates that irrespective of data frequency and forecasting horizon, S&P 100 VIX index of implied volatility is more accurate for out-of-sample forecasting than realized volatility.[4] In sharp conflict with this mounting evidence, Canina and Figlewski (1993) suggest that S&P 100 implied volatility is neither informative on realized volatility nor accurate in forecasting future volatility.

An international perspective on the informational content of implied volatility is added to the empirical literature using the Nikkei 225 option prices and the parsimonious Black-Scholes model, which despite inherent inconsistencies, remains the cornerstone of theoretical and empirical studies in implied volatility. Before discussing the results of our findings, the methodology followed in the construction of the implied volatility benchmark is explained in the next section.

3. Implied volatility index: data and methodological issues

The volatility implicit in the daily closing prices of European options on Nikkei 225 stock index is estimated using the theoretical call valuation equation (2) and put price derived from put-call parity. The Japanese index options expire on the second Friday of the contract month, covering a period of 144 maturities starting from the inception of options trading on OSE in June 1989 through June 2001.

The risk-free interest rate is approximated by the continuous yield on the three-month or one-month Certificate of Deposit with the closest maturity to the expiration date. The dividend yield is estimated as a fraction of current stock index level, using the annual average yield on all stocks listed on TSE. To facilitate comparison across international markets, we use the daily closes of S&P 100 implied volatility index based on average bid-ask option quotes, which are obtained from Thomson Financial Datastream database. There are discrepancies in the sample sizes across markets due to differences in trading holidays and focus is made on the newly constructed index of implied volatility in the Japanese market.

The construction of the implied volatility index can be viewed as another variant of implied volatility weighting schemes.[5] The calculus of VIX, which is described in further detail in Whaley (2000), involves a series of averaging and interpolation exercises and takes into account both the term structure of implied volatility and volatility smiles and sneers. Theoretically, the index measures the market volatility implied by the price of a hypothetical option with 30 calendar days to expiration and with exercise price exactly equal to the underlying asset price. The VIX benchmark is constructed as a weighted average of volatilities implied by the prices of options in the at-the-money neighborhood and nearest maturities. The implied volatility is calculated using the remaining time to maturity expressed in calendar days τ_c, and subsequently approximated using $v_t = v_t \sqrt{\tau_c / \tau}$ into volatility over $\tau = \tau_c - 2\,\mathrm{int}(\tau_c / 7)$ trading days.[6]

To mitigate measurement problems deriving from short-lived options which are usually associated with extreme values of implied volatility, options with less than eight calendar days to expiration (i.e., approximately six trading days) are eliminated. Focus is made also on near-the-money options, which are theoretically most sensitive to changes in volatility and unlike deep in- or deep-out-of-the-money, they are associated with higher time premium. Upon identifying the upper $K = U$ and lower $K = L$ exercise prices immediately above and below the daily stock index price, the selection of the corresponding nearest-maturity $m = 1$ call (C_{U1}, C_{L1}) and put

(P_{U1}, P_{L1}) options together with the second near-maturity $m = 2$ call (C_{U2}, C_{L2}) and put (P_{U2}, P_{L2}) options results in eight estimates that differ across option types, maturities and exercise prices.

These volatility measures $v_t(K, m)$ with exercise price $K = U, L$ and maturity $m = 1, 2$ need to be averaged at each of the three levels to construct the VIX index. First, the simple averaging $v_t(K, m) = \{v_t(C, K, m) + v_t(P, K, m)\}/2$ deals away with call and put notations. The remaining two couples of implied volatilities from the nearest-maturity $[v_t(U,1), v_t(L,1)]$ and second-nearest-maturity option $[v_t(U,2), v_t(L,2)]$ are also used in the interpolation process described by equation (4), which is aimed at adjusting for differences between the current stock index level and exercise prices. This process results in the approximation of the implied volatility of the hypothetical at-the-money option for each expiration month.

$$v_t(m) = v_t(U, m)\frac{U - S}{U - L} + v_t(L, m)\frac{S - L}{U - L} \qquad \text{for } m = 1, 2. \tag{4}$$

Finally, an extrapolation process is required to adjust for differences in periods of time τ_{n1} and τ_{n2} remaining to the expiration of the nearby and second-nearest maturity options, respectively. The implied volatility index is made to reflect the volatility implicit in the price of a single theoretical option with 22 trading-days (approximately 30 calendar days or a month) remaining to expiration.

$$v_t = v_t(1)\frac{\tau_2 - 22}{\tau_2 - \tau_1} + v_t(2)\frac{22 - \tau_1}{\tau_2 - \tau_1} \tag{5}$$

Apart from Black-Scholes misspecification and usual problems of non-synchronous pricing of stock and options, the implementation of this methodology using data on the Japanese options market poses some difficulties. The tendency for daily trading on the European-style Nikkei 225 stock index options on OSE to concentrate on near-maturity, second- and third-month expirations should actually facilitate the implementation of VIX calculation. But, the likelihood of non-convergence for the near-maturity near-the-money options is found to be higher with respect to early periods of thin trading of Nikkei 225 options on OSE. The problem of non-convergence of implied volatility numerical calculations, which may be attributed to option mispricing and error-in-variables problems, can further complicate the implementation of VIX methodology by undermining either the averaging or interpolation process.

The methodology does not allow for the upper or lower level of exercise prices on a given trading day to differ across call and put options. In reconstructing the volatility index, it remains silent on the appropriate way to deal with cases where no put option can be found to match exactly the exercise price of calls at either the upper or lower levels. Substituting the non-converging option price with that of the next-near-the-money option effectively compromises the simple averaging across option types because of the discrepancy in exercise price boundaries across call and put options. To minimize the loss of information, we adopt the simpler approach of referring in turn to the opening, highest or lowest price when numerical convergence is achieved with respect to closing prices.[7] In the case where $v_t(1) = 0$ (or $v_t(2) = 0$), the value of VIX is approximated on the basis of equation (5) and under the usual underlying assumption that volatility is related to the square root of time, with the point estimate $v_t = v_t(2)\sqrt{\tau_2/22}$ (or $v_t = v_t(1)\sqrt{\tau_1/22}$). The resulting time-series of daily estimates of the implied volatility index provides an *ex ante* estimate of short-term volatility. The next sections assess the distributional properties and usefulness of implied volatility in forecasting realized volatility.

4. Distributional and stochastic properties of implied volatility

This section examines the behavior of daily Nikkei 225 and S&P 100 VIX indices, as well as their distributional characteristics over monthly options expirations. Figure 1 exhibits the daily volatility series, which reflect two distinct trends, apart from the short period following the inception of option trading on OSE, during which implied volatility has been lower in the Japanese than in US options markets. This period of rising stock prices and lower volatility precedes the burst of the Japanese stock market bubble by the end of the 1980s. The first pattern, which is obvious over the period extending into the mid-1990s, reflects episodic surges and falls in Japanese market volatility, which are in sharp contrast with the smoother and constant path of implied volatility in US markets. This trend is reversed however when implied volatility in the US market increased and the alignment of volatility paths in both markets persisted until the end of the sample period. This parity in the implied volatility levels is interesting is maintained despite the respective booms and slumps in the US and Japanese economies.

From the distributional moments of price, return and implied volatility in both markets are reported in Panel A of Table 1, there is evidence that implied volatility is on average higher and more volatile in Japanese stock markets than US markets. Upon conversion of annual implied volatility to daily basis (dividing by the square root of 250), the median of Nikkei 225 implied volatility equal to 1.67% is found to be comparable to the sample standard deviation of returns 1.48%. The evidence, which applies also to the US markets, suggests that implied volatility can constitute a good proxy of realized volatility.

Before examining the time series properties, determinants and forecasting ability of implied volatility, it is noted from unit root test results, that with the exception of stock prices, all daily and monthly return and volatility series are stationary. Judging from the distributional moments and behavior of implied volatility over time, it seems that shocks to volatility are likely to persist. The stochastic dynamics of the implied volatility series can be examined using the class of GARCH econometric models

$$v_t = \vartheta_0 + \sum_{k=1}^{n} \vartheta_k v_{t-k} + e_t \tag{6}$$

$$\chi_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i e_{t-i}^2 + \sum_{j=1}^{p} \beta_j \chi_{t-j}^2 \tag{7a}$$

As with the parsimonious GARCH(1,1) process, when the roots of $\phi_1 = \alpha_1 + \beta_1$ lie outside the unit circle (i.e.,), the conditional variance of the implied volatility index converges towards the long run unconditional variance. By redefining the constant in the conditional variance equation in the

GARCH(1,1) process as function of the unconditional variance $\alpha_0 = \varpi(1-\phi_1)$, equation (8a) can be rewritten as

$$\chi_t^2 = \varpi + \alpha_1(e_{t-1}^2 - \varpi) + \beta_1(\chi_{t-1}^2 - \varpi) \qquad (7b)$$

It is possible to allow for reversion to a long-term time varying ω_t instead of a constant mean ϖ by expressing the conditional variance in the component GARCH(1,1) model as

$$\begin{aligned} \chi_t^2 - \omega_t &= \alpha_1(e_{t-1}^2 - \omega_{t-1}) + \beta_1(\chi_{t-1}^2 - \omega_{t-1}) \\ \omega_t &= \varpi + \delta(\omega_{t-1} - \varpi) + \gamma(e_{t-1}^2 - \chi_{t-1}^2) \end{aligned} \qquad (7c)$$

In this nested setting, the long run component ω_t defined in the second part of the system of equations (7c) converges to ϖ with powers of δ while the transitory component $\chi_t^2 - \omega_t$ described in the first equation converges to zero with powers of ϕ_1. The estimation results of the component GARCH modeling of implied volatility in Japanese and US markets are reported in Panel A of Table 2. The diagnostic tests indicate that these models are not associated with serial correlation in the squared standardized residuals and that there are no traces of ARCH effects. There is evidence that the value ϖ towards which the time-varying long-run volatility of the Japanese implied volatility index converges is contrary to evidence from US markets, statistically insignificant. In both markets, convergence takes place at a slow rate determined with powers of δ, which is found to be typically close to unity.

From the transitory equation, the insignificance of α_1 estimate for the Japanese market suggests that differences between innovations in volatility and the permanent component do not affect the short run movements of implied volatility. The influential force behind the convergence of the transitory component towards zero lies in past levels of implied volatility. In contrast, both past levels of volatility and past innovations in US implied volatility constitute significant determinants of the transitory component. Judging from the magnitude of the transitory coefficients, the dynamics of short-term convergence towards zero are inherently different across these markets. The reversion of the transitory component towards zero via powers of ϕ_1, tends to occur in the US market at faster rates than in the Japanese market. The evidence is important for international portfolio hedging purposes as it aids in understanding options investors' formation of expectations about long-term levels and short-term movements in stock market volatility.

5. Conditional variance, implied volatility and leverage effects

In light of reported evidence reported on the stochastic properties of implied volatility, it is important to explore its relationship with alternative estimates of asset volatility. Judging from the comparative behavior of daily returns and implied volatility estimates illustrated in Figure 2, it is likely that periods of increased implied volatility coincide with episodes of higher variability of returns. This adds to the graphical evidence from Figure 1 that these periods are also associated with declining stock prices. To examine this stylized tendency for the volatility of equity returns to cluster and persist and its asymmetric behavior during bullish and bearish markets, two variants of asymmetric GARCH modeling are considered. The first variant represented by the threshold ARCH (TARCH) model developed by Zakoian (1994), accounts for asymmetric shocks to volatility and describes the mean equation and conditional variance function respectively as

$$r_t = \mu_0 + \sum_{k=1}^{n} \mu_k r_{t-k} + \psi_t \tag{8}$$

$$\sigma_t^2 = \upsilon_0 + \sum_{i=1}^{q} (\alpha_i \psi_{t-i}^2 + \gamma_i \psi_{t-i}^2 q_{t-i}) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \ , \tag{9a}$$

where ψ_t are normally distributed errors. The dummy variable q_t is equal to unity in the case of good news $(\psi_t > 0)$ and zero otherwise. The news impact on volatility is function of the sign and magnitude of γ. The impact is limited to α in the case of good news and amounts to $\alpha + \gamma$ with respect to bad news. There is evidence of asymmetries when $\gamma \neq 0$ and of leverage effects when $\gamma > 0$. When the leverage effect is assumed to be exponential, an alternative modeling of non-asymmetric responses to shocks is considered. The exponential GARCH (EGARCH) model proposed by Nelson (1991) expresses the variance equation as

$$\log\sigma_t^2 = \upsilon_0 + \sum_{i=1}^{q}(\alpha_i\left|\frac{\psi_{t-i}}{\sigma_{t-i}}\right| + \gamma_i\frac{\psi_{t-i}}{\sigma_{t-i}}) + \sum_{j=1}^{p}\beta_j\log\sigma_{t-j}^2 \qquad (9b)$$

In this exponential mapping, the increase in volatility associated with bad news is higher than that generated by good news when γ is negative and significant. Table 2 panel B reports Model I estimates, which result from TARCH and EGARCH modeling of Japanese and US stock market returns. Independent of the asymmetric GARCH variant, there is evidence of volatility persistence. The results indicate also the existence of leverage effects as γ is found to be positive and significant for TARCH models and negative and significant for EGARCH models. This constitutes evidence that bad news is likely to generate a larger increase in market volatility than good news.

To examine its informational content of implicit volatility, Day and Lewis (1992) and Lamoureux and Lastrapes (1993) propose its inclusion as an exogenous variable in the conditional variance equation. Instead of the GARCH in mean modeling used in the above studies, testing the hypothesis that implied volatility does not contain information beyond that implied by realized returns is performed using TARCH and EGARCH variance equations (10a) and (10b), respectively.

$$\sigma_t^2 = \upsilon_0 + \sum_{i=1}^{q}(\alpha_i\psi_{t-i}^2 + \gamma_i\psi_{t-i}^2 q_{t-i}) + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 + \xi v_{t-1}^2 \qquad (10a)$$

$$\log\sigma_t^2 = \upsilon_0 + \sum_{i=1}^{q}(\alpha_i\left|\frac{\psi_{t-i}}{\sigma_{t-i}}\right| + \gamma_i\frac{\psi_{t-i}}{\sigma_{t-i}}) + \sum_{j=1}^{p}\beta_j\log\sigma_{t-j}^2 + \xi\log v_{t-1}^2 \qquad (10b)$$

The significance of the coefficient associated with implied volatility ξ constitutes evidence that this variable contains incremental information content on return volatility. Judging from the estimates of Model II reported in Table 2 Panel B, this coefficient is found to be positive and significant in both markets and irrespective of the TARCH or EGARCH modeling. As far as the US market is concerned, the significance of ξ is accompanied by a significant change in the sign of GARCH terms β_1. Though the significance of leverage effects measured by the news impact coefficient γ fades away, neither quadratic nor exponential estimates do change in sign. The log likelihood ratio tests suggest that irrespective of the market under examination, the inclusion of implied volatility results in a significant improvement in model estimation. This is evidence that implied volatility impounds useful information on the persistence of return volatility as well as on its asymmetric response to bad and good news.

It is possible to visualize the leverage effect using the news impact curve, which reflects the sensitivity of the conditional variance to innovations standardized by market volatility ψ / σ, estimated around the median of the conditional variance. Figure 3 illustrates the impact functions of the exponential modeling of Japanese and US market volatility with and without implied volatility. It appears that increases in volatility differ in magnitude across markets, with the stronger impact associated with Japanese market volatility. The slope of the curves with respect to bad news is steeper than for good news. Modeling using implied volatility as regressor in the conditional variance equation aids in explaining part of the impact of bad news on Japanese volatility and the entire impact on US market volatility. Expectations of market declines may generate a buying pressure on puts, thereby driving their prices up and increasing the level of implied volatility. The evidence is important in understanding the important aspects of the relationship between implied volatility and realized volatility, which is the subject of the next section.

6. Realized volatility and the forecasting ability of implied volatility

Having assessed the distributional properties and stochastic dynamics of the implied volatility, it is the usefulness of this forward-looking measure of volatility in forecasting realized volatility that can be now examined. The standard deviation of log returns provides an *ex post* estimate of realized volatility at the end of the option maturity month m as

$$\sigma_m^h = \sqrt{\tau_m^{-1} \sum_{t=1}^{t+\tau_m} (r_t - \bar{r}_m)^2} \tag{11}$$

where $\bar{r}_m = \tau_m^{-1} \sum_{t=1}^{t+\tau_m} r_t$ is the mean of spot returns over the maturity month and τ_m refers to the number of remaining days to expiration. It is possible to test as in Christensen and Prabhala (1998), the significance of the relationship between implied and realized volatilities using the following regression model.

$$\sigma_m^h = \lambda_0 + \lambda_v v_m + u_m \,, \tag{12}$$

where σ_m^h denotes historical volatility defined in equation (12) and v_m is the annual implied volatility estimated at the beginning of the expiration month (thirty calendar days before maturity). The residuals u_m are white noise when implied volatility constitutes an efficient estimate of observed volatility. From Figure 4, it appears that there is a strong tendency for Nikkei 225 implied volatility index to constitute an upper boundary for realized volatility on the spot market. The evidence is consistent with results from Fleming (1998) that implied volatility is an upward biased volatility forecast. Because implied volatility is shown by Jensen’s inequality (3) to be an upward biased estimate of actual volatility, the intercept is likely to be also upwardly biased but the estimate of the slope coefficient does not necessarily suffer from such a bias. The slope of the estimated regression model reported in Table 3 (Model I) is found to be positive and significant suggesting that useful information on the future variability of returns is embedded in actual option prices.

Additional information is conveyed by the difference between lagged estimates of implied volatility and realized volatility over the preceding expiration month. Model II reports the negative and significant coefficient associated with this differential, which if viewed as the forecasting error, suggests that changes in implied volatility are embedded in an ongoing adaptive process towards the observed variability of returns. As suggested by the additional coefficient estimate in Model III, actual volatility is also negatively related with average spot returns, indicating a tendency to increase during periods of falling prices and increasing returns. This result is consistent with evidence on leverage effects analyzed in Section 5. Whereas the relation between market conditions and realized volatility is found to be sample-independent, past deviations of implied volatility from historical levels have become irrelevant as they decreased in magnitude over the late 1990s. Neither the sign nor the significance of the implied volatility index in forecasting the future level of market volatility seems to be altered. This evidence is inconsistent with results by Canina and Figlewski (1993) but in line with the conclusions by Day and Lewis (1992) and Lamoureux and Lastrapes (1993), suggesting that implied volatility is significantly informative on the future variability of returns.

7. Risk factors and the predictability of implied volatility

In light of evidence on the stochastic dynamics of implied volatility and its relationship with leverage effects, it is important to examine the issue of whether changes in implied volatility are predictable. Based on regression analysis, there is evidence by Harvey and Whaley (1992) that there are predictable variations in daily fluctuations of implied volatility, the statistical significance of which does not translate into economically profitable trading strategies. When predictability can be assessed on the basis of its relationship with various risk factors, a structural analysis of implied volatility can be made drawing upon Merton’s (1973b) approach to intertemporal portfolio theory and the intertemporal capital asset pricing model. In pricing securities in the dynamic market framework, there are additional sources of risk stemming from uncertainties caused by shifts in the investment opportunity set. Equity risk is function of the stochastic investment opportunity set, i.e., changes in expected returns and covariance of returns related to state variables such as the expected inflation rate, the expected productivity of capital and proxies of the level of uncertainty about economic activity.

Assuming that the state variables describing the instantaneous investment opportunities follow a vector H of Markov diffusion process with drift μ_H and volatility σ_H vector components

$$dH = \mu_H dt + \sigma_H dz_H , \tag{13}$$

with dz_H denoting the standard Wiener process. The dynamics of the pricing kernel k such that $E[d(kS)] = 0$, can also be described by a diffusion process that is a linear function of the state variables dynamics

$$dk/k = -r_f(H) - \zeta(H)dz_k\,, \tag{14}$$

where $r_f(H)$ is the instantaneously riskless rate, $\zeta(H)$ is the risk premium per unit of covariance with the pricing kernel. Recognizing the asset volatility as σ_S and the correlation with the pricing kernel according to $\rho_{Sk}dt = dz_S dz_k$, the expected returns can be expressed as[8]

$$E[dS/S] \equiv \mu_S dt = r_f(H)dt + \zeta(H)\rho_{Sk}\sigma_S dt \tag{15}$$

When the pricing kernel is perfectly correlated with the market portfolio returns, the simple CAPM ensues. The ICAPM derives from the expression of the pricing kernel innovations as a linear function of market returns and stochastic changes in the opportunity set so that covariances with the state variables also matter in the definition of systematic risk. Risk premia are thus determined according to a multi-beta intertemporal capital asset-pricing model. As noted by Harvey and Whaley (1992), when implied volatility follows a random walk, its stochastic changes should be orthogonal to any set of information variables, which includes innovations in the state variables under the present intertemporal setting.[9] Whether fluctuations in implied volatility account for changes in the stochastic investment opportunity set is essentially an empirical issue. Consistent with the assumptions underlying the ICAPM, implied volatility can be expressed as a function of the vector of stochastic state variables

$$v \equiv v(H) \tag{16}$$

The vector of state variables includes in the present study, proxies for uncertainty in the domestic real economy and the international financial economy. The leading diffusion index D, which is compiled by the Economic and Social Research Institute in the Japanese Cabinet Office aggregates in percentage terms, the directions of change in a selection of economic time series to anticipate turning points in the business cycle. It assesses the extent to which business fluctuations spread across the economic sectors, generally approaching zero to signal recessions and 100 to anticipate economic recoveries. As a measure of the dispersion of change across economic indicators, it is widely believed to contain useful information on future structural changes in the real economy. It averages a value of 50 (no turning points) over the sample period but as exhibited in Figure 5, its behavior over time reflects cyclical patterns of anticipated fluctuations in business conditions. Its decline in the early 1990s is consistent with the beginning of the prolonged economic recession as it indicates the predominance of declining industries. Judging from the observed patterns, the leading diffusion index is likely to be negatively correlated with estimates of implied volatility.

The variations in the international financial investment opportunity set are, as noted by Brennan, Wang and Xia (2001), not necessarily related to changes in the domestic output. Uncertainty relates to unanticipated fluctuations in exchange rates, monetary and fiscal impulses, and inflationary and deflationary pressures, among others. These additional sources of risk in the international financial economy are assessed hereafter using the volatility of the Morgan Stanley Capital International world price index. An increase in the volatility of the international equity index can signal investors' perceptions of higher levels of uncertainty in international financial markets that translates into an increase in implied volatility. The graphical evidence from Figure 5 suggests that implied volatility tends to move in tandem with the usually lower levels of MSCI world index volatility.

To the extent that the relation between implied volatility in the Japanese options market and the stochastic dynamics of these state variables is linear, the implied volatility function expressed by equation (16) in an intertemporal setting is amenable to econometric estimation according to a two-factor model of risk components[10]

$$v_m = \pi_0 + \pi_D D_{m-1} + \pi_M \sigma_{M,m-1} + u_m , \qquad (17)$$

where v is the ex ante estimate of implied volatility with one-month remaining to maturity, whereas u represents measurement and estimation errors. Whereas the implied volatility index is represented by the point estimate at the beginning of expiration months, the standard deviation of MSCI world index is calculated using log returns until maturity. The test results of regression model (17) reported in Table 4 Model I indicate that the estimates of uncertainty underlying the real and financial economic conditions contain useful information on investors' expectations of implied volatility. The slope coefficients take the expected signs and are statistically significant, thereby confirming the evidence from the scatter diagrams and simple regression lines in Figure 5. There is evidence that future levels of implied volatility are negatively related to the leading diffusion index level signaling shifts in the prospects about the domestic real economy. Implied volatility is also found to increase significantly with rising levels of volatility in international equity markets.

There are however signs of serial correlation in the residuals suggesting that implied volatility is not fully reflective of the dynamics of the stochastic investment opportunity set described by these proxies of uncertainty in the real and financial economy. The estimation of an autoregressive AR(1) model using nonlinear regression techniques implies Model II results, which satisfy the stationarity condition as the inverted roots lie inside the unit circle. The expected signs and statistical significance of the regression coefficients are confirmed. As a test of model stability, the AR model is estimated with respect to two non-overlapping sample periods. There is evidence of an increasing sensitivity of implied volatility to perceptions of uncertainty over the Japanese real economic conditions. The prospects over the international financial economy continue to play a significant role irrespective of sample periods. These results constitute evidence that expectations of volatility in options markets are not only function of perceptions of uncertainty about the domestic economy but over the international financial economy as well.

8. Conclusions

This study made an attempt to examine the relationship between market volatility and its estimates implicit in options prices and provides new evidence on many issues. First, a new benchmark of implied volatility in the Japanese market was constructed following the methodology of the S&P 100 implied volatility index. The Nikkei 225 implied volatility index allows for the comparison of the level of implied volatility across international markets. There is a tendency for closer alignment of implied volatility levels over the late 1990s despite sharp contrasts between the booming US economy and slumping Japanese economy. Second, the new evidence on the stochastic dynamics of implied volatility sheds lights on options investors' formation of expectations about long-term levels and short-term movements in stock market volatility. Convergence towards the time-varying long-run volatility component occurs at slow rates in both the US and Japanese markets. But the dynamics of short-term convergence of implied volatility's transitory component towards zero are inherently different across markets as it tends to occur in the US market at faster rates than in the Japanese market.

Third, it is shown that using implied volatility in modeling the asymmetric behavior and clustering properties of market volatility implies significant improvements in model estimation. Implied volatility adds useful insights into the leverage effects as its inclusion into the conditional models of volatility explains part of the impact of bad news in Japanese markets and the entire impact in US markets. Fourth, there is evidence that though upwardly biased, *ex ante* implied volatility estimates contain useful information on future levels of realized volatility in the Japanese market. Fifth, the results indicate that volatility expectations in the Japanese options markets are function of uncertainty about the domestic real economy and international financial economy. Implied volatility is found to be an increasing function of the level of volatility in international equity markets and to be sensitive to potential shifts in the business cycle signaled by economic indicators of dispersion of changes such as the leading diffusion index. These results add an international perspective to the limited literature on the informational content of implied volatility in US markets. Whereas investors' beliefs about future volatility are not necessarily based solely on the price history, further analyses of the importance of market sentiment, and the impact of non-quantifiable news on changing patterns of implied volatility are thus warranted.

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Tables

Table 1. Summary statistics and unit root test results

	Descriptive Statistics					*Unit root tests*			
	Mean	Median	Std. Dev.	Skewness	Kurtosis	ADF statistic	Lag order	LB(1)	LB(k)
Panel A				Daily observations					
Nikkei 225 Price	20214.40	18975.10	5451.99	1.5490	5.1406	-2.156[b]	2	0.0055	0.6755
								(0.941)	(0.984)
- Return	-0.0003	-0.0004	0.0148	0.3494	6.8814	-41.531[c]	1	0.0000	0.9392
								(0.995)	(0.967)
- Volatility index	0.2718	0.2633	0.0907	0.5750	4.1256	-6.548[b]	2	0.0123	6.458
								(0.912)	(0.264)
S&P 100 Price	359.08	255.65	209.74	0.8354	2.2288	-1.917[a]	3	0.0019	1.4425
								(0.965)	(0.920)
- Return	0.0005	0.0003	0.0101	-0.3711	8.0955	-39.485[c]	1	0.0288	5.0802
								(0.865)	(0.406)
- Volatility index	0.1973	0.1863	0.0637	0.8300	3.7624	-4.167[a]	7	0.0007	0.0401
								(0.978)	(1.000)
Panel B				Monthly expirations					
VIX -30-day Forward	0.2754	0.2654	0.0919	0.6724	3.6646	-4.405[b]	3	0.0198	5.9433
								(0.888)	(0.919)
- Average	0.2722	0.2631	0.0833	0.2439	2.9852	-4.102[b]	0	0.8936	6.9319
								(0.344)	(0.862)
Std. Dev. - Nikkei Spot	0.2166	0.1950	0.0949	1.2150	5.1522	-6.902[b]	0	0.0476	4.0825
								(0.827)	(0.982)
- MSCI world index	0.1123	0.1003	0.0482	1.2493	4.8342	-4.811[b]	1	0.0174	13.091
								(0.895)	(0.362)
Diffusion index	3.7778	3.9551	0.6140	-2.2172	12.0367	-4.270[b]	1	0.0005	11.217
								(0.981)	(0.510)

Notes: The number of daily and monthly observations is 2953 and 144, respectively. ADF statistics refer to the augmented Dickey-Fuller results for unit root tests. The appropriate lag order is determined using Schwarz information criterion and additional lags are included to eliminate ARCH effects in the residuals. For level tests of daily series, MacKinnon 1% critical values for rejection of the unit root hypothesis are [(a)] –3.967 for tests with intercept and trend terms, [(b)] –3.436 for tests with intercept, and [(c)] –2.567 for tests with neither term. For level tests of monthly series with intercept, MacKinnon 1, 5 and 10% critical values for rejection of the unit root hypothesis are 3.477, -2.882 and –2.577, respectively. LB(k) refers to Ljung-Box test of serial correlation in the residuals. LM(k) is the Lagrange-multiplier test of heteroskedasticity up to order k. The tests are distributed as $\chi^2(k)$ on the null, with k equal to 5 and 12 for daily and monthly series, respectively. Statistics between brackets represent probability values. For monthly expirations, standard deviations of returns on the Nikkei 225 spot and MSCI world price index are annualized statistics. The leading diffusion index is expressed in logarithm.

Table 2. Stochastic dynamics of implied volatility and leverage effects

Panel A	*Component GARCH modeling of implied volatility*									
	ϑ_0	ϑ_1	ϖ	δ	γ	α_1	β_1	LB(10)	LM(10)	Adj. R^2
Nikkei 225	0.003	0.800	0.015	0.998	0.214	-0.030	-0.857	8.163	0.311	0.863
	(0.047)	(0.000)	(0.935)	(0.000)	(0.000)	(0.161)	(0.000)	(0.613)	(0.979)	
S&P 100	0.003	0.867	0.000	0.978	0.053	0.142	0.687	7.745	0.093	0.954
	(0.000)	(0.000)	(0.003)	(0.000)	(0.209)	(0.019)	(0.000)	(0.654)	(1.000)	
Panel B	*Asymmetric GARCH modeling of return volatility*									
TARCH models	μ_0	υ_0	α_1	α_2	β_1	γ_1	ξ	LB(12)	LM(12)	LogL
Model I										
Nikkei 225	0.000	0.000	0.023		0.901	0.134		8.779	0.395	8579.09
	(0.388)	(0.000)	(0.057)		(0.000)	(0.000)		(0.722)	(0.966)	
S&P 100	0.000	0.000	-0.053	0.064	0.905	0.129		19.309	0.602	9729.22
	(0.027)	(0.000)	(0.007)	(0.006)	(0.000)	(0.000)		(0.081)	(0.842)	
Model II										
Nikkei 225	0.000	0.000	-0.037		0.334	0.165	0.370	8.357	0.966	8668.31
	(0.962)	(0.239)	(0.000)		(0.077)	(0.000)	(0.001)	(0.757)	(0.479)	
S&P 100[a]	0.001	0.000	-0.029	0.039	-0.359	0.056	0.808	19.636	0.816	9906.74
	(0.000)	(0.001)	(0.121)	(0.068)	(0.183)	(0.123)	(0.000)	(0.074)	(0.634)	
EGARCH models	μ_0	υ_0	α_1		β_1	γ_1	ξ	LB(12)	LM(12)	LogL
Model I										
Nikkei 225	0.000	-0.318	0.153		0.977	-0.099		9.453	0.471	8594.71
	(0.263)	(0.000)	(0.000)		(0.000)	(0.000)		(0.664)	(0.933)	
S&P 100	0.001	-0.282	0.116		0.979	-0.084		19.188	0.539	9738.41
	(0.022)	(0.000)	(0.000)		(0.000)	(0.000)		(0.084)	(0.890)	
Model II										
Nikkei 225	0.000	-0.432	0.133		0.797	-0.146	0.174	9.895	0.809	8630.49
	(0.367)	(0.005)	(0.000)		(0.000)	(0.000)	(0.070)	(0.625)	(0.641)	
S&P 100[b]	0.001	2.808	-0.063		-0.523	-0.028	1.954	20.128	0.736	9924.12
	(0.000)	(0.000)	(0.153)		(0.042)	(0.471)	(0.000)	(0.065)	(0.717)	

Notes: Component GARCH(1,1) model estimation using Bollerslev-Wooldrige robust standard errors and covariance with lag order (7) determined using Schwartz information criterion. Model I refers to equations (9a) and (9b) which do not include implied volatility as independent variable in the conditional variance of TARCH ad EGARCH models, respectively. Model II includes implied volatility as regressor in the conditional variance equations (10a) and (10b). Model estimation using maximum likelihood ARCH (Marquardt) method and Bollerslev-Wooldridge robust standard errors & covariance. Model selection is based on the Schwartz information criterion and additional lags included in the mean equation to eliminate ARCH effects. [a] ([b]) Lag order of three (four) in the mean equation. LB(k) refers to Ljung-Box test of serial correlation in the residuals. LM(k) is the Lagrange-multiplier test of heteroskedasticity up to order k. The tests are distributed as $\chi^2(k)$ on the null. LogL refers to Log Likelihood statistics.

Table 3. Realized volatility and the forecasting ability of the implied volatility index

	λ_0	λ_v	$\lambda_{v,h}$	λ_S	LB(12)	LM(12)	Adj. R^2
Model I	0.032	0.670			14.944	0.723	0.416
	(0.094)	(0.000)			(0.245)	(0.727)	
Model II	0.054	0.630	-0.179		12.018	0.713	0.427
	(0.011)	(0.000)	(0.025)		(0.444)	(0.735)	
Model III	0.038	0.671	-0.151	-7.881	15.104	0.989	0.491
	(0.059)	(0.000)	(0.044)	(0.000)	(0.236)	(0.463)	
- Sub-period A	0.054	0.606	-0.169	-9.334	9.646	0.769	0.480
1989/06~1995/06	(0.051)	(0.000)	(0.098)	(0.002)	(0.647)	(0.678)	
- Sub-period B	-0.015	0.858	-0.061	-7.208	9.725	0.670	0.515
1995/07~2001/06	(0.648)	(0.000)	(0.612)	(0.002)	(0.640)	(0.771)	

Notes: Number of monthly observations is 144. Estimation of the regression model (12), where λ_v, $\lambda_{v,h}$ and λ_S denote the coefficient estimates associated with the independent variables v_m, $v_{m-1} - \sigma^h_{m-1}$ and $\bar{r}_m$, respectively. LB(k) refers to Ljung-Box test of serial correlation in the residuals. LM(k) is the Lagrange-multiplier test of heteroskedasticity up to order k. The tests are distributed as $\chi^2(k)$ on the null.

Table 4. Structural analysis of implied volatility

	π_0	π_D	π_M	AR(1)	Inverted AR Roots	LB(12)	LM(12)	Adj. R^2
Model I	0.310	-0.037	0.947			72.815	0.887	0.355
	(0.000)	(0.000)	(0.000)			(0.000)	(0.562)	
Model II	0.293	-0.026	0.743	0.519	0.52	10.358	1.490	0.529
	(0.000)	(0.014)	(0.000)	(0.000)		(0.499)	(0.138)	
- Sub-period A	0.231	-0.017	1.040	0.547	0.55	9.812	1.187	0.571
1989/06~1995/06	(0.000)	(0.236)	(0.000)	(0.000)		(0.547)	(0.321)	
- Sub-period B	0.395	-0.045	0.507	0.396	0.40	16.116	0.935	0.477
1995/07~2001/06	(0.000)	(0.003)	(0.001)	(0.001)		(0.137)	(0.521)	

Notes: Estimation of the regression model (17) where π_D and π_M represent the slope coefficients associated with past levels of the diffusion index, and of the standard deviation of MSCI world index, respectively. LB(k) refers to Ljung-Box test of serial correlation in the residuals. LM(k) is the Lagrange-multiplier test of heteroskedasticity up to order k. Probability values reported in brackets. For AR models, the regression statistics are based on the one-period ahead forecast errors.

Figures

Figure 1. The behavior of Nikkei 225 spot and implied volatility index in Japanese and US markets

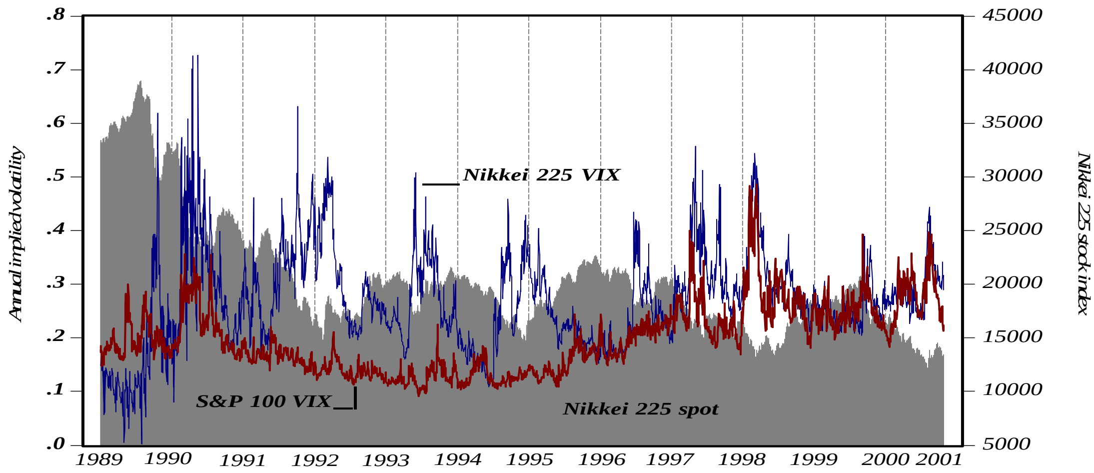


Figure 2. The time series of daily implied volatility index and return on Nikkei 225 stock index

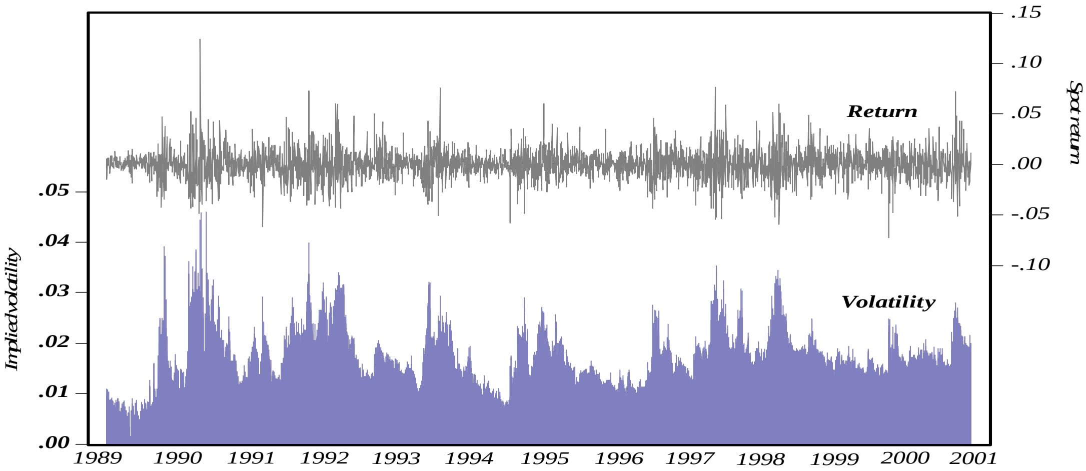


Figure 3. Asymmetric news impact functions from conventional and implied volatility-based TARCH and EGARCH modeling of Nikkei 225 and S&P 100 stock index returns

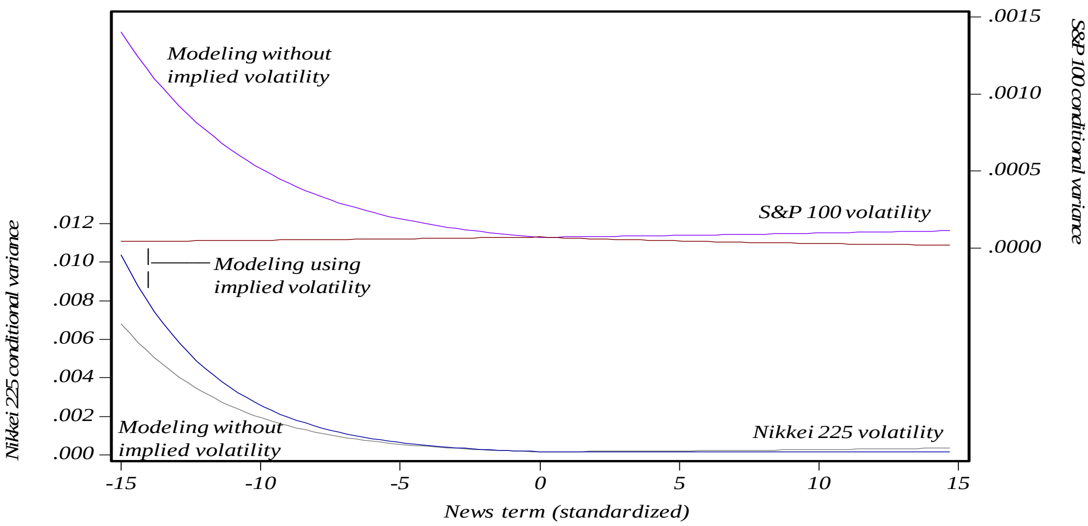


Figure 4. The behavior of realized and implied volatilities over monthly Nikkei 225 options expirations

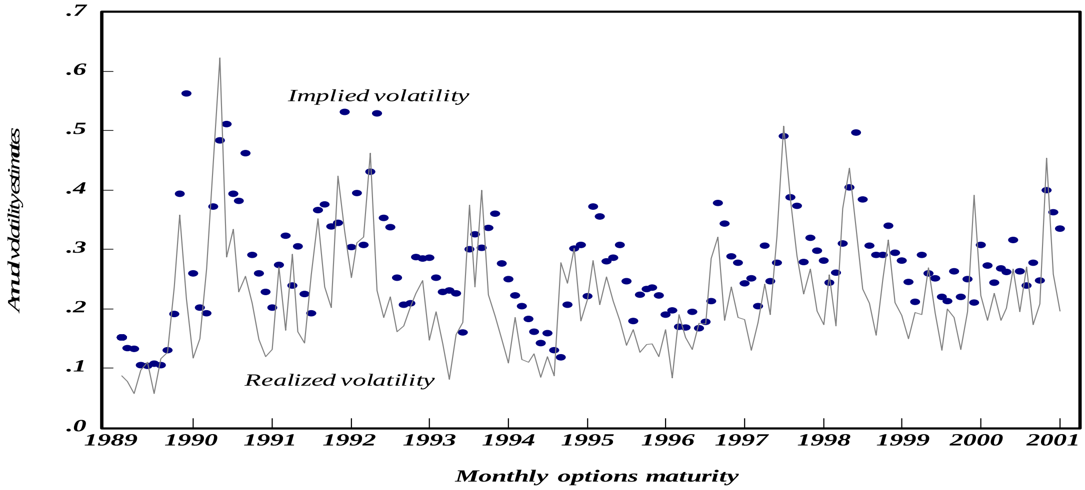
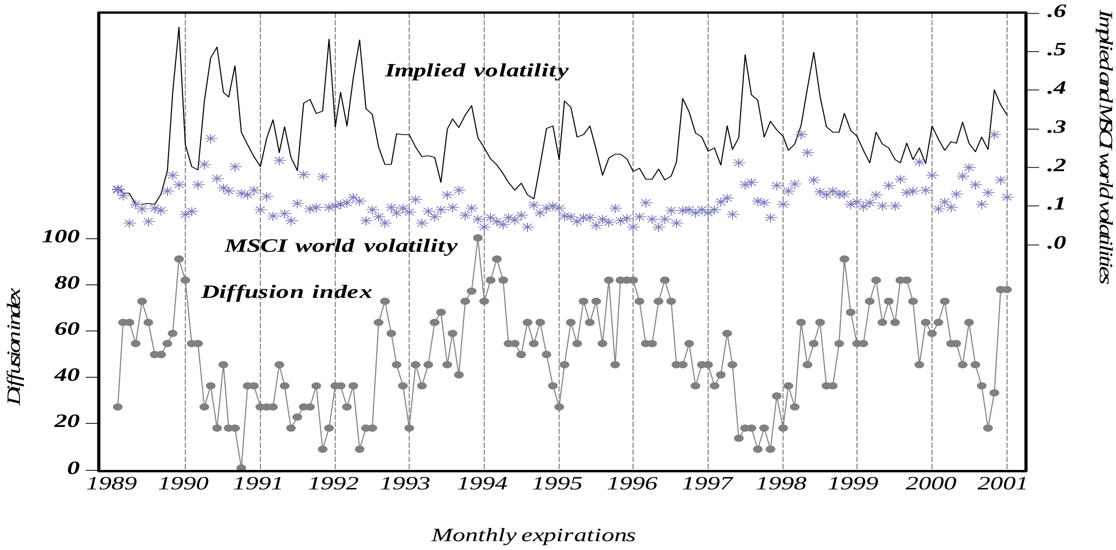
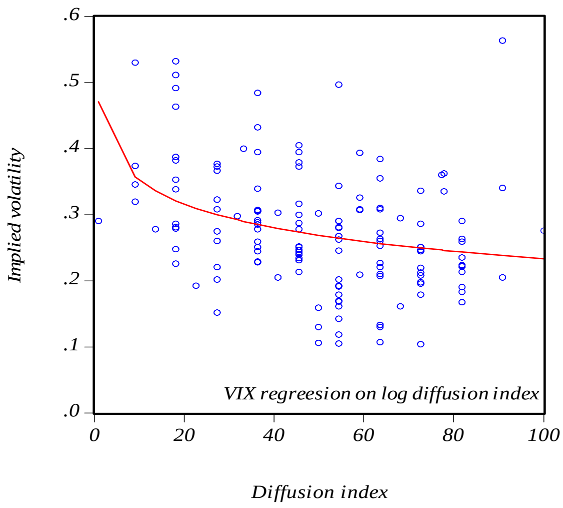
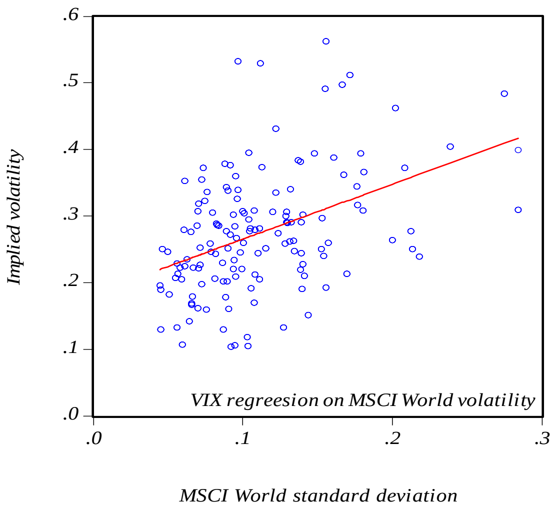


Figure 5. The implied volatility in Japanese options market and its relation with the diffusion index and the volatility of MSCI world price index







Footnotes

[1] Nikkei 225 stock average futures are traded on the Singapore Exchange Derivatives Trading Division (formerly, on the Singapore International Monetary Exchange) since September 3, 1986, on Osaka Securities Exchange since September 3, 1988 and on Chicago Mercantile Exchange together with futures options since September 25, 1990.

[2] In the absence of closed-form solutions, simple approximations of implied volatility values from option premium can be made as in Corrado and Miller (1996) and Chambers and Nawalkha (2001), inter alia.

[3] This brief review of recent literature focuses on studies in implied volatility. A broader survey of research in option pricing is thus beyond its scope but reference is made to useful reviews of tests of option pricing models by Bates (1996) and trends in options research by Chance (1999).

[4] Additional empirical evidence by Jorion (1995) from currency options markets is indicative of higher forecasting ability of implied volatility over GARCH conditional volatility.

[5] These weighting schemes include inter alia, the equal weighting, the elasticity-based approach and the weighting on the basis of the value of vega or partial derivative with respect to volatility. The VIX methodology is closer to the latter scheme as it relies on at- and near–the-money options, which tend also to be the most liquid options.

[6] The expression of implied volatility in trading days rather than calendar days is consistent with empirical evidence in Davidson, Kim, Ors and Szakmary (2001) that volatility in financial markets is more directly related to trading days for a wide variety of underlying assets.

[7] The approach is based on the tacit assumption that investors' expectations about volatility do not change significantly during trading sessions. Its implementation results in as few as 5 and 4 cases, where the opening and highest prices are used for call and put options, respectively.

[8] For further discussion on the mathematics and applications of the ICAPM, see Merton (1973b) and Breeden (1979), inter alia.

[9] The intertemporal approach is consistent with Black-Scholes option pricing, as the replication portfolio remains riskless over a short period of time when the option and underlying asset are subject to the same sources of uncertainty.

[10] The sensitivity of implied volatility to risk factors can be also examined within the theoretical framework of the arbitrage pricing model as in the study by Franks and Schwartz (1991) who regress average implied volatility on economic variables such as real interest rates, oil prices and exchange rates.