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Abstract

I construct a model of a dynamic economy in which the government collects taxes and injects them into banks' capitals for stabilization of financial system. In theoretical part, I derive loan demand and supply functions from dynamic optimizing problems of households and banks. Carrying out a simulation, I show that the injection improves welfare in some first periods, although it rather aggravates in the long term. I also show that the injection induces efficient investments in some first periods, while, it induces inefficient investments in the long term.

Keywords: Public funds injection; Stabilization of financial system; Welfare analysis

JEL classification number: E17; E59; E65

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1 Introduction

The purpose of this paper is to evaluate the effects of public funds injection into private banks. In Japan, to mitigate a systemic risk, huge amounts of public funds were injected in 1998 and 1999, and currently there is a heated dispute over whether to inject them or not. The discussion of public funds injection so far often focuses on moral hazard problem of a bank manager (see Osano, 2001).

Another problem of the injection, which has not been focused on so far, is who bears its cost. This paper considers this unheeded cost and assumes it is financed by taxes. The actual injection in Japan was carried out by borrowing 7.5 trillion yen (63 billion dollar) from the Bank of Japan and the private financial institutions, and investing in the preferred stocks and subordinated debentures which the recipient banks newly issued[1]. Seemingly, people do not have any burdens. However, this is not true because the government fully guarantees its debts. If the preferred stocks or subordinated debentures become valueless by the failures of the recipient banks, those losses will be compensated by taxes.

The injection may also increase bank lendings and lower a lending rate of interest. This happens because the injected public funds can be used for the lendings and a blip of the bank's capital ratio, the ratio of capitals to assets, lowers its credit cost under the Basel Accord. Figure 1 implies this is actually the case: when the public funds were injected in 1999, the lending attitude of financial institutions in Japan were greatly improved for accommodative.

To analyze how these effects influence the macroeconomy through time, I construct a model of a dynamic economy in which the government collects taxes and injects them into banks' capitals as a financial system stabiliza-

[1]The breakdown of the borrowed money was as follows: about 1.2 trillion yen (10 billion dollar) from the Bank of Japan, and about 6.3 trillion yen (53 billion dollar) from the private financial institutions.

tion measure. Since it is too difficult to solve analytically the influences of the public funds injection on the economy, I carry out a simulation by using the theoretical model and the empirically-known parameters. From this simulation, it is shown that the injection improves welfare in some first periods; however, it rather aggravates in the long term. It is also shown that the injection induces efficient investments in some first periods, while, it induces inefficient investments in the long term.

To the best of our knowledge, there are few studies on public funds injection into banks. Kobayashi(2002) makes a survey of financial system stabilization measures, including public funds injection. Osano(2001) studies the optimal scheme of injection to prevent a bank from taking moral hazard action.

The rest of this paper is organized as follows. Section 2 derives the borrowing-demand function and the bank's loan-supply function. Section 3 analyzes the effects of the tax-financed public funds injections into banks. Section 4 carries out the simulation. Section 5 concludes.

2 The model

2.1 The household production

Consider a discrete-time economy in which agents borrow from a bank and carry out investments, productions, and consumptions. Hereafter, we call those agents household productions. They are homogeneous with population size one. Each household production is risk-averse, and has the following CRRA type expected utility function at date t:

$$E_t[\sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma}] , \tag{1}$$

where c_{t+s} is his or her consumption at date $t+s$, and E_t denotes expectations formed at date t. The subjective discount factor β takes the value between zero and one strictly. σ expresses the degree of relative risk aversion. These parameters are common to all agents.

Each household production has the following Cobb-Douglas type production function:

$$y_t = f(k_{t-1}) = k_{t-1}^{\alpha} \ , \tag{2}$$

where k_{t-1} is the capital used for production at date $t-1$, and y_t is the output (GDP) at date t. α is a capital share relative to the output and lies between zero and one.

Each household production can borrow from a bank. The amount of the loan made at date $t-1$ and the gross rate of interest at date $t-1$ are denoted by b_{t-1} and R_{t-1}, respectively. At date t, he or she consumes c_t, repays $R_{t-1}b_{t-1}$, and invests $k_t - k_{t-1}$. Since these expenditures are financed by the output y_t and the new loan b_t, each household production faces the following budget constraint:

$$f(k_{t-1}) + b_t \geq c_t + R_{t-1}b_{t-1} + (k_t - k_{t-1}) \ . \tag{3}$$

All these variables are measured by the consumption goods.

Each household production maximizes expected discounted utility (1) subject to intertemporal budget constraint (3) and the perfect foresight assumption $E_t[x_{t+s}] = x_{t+s}$:

$$\begin{aligned}
\max_{\{\mathbf{c},\mathbf{b},\mathbf{k}\}} : \ E_t[\sum_{s=0}^{\infty} \beta^s u(c_{t+s})] \ &= \ \sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma} \\
s.t. \quad f(k_{t-1}) + b_t &\geq c_t + R_{t-1}b_{t-1} + (k_t - k_{t-1}) \ .
\end{aligned}$$

By appealing to the Bellman's principle of optimality, the solutions are obtained as follows[2]:

$$u'(c_{t-1}) \ = \ \beta R_{t-1} u'(c_t) \quad \text{or} \quad c_t = (\beta R_{t-1})^{\frac{1}{\sigma}} c_{t-1} \ , \tag{4}$$

$$f'(k_t) \ = \ R_t - 1 \quad \text{or} \quad k_t = (\frac{R_t - 1}{\alpha})^{\frac{1}{\alpha-1}} \ , \tag{5}$$

[2]Refer to Appendix A for the detailed derivation.

$$
\begin{aligned}
b_t &= (\beta R_{t-1})^{\frac{1}{\sigma}} c_{t-1} + R_{t-1} b_{t-1} + (\frac{R_t - 1}{\alpha})^{\frac{1}{\alpha-1}} \\
&\quad - (\frac{R_{t-1} - 1}{\alpha})^{\frac{1}{\alpha-1}} - (\frac{R_{t-1} - 1}{\alpha})^{\frac{\alpha}{\alpha-1}} \ . \qquad (6)
\end{aligned}
$$

Equation (4) claims that he or she makes an optimal consumption choice. Equation (5) claims that he or she makes an optimal investment choice. Equation (6) is interpreted as the borrowing-demand function. Equation (6) is derived by substituting optimal paths (4) and (5) for budget constraint (3) with equality.

From equations (2) and (5), the optimal output y_t can be obtained as

$$
y_t = f(k_{t-1}) = k_{t-1}^{\alpha} = (\frac{R_{t-1} - 1}{\alpha})^{\frac{\alpha}{\alpha-1}} \ . \qquad (7)
$$

To ensure long-term rationality of household productions, we rule out exploding bubbles in the gross rate of interest:

$$
\lim_{s \to \infty} E_t[\beta^{-s} R_{t+s}] = 0 \ . \qquad (8)
$$

Before we turn to dynamics, it is useful to look at the steady-state equilibrium. The steady-state variables should satisfy

$$
\begin{aligned}
R^* &= 1/\beta \ , \\
k^* &= (\frac{1-\beta}{\alpha\beta})^{\frac{1}{\alpha-1}} \ , \\
c^* &= (\frac{\beta-1}{\beta}) b^* + k^{*\alpha} = (\frac{\beta-1}{\beta}) b^* + (\frac{1-\beta}{\alpha\beta})^{\frac{\alpha}{\alpha-1}} \ , \\
y^* &= k^{*\alpha} = (\frac{1-\beta}{\alpha\beta})^{\frac{\alpha}{\alpha-1}} \ .
\end{aligned}
$$

Now I consider the dynamics. Since optimal paths (4)-(6) are non-linear, it is difficult to derive an analytical solution. Then, I analyze this system by linearizing around the steady state. Denoting the percentage deviation from the steady state of an optimal path $\{X_t\}$ as $\widehat{X}_t \equiv (X_t - X^*)/X^*$, the linear approximations of equations (4)-(6) are

$$
\widehat{c}_t = \frac{1}{\sigma} \widehat{R}_{t-1} + \widehat{c}_{t-1} \ , \qquad (9)
$$

$$\widehat{k}_t = -a_1 \widehat{R}_t \ , \tag{10}$$

$$\widehat{b}_t = a_2 \widehat{c}_{t-1} + \frac{1}{\beta} \widehat{b}_{t-1} - a_1 a_3 \widehat{R}_t + a_4 \widehat{R}_{t-1} \ , \tag{11}$$

$$\widehat{y}_t = -\alpha a_1 \widehat{R}_{t-1} \ , \tag{12}$$

where

$$a_1 \equiv \frac{1}{(1-\alpha)(1-\beta)} \ (>0) \ , \ a_2 \equiv (\frac{c^*}{b^*}) \ , \ a_3 \equiv (\frac{k^*}{b^*}) \ , \ a_4 \equiv \frac{1}{\sigma} a_2 + \frac{1}{\beta} + a_1 a_3 - \frac{1}{(\alpha-1)\beta} a_3 \ .$$

2.2 The representative bank

The balance sheet identity of a bank at date t is given as

$$CASH_t + L_t = \overline{D_t} + \overline{CAP_t} \ , \tag{13}$$

where $CASH_t$ is the amount of cash (riskless assets)[3] at date t, L_t is the amount of loans (risky assets) at date t, $\overline{D_t}$ is the amount of debts at date t, and $\overline{CAP_t}$ is the amount of capitals at date t. We assume that the bank raises funds of debts from the outside perfect financial market. Among these variables, $CASH_t$ and L_t are endogenous variables, and $\overline{D_t}$ and $\overline{CAP_t}$ are exogenous variables[4].

At date t, the bank aims to maximize its next date profit Π_{t+1} under balance sheet identity (13)[5]:

$$\max_{L_t} : \Pi_{t+1} = (R_t - 1)L_t - (\overline{R_t^D} - 1)\overline{D_t} - C(FL_t, BIS_t) \ , \tag{14}$$

where R_t is the gross rate of interest on the loans from date t to date $t+1$, which is assumed to to be the same as that on the borrowings of

[3]Actually, banks often hold the government bonds as riskless assets. Here, for simplicity, I assume banks to hold cash as riskless assets.

[4]In order to focus on the bank lendings, I assume that the amount of debts is exogenous.

[5]Readers may wonder why the bank does not aim to maximize its profit over the "infinite" horizons. We have two reasons to adopt the simplified optimization. First, the dynamic paths become too complicated if we adopt the infinite problem. Second, each approach yields the similar results.

the household productions, $\overline{R_t^D}$ is the gross rate of interest on the debts from date t to date $t+1$, C_t is the cost function of the bank at date t, $FL_t \equiv L_t - L_{t-1}$ is a net increase of the loans from date $t-1$ to date t, and $BIS_t \equiv \overline{CAP_t}/L_t$ is the bank's capital ratio at date t. Among these variables, R_t is an endogenous variable, and $\overline{R_t^D}$ is an exogenous variable.

The cost function of the bank at date t, C_t, has two elements: adjustment costs and credit costs (BIS costs). The bank bears adjustment costs when it increases new loans (see Ogawa and Kitasaka, 2000, and E.Elyasiani et al., 1995). The cost function C_t satisfies the following conditions:

$$\begin{cases} \dfrac{\partial C_t}{\partial FL_t} & > 0 \ , \\ \dfrac{\partial^2 C_t}{\partial FL_t^2} & \geq 0 \ . \end{cases}$$

The bank also incurs a penalty from the financial market when the bank's capital ratio becomes smaller. Hereafter, we call this penalty credit costs (BIS costs) (see Ito and Sasaki, 2000). The cost function C_t also satisfies the following conditions:

$$\begin{cases} \dfrac{\partial C_t}{\partial BIS_t} & < 0 \ , \\ \dfrac{\partial^2 C_t}{\partial BIS_t^2} & \geq 0 \ . \end{cases}$$

I specify the cost function C_t, which satisfies the above conditions, as

$$C(FL_t, BIS_t) = \gamma_0 FL_t + \frac{\gamma_1}{2} FL_t^2 + \frac{\gamma_2}{2} \frac{1}{BIS_t^2} \qquad (\gamma_0, \gamma_1, \gamma_2 > 0) \ . \tag{15}$$

The first order condition of this profit maximization problem is

$$(R_t - 1) = \frac{\partial C_t}{\partial FL_t} \frac{\partial FL_t}{\partial L_t} + \frac{\partial C_t}{\partial BIS_t} \frac{\partial BIS_t}{\partial L_t} \ . \tag{16}$$

Equation (16) claims that the marginal revenue and marginal cost are equal[6]. The optimal path is obtained from equation (16) as

$$L_t \quad = \quad \frac{\gamma_1}{\gamma_1 + \frac{\gamma_2}{\overline{CAP_t}^2}} L_{t-1} + \frac{1}{\gamma_1 + \frac{\gamma_2}{\overline{CAP_t}^2}} R_t - \frac{\gamma_0 + 1}{\gamma_1 + \frac{\gamma_2}{\overline{CAP_t}^2}} \ , \tag{17}$$

[6]The second order condition, $\frac{\partial^2 \Pi_{t+1}}{\partial L_t^2} \leq 0$, is also satisfied.

which is interpreted as the loan-supply function.

Now, let us consider the dynamics. The linear approximation of the loan-supply function and the steady-state value of L_t are calculated as follows respectivley:

$$\begin{aligned}\widehat{L}_t &= \delta_1\widehat{L}_{t-1} + \delta_2\widehat{R}_t + \delta_3\widehat{\overline{CAP}}_t \ , \qquad (18)\\ L^* &= \frac{\overline{CAP}^{*2}}{\gamma_2}(\frac{1}{\beta} - 1 - \gamma_0) \ ,\end{aligned}$$

where

$$\delta_1 \equiv \frac{\gamma_1}{\gamma_1 + \frac{\gamma_2}{\overline{CAP}^{*2}}} \ (< 1) \ , \ \delta_2 \equiv \frac{1}{\beta L^*(\gamma_1 + \frac{\gamma_2}{\overline{CAP}^{*2}})} \ , \ \delta_3 \equiv \frac{2\gamma_2}{\overline{CAP}^{*2}\gamma_1 + \gamma_2} \ .$$

2.3 The market equilibrium

The market equilibrium condition of the bank loans and borrowings is $L_t = b_t$, which determins R_t. The linear approximation of the above equation is $\widehat{L}_t = \widehat{b}_t$, which determins $\widehat{R}_t$:

$$\widehat{R}_t = \frac{1}{\delta_4}\{a_2\widehat{c}_{t-1} + (\frac{1}{\beta} - \delta_1)\widehat{b}_{t-1} + a_4\widehat{R}_{t-1}\} \ , \qquad (19)$$

where

$$\delta_4 \equiv \delta_2 + a_1a_3 \ (> 0) \ .$$

Equation (19) shows that a contemporaneous lending rate of interest depends on the various "lagged" variables.

3 Tax-financed public funds injection into the bank

In this section, I evaluate the effects of tax-financed public funds injection on the economy. To examin, I adopt the technique employed in Kiyotaki and Moor (1997): the injection and taxation are considered as unanticipated shocks to the steady-state equilibrium, and the response to that impulse is considered.

Suppose that the economy is in the steady state in which a bank's capital ratio is relatively low. Then, in order to improve the bank's fragility and stabilize the financial system, the government collects taxes from the household productions and injects them into the bank's capital at date $t = T$. The taxes are collected by a lump-sum manner.

1. At date $t = T$

Each household production pays a lump sum tax τ to the government, and the government injects them to the bank. Moreover, I assume that the proportion $\lambda(0 \leq \lambda \leq 1)$ of the injected public funds are used for the loans, and the rest $1 - \lambda$ of those are left as cash. Denoting the ratio of the tax to the output (GDP) as $\widehat{\tau}(\equiv \tau/y^*)$ and noting that $\widehat{X}_{T-1} = 0$ holds, equations (9) $-$ (12) and (18) at date $t = T$ become

$$
\begin{aligned}
\widehat{c}_T &= 0 \, , \\
\widehat{k}_T &= -a_1 \widehat{R}_T \, , \\
\widehat{b}_T &= -a_1 a_3 \widehat{R}_T + (\frac{y^*}{b^*})\widehat{\tau} \, , \\
\widehat{L}_T &= \delta_2 \widehat{R}_T + \delta_3 \widehat{\overline{CAP}}_T + \lambda(\frac{y^*}{L^*})\widehat{\tau} \, , \\
\widehat{\overline{CAP}}_T &= (\frac{y^*}{\overline{CAP}^*})\widehat{\tau} \, .
\end{aligned}
$$

The percentage deviation of the gross rate of interest at date $t = T$, $\widehat{R}_T$, is calculated by the market equilibrium condition $\hat{b}_T = \hat{L}_T$. The result is

$$
\widehat{R}_T = \frac{y^*}{\delta_4}\{(\frac{1-\lambda}{b^*}) - (\frac{\delta_3}{\overline{CAP}^*})\}\widehat{\tau} \, . \tag{20}
$$

The first term $(1 - \lambda)/b^*$ expresses the taxation effect: the lending rate of interest rises at date $t = T$. The second term $(\delta_3/\overline{CAP}^*)$ is the injection effect: the lending rate of interest falls at date $t = T$. The total effect on the lending rate of interest at date $t = T$, which is the sum of the first and second effects, depends on parameters.

2. At date $t \geq T+1$

Equations (19) $-$ (12) directly apply at date $t \geq T+1$. Equation (18) at date $t \geq T+1$ becomes

$$\widehat{L}_t \quad = \quad \delta_1 \widehat{L}_{t-1} + \delta_2 \widehat{R}_t + \delta_3 (\frac{y^*}{\overline{CAP}^*}) \widehat{\tau} \ .$$

The percentage deviation of the gross rate of interest at date $t \geq T+1$, $\widehat{R}_t$, is calculated by the market equilibrium condition $\hat{b}_t = \hat{L}_t$. The result is

$$\widehat{R}_t = \frac{1}{\delta_4} \{ a_2 \widehat{c}_{t-1} + (\frac{1}{\beta} - \delta_1) \widehat{b}_{t-1} + a_4 \widehat{R}_{t-1} - \delta_3 (\frac{y^*}{\overline{CAP}^*}) \widehat{\tau} \} \ . \tag{21}$$

4 Simulation

It is too difficult to solve analytically the influences of the public funds injection on the economy. Thus, we carry out a simulation to show numerically how the economy developes through time[7]. In order to carry out the simulation, we need to determine values of parameters in a model. As the values of the parameters, I adopt the values compatible with the real economy and those based on empirical studies.

Six baseline (deep) parameters appear in this model: α, β, γ_0, γ_1, γ_2, λ and σ. For α(the capital share relative to the output) and β(the subjective discount rate), Cooley and Prescott (1995) reported $\alpha = 0.40$ and $\beta = 0.98$, where one period is taken as three months. In this simulation, I adopt these values and the interval.

For the parameters in the cost function of the bank, the values of $\gamma_0 = \gamma_1 = 1.4 \times 10^{-2}$ and $\gamma_2 = 3.6 \times 10^{-3}$ imply that $((\overline{R^D}^* - 1)\overline{D}^*)/C(FL^*, BIS^*) =$ (costs of paying the interest on debts)/(adjustment costs + credit costs)=9[8], BIS^*(capital ratio) $\equiv \overline{CAP}^*/L^* = 0.08$, $\overline{R^D}^* = 1.00115$ [9] and $\overline{D}^* + \overline{CAP}^* =$

[7] For the general instructions of the simulation methods, refer to Kawasaki(1999).

[8] This calibration reflects the fact that about 90 % of the total costs are used for paying interest on debts in the Japanese banks.

[9] We take $\overline{R^D}^*$ as the average deposit rates from 1995 to 2001 in Japan. As for the amount and term, I adopt "no less than 10 million yen" and "3 months less than 6 months".

L^*. We assume $\lambda = 0.5$ and $\sigma = 1$. Table 1 collects the values of these baseline (deep) parameters.

When the values of the baseline (deep) parameters in Table 1 are known, the steady-state values of R^*, k^* and y^* can be determined. The results are $R^* = 1.018$, $k^* = 1.7 \times 10^2$ and $y^* = 7.8$. The values of $\overline{CAP}^*$=9.4 and $b^* = L^* = 1.2 \times 10^2$ imply that BIS^*(capital ratio) $\equiv \overline{CAP}^*/L^*$=0.08 and b^*/k^*=0.7 . And, when these values are known, we can calculate the amount of the consumption in the steady state c^*. The result is $c^* = 5.6$. Finally, we assume that the tax rate in the steady state, $\widehat{\tau}^* = \tau^*/y^*$, is 0.02. Table 2 collects the steady-state values of these variables. The values of $a_1 \sim a_4$ and $\delta_1 \sim \delta_4$ can be calculated from the values in Tables 1 and 2. Table 3 presents the results.

We are now in a position to investigate stability of the dynamics around the steady state. Using the results of Tables 1 and 3, it can be shown that one eigenvalue is larger than 1, which corresponds to an explosive path; and that the other two eigenvalues lie strictly between -1 and 1, which corresponds to stable paths[10]. Therefore, this dynamics has a saddle-stable path. We take the gross rate of interest to be a jump variable so that the dynamic path

Table 1: Values of the baseline (deep) parameters

α	β	γ_0	γ_1	γ_2	λ	σ
0.40	0.98	1.3×10^{-2}	1.3×10^{-2}	3.6×10^{-3}	0.5	1

Table 2: Steady-state values of the variables

R^*	k^*	y^*	$b^*(= L^*)$	c^*	$\overline{CAP}^*$	BIS^*	$\widehat{\tau}^*$
1.018	1.7×10^2	7.8	1.2×10^2	5.6	9.4	0.08	0.02

Table 3: Values of the coefficients

a_1	a_2	a_3	a_4	δ_1	δ_2	δ_3	δ_4
91.7	4.8×10^{-2}	1.4	1.3×10^2	0.997	0.63	5.9×10^{-3}	1.3×10^2

[10]Refer to Appendix B for the derivation.

lies on a two-dimensional stable manifold:

$$\widehat{R}_T = \frac{y^*}{\delta_4}\{(\frac{1-\lambda}{b^*}) - (\frac{\delta_3}{CAP^*})\}\widehat{\tau} + \widehat{R}_{jump} \ ,$$

where $\widehat{R}_{jump}$ is a jump component of $\widehat{R}_T$, which is chosen so as to satisfy the long-term rationality condition (8). Actually, we can find the value of $\widehat{R}_{jump}$ such that $\widehat{R}_{T+s}$ converges to some finite value for large s.

Figure 2 shows how the lending rate of interest developes through time. At the initial date, it jumps upward greatly, which can be understood as an overshooting phenomenon of the taxation effect. After the initial date, it gradually falls toward the new steady-state equilibrium, which is about 0.002 % less than the original steady state. One possible explanation of this gradual decline is that the injection into the bank's capitals reduces the credit cost permanently. Fourth term in the R.H.S. of equation (21), $-\delta_3(\frac{y^*}{CAP^*})\widehat{\tau}$, captures this effect.

Figure 3 shows how the borrowings develope through time. At the initial date, they jump downward greatly, which is triggerd by the upward jump of the lending rate of interest. After the initial date, they gradually increase toward the new steady-state equilibrium, which exceeds the original steady state by about 1.0 %.

Figure 4 shows how the outputs (GDP) develope through time. At the second quarters, they jump downward greatly, which is triggerd by the upward jump of the lending rate of interest. After the second quarters, they gradually increase toward the new steady-state equilibrium, which exceeds the original steady state by about 0.8 %.

Figure 5 shows how the capital stocks develope through time. At the initial date, they jump downward greatly, which is triggerd by the upward jump of the lending rate of interest. After the initial date, they gradually increase toward the new steady-state equilibrium, which exceeds the original steady state by about 0.2 %.

Finally, Figure 6 shows how the consumption developes through time. From the initial date to the 40th quarters, the consumption exceeds the original steady state: the injection improves welfare in some first periods. However, after the 40th quarters, it gradually decreases toward the new steady-state equilibrium, which is about 0.25 % less than the original steady state: the injection rather aggravates welfare in the long term.

The intuition for why this is the case is given by considering equation (9). The first term $\frac{1}{\sigma}\widehat{R}_{t-1}$ in the R.H.S. of equation (9) captures the substitution effect by interest rate fluctuations: when the interest rate at date $t-1$, $\widehat{R}_{t-1}$, rises, an agent finds it profitable to consume more at the next date t because the gain from tilting its consumption from date $t-1$ to date t also increases. The second term $\widehat{c}_{t-1}$ in the R.H.S. of equation (9) captures the effect of diminishing maginal utility: when an agent consumes more at date t, she finds it profitable to consume more at next date t because the utility gain from consuming more at date t decreases. Therefore, the consumption fluctuates parallel to the interest rate with some lags.

Putting Figure 5 and 6 together yields another important implications. From 14th quarters to 40th, the increase of the capital stocks results in the increase of the consumption: the injection induces efficient investments. However, after the 40th quarters, the increase of the capital stocks never results in the increase of the consumption: it induces inefficient investments.

5 Conclusion

I have analyzed how tax-financed public funds injection into banks influence a macroeconomy. First, I carried out theoretical analyses to derive loan demand and supply functions from dynamic optimizing problems of households and banks. Then, I carried out a simulation by using the theoretical model and the empirically-known parameters. From the simulation, it is shown that the injection improves welfare in some first periods; however, it rather aggravates in the long term. It is also shown that the injection induces efficient investments in some first periods, while, it induces inefficient investments in the long term. These results have important policy implications: public funds injection itself can mitigate a systemic risk, however, it can aggravate welfare and induce inefficient investments in the long term. Therefore, the government should take these things into consideration when it carries out public funds injection.

Finally, in the future research, it would be very interesting to discuss to what extent the public funds injection can mitigate a systemic risk. This discussion is important when we consider the total quantitative evaluation of the public funds injection.

Appendix A: derivation of the optimal paths of the household production

Under the perfect foresight assumption $E_t[x_{t+s}] = x_{t+s}$, the utility maximization problem of the household production over infinite terms is

$$\max_{\{\mathbf{c},\mathbf{b},\mathbf{k}\}} : \; E_t[\sum_{s=0}^{\infty} \beta^s u(c_{t+s})] \; = \; \sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma}$$

$$s.t. \quad f(k_{t-1}) + b_t \geq c_t + R_{t-1}b_{t-1} + (k_t - k_{t-1}) \; .$$

This maximization problem over infinite terms can be solved by the Bellman's principle of optimality. First, the variables decided at date $t-1$ and known at date t can be proxied by a state variable ω_t at date t. In this problem, ω_t is chosen as

$$\omega_t \equiv f(k_{t-1}) + k_{t-1} - R_{t-1}b_{t-1} \; . \tag{22}$$

The discounted present value of lifetime utility from date t, $\sum_{s=0}^{\infty} \beta^s u(c_{t+s})$, is determined by ω_t, and let $V(\omega_t)$ denote the value function of this lifetime utility. Then, the above maximization problem over infinite terms can be reduced to a two-period problem with the state variable ω_t and value function $V(\omega_t)$ (Bellman equation):

$$V(\omega_t) = \max_{\{c_t,b_t,k_t\}} [u(c_t) + \beta V(\omega_{t+1})] \tag{23}$$

$$s.t. \quad \omega_t \equiv f(k_{t-1}) + k_{t-1} - R_{t-1}b_{t-1} = c_t + k_t - b_t \; . \tag{24}$$

When ω_{t+1} in equation (23) is written in terms of c_t, b_t and k_t by using equation (24) at date $t+1$, the above maximization problem becomes

$$V(\omega_t) = \max_{\{c_t,b_t,k_t\}} [u(c_t) + \beta V(f(k_t) + k_t - R_t b_t)] \;\; s.t. \;\; \text{Eq.}(44) \; . \tag{25}$$

Although equation (25) is the maximization problem with respect to three variables c_t, b_t and k_t, this problem can be reduced to that of two variables c_t and k_t by eliminating b_t:

$$V(\omega_t) = \max_{\{c_t,k_t\}} [u(c_t) + \beta V(f(k_t) + k_t - R_t(c_t + k_t - \omega_t))] \; . \tag{26}$$

Hereafter, we use this simplified equation (26).

The first order conditions with respect to c_t and k_t are obtained as follows respectively:

$$\begin{aligned} u'(c_t) + \beta V_\omega(\omega_{t+1}) \frac{\partial \omega_{t+1}}{\partial c_t} &= u'(c_t) + \beta V_\omega(\omega_{t+1})(-R_t) = 0 \ , && (27) \\ \beta V_\omega(\omega_{t+1}) \frac{\partial \omega_{t+1}}{\partial k_t} &= \beta V_\omega(\omega_{t+1})(f'(k_t) + 1 - R_t) = 0 \ . && (28) \end{aligned}$$

And, the following equation is obtained by differentiating both sides of equation (26) with respect to ω_t (envelope theorem):

$$V_\omega(\omega_t) = \beta V_\omega(\omega_{t+1}) \frac{\partial \omega_{t+1}}{\partial \omega_t} = \beta V_\omega(\omega_{t+1}) R_t \ . \quad (29)$$

Substituting $V_\omega(\omega_{t+1})$ in equation (29) for equation (27) yields $u'(c_{t-1}) = \beta R_{t-1} u'(c_t)$, which is the same as equation (4). And equation (28) is equivalent to $f'(k_t) = R_t - 1$, which is the same as equation (5).

Appendix B: calculating the eigenvalues of the dynamic system

The independent variables are only c_t, R_t and b_t. From equations (15), (27) and (30) together with $\widehat{b}_t = \widehat{L}_t$, the transition equations are surmarized into the following matrix form:

$$\begin{pmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{b}_t \end{pmatrix} = \begin{pmatrix} 1 & 1/\sigma & 0 \\ a_2/\delta_4 & 1/\delta_4(1/\beta - \delta_1) & a_4/\delta_4 \\ (a_2\delta_2)/\delta_4 & \delta_1 + \delta_2/\delta_4(1/\beta - \delta_1) & (a_4\delta_2)/\delta_4 \end{pmatrix} \begin{pmatrix} \widehat{c}_{t-1} \\ \widehat{R}_{t-1} \\ \widehat{b}_{t-1} \end{pmatrix} .$$

Using the results of Tables 1 and 3, we can calculate the eigenvalues of this transition matrix. The results are 1.3628, 0.9994 and -0.7321.

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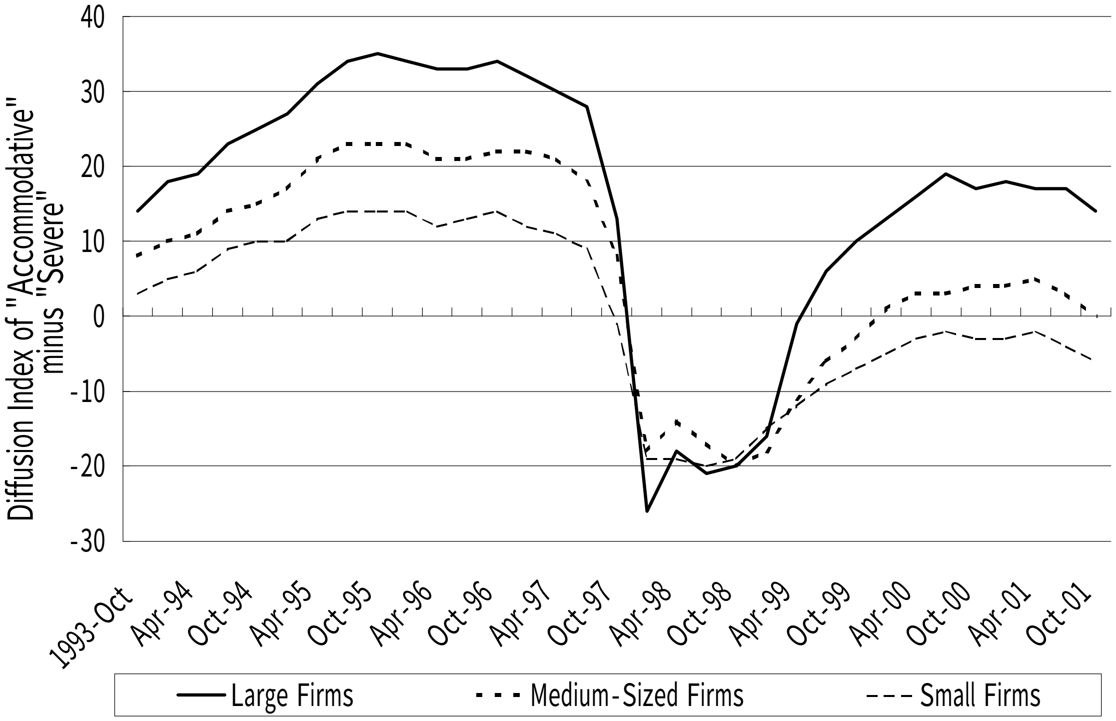

Fig.1. Lending Attitude of Financial Institutions in Japan

(Source: TANKAN (Short-term Economic Survey of All Enterprises in Japan) 2001, the Bank of Japan)

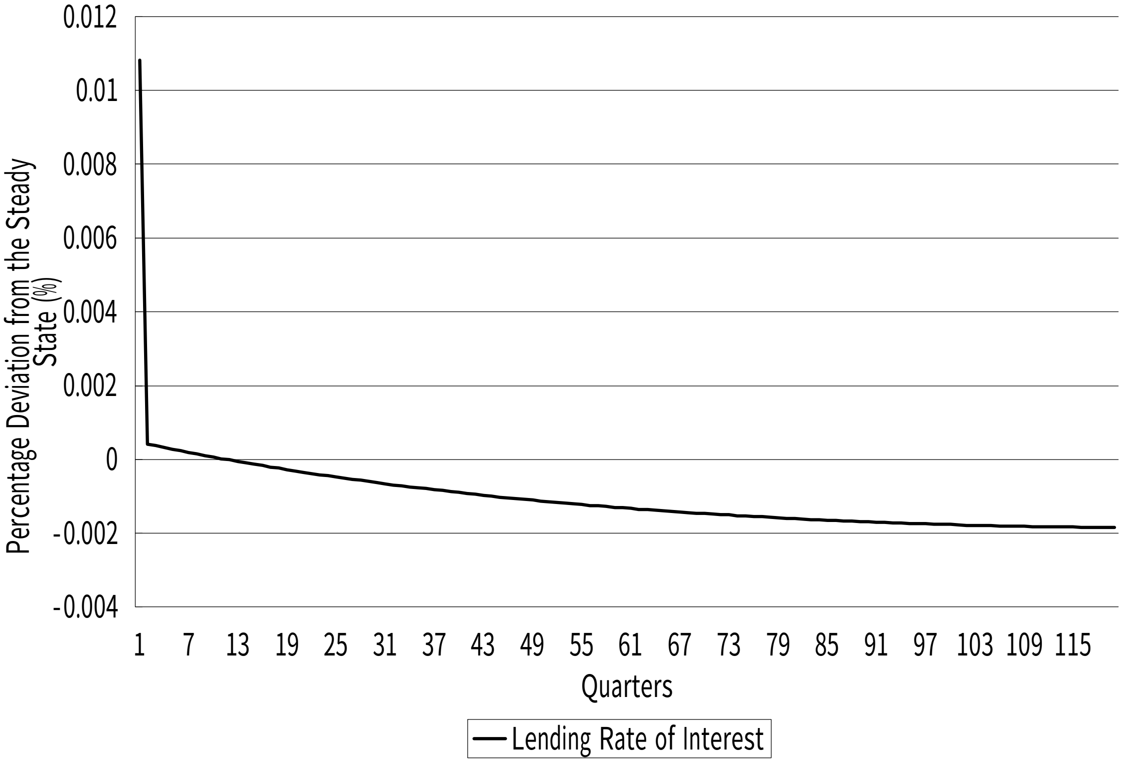

Fig.2. Dynamics of the Lending Rate of Interest

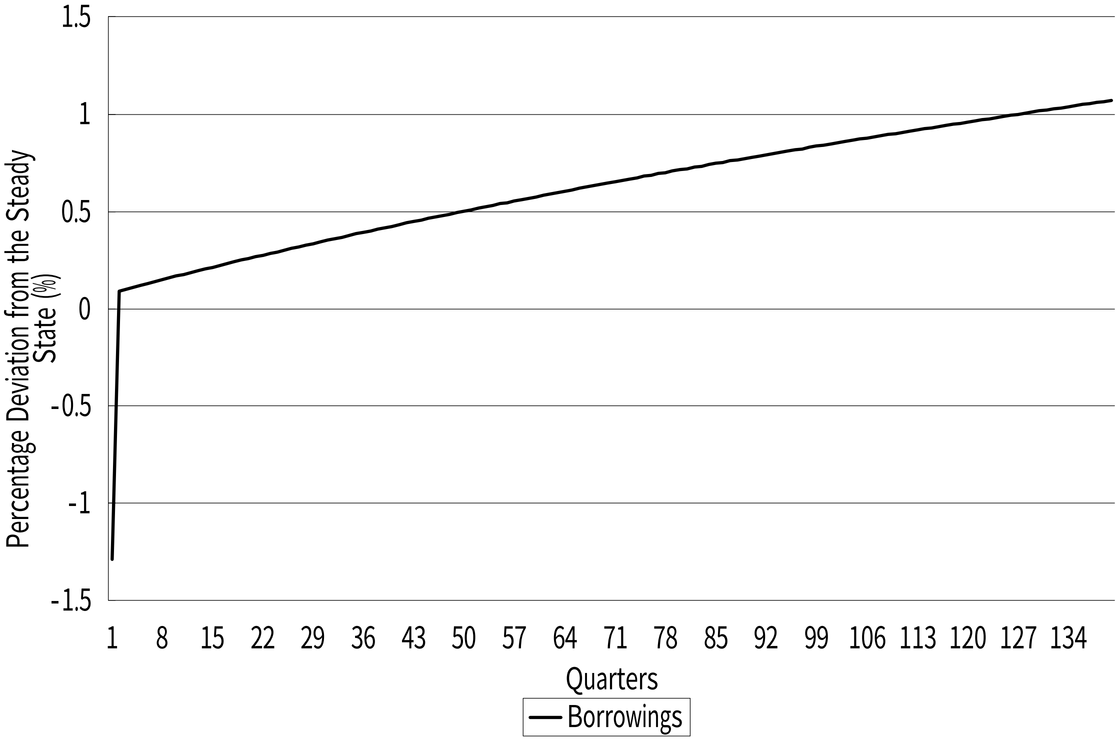

Fig.3. Dynamics of the Borrowings

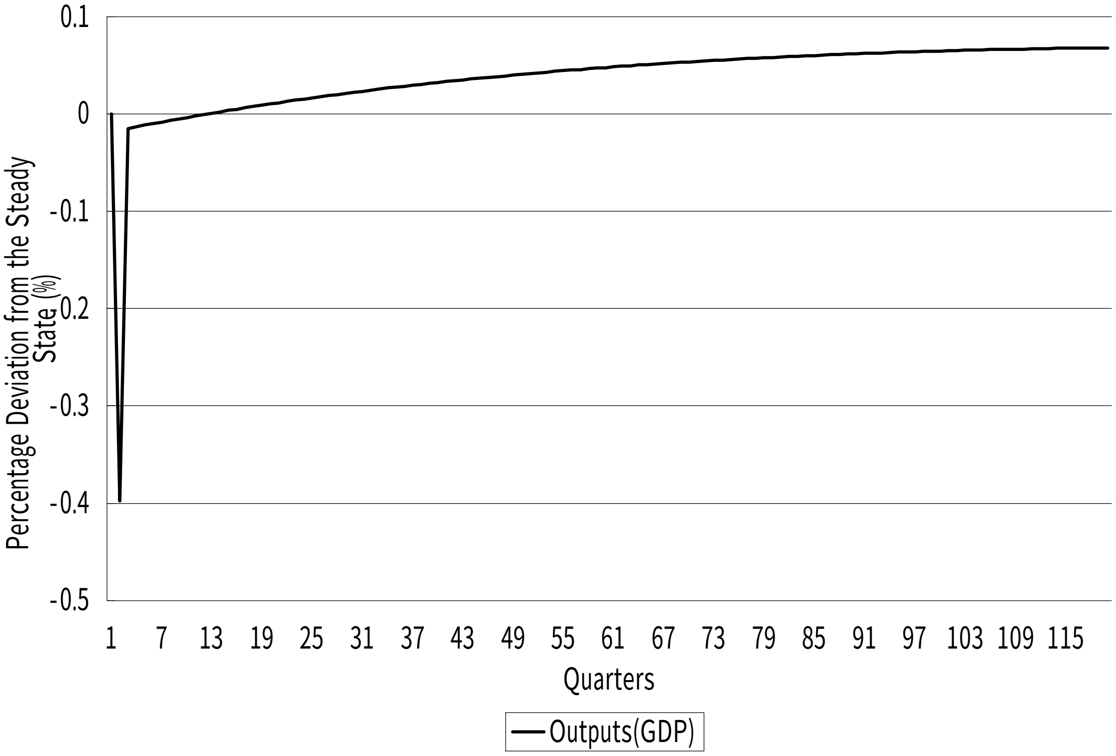

Fig.4. Dynamics of the Outputs(GDP)

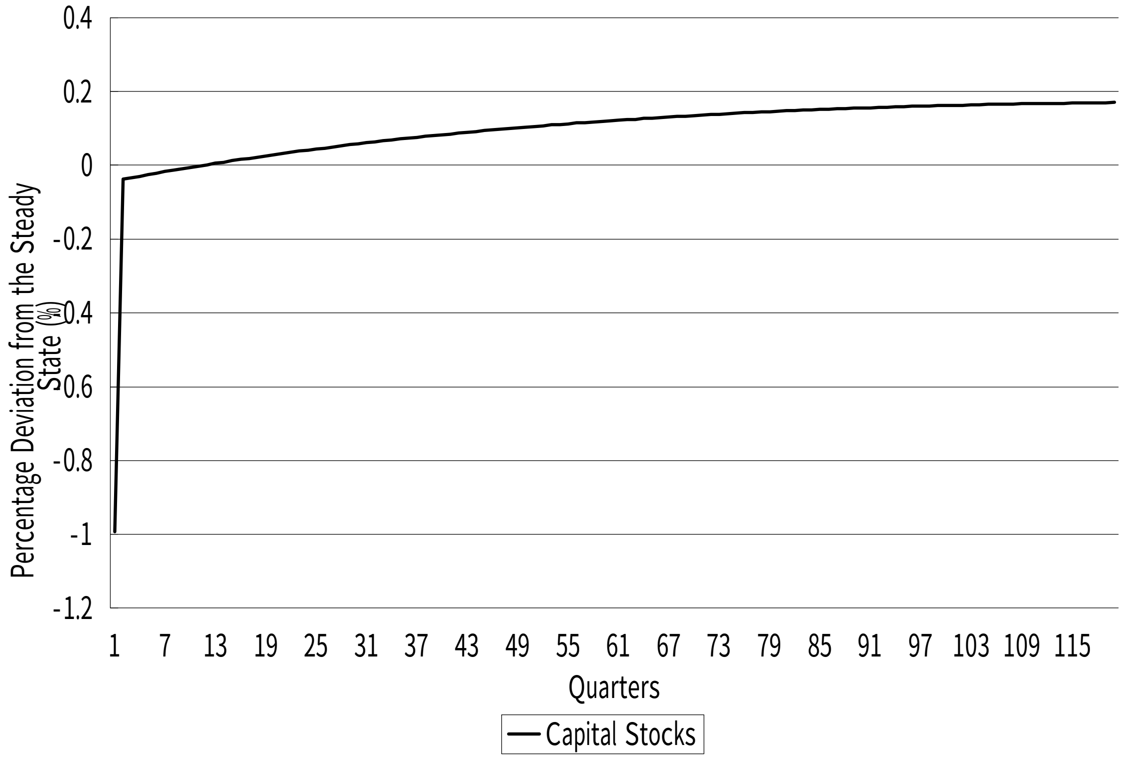

Fig.5. Dynamics of the Capital Stocks

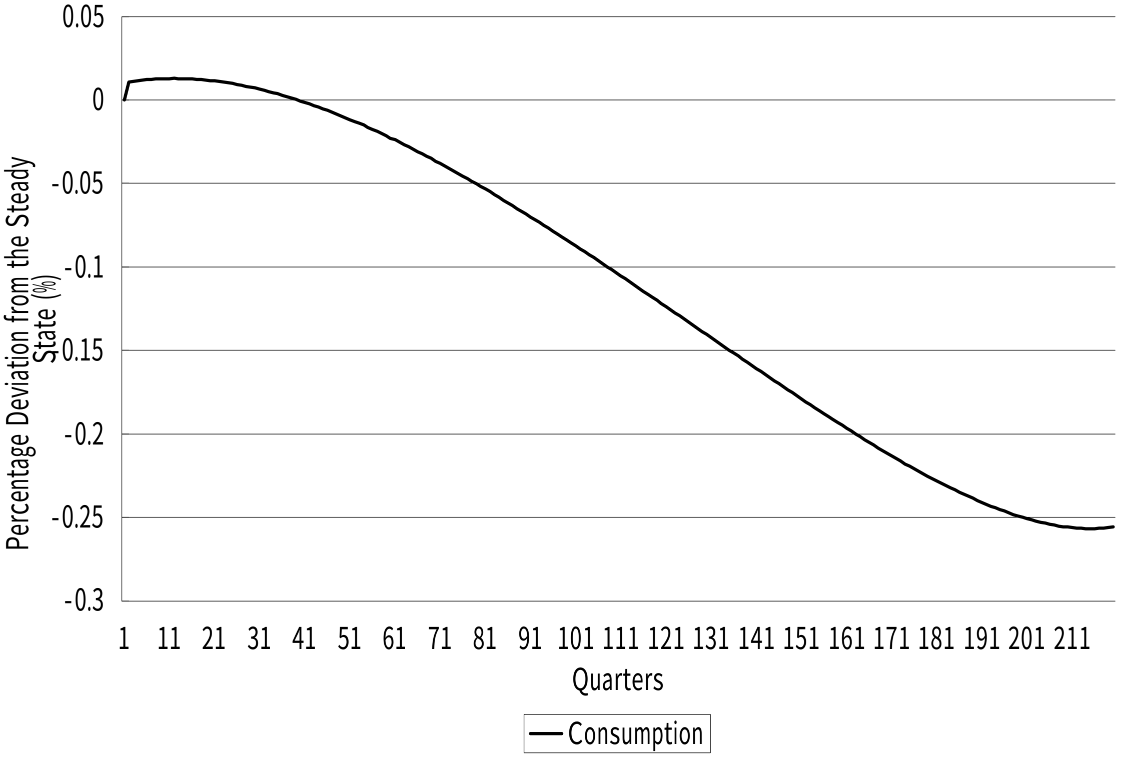

Fig.6. Dynamics of the Consumption