Discussion Papers In Economics And Business

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Two-Level Supply Chain

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February 2004

この研究は「大学院経済学研究科・経済学部記念事業」
基金より援助を受けた、記して感謝する。

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Abstract

In many supply chains, the variance of orders may be considerably larger than that of sales, and this distortion tends to increase as one moves up a supply chain, this is known as "Bullwhip Effect". The Bullwhip phenomenon has recognized in many diverse markets. Procter & Gamble found that the diaper orders issued by the distributors have a degree of variability that cannot be explained by consumer demand fluctuations (Lee, Padamanabhan and Wang 1997a). Lee, Padamanabhan and Wang (1997a, b) developed a framework for explaining this phenomenon. Lee, So, and Tang (2000) showed that, within the context of a two-level supply chain consisting of single manufacturer and single retailer with $AR(1)$ end demand, the manufacturer would benefit when the retailer shared its demand information. This paper considers the effect of partial information sharing, within the framework of Lee, So and Tang, in one manufacturer and n retailers model, focusing on the variance of the manufacturer's "demand" (the retailers' order quantity).

JEL Classification Numbers: C61, M11.
Keywords: Supply Chain Management, Information Sharing, Inventory.

1 Introduction

The term supply chain refers to a system consisting of material suppliers, production facilities, distribution services, and customers who are all linked together via the downstream flow of materials (deliveries) and the upstream feedback flow of orders (Stevens 1989; Disney and Towill 2003). In many supply chains, the variance of orders may be considerably larger than the variance of sales, and this distortion tends to increase as one moves upstream, this is so-called "Bullwhip Effect". The Bullwhip phenomenon has recognized in many diverse markets. Procter & Gamble found that the diaper orders issued by the distributors have a degree of variability that cannot be explained by consumer demand fluctuations (Lee, Padamanabhan and Wang 1997a). This phenomenon was extensively analyzed by Lee, Padamanabhan and Wang (1997a, b), which have pointed out five fundamental causes, demand signal processing (information sharing), order batching, rational game, price variations and long lead time. Thus, the members in a supply chain have been facing such a phenomenon that causes the increasing of the average inventory and the total expected cost. In order to avoid the bullwhip effect, we should eliminate or decrease those causes.

To share the information, however, how should the members of a supply chain do? In practice, each company's information system should support both proprietary and shared data. Since it is needed to manage the company, the proprietary data would be accessible only those employees who have legitimate internal business needs. The shared data should be available through appropriate information interfaces to customers, logistics suppliers, or any other party having a need to know, through a contract or a standard to which all parties agree (Coyle, Bardi and Langley 1996; Stefansson 2002). Advances in Information Technology made it possible to process information at different locations in the supply chain and thus enabled the application of advanced planning. Cheap and large storage devices allow to store and retrieve historical mass data, like past sales (Stadlter and Kilger 2002). Stefansson (2002) showed in the case studies that, both small- and medium-sized enterprises in transportation and production companies do not have basic information systems to implement some kind of advanced communication system; they communicate only by phone and fax. For the small- and medium-sized enterprises, they use Internet to communicate with each other, since they cannot afford for the EDI – Electronic data interchange (Jonsson 1998; Stefansson 2002).

Lee, So, and Tang (2000), hereafter referred to as LST, showed that, within the context of a two-level supply chain consisting of a manufacturer a retailer with an $AR(1)$ end demand, the manufacturer would experience great savings when the retailer shares its demand information. For the reason that it is complex to analyze the value of information sharing analytically for the case when the manufacturer utilizes historical order quantities to estimate the actual demand, LST did not utilize the retailer's historical order to infer the actual value of demand.

Yu, Yan and Cheng (2002) also showed that increasing information sharing among the members in a decentralized supply chain will lead to Pareto improvement in the performance of the entire chain. Specifically, the manufacturer can obtain benefits in terms of reductions in inventory levels and cost saving.

Recenty, Raghunathan and Yeh (2001) extended the LST's model to the n-retailer's model by using continuous replenishment program (CRP) – retailers share their real-time inventory data with the manufacturer and the manufacturer continuously replenishes inventory of each participant retailer by his/her own decision. They showed that the CRP can reduce the expected inventory holding costs of both the manufacturer and the retailer participants.

Raghunathan (2003) also performed the similar analyses, based on Raghunathan and Yeh (2001), to study the effect of information sharing. Using Shapley value concept from game theory to analyze the expected manufacturer and retailer shares of the surplus generated from information sharing, to find out the optimal number of the retailers in such a partnership. Raghunathan assumed that the replenishment lead times are zero and that the retailers are identical with respect to demand variances and correlation.

This paper is also based on the LST's model and considers the general case of the one-manufacturer and multi-retailer's model. This paper differs from those of previous studies such as in Raghunathan and Yeh (2001) and Raghunathan (2003) since, stressing on the variance of the manufacturer's "demand" (the retailers' order quantity) in the situation where (1) the replenishment lead times exist, (2) the retailers are non identical and (3) their demands are correlated, we obtain the results in various cases (without information sharing, with partial information sharing, and with full information sharing).

The rest of the paper is organized as follows. In Section 2, we shall briefly review only a part of LST's work which will be needed in the following. In Section 3, based on the LST model, we investigate the effect of partial information sharing in a general case – the n-retailer's model. In Section 4, we shall show the numerical examples of the special case of $n = 3$. The concluding remark is given at the final section.

2 LST Model

We shall briefly review the LST's model which includes a two-level supply chain consisting of one retailer and one manufacturer. External demand for a single item occurs at the retailer, where the underlying demand process faced by the retailer, is a simple auto correlated $AR(1)$ process.

LST consider a periodic review system in which each site reviews its inventory level and replenishes its inventory from the upstream site every period, and assume that the replenishment lead time L from the external supplier to the manufacturer and l from the manufacturer to the retailer are constant.

LST describe the ordering process as follows. First, before the end of time period t, $t = 1, 2, 3, \ldots$, after demand D_t has been realized, the retailer observes the inventory level and places an order of size Y_t with the manufacturer to replenish his/her inventory. It is assumed that there is no time lag during the order is placed. The retailer will receive the shipment of his/her order at the beginning of time period $t + l + 1$. Excess demand is backlogged.

Second, at the end of time period t, the manufacturer receives and ships the required order quantity Y_t to the retailer. If the manufacturer does not have enough stock to fill this order, then it is assumed that the manufacturer will meet the shortfall by obtaining some units from an "alternative" source with additional cost which is considered as the penalty cost to this shortfall. LST consider the case in which the manufacturer is solely responsible for the penalty cost and for resupplying this alternative source. Thus, the inventory system at the manufacturer resembles a system with back orders and the manufacturer guarantees supply to the retailer.

Third, it is assumed that no fixed order cost is incurred when placing an order, and the unit inventory holding cost and shortage cost are stationary over time. Let h and p denote the unit holding and shortage costs per time period for the retailer, respectively. Let H and P be the unit holding and shortage (or back order) costs per time period at the manufacturer, respectively. The shortage cost at the manufacturer represents the penalty cost to the manufacturer for obtaining items from the alternative source.

Last, suppose that the partners expect to operate the business for a long time, both the retailer and the manufacturer would adopt the order-up-to level policy, which leads to minimize the total discounted holding and shortage costs over the infinite horizon (Heyman and Sobel 1984).

The notation used by LST is summarized as follows:

t Time period, $t = 1, 2, 3, \ldots$

D_t Demand faced by retailer at time period t.

S_t Retailer's order-up-to level for time period t.

Y_t Retailer's order quantity at the end of time period t.

T_t Manufacturer's order-up-to level for time period.

I_t Manufacturer's average (on-hand) inventory level.

h	Unit holding cost per time period for the retailer.
H	Unit holding cost per time period for the manufacturer.
p	Unit shortage cost per time period for the retailer.
P	Unit shortage cost per time period for the manufacturer.
l	Replenishment lead time for retailer.
L	Replenishment lead time for the manufacturer.

Let D_t be the $AR(1)$ demand process at the retailer, where

$$D_t = d + \alpha D_{t-1} + \varepsilon_t, \tag{2.1}$$

$d > 0$, $-1 < \alpha < 1$, and ε_t is i.i.d normally distributed with mean zero and variance σ^2. LST assume that σ is significantly smaller than d, so that the probability of a negative demand is negligible.

Retailer's Ordering Decision

At the end of time period t, the retailer orders Y_t, where

$$Y_t = D_t + (S_t - S_{t-1}). \tag{2.2}$$

Eq. (2.2) describes that, the order quantity Y_t is equal to the demand during period t plus the changes being made in the order-up-to levels. It is possible for Y_t to be negative. However, by assuming that σ is significantly smaller than d, the probability of having $Y_t < 0$ is negligible (Lee, Padamanabhan and Wang 1997b).

By using the recursive relationship of D_t given in Eq. (2.1), the total demand over the lead time l can be written by

$$\sum_{s=1}^{l+1} D_{t+s} = \frac{d}{1-\alpha}\left(l+1-\alpha\gamma^l\right) + \alpha\gamma^l D_t + \sum_{s=1}^{l+1} \gamma^{l+1-s}\varepsilon_{t+s},$$

where $\gamma^l = \left(1-\alpha^{l+1}\right)/(1-\alpha)$.

Let m_t and v_t be the conditional expectation and the conditional variance (conditioned on D_t) of the total demand over the lead time l, respectively, where

$$m_t = \frac{d}{1-\alpha}\left(l+1-\alpha\gamma^l\right) + \alpha\gamma^l D_t, \tag{2.3}$$

$$v_t = \sigma^2 \sum_{s=1}^{l+1}\left(\gamma^{l+1-s}\right)^2. \tag{2.4}$$

Thus, the retailer's order-up-to level S_t is given as

$$S_t = m_t + k^*\sqrt{v_t}, \tag{2.5}$$

where $k^* = \Phi^{-1}\left[p/(p+h)\right]$ for the standard normal distribution function Φ. From Eq. (2.2) and the above expression for S_t, the retailer's order quantity Y_t can be written as:

$$Y_t = D_t + \alpha\gamma^l\left(D_t - D_{t-1}\right). \tag{2.6}$$

Yu et al. (2002) show that $V(Y_t) \geq V(D_t)$ for any $0 \leq \alpha < 1$. This indicates that the Bullwhip Effect occurs while the auto correlation coefficient $0 \leq \alpha < 1$. Thus, in what follows we assume that $0 \leq \alpha < 1$ to study the value of information sharing, when examining the manufacturer ordering process.

Manufacturer's Ordering Decision

It is assumed that the manufacturer is aware of the fact that the demand process D_t is followed an $AR(1)$ process with known parameter d, ρ, and σ. This assumption is reasonable, as such information about the underlying demand process can be communicated to the manufacturer through periodic discussion with the retailer, or the manufacturer can be provided with historic demand data from which such inventory can be readily deduced with sufficient accuracy.[1] After the manufacturer receives and ships the retailer's order Y_t at the end of time period t, the manufacturer immediately places an order with his/her supplier at the end of time period t so as to bring his/her inventory position to an order-up-to level T_t. This order will arrive at the beginning of time period $t + L + 1$ to be ready for the retailer's order placed at the end of time period $t + L + 1$.

In order to determine his/her order-up-to level T_t, the manufacturer needs to anticipate his/her total "demand" (shipment quantity) over the manufacturer's lead time L. Since the manufacturer's "demand" corresponds to the retailer's order quantity, the total shipment quantity over the manufacturer's lead time L, denoted by B_t, is equal to the total orders placed by the retailer over the time period $t + 1, \ldots, t + L + 1$.

From Eqs. (2.1) and (2.6), it follows that

$$Y_{t+1} = d + \alpha Y_t + \gamma^{l+1}\varepsilon_{t+1} - \alpha\gamma^l\varepsilon_t. \tag{2.7}$$

By repeating Eq. (2.7) above, Y_{t+s} can be derived as

$$Y_{t+s} = \gamma^{s-1}d + \alpha^s Y_t + \gamma^{l+1}\varepsilon_{t+s} + \alpha^{l+1}\sum_{k=1}^{k=s-1}\alpha^{s-k}\varepsilon_{t+k} - \alpha^s\gamma^l\varepsilon_t. \tag{2.8}$$

The total shipment quantity over the manufacturer's lead time L for any given Y_t is given as

$$B_t = \sum_{s=1}^{L+1} Y_{t+s} = \frac{d}{1-\alpha}\left(L + 1 - \alpha\gamma^L\right) + \alpha\gamma^L Y_t - \alpha\gamma^{L+l}\varepsilon_t + \sum_{s=1}^{L+1}\gamma^{L+l+2-s}\varepsilon_{t+s}. \tag{2.9}$$

To determine the manufacturer's order-up-to level T_t that minimizes the total expected inventory holding and shortage costs in period $t + L + 1$, the manufacturer needs to find the distribution of B_t. Based on difference in information sharing and ordering, the information sharing-based relationship between the retailer and the manufacturer can be derived to two cases.

When there is no information sharing, the manufacturer receives only information about the retailer's order quantity Y_t. In this case, the error term ε_t has already been realized, but is unknown to the manufacturer when he determines his/her order-up-to level T_t at the end of period t. Thus, under such ideal situation, the manufacturer can obtain the conditional expectation conditioned on Y_t over the manufacturer's lead time L as:

$$M_t = E\left(B_t \left| Y_t\right.\right) = \frac{d}{1-\alpha}\left(L + 1 - \alpha\gamma^L\right) + \alpha\gamma^L Y_t, \tag{2.10}$$

and the conditional variance conditioned on Y_t, as

$$V_t = Var\left(B_t \left| Y_t\right.\right) = \sigma^2\sum_{s=1}^{L+1}\left(\gamma^{L+l+2-s}\right)^2 + \left(\alpha\gamma^{L+l}\right)^2\sigma^2. \tag{2.11}$$

Thus, the manufacturer's optimal order-up-to level T_t under the case of no information sharing is,

$$T_t = M_t + K^*\sqrt{V_t}, \tag{2.12}$$

[1]See Lee et al. 2000.

with $K^* = \Phi^{-1}[P/(P+H)]$ for the standard normal distribution function Φ.

With information about customer's demand in period t, the manufacturer now knows both the retailer's order quantity Y_t and the error term ε_t (through the sharing of information about D_t) when he determines the order-up-to level T_t at the end of the time period t . Thus, the manufacturer can obtain the conditional expectation and the conditional variance conditioned on Y_t and ε_t over the manufacturer's lead time L, respectively, as:

$$M_t' = E(B_t | Y_t, \varepsilon_t) = \frac{d}{1-\alpha}\left(L+1-\alpha\gamma^L\right) + \alpha\gamma^L Y_t - \alpha\gamma^{L+l}\varepsilon_t, \tag{2.13}$$

$$V_t' = Var(B_t | Y_t, \varepsilon_t) = \sum_{s=1}^{L+1}\left(\gamma^{L+l+2-s}\right)^2 \sigma^2. \tag{2.14}$$

Thus, the optimal order-up-to level T_t' in the case of information sharing is

$$T_t' = M_t' + K^*\sqrt{V_t'}. \tag{2.15}$$

Observe that V_t and V_t' are independent of t and are increasing in l, L, α for any $\alpha \geq 0$. In addition, note from Eqs. (2.11) and (2.14) that $V_t \geq V_t'$. Thus, information sharing would reduce the variance of the total shipment quantity over the manufacturer's lead time.

The approximate manufacturer average on-hand inventory in the case when there is information sharing and there is no information sharing, can be given respectively as follows:

$$\begin{aligned} I_t &= \frac{d}{2(1-\rho)} + K^*\sqrt{V_t}, \\ I_t' &= \frac{d}{2(1-\rho)} + K^*\sqrt{V_t'}. \end{aligned}$$

From the two expressions above, the manufacturer's average inventory when there is no information sharing is larger than that when there is information sharing, i.e.,

$$I_t \geq I_t'. \tag{2.16}$$

3 Single Manufacturer and n-Retailer Model

In this section, we consider a two-level supply chain that consists of a manufacturer and n retailers. As in Section 2, the external demand for a single item which occurs at each retailer is assumed to be a simple $AR(1)$ demand process. Let $D_{i,t}$ be the retailer i's demand for period t

$$D_{i,t} = d_i + \alpha D_{i,t-1} + \varepsilon_{i,t}, \quad \text{for any } i = 1, 2, \ldots, n. \tag{3.1}$$

We assume that $-1 < \alpha < 1$ and that, for any i $(i = 1, 2, \ldots, n)$ $d_i > 0$ and σ_i is significantly smaller than d_i. For a given t, the random element of demand at all retailers, $\boldsymbol{\varepsilon}_t = (\varepsilon_{i,t})$, follow an n-variate normal distribution with mean vector zero and variance-covariance matrix $\boldsymbol{\Sigma}$, which is assumed to be positive definite. The correlation coefficient between $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ for $i \neq j$ is $\rho_{i,j}$. $\varepsilon_{i,t}$ and $\varepsilon_{i,s}$ are independent where $t \neq s$.

For the convenience, let $k, r = 1, 2, \ldots, q$, and $k', r' = (q+1), (q+2), \ldots, n$. In order to calculate the conditional mean and the conditional covariance conditioned on $\boldsymbol{\varepsilon}_{2,t}$, partition $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\Sigma}$ as:[2]

$$\underset{n\times 1}{\boldsymbol{\varepsilon}_t} = \left(\begin{array}{c} \underset{q\times 1}{\boldsymbol{\varepsilon}_{1,t}} \\ \hline \underset{(n-q)\times 1}{\boldsymbol{\varepsilon}_{2,t}} \end{array}\right), \quad \underset{n\times n}{\boldsymbol{\Sigma}} = \left(\begin{array}{c|c} \underset{q\times q}{\boldsymbol{\Sigma}_{11}} & \underset{q\times(n-q)}{\boldsymbol{\Sigma}_{12}} \\ \hline \underset{(n-q)\times q}{\boldsymbol{\Sigma}_{21}} & \underset{(n-q)\times(n-q)}{\boldsymbol{\Sigma}_{22}} \end{array}\right),$$

[2]See Johnson and Wichern 1988 for more details.

where

$$\boldsymbol{\varepsilon}_t = \left(\begin{array}{c} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{q,t} \\ \hline \varepsilon_{q+1,t} \\ \vdots \\ \varepsilon_{n,t} \end{array}\right), \boldsymbol{\Sigma} = \left(\begin{array}{ccc|ccc} \rho_{1,1}\sigma_1\sigma_1 & \cdots & \rho_{1,q}\sigma_1\sigma_q & \rho_{1,q+1}\sigma_1\sigma_{q+1} & \cdots & \rho_{1,n}\sigma_1\sigma_n \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \rho_{q,1}\sigma_q\sigma_1 & \cdots & \rho_{q,q}\sigma_q\sigma_q & \rho_{q,q+1}\sigma_q\sigma_{q+1} & \cdots & \rho_{q,n}\sigma_q\sigma_n \\ \hline \rho_{q+1,1}\sigma_{q+1}\sigma_1 & \cdots & \rho_{q+1,q}\sigma_{q+1}\sigma_q & \rho_{q+1,q+1}\sigma_{q+1}\sigma_{q+1} & \cdots & \rho_{q+1,n}\sigma_{q+1}\sigma_n \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \rho_{n,1}\sigma_n\sigma_1 & \cdots & \rho_{n,q}\sigma_n\sigma_q & \rho_{n,q+1}\sigma_n\sigma_{q+1} & \cdots & \rho_{n,n}\sigma_n\sigma_n \end{array}\right),$$

and $\rho_{i,j} = 1$ for $i = j$, $\rho_{i,j} = \rho_{j,i}$ for $i \neq j$.

The conditional mean conditioned on $\varepsilon_{2,t}$ is getting from the form of $\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}$.

$$\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1} = \begin{pmatrix} \beta_{1,q+1} & \beta_{1,q+2} & \cdots & \beta_{1,n} \\ \beta_{2,q+1} & \beta_{2,q+2} & \cdots & \beta_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{q,q+1} & \beta_{q,q+2} & \cdots & \beta_{q,n} \end{pmatrix},$$

where $\beta_{r,r'} = 1/|\boldsymbol{\Sigma}_{22}| \sum_{k'=q+1}^{n} \rho_{r,k'}\sigma_r\sigma_{k'}A_{r',k'}$, and $A_{r',k'}$ is the cofactor of $\rho_{r',k'}\sigma_{r'}\sigma_{k'}$ of matrix $\boldsymbol{\Sigma}_{22}$.

Thus, the conditional mean conditioned on $\boldsymbol{\varepsilon}_{2,t}$ is of the form:

$$E\left(\varepsilon_{k,t} \,\middle|\, \sum_{k'=q+1}^{n} \varepsilon_{k',t}\right) = \sum_{k'=q+1}^{n} \beta_{k,k'}\varepsilon_{k',t}. \tag{3.2}$$

Moreover, the conditional covariance conditioned on $\boldsymbol{\varepsilon}_{2,t}$ is

$$\boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21} = \begin{pmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,q} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,q} \\ \vdots & \vdots & \ddots & \vdots \\ c_{q,1} & c_{q,2} & \cdots & c_{q,q} \end{pmatrix}, \tag{3.3}$$

where $c_{k,r} = \rho_{k,r}\sigma_k\sigma_r - \sum_{k'=q+1}^{n} \beta_{k,k'}\rho_{k',r}\sigma_{k'}\sigma_r$.

Note that each retailer's ordering decision is the same as in the previous section. As mentioned earlier, when we study the manufacturer ordering process we consider only the case which $0 \leq \alpha < 1$.

Manufacturer's Ordering Decision

As in the previous section, for any $i = 1, 2, \ldots, n$, the manufacturer can deduce the retailer i's order quantity on the period $t + 1$ as

$$Y_{i,t+1} = d_i + \alpha Y_{i,t} + \gamma^{l+1}\varepsilon_{i,t+1} - \alpha\gamma^l\varepsilon_{i,t}.$$

By repeating the above equation, the retailer i's order quantity on the period $t + s$ is described as,

$$Y_{i,t+s} = \gamma^{s-1}d_i + \alpha^s Y_{i.t} + \gamma^{l+1}\varepsilon_{i,t+s} + \alpha^{l+1} \sum_{k=1}^{k=s-1} \alpha^{s-k}\varepsilon_{i,t+k} - \alpha^s\gamma^l\varepsilon_{i,t}.$$

Thus, the retailer i's total shipment quantity over the manufacturer's lead time L for any given $Y_{i,t}$ is

$$\sum_{s=1}^{L+1} Y_{i,t+s} = \frac{d_i}{1-\alpha}\left(L + 1 - \alpha\beta^L\right) + \alpha\gamma^L Y_{i,t} - \alpha\gamma^{L+l}\varepsilon_{i,t} + \sum_{s=1}^{L+1} \gamma^{L+l+2-s}\varepsilon_{i,t+s}.$$

The sum of purchase orders of the n retailers is the aggregate order for the supplier in each period. As the result, all retailers' total shipment quantity over the manufacturer's lead time L can be given by

$$B_{n,t} = \frac{1}{1-\alpha}\left(L+1-\alpha\gamma^{L}\right)\sum_{i=1}^{n} d_i + \alpha\gamma^{L}\sum_{i=1}^{n} Y_{i,t} - \alpha\gamma^{L+l}\sum_{i=1}^{n}\varepsilon_{i,t} + \sum_{s=1}^{L+1}\gamma^{L+l+2-s}\sum_{i=1}^{n}\varepsilon_{i,t+s}.$$

According to difference in information sharing and ordering coordination, we formulate the information sharing-based relationship between the retailers and the manufacturer as three cases: with partial information sharing, without information sharing, and with full information sharing. The manufacturer needs to find the distribution of $B_{n,t}$ according to each case in order to determine his/her inventory order-up-to level T_t which minimizes the total expected inventory holding and shortage costs in period $t+L+1$.

The Case of Partial Information Sharing
We assume that only $(n-q)$ retailers share their demand information with the manufacturer and that the remaining q retailers do not share any information with the manufacturer.

In what follows, with no loss of generality, we assume that the last $(n-q)$ retailers share their information sharing. Thus, through the sharing of information about $\sum_{k'=q+1}^{n} D_{k',t}$, the manufacturer knows the $(n-q)$ retailers' error term $\sum_{k'=q+1}^{n}\varepsilon_{k',t}$.

From Eqs. (3.2) and (3.3), the conditional expectation and conditional variance that conditioned on $\sum_{k'=q+1}^{n}\varepsilon_{k',t}$ over the manufacturer's lead-time L can be given, respectively, as

$$M_{n,t}^{(n-q)} = E\left(B_{n,t}\middle|\sum\nolimits_{k'}^{n}\varepsilon_{k',t}\right) = M_{n,t} - \alpha\gamma^{L+l}\left(\sum_{k=1}^{q}\sum_{k'=q+1}^{n}\beta_{k,k'}\varepsilon_{k',t} + \sum_{k'=q+1}^{n}\varepsilon_{k',t}\right), \tag{3.4}$$

$$V_{n,t}^{(n-q)} = Var\left(B_{n,t}\middle|\sum\nolimits_{k'}^{n}\varepsilon_{k',t}\right) = V_n + \left(\alpha\gamma^{L+l}\right)^2\sum_{k=1}^{q} c_{k,k} + 2\left(\alpha\gamma^{L+l}\right)^2\sum_{k<r}\sum c_{k,r}, \tag{3.5}$$

where

$$M_{n,t} = \frac{1}{1-\alpha}\left(L+1-\alpha\gamma^{L}\right)\sum_{i=1}^{n} d_i + \alpha\gamma^{L}\sum_{i=1}^{n} Y_{i,t},$$

and

$$V_n = \sum_{s=1}^{L+1}\left(\gamma^{L+l+2-s}\right)^2\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{i,j}\sigma_i\sigma_j.$$

The Case of No Information Sharing
In this case, the manufacturer does not receive any information from the retailers. Thus, the respective conditional expectation and conditional variance that conditioned on $\sum_{i=1}^{n} Y_{i,t}$ over the manufacturer's lead-time L are

$$M_{n,t}^{no} = M_{n,t}, \tag{3.6}$$

$$V_{n,t}^{no} = V_n + \left(\alpha\gamma^{L+l}\right)^2\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{i,j}\sigma_i\sigma_j. \tag{3.7}$$

The Case of Full Information Sharing
We assume that all retailers will share their demand information with the manufacturer. Thus, the conditional expectation and conditional variance over the manufacturer's lead-time L are

$$M_{n,t}^{full} = E\left(B_{n,t} \left| \sum\nolimits_i^n \varepsilon_{i,t}\right.\right) = M_{n,t} - \alpha\gamma^{L+l}\sum_{i=1}^{n}\varepsilon_{i,t}, \tag{3.8}$$

$$V_{n,t}^{full} = Var\left(B_{n,t} \left| \sum\nolimits_i^n \varepsilon_{i,t}\right.\right) = V_n, \tag{3.9}$$

respectively. We can easily see the following

Proposition 1.

(1) $\quad V_{n,t}^{no} = V_{n,t}^{(n-q)} = V_{n,t}^{full} = (L+1)\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{i,j}\sigma_i\sigma_j,\ for\ \alpha = 0.$

(2) $\quad When\ t \to \infty,\ E\left(M_{n,t}^{no}\right) = E\left(M_{n,t}^{(n-q)}\right) = E\left(M_{n,t}^{full}\right) = \frac{L+1}{1-\alpha}\sum_{i=1}^{n} d_i.$

Proof. Obvious.

Proposition 2. *Let $V_{n,t}^{(1)}$ and $V_{n,t}^{(2)}$ be the conditional variance over the manufacturer's lead-time L in case where the last one retailer and the last two retailers respectively share their customer demand information with the manufacturer. Then*

(1) $\quad V_{n,t}^{no} - V_{n,t}^{(1)} = \left(\alpha\gamma^{L+l}\right)^2\left(\sum_{i=1}^{n}\rho_{i,n}\sigma_i\right)^2 \geq 0,$

(2) $\quad V_{n,t}^{(1)} - V_{n,t}^{(2)} = \left(\alpha\gamma^{L+l}\right)^2\frac{1}{1-\rho_{n-1,n}^2}\left(\sum_{i=1}^{n}\sigma_i\left(\rho_{i,n-1} - \rho_{i,n}\rho_{n-1,n}\right)\right)^2 \geq 0,$

(3) $\quad V_{n,t}^{no} - V_{n,t}^{full} = \left(\alpha\gamma^{L+l}\right)^2\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{i,j}\sigma_i\sigma_j \geq 0.$

Proof. See Appendix.

Remark 1. If all demands across retailers are independent, then

$$V_{n,t}^{(n-q)} - V_{n,t}^{(n-q+1)} = \left(\alpha\gamma^{L+l}\right)^2\sigma_q^2, \quad \text{for } q = 1, 2, \ldots, n.$$

From Silver and Peterson (1985) and LST, the approximate average inventory can be given as,

$$I = \frac{d}{2(1-\alpha)} + K^*\sqrt{V},$$

and from Proposition 2, we have

$$V_{n,t}^{no} \geq V_{n,t}^{(1)} \geq V_{n,t}^{(2)}.$$

Thus,

$$I_{n,t}^{no} \geq I_{n,t}^{(1)} \geq I_{n,t}^{(2)}. \tag{3.10}$$

The manufacturer's average inventory level decreases with an increase of information sharing, and the manufacturer can obtain a reduction in its inventory level.

4 Numerical Examples

This section considers the special case of previous section specified on a simple two-level supply chain consisting one manufacturer and three retailers model in more details. From Eq. (3.5), we have

$$V_{3,t}^{no} = V_3 + \left(\alpha\beta^L\beta^l\right)^2 \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\rho_{12}\sigma_1\sigma_2 + 2\rho_{13}\sigma_1\sigma_3 + 2\rho_{23}\sigma_2\sigma_3\right), \tag{4.1}$$

$$V_{3,t}^{(1)} = V_3 + \left(\alpha\beta^L\beta^l\right)^2 \left[\sigma_1^2\left(1-\rho_{13}^2\right) + \sigma_2^2\left(1-\rho_{23}^2\right) + 2\sigma_1\sigma_2\left(\rho_{12}-\rho_{13}\rho_{23}\right)\right], \tag{4.2}$$

$$V_{3,t}^{(2)} = V_3 + \left(\alpha\beta^L\beta^l\right)^2 \frac{\sigma_1^2}{1-\rho_{23}^2}\left(1 + 2\rho_{12}\rho_{13}\rho_{23} - \rho_{12}^2 - \rho_{13}^2 - \rho_{23}^2\right), \tag{4.3}$$

$$V_{3,t}^{full} = \sum_{s=1}^{L+1}\left(\beta^{L+l+2-s}\right)^2 \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\rho_{12}\sigma_1\sigma_2 + 2\rho_{13}\sigma_1\sigma_3 + 2\rho_{23}\sigma_2\sigma_3\right), \tag{4.4}$$

where $V_3 = \sum_{s=1}^{L+1}\left(\beta^{L+l+2-s}\right)^2\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\rho_{12}\sigma_1\sigma_2 + 2\rho_{13}\sigma_1\sigma_3 + 2\rho_{23}\sigma_2\sigma_3\right)$.

From Eqs. (3.5), (3.7), (3.9), and Proposition 1 it follows that

(1) $\quad V_{3,t}^{no} - V_{3,t}^{(1)} = \left(\alpha\beta^L\beta^l\right)^2\left(\sigma_3 + \rho_{13}\sigma_1 + \rho_{23}\sigma_2\right)^2,$

(2) $\quad V_{3,t}^{(1)} - V_{3,t}^{(2)} = \left(\alpha\beta^L\beta^l\right)^2 \frac{1}{1-\rho_{23}^2}\left[\sigma_1\left(\rho_{12}-\rho_{13}\rho_{23}\right) + \sigma_2\left(1-\rho_{23}^2\right)\right]^2,$

(3) $\quad V_{3,t}^{(2)} - V_{3,t}^{full} = \left(\alpha\beta^L\beta^l\right)^2 \frac{\sigma_1^2}{1-\rho_{23}^2}\left(1 + 2\rho_{12}\rho_{13}\rho_{23} - \rho_{12}^2 - \rho_{13}^2 - \rho_{23}^2\right).$

From Eq. (2.16) we have,

$$I_{3,t} = \frac{\sum_{i=1}^{3} d_i}{2\left(1-\alpha\right)} + K^*\sqrt{V_{3,t}}. \tag{4.5}$$

Corollary 1. *From the above (1), (2) and (3), we can easily show:*

(1) *The value of* $V_{3,t}^{(p)}$ *is decreasing in* p, *i.e.,* $V_{3,t}^{no} \geq V_{3,t}^{(1)} \geq V_{3,t}^{(2)} \geq V_{3,t}^{full}$.

(2) *For any* $0 \leq \rho_{i,j} = \rho < 1$ *and* $\sigma_1 \leq \sigma_2 \leq \sigma_3$ *we have, the marginal value of* $V_{3,t}^{(p)}$ *is decreasing, i.e.,* $\left(V_{3,t}^{no} - V_{3,t}^{(1)}\right) \geq \left(V_{3,t}^{(1)} - V_{3,t}^{(2)}\right) \geq \left(V_{3,t}^{(2)} - V_{3,t}^{full}\right)$.

Corollary 2. *For* $\alpha = 0$ *we have*

$$V_{3,t}^{no} = V_{3,t}^{(1)} = V_{3,t}^{(2)} = V_{3,t}^{full} = (L+1)\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\rho_{12}\sigma_1\sigma_2 + 2\rho_{13}\sigma_1\sigma_3 + 2\rho_{23}\sigma_2\sigma_3\right).$$

Proposition 3. *From Proposition 1 and Corollary 1, the manufacturer's optimal order-up-to level is decreasing with increasing level of information integration, i.e.,*

$$\lim_{t\to\infty} E\left(T_{3,t}^{no}\right) \geq \lim_{t\to\infty} E\left(T_{3,t}^{(1)}\right) \geq \lim_{t\to\infty} E\left(T_{3,t}^{(2)}\right) \geq \lim_{t\to\infty} E\left(T_{3,t}^{full}\right).$$

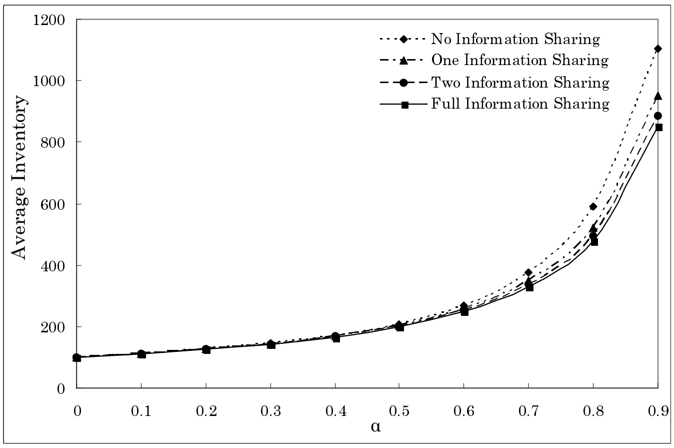
From the previous Corollary 1, and Eq. (4.5) we can deduce the following,

Proposition 4. *The manufacturer's average inventory level is decreasing with increasing level of information integration, i.e.,*

$$I^{no}_{3,t} \geq I^{(1)}_{3,t} \geq I^{(2)}_{3,t} \geq I^{full}_{3,t}.$$

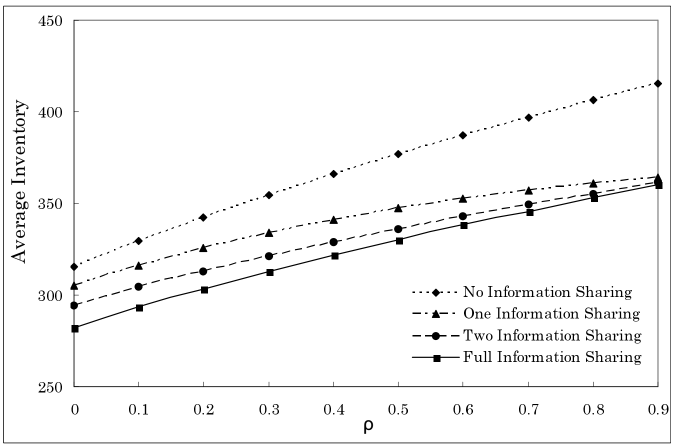
We present some numerical examples to illustrate the manufacturer's inventory levels, associated with information sharing as a function of the demand process characteristics σ_i, α, and $\rho_{i,j}$, based on Eqs. (4.1), (4.2), (4.3), (4.4), and (4.5). We exclusively focus on the case where $\alpha \geq 0$. For the convenience, we assume that all retailers are identical. The parameters are set as follows: the demand process is specified by $d_i = d = 100$, and the manufacturer's cost parameters are given as $P = 25$, $H = 1.25$.

Figure 1: The Impact of α on Average Manufacturer Inventory.



When analyzing the impact of α, ρ and σ, we set $l = 7$ and $L = 5$. In Figures 1 through 5 below, the vertical axis denotes the manufacturer's (approximate) average inventory which can reflect production costs, α represents auto correlation coefficient, ρ the correlation coefficient across the retailers, and σ the standard deviation of each retailer's demand. Firstly, we examine the impact of α, by setting $\sigma_i = \sigma = 5$ and $\rho_{i,j} = \rho = 0.5$ and letting α vary from 0 to 0.9. Figure 1 depicts the

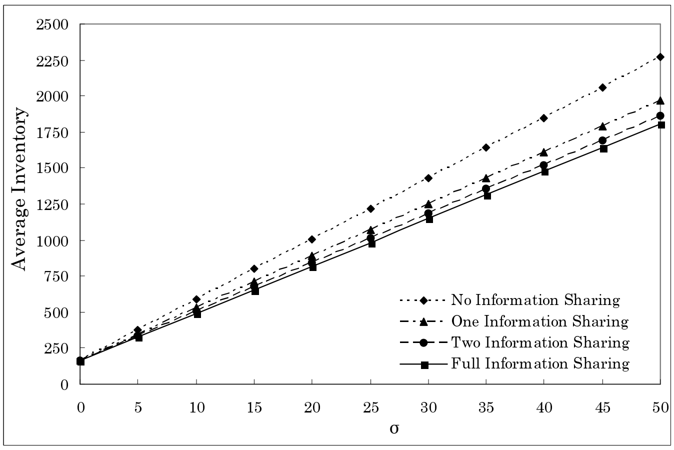
Figure 2: The Impact of ρ on Average Manufacturer Inventory.



average manufacturer's inventory when α varies from 0 to 0.9, with all cases of information sharing pattern. We observe that the average manufacturer's inventory increases as α increases, and that for the greater value of α, the graphs increase sharply. And, the impact of information sharing on the manufacturer's average inventory is very small when α is smaller (especially, $\alpha < 0.5$).

In Figure 2, we analyze the impact of ρ, by setting $\sigma_i = \sigma = 5$, $\alpha = 0.7$, and varying ρ from 0 to 0.9. It shows that the average manufacturer's inventory goes up as ρ increases. Note that, when the value of ρ is going larger the benefit of information sharing can also be obtained greater.

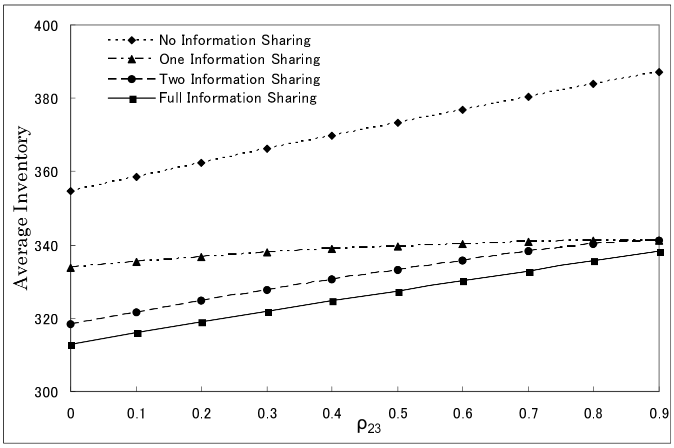
Figure 3: The Impact of σ on Average Manufacturer Inventory.



In Figure 3, we see the impact of σ, by setting $\rho_{i,j} = \rho = 0.5$, $\alpha = 0.7$ and varying σ from 0 to 50. It shows that, the impact of information sharing is significantly great as σ increases.

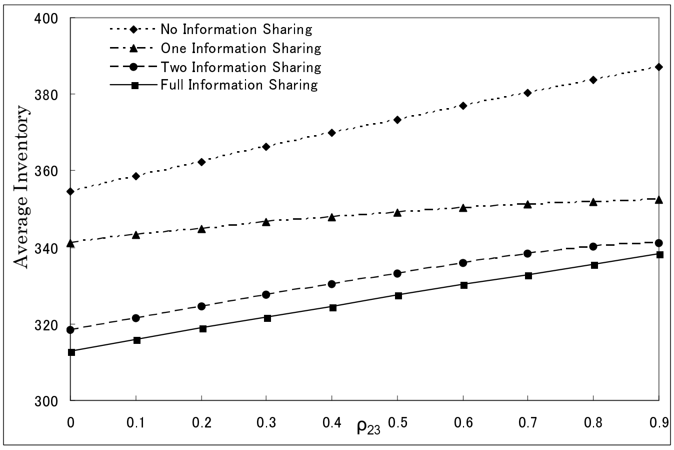
The last two figures depict the average manufacturer inventory when we set the correlation coefficient ρ as different value. Figure 4 pictures the average manufacturer inventory by setting $\rho_{12} = 0.3$

Figure 4: The Changing of Average Inventory while $\rho_{12} = 0.3$ and $\rho_{13} = 0.6$.



and $\rho_{13} = 0.6$ with the variation of ρ_{23} from 0 to 0.9. Figure 5 depicts the average manufacturer inventory by setting $\rho_{12} = 0.6$ and $\rho_{13} = 0.3$ and ρ_{23} is varied from 0 to 0.9. Note that, the value of ρ_{23} must not be larger than 0.9432 to satisfy the positive definite of $\mathbf{\Sigma}$. In Figures 4 and 5, graphs do not change except for the case of "one information sharing". We can see that, as ρ_{23} increases, the value of information sharing increases.

Figure 5: The Changing of Average Inventory while $\rho_{12} = 0.6$ and $\rho_{13} = 0.3$.



5 Concluding Remark

This paper attempts to quantify the benefits of information sharing. By assuming that the external demand for the retailers is autocorrelated $AR(1)$, our analytical study and numerical examples show that when demand information is shared, the manufacturer can obtain average inventory reduction. Moreover, our results suggest that, the manufacturer may get the larger reduction (in terms of average inventory level), in case where the autocorrelation coefficient and correlation coefficient between the retailers are higher, and the replenishment lead time is longer. Also this paper mainly analyzes the manufacturer's ordering process which leads to obtain more considerable improvement in performance, and there is no change in the retailers inventory level. Therefore, the manufacturer should make some concessions instead, such as reducing the retailer replenishment lead time, in order to encourage retailers to share their customers' demand information.

Appendix

Proof of Proposition 2.

When $q = n-1$ then $k' = n$ therefore, $|\mathbf{\Sigma}_{22}| = \sigma_n^2$ and $A_{n,n} = 1$. It follows that,

$$\beta_{k,k'} = \beta_{k,n} = \frac{1}{|\mathbf{\Sigma}_{22}|}\sum_{k}^{n} \rho_{k,n}\sigma_k\sigma_n A_{n,n} = \frac{1}{|\mathbf{\Sigma}_{22}|}\rho_{k,n}\sigma_k\sigma_n A_{n,n} = \frac{1}{\sigma_n^2}\rho_{k,n}\sigma_k\sigma_n.$$

Thus,

$$\begin{aligned} V_{n,t}^{(1)} &= V_n + \left(\alpha\gamma^{L+l}\right)^2 \left[\sum_{k=1}^{n-1}\left(\sigma_k^2 - \beta_{k,n}\rho_{k,n}\sigma_k\sigma_n\right) + 2\sum_{k<r}^{n-1}\sum\left(\rho_{k,r}\sigma_k\sigma_r - \beta_{k,n}\rho_{r,n}\sigma_r\sigma_n\right)\right] \\ &= V_n + \left(\alpha\gamma^{L+l}\right)^2 \left[\sum_{k=1}^{n-1}\left(\sigma_k^2 - \rho_{k,n}^2\sigma_k^2\right) + 2\sum_{k<r}^{n-1}\sum\left(\rho_{k,r}\sigma_k\sigma_r - \rho_{k,n}\rho_{r,n}\sigma_k\sigma_r\right)\right]. \end{aligned}$$

When $q = n-2$ then $k' = \{n-1, n\}$. Thus, $|\mathbf{\Sigma}_{22}| = \sigma_{n-1}^2\sigma_n^2\left(1-\rho_{n-1,n}^2\right)$ and $A_{n-1,n-1} = \sigma_n^2$, $A_{n,n} = \sigma_{n-1}^2$, $A_{n-1,n} = -\rho_{n-1,n}\sigma_{n-1}\sigma_n$ and

$$\beta_{k,n-1} = \frac{\sigma_k}{\sigma_{n-1}^2\left(1-\rho_{n-1,n}^2\right)}\left(\rho_{k,n-1}\sigma_{n-1} - \rho_{k,n}\rho_{n-1,n}\sigma_{n-1}\right),$$

$$\beta_{k,n} = \frac{\sigma_k}{\sigma_n^2\left(1-\rho_{n-1,n}^2\right)}\left(\rho_{k,n}\sigma_n - \rho_{k,n-1}\rho_{n-1,n}\sigma_n\right).$$

Therefore, we have

$$
\begin{aligned}
V_{n,t}^{(2)} &= V_n + \left(\alpha\gamma^{L+l}\right)^2 \Bigg[\sum_{k=1}^{n-2}\left(\sigma_k^2 - \beta_{k,n-1}\rho_{k,n-1}\sigma_k\sigma_{n-1} - \beta_{k,n}\rho_{k,n}\sigma_k\sigma_n\right) \\
&\quad + 2\sum_{k<r}^{n-2}\sum\left(\rho_{k,r}\sigma_k\sigma_r - \beta_{k,n-1}\rho_{r,n-1}\sigma_r\sigma_{n-1} - \beta_{k,n}\rho_{r,n}\sigma_r\sigma_n\right)\Bigg] \\
&= V_n + \left(\alpha\gamma^{L+l}\right)^2 \Bigg[\sum_{k=1}^{n-2}\left(\sigma_k^2 - \frac{\sigma_k^2}{1-\rho_{n-1,n}^2}\left(\rho_{k,n-1}^2 + \rho_{k,n}^2 - 2\rho_{k,n-1}\rho_{k,n}\rho_{n-1,n}\right)\right) \\
&\quad + 2\sum_{k<r}^{n-2}\sum\left(\rho_{k,r}\sigma_k\sigma_r - \frac{\sigma_k\sigma_r}{1-\rho_{n-1,n}^2}\left(\rho_{k,n-1}\rho_{r,n-1} + \rho_{k,n}\rho_{r,n} - \rho_{r,n-1}\rho_{k,n}\rho_{n-1,n} - \rho_{k,n-1}\rho_{r,n}\rho_{n-1,n}\right)\right)\Bigg].
\end{aligned}
$$

(1):

$$
\begin{aligned}
V_{n,t}^{no} - V_{n,t}^{(1)} &= \left(\alpha\gamma^{L+l}\right)^2 \Bigg(\sum_{k=1}^{n}\sigma_k^2 + 2\sum_{k<r}^{n}\sum\rho_{k,r}\sigma_k\sigma_r - \sum_{k=1}^{n-1}\sigma_k^2 + \sum_{k=1}^{n-1}\rho_{k,n}^2\sigma_k^2 - 2\sum_{k<r}^{n-1}\sum\rho_{k,r}\sigma_k\sigma_r \\
&\quad + 2\sum_{k<r}^{n-1}\sum\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r\Bigg) \\
&= \left(\alpha\gamma^{L+l}\right)^2 \left(\sigma_n^2 + 2\sum_{k=1}^{n-1}\rho_{k,n}\sigma_k\sigma_n + \sum_{k=1}^{n-1}\rho_{k,n}^2\sigma_k^2 + 2\sum_{k<r}^{n-1}\sum\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r\right) \\
&= \left(\alpha\gamma^{L+l}\right)^2 \left(\sigma_n + \sum_{k=1}^{n-1}\rho_{k,n}\sigma_k\right)^2 \\
&= \left(\alpha\gamma^{L+l}\right)^2 \left(\sum_{k=1}^{n}\rho_{k,n}\sigma_k\right)^2 \geq 0.
\end{aligned}
$$

(2):

$$
\begin{aligned}
V_{n,t}^{(1)} - V_{n,t}^{(2)} &= \left[\sum_{k=1}^{n-1}\left(\sigma_k^2 - \rho_{k,n}^2\sigma_k^2\right) + 2\sum_{k<r}^{n-1}\sum\left(\rho_{k,r}\sigma_k\sigma_r - \rho_{k,n}ho_{r,n}\sigma_k\sigma_r\right)\right] - \sum_{k=1}^{n-2}\sigma_k^2\Bigg[1 - \frac{1}{1-\rho_{n-1,n}^2}\Big(\rho_{k,n-1}^2 + \rho_{k,n}^2 \\
&\quad - 2\rho_{k,n-1}\rho_{k,n}\rho_{n-1,n}\Big)\Bigg] - 2\sum_{k<r}^{n-2}\sum\sigma_k\sigma_r\Bigg[\rho_{k,r} - \frac{1}{1-\rho_{n-1,n}^2}\Big(\rho_{k,n-1}\rho_{r,n-1} + \rho_{k,n}\rho_{r,n} \\
&\quad - \rho_{r,n-1}\rho_{k,n}\rho_{n-1,n} - \rho_{k,n-1}\rho_{r,n}\rho_{n-1,n}\Big)\Bigg].
\end{aligned}
$$

Let

$$
\begin{aligned}
A &= \sum_{k=1}^{n-1}\sigma_k^2\left(1-\rho_{k,n}^2\right) + 2\sum_{k<r}^{n-1}\sum\rho_{k,r}\sigma_k\sigma_r - 2\sum_{k<r}^{n-1}\sum\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r - \sum_{k=1}^{n-2}\sigma_k^2 - 2\sum_{k<r}^{n-2}\sum\rho_{k,r}\sigma_k\sigma_r \\
&= \sigma_{n-1}^2 - \sum_{k=1}^{n-1}\left(\rho_{k,n}\sigma_k\right)^2 + \left(\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,r}\sigma_k\sigma_r - \sum_{k=1}^{n-1}\sigma_k^2\right) - \left[\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r - \sum_{k=1}^{n-1}\left(\rho_{k,n}\sigma_k\right)^2\right] \\
&\quad - \left(\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,r}\sigma_k\sigma_r - \sum_{k=1}^{n-2}\sigma_k^2\right) \\
&= 2\sigma_{n-1}\sum_{r=1}^{n-1}\rho_{r,n-1}\sigma_r - \sigma_{n-1}^2 - \sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r,
\end{aligned}
$$

and let

$$B = \frac{1}{1-\rho_{n-1,n}^2}\Bigg[\sum_{k=1}^{n-2}\sigma_k^2\Big(\rho_{k,n-1}^2+\rho_{k,n}^2-2\rho_{k,n-1}\rho_{k,n}\rho_{n-1,n}\Big)+2\sum_{k<r}^{n-2}\sum\sigma_k\sigma_r\Big(\rho_{k,n-1}\rho_{r,n-1}+\rho_{k,n}\rho_{r,n}$$
$$-\rho_{r,n-1}\rho_{k,n}\rho_{n-1,n}-\rho_{k,n-1}\rho_{r,n}\rho_{n-1,n}\Big)\Bigg]$$
$$= \frac{1}{1-\rho_{n-1,n}^2}\Bigg[\sum_{k=1}^{n-2}\sigma_k^2\left(\rho_{k,n-1}^2+\rho_{k,n}^2-2\rho_{k,n-1}\rho_{k,n}\rho_{n-1,n}\right)-\sum_{k=1}^{n-2}\sigma_k^2\left(\rho_{k,n-1}^2-2\rho_{k,n-1}\rho_{k,n}\rho_{n-1,n}+\rho_{k,n}^2\right)$$
$$+\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\sigma_k\sigma_r\Big(\rho_{k,n-1}\rho_{r,n-1}+\rho_{k,n}\rho_{r,n}-\rho_{r,n-1}\rho_{k,n}\rho_{n-1,n}-\rho_{k,n-1}\rho_{r,n}\rho_{n-1,n}\Big)\Bigg]$$
$$= \frac{1}{1-\rho_{n-1,n}^2}\Bigg(\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n-1}\sigma_k\rho_{r,n-1}\sigma_r+\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r-\rho_{n-1,n}\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n}\sigma_k\rho_{r,n-1}\sigma_r$$
$$-\rho_{n-1,n}\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n-1}\sigma_k\rho_{r,n}\sigma_r\Bigg).$$

Thus,

$V_{n,t}^{(1)}-V_{n,t}^{(2)}=$

$$= \frac{1}{1-\rho_{n-1,n}^2}\Bigg\{2\sigma_{n-1}\sum_{r=1}^{n-1}\rho_{r,n-1}\sigma_r-\sigma_{n-1}^2-\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r-2\rho_{n-1,n}^2\sigma_{n-1}\sum_{r=1}^{n-1}\rho_{r,n-1}\sigma_r+\rho_{n-1,n}^2\sigma_{n-1}^2$$
$$+\rho_{n-1,n}^2\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r+\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n-1}\sigma_k\rho_{r,n-1}\sigma_r+\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r-\rho_{n-1,n}\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n}\sigma_k\rho_{r,n-1}\sigma_r$$
$$-\rho_{n-1,n}\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n-1}\sigma_k\rho_{r,n}\sigma_r\Bigg\}$$
$$= \frac{1}{1-\rho_{n-1,n}^2}\Bigg\{\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n-1}\sigma_k\rho_{r,n-1}\sigma_r-2\rho_{n-1,n}\sigma_{n-1}\sum_{r=1}^{n-1}\rho_{r,n}\sigma_r+\rho_{n-1,n}^2\sigma_{n-1}^2-2\rho_{n-1,n}^2\sigma_{n-1}\sum_{r=1}^{n-1}\rho_{r,n-1}\sigma_r$$
$$+\rho_{n-1,n}^2\sigma_{n-1}^2+\rho_{n-1,n}^2\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r\rho_{n-1,n}\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n}\sigma_k\rho_{r,n-1}\sigma_r-\rho_{n-1,n}\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n-1}\sigma_k\rho_{r,n}\sigma_r\Bigg\}$$
$$= \frac{1}{1-\rho_{n-1,n}^2}\Bigg\{\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n-1}\sigma_k\rho_{r,n-1}\sigma_r-2\rho_{n-1,n}\sigma_{n-1}\sum_{r=1}^{n-1}\rho_{r,n}\sigma_r-\rho_{n-1,n}\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n}\sigma_k\rho_{r,n-1}\sigma_r+\rho_{n-1,n}^2\sigma_{n-1}^2$$
$$+\rho_{n-1,n}^2\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r-\rho_{n-1,n}\sum_{k=1}^{n-2}\sum_{r=1}^{n-2}\rho_{k,n-1}\sigma_k\rho_{r,n}\sigma_r\Bigg\}$$
$$= \frac{1}{1-\rho_{n-1,n}^2}\Bigg\{-\rho_{n-1,n}\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n}\sigma_k\rho_{r,n-1}\sigma_r+\rho_{n-1,n}^2\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n}\sigma_k\rho_{r,n}\sigma_r+\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n-1}\sigma_k\rho_{r,n}\sigma_r$$
$$-\rho_{n-1,n}\sum_{k=1}^{n-1}\sum_{r=1}^{n-1}\rho_{k,n-1}\sigma_k\rho_{r,n}\sigma_r\Bigg\}$$
$$= \frac{1}{1-\rho_{n-1,n}^2}\Bigg\{\Bigg(\sum_{k=1}^{n-1}\rho_{k,n-1}\sigma_k\Bigg)^2+\Bigg(\rho_{n-1,n}\sum_{k=1}^{n-1}\rho_{k,n}\sigma_k\Bigg)^2-2\rho_{n-1,n}\sum_{k=1}^{n-1}\sum_{k=1}^{n-1}\Big(\rho_{k,n}\sigma_k\Big)\Big(\rho_{k,n-1}\sigma_k\Big)\Bigg\}$$
$$= \frac{1}{1-\rho_{n-1,n}^2}\left(\sum_{k=1}^{n-1}\sigma_k\Big(\rho_{k,n-1}-\rho_{k,n}\rho_{n-1,n}\Big)\right)^2$$
$$= \frac{1}{1-\rho_{n-1,n}^2}\left(\sum_{k=1}^{n}\sigma_k\Big(\rho_{k,n-1}-\rho_{k,n}\rho_{n-1,n}\Big)\right)^2\geq 0.$$

(3): Since $\mathbf{\Sigma}$ is positive definite, $\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{i,j}\sigma_i\sigma_j>0$. □

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