# Discussion Papers In Economics And Business

Welfare Analysis of Debt Policy during Recessions

Takayuki Ogawa

Discussion Paper 04-14

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

# Welfare Analysis of Debt Policy during Recessions

Takayuki Ogawa

Discussion Paper 04-14

July 2004

この研究は「大学院経済学研究科・経済学部記念事業」
基金より援助を受けた、記して感謝する。

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

# Welfare Analysis of Debt Policy during Recessions*

Takayuki Ogawa†

*Osaka University, Japan*

**Abstract**

This paper develops an overlapping-generations model with nominal wage rigidities and examines the welfare effects of debt policy during recessions. Issues of public debt stimulate aggregate consumption demand and create employment. Future generations then face both increased wage incomes and higher taxes. If the amount of outstanding bonds is already large, debt policy deteriorates the welfare of future generations by levying heavy taxes. By contrast, if the outstanding bond issue is relatively small, debt policy can be Pareto improving by creating more employment. Therefore, the welfare implications of debt policy during recessions can be discriminated from those during booms.

**Keywords**: debt policy, overlapping generations, welfare effects.

**JEL Classification**: E12, E24, H63.

---

*I thank Shinsuke Ikeda, Yoshiyasu Ono, and Tadashi Shigoka for their helpful comments. This research is financially supported by the Grant-in-Aid for JSPS Fellows, the Ministry of Education, Culture, Sports, Science and Technology.

†Mailing Address: Institute of Social and Economic Research, Osaka University, 6-1 Mihogaoka, Ibaraki, Osaka 567-0047, JAPAN. E-mail Address: tkogawa@iser.osaka-u.ac.jp

# I. Introduction

Tobin (1980) asserts that in conditions of under-employment further issues of public debt stimulate aggregate demand and increase output and employment. Inspired by Tobin, Rankin (1986) demonstrates that when there are demand shortages and unemployment, such Keynesian debt policy brings higher steady-state social welfare. And more recently, Ono (2001) contends that, in the context of liquidity traps, current debt issue never places a burden on future generations. By way of contrast with the Keynesian scenario, Modigliani (1961) argues that debt issue puts a strain on future generations, even with under-employment situations. Accordingly, whether debt policy during recessions is beneficial to future generations remains a controversial issue. By introducing nominal wage rigidities into a monetary overlapping-generations model *à la* Weil (1987), this paper clarifies the welfare implications of debt policy during recessions.

In considering the problem of a burden on future generations of public debt, the definition of *future generations* is crucial. According to Lerner (1948), future generations are defined as people who exist at future dates irrespective of birth dates. On the other hand, Bowen *et al.* regard people who were born at the same date as the same generation, and define future generations as people who will be born after the present date. The analyses that approach the problem by using overlapping-generations models, including the current paper, follow the latter definition.[1]

Another important point involved in analysing this problem is whether the analysis uses an overlapping-generations model *à la* Diamond (1965) or one *à la* Blanchard (1985) and Weil (1987, 1989). In the Diamond model, people undoubtedly die and hence the

[1]Refer to Iwamoto (2002).

current generation that enjoys benefit from debt issue never exists at far future dates. Therefore, the focus of the analysis is mainly drawn to the future level of output. Assuming full employment, Diamond (1965) shows that debt issue interferes with capital accumulation and lowers the steady-state level of output and welfare (if the economy is dynamically efficient). By contrast, developing the Diamond model with fixed prices and wages, Rankin (1986) finds that debt issue raises the steady-state level of output and welfare.[2] Consequently, in the Diamond situation, debt policy is harmful to future generations during booms, whereas it is beneficial during recessions (as long as the economy is dynamically efficient).[3]

On the other hand, the Blanchard–Weil model examines the situation where the current generation exists even at far future dates.[4] Then, the literature on the lines of Blanchard and Weil focuses mainly on the income redistribution effect between different generations of government policy. As shown in Blanchard (1985) and Weil (1989), debt finance means an income transfer from future generations to existing generations. It is

[2]Rankin (1986) additionally finds that debt issue reduces welfare despite raising output, if consumers supply labour elastically and the economy is sufficiently close to Walrasian equilibrium.

[3]Ono (2001) stresses that debt policy during recessions never places a burden on all future generations even if the transitional dynamics are considered in the Diamond situation. Sen (2002) points out that, in the Diamond model with monopolistic competition, debt finance can be Pareto improving depending on production technology.

[4]Blanchard assumes that people face a constant instantaneous probability of death throughout their lifetimes and thus the average rest of lives at some date is identical for all generations. Buiter (1988) concludes that, in the Blanchard model, the role of finite lifetimes is negligible and the appearance of new generations is rather more important, like the Weil model. Weil assumes that infinitely-lived agents appear at a constant rate. One justification of such a situation is primogeniture through inheritance of bequest. See Weil (1989) for other justifications.

because part of taxes associated with debt issue is levied on future generations, which are not connected with existing generations. Based on this intergenerational income redistribution effect, debt policy during booms is beneficial to existing generations and harmful to future generations (unless the economy is dynamically inefficient).[5]

The purpose of this paper is to examine the welfare implications of debt policy during recessions in the context of Blanchard and Weil. Taking into account both the intergenerational income redistribution effect and the existence of unemployment, the result differs from that of Diamond (1965), Rankin (1986), Blanchard (1985), and Weil (1989). A further issue of public debt stimulates aggregate consumption demand and decreases unemployment, thereby supporting the Keynesian scenario. However, future generations face both this beneficial effect of increased wage incomes and the harmful effect of higher taxes. If the amount of outstanding bonds is relatively small and there is high population growth, debt policy enhances the welfare of future generations through creating more employment and is Pareto improving. Conversely, debt policy diminishes the welfare of future generations by levying heavy taxes if the outstanding issue is already large.

Section II presents the structure of the model. By using a simplified model without capital accumulation, Section III clarifies the essence of debt policy during recessions. Section IV considers capital accumulation. It shows that, as Modigliani (1961) is concerned, debt issue discourages investment and can lead to low levels of steady-state output if the prevailing interest rate lies at relatively low levels. This possibility barely changes the prime welfare implication obtained in Section III, because the economy is in dynamic-

[5]Saint-Paul (1992) also demonstrates that in an endogenous growth model with overlapping generations and production externalities, debt policy reduces the growth rate and welfare of some future generations.

inefficiency at low levels of the interest rate. Section V summarizes and concludes.

# II. The Model

## II.A. Consumers

The structure of the model is the same as that of Weil (1987), i.e., a hybrid of a money-in-utility model and an overlapping-generations model. Economy starts at time 0. The size of initial population is normalized to unity. At each point in time, new infinitely-lived consumers appear at the rate $n(>0)$. The total population at time $t$ is hence obtained as $N(t) = e^{nt}$.

Given the time paths of the real interest rate $\{r(t)\}_{t=s}^{\infty}$, the nominal interest rate $\{R(t)\}_{t=s}^{\infty}$, the real wage rate $\{w(t)\}_{t=s}^{\infty}$, and the lump-sum tax $\{\tau(s,t)\}_{t=s}^{\infty}$, a consumer representative of the cohort born at time $s(\geq 0)$ chooses $\{c(s,t), k(s,t), b(s,t), m(s,t)\}_{t=s}^{\infty}$ to maximize the lifetime utility,

$$U(s,s) \equiv \int_s^{\infty} [(1-\alpha)\ln c(s,t) + \alpha \ln m(s,t)] e^{-\rho(t-s)} dt, \tag{1}$$

subject to

$$\frac{da(s,t)}{dt} = r(t)a(s,t) + w(t)x(s,t) - c(s,t) - R(t)m(s,t) - \tau(s,t), \tag{2}$$

$$a(s,t) \equiv k(s,t) + b(s,t) + m(s,t), \tag{3}$$

$$c(s,t) \geq 0, \quad m(s,t) \geq 0, \tag{4}$$

$$a(0,0) > 0, \quad a(s,s) = 0 \ \text{ for } \forall s > 0, \tag{5}$$

$$\lim_{t\to\infty} a(s,t) e^{-\int_s^t r(v)dv} \geq 0, \tag{6}$$

where $\rho(>0)$ is the subjective discount rate and $c(s,t)$ is consumption at time $t(\geq s)$ of

a consumer born at time $s$.[6] Total non-human wealth $a(s,t)$ consists of capital $k(s,t)$, public bonds $b(s,t)$, and real money balances $m(s,t)$. The constant parameter $\alpha$ satisfies $\alpha \in (0,1)$. For simplicity, labour endowment of each consumer is normalized to unity. To consider unemployment, however, the realized labour supply is denoted by $x(s,t)$. Equation (5) indicates that the initial generation is endowed with positive non-human wealth; whereas, for lack of bequest motives of pre-existing generations, none of new generations receives non-human wealth at birth.

The first-order conditions are

$$\frac{dc(s,t)}{dt} = [r(t) - \rho]c(s,t), \tag{7}$$

$$R(t) = \frac{\alpha}{1-\alpha}\frac{c(s,t)}{m(s,t)}, \tag{8}$$

and the above constraints. Equations (2) and (6)–(8) yield the individual consumption function

$$c(s,t) = \rho(1-\alpha)[a(s,t) + h(s,t)], \tag{9}$$

where $h(s,t)$ is the net human wealth defined as

$$h(s,t) = \int_t^\infty [w(z)x(s,z) - \tau(s,z)]e^{-\int_t^z r(v)dv}dz. \tag{10}$$

Define the *aggregate consumption per capita* $c(t)$ as $c(t) = \frac{c(0,t)+\int_0^t c(s,t)ne^{ns}ds}{N(t)}$. Same definitions hold for other variables: $a(t)$, $k(t)$, $b(t)$, $m(t)$, $x(t)$, $\tau(t)$, and $h(t)$. By applying this aggregation rule to (8) and (9), I have

$$R(t) = \frac{\alpha}{1-\alpha}\frac{c(t)}{m(t)}, \tag{11}$$

$$c(t) = \rho(1-\alpha)[a(t) + h(t)]. \tag{12}$$

[6]To characterize the instantaneous utility function by a more general CES form does not add further insights.

## II.B. Production

Using physical capital and labour, competitive firms produce a commodity used for consumption and investment. Firms have identical production technology. For analytical tractability, the production function is specified by the Cobb-Douglas form: $y(t) = \theta k(t)^{\beta} l(t)^{1-\beta}$, which is expressed in aggregate per capita. $y(t)$ is the aggregate output per capita and $l(t)$ the aggregate labour demand per capita. $\theta(> 0)$ and $\beta$ respectively measure productivity and capital share in output. In particular, this paper assumes $\beta \in \left[0, \frac{1}{2}\right)$ of which analytical meaning is presented in Section IV.[7] The optimization by firms provides

$$r(t) = \beta\theta \left[\frac{k(t)}{l(t)}\right]^{\beta-1}, \tag{13}$$

$$w(t) = (1-\beta)\theta \left[\frac{k(t)}{l(t)}\right]^{\beta}. \tag{14}$$

## II.C. Government

The government budget constraint is

$$\frac{db(t)}{dt} = [r(t) - n]b(t) - \mu m(t) - \tau(t), \tag{15}$$

which is expressed in aggregate per capita. $\mu$ is the expansion rate of nominal money supply. This paper assumes $\mu = n$ to obtain the steady state of $m(t)$, so that the nominal value of $m(t)$ remains at $\bar{M}(> 0)$. I omit the role of fiscal spending and distortional taxes for purpose of comparison with the literature.

Following the existing studies, I characterize *debt policy* as follows. At time 0, government increases the nominal value of $b(t)$ through a *helicopter drop*; and after time 0, aims

[7] This range regarding $\beta$ includes its plausible values used in the standard RBC literature. For example, King and Rebelo (1999) set $\beta$ equal to one-thirds.

to keeping it constant at that value, $\bar{B}(>0)$.

# III. Debt Policy during Recessions

## III.A. Simple Setting

To elicit the essence of debt policy during recessions, I establish the following simple setting. First, throughout this paper, the nominal wage rate is permanently fixed at a given level, $\bar{W}$.[8] Hence, unemployment may arise.

Second, throughout the paper, I assume the distribution rule that labour demand and lump-sum taxes are identical across agents: $x(s,t) = \min\big(1, l(t)\big)$ and $\tau(s,t) = \tau(t)$.[9] Then, $h(s,t)$ is an age-independent variable: $h(s,t) = h(t)$.

Third, this section imposes $\beta = 0$, so that the role of capital accumulation is set aside and labour is a unique production input. Given the nominal wage rigidity, the commodity market is in equilibrium if and only if $\frac{\bar{W}}{P(t)} = \theta$, where $P(t)$ denotes the price level. $P(t)$ is instantaneously adjusted to satisfy this value, which is henceforth labelled $\bar{P}$. As the inflation rate is always zero, $m(t)$ remains at $\frac{\bar{M}}{\bar{P}}(>0)$ and $b(t)$ at $\frac{\bar{B}}{\bar{P}}(>0)$. The real interest rate equals the nominal interest rate: $r(t) = R(t)$.

[8] For example, Blanchard and Kiyotaki (1987) show nominal price/wage rigidity to arise as a consequence of monopolistic agents' behaviour in the presence of menu costs. See Mankiw and Romer (1991) for microfoundations of price/wage rigidities.

[9] I can employ the age-dependent income schedule used in Blanchard (1985), but it does not alter the main result of this paper.

## III.B. Steady State and Welfare

Keeping the distribution rule in mind, (2), (5), (8), (10), and (12) together derive the dynamic equation for $c(t)$,

$$\frac{dc(t)}{dt} = [r(t) - \rho]c(t) - \rho n(1-\alpha)a(t). \tag{16}$$

This equation implies that the amount of assets affects aggregate consumption streams for lack of bequest motives.[10] From (11), (16), and $a(t) = \frac{\bar{B}}{P} + \frac{\bar{M}}{P}$, the aggregate dynamic system is characterized by

$$\frac{dr(t)}{dt} = r(t)^2 - \rho r(t) - \rho n\alpha \frac{\bar{B} + \bar{M}}{\bar{M}}.$$

As seen in Figure 1, there is a unique steady state, $r^*$, with $r(t) > 0$, which is determined by

$$r^{*2} - \rho r^* - \rho n\alpha \frac{\bar{B} + \bar{M}}{\bar{M}} = 0. \tag{17}$$

The steady state associated with $r^*$ is unstable. If $r(t)$, or equivalently $c(t)$, evolves along the divergent paths, either the intertemporal budget constraint of the consumer or the non-negativity constraint on consumption is violated. Therefore, $r^*$ must be chosen at the initial point in time. In Figure 1, $\Theta \equiv \frac{\alpha}{1-\alpha}\frac{\theta \bar{P}}{M}$ represents the interest rate associated with full employment. To take account of unemployment, I set $\Theta$ above $r^*$.

As in Blanchard (1985) and Weil (1987, 1989), the non-negativity on individual consumption requires $r^* \leq \rho + n$.[11] Substituting $\bar{B} = 0$ into (17) gives the lower bound of $r^*$, which is labelled $\underline{r}^*$ and satisfies $\underline{r}^* > \rho$. As a result, $r^*$ must be in the following range:

[10] Refer to Blanchard (1985) and Weil (1989) for this point.

[11] From (5) and (9), the non-negativity of individual consumption is satisfied if and only if $h(s,s)$ is non-negative for all $s > 0$. Equation (12) implies that $h(s,s) \geq 0$ is equivalent to $c(t) \geq \rho(1-\alpha)\left(\frac{\bar{B}}{P} + \frac{\bar{M}}{P}\right)$, which, together with (17), generates this condition.

**Condition 1.** $\underline{r}^* < r^* \leq \rho + n$.

From (15), the steady-state lump-sum tax $\tau^*$ is

$$\tau^* = r^* \frac{\bar{B}}{\bar{P}} - n\left(\frac{\bar{B}}{\bar{P}} + \frac{\bar{M}}{\bar{P}}\right). \tag{18}$$

Next, I derive welfare. In the absence of the transition process, substituting (8) into (1) yields

$$U(s,s) = \int_s^\infty \ln c(s,t) e^{-\rho(t-s)} dt - \frac{\alpha}{\rho} \ln r^* + \frac{\alpha}{\rho} \ln \frac{\alpha}{1-\alpha}.$$

The first term in the right-hand side of this equation represents the *gross* lifetime utility derived from both consumption and holding money. The second term implies the opportunity costs of holding money. By use of (7) and (9), welfare is reduced to[12]

$$U(s,s) = \frac{1}{\rho} \ln r^* [a(s,s) + h(s,s)] - \frac{\alpha}{\rho} \ln r^* + \frac{1}{\rho} \ln \alpha^\alpha (1-\alpha)^{1-\alpha}. \tag{19}$$

Letting $c^*$ be the steady-state consumption, $h(s,s)$ is obtained as $h(s,s) = \frac{c^* - \tau^*}{r^*}$ from (10) and the equilibrium condition in the commodity market. Equation (19) is eventually rewritten as

$$U(s,s) = \frac{1}{\rho} \ln [r^* a(s,s) + c^* - \tau^*] - \frac{\alpha}{\rho} \ln r^* + \frac{1}{\rho} \ln \alpha^\alpha (1-\alpha)^{1-\alpha}, \tag{20}$$

which says that the gross lifetime utility is increasing with respect to the net income at birth, that is, $r^* a(s,s) + c^* - \tau^*$.

## III.C. Comparative Statics

This subsection examines the effects of an increase in $\bar{B}$. First, I show its implication as the Keynesian demand-management policy, which is emphasized by Tobin (1980).

[12] See e.g., Mino and Shibata (2000) and Saint-Paul (1992) for details of the derivation. In deriving this formula, I used the approximate expression of $\ln(1+\epsilon) = \epsilon$.

**Lemma 1.** *Debt policy raises the interest rate.*

*Proof.* Differentiating (17) totally and using Condition 1, I get $\frac{dr^*}{d\bar{B}} = \frac{\rho n \alpha}{(2r^* - \rho)M} > 0$. ■

**Proposition 1.** *Debt policy stimulates aggregate consumption demand.*

*Proof.* Use of (11) and Lemma 1 gives $\frac{dc^*}{d\bar{B}} = \frac{\rho n(1-\alpha)}{(2r^* - \rho)\bar{P}} > 0$. ■

Consideration of the intergenerational income redistribution effect of debt finance is sufficient to explain this proposition. That is, unless wage income changes, debt policy increases total wealth of existing generations and decreases that of future generations through higher taxes. Therefore, existing generations consume more and future generations less. The total effect on aggregate consumption is definitely positive because part of future generations that consume less does not yet appear in the economy. Consequently, the larger amount of assets induces more aggregate consumption demand for lack of a consumption-smoothing motive across generations. Since the resultant rise in wage income does not disturb this explanation, increasing non-human wealth through debt policy becomes an effective demand-management policy.

As implied by (20), the level of the net income at birth is crucial for welfare. I next examine the effect on $r^* a(s,s) + c^* - \tau^*$.

**Lemma 2.** *Debt policy raises the net income of the initial generation at birth.*

*Proof.* Taking notice of $a(0,0) = \frac{\bar{B}}{\bar{P}} + \frac{\bar{M}}{\bar{P}}$ and using (18) and Lemma 1, I get $\frac{d[r^* a(0,0) - \tau^*]}{d\bar{B}} = \frac{\bar{M}}{\bar{P}} \frac{dr^*}{d\bar{B}} + \frac{n}{\bar{P}} > 0$. Together with Proposition 1, $\frac{d[r^* a(0,0) + c^* - \tau^*]}{d\bar{B}} > 0$. ■

The interpretation of Proposition 1 is also applied to this lemma. Because part of taxes associated with debt issue is levied on future generations, which are not connected with existing generations, public bonds become net wealth for existing generations. That

is, debt policy raises $r^*a(0,0)-\tau^*$. Combined with the demand-stimulating effect (Proposition 1), the net incomes of existing generations undoubtedly rise.

Meanwhile, I have the following:

**Lemma 3.** *Debt policy raises the net incomes of future generations at birth if* $\underline{r}^* < r^* < \frac{2}{3}(\rho+n)$ *and reduces those if* $\max\left(\underline{r}^*, \frac{2}{3}(\rho+n)\right) < r^* \leq \rho+n$.

*Proof.* Notice $a(s,s)=0$. Differentiating (18) totally and combining it with Proposition 1 yield $\frac{d(c^*-\tau^*)}{d\bar{B}} = \frac{-3r^*\left[r^*-\frac{2}{3}(\rho+n)\right]}{(2r^*-\rho)\bar{P}}$ in which $\bar{B}$ is eliminated by use of (17). Condition 1 ensures the denominator to be positive. The numerator is positive if $0 < r^* < \frac{2}{3}(\rho+n)$ and negative if $\frac{2}{3}(\rho+n) < r^*$. Taking into account Condition 1, the proof is completed.■

The intuitive interpretation of this lemma is as follows. Since $\frac{d^2c^*}{d\bar{B}^2} = \frac{-2\rho n(1-\alpha)}{(2r^*-\rho)^2\bar{P}}\frac{dr^*}{d\bar{B}} < 0$ is satisfied, I find that the marginal effect on aggregate demand decreases.[13] On the other hand, the relation between $\tau^*$ and $\bar{B}$ satisfies

$$\frac{d\tau^*}{d\bar{B}} = \frac{\bar{B}}{\bar{P}}\frac{dr^*}{d\bar{B}} + \frac{r^*-n}{\bar{P}}, \tag{21}$$

$$\frac{d^2\tau^*}{d\bar{B}^2} = \frac{2[3r^*(r^*-\rho)+\rho(\rho+n\alpha)]}{(2r^*-\rho)^2\bar{P}}\frac{dr^*}{d\bar{B}} > 0. \tag{22}$$

The first term in the right-hand side of (21) represents increased tax by a rise in the interest rate. The second term in the right-hand side of (21) is negative when $r^* < n$ but positive when $r^* > n$. As illustrated in Figure 2(a) and 2(b), there are two possible cases concerning the schedule of $\tau^*$. In the figures, $\bar{B}^P \equiv \frac{\bar{M}}{\rho\alpha}[n-\rho(1+\alpha)]$ is the $\bar{B}$ corresponding to $r^*=n$, and $\bar{B}^U \equiv \frac{\bar{M}}{\rho\alpha}[n+\rho(1-\alpha)]$ represents the upper bound of $\bar{B}$ that gives $r^*=\rho+n$. Figure 2(a) depicts the case with a Ponzi scheme (i.e., $\bar{B}^P > 0$),

[13] The source of this result is that an expansion in consumption raises the interest rate simultaneously. As $c^*$ (or $r^*$) is determined so as to clear (17), a small rise in $c^*$ (or $r^*$) is enough to offset an increase in $\bar{B}$ when $c^*$ (or $r^*$) has a large value.

in the words of Weil (1989). At $\bar{B} \in (0, \bar{B}^P)$, $r^* < n$ is satisfied and lump-sum transfers exceed seigniorages ($\tau^* < -n\frac{\bar{M}}{P}$). At $\bar{B} \in (\bar{B}^P, \bar{B}^U]$, I have $r^* > n$ and $\tau^* > -n\frac{\bar{M}}{P}$, so that $\tau^*$ is increasing with respect to $\bar{B}$. Figure 2(b) describes the case without a Ponzi scheme (i.e., $\bar{B}^P \leq 0$). In this case, debt issue always involves higher taxes.[14]

For these reasons, the schedule of $c^* - \tau^*$ falls into two patterns, Figure 3(a) and 3(b). In the figures, $\bar{B}^M \equiv \frac{\bar{M}}{9\rho n\alpha}(-2\rho^2 + 4n^2 + 2\rho n - 9\rho n\alpha)$ is the $\bar{B}$ that attains $r^* = \frac{2}{3}(\rho + n)$, and satisfies that $\frac{d\bar{B}^M}{dn} = \frac{2}{9}\frac{\bar{M}(\rho^2+2n^2)}{\rho n^2\alpha} > 0$. If population growth is relatively low, I obtain Figure 3(a) with $\bar{B}^M \leq 0$, where debt policy always reduces the net incomes of future generations by levying heavy taxes. On the contrary, if there is enough high population growth, I get Figure 3(b) with $\bar{B}^M > 0$. Because high population growth brings the large demand-stimulating effect, debt policy raises the net incomes of future generations at $\bar{B} \in (0, \bar{B}^M)$. However, the demand-stimulating effect decreases with issuing of bonds. When $\bar{B}$ reaches $\bar{B}^M$, debt policy reduces $c^* - \tau^*$ like Figure 3(a).

**Remark 1.** It is easy to prove that $\bar{B}^P > 0$ is not necessary condition but sufficient condition for $\bar{B}^M > 0$. At $\bar{B} \in (\bar{B}^P, \bar{B}^M)$, debt policy raises the net incomes of all generations despite a non-Ponzi scheme.

Lemma 2 and 3 directly derive the welfare implication:

**Theorem 1.** *If* $\underline{r}^* < r^* < \frac{2-\alpha}{3-\alpha}(\rho + n)$*, debt policy is Pareto improving.*

*If* $\max\left(\underline{r}^*, \frac{2-\alpha}{3-\alpha}(\rho + n)\right) < r^* \leq \rho + n$*, debt policy improves welfare of the initial generation but deteriorates that of future generations.*

*Proof.* See Appendix I.

[14] An increase in $\bar{B}$ raises the interest rate at the same time. This is the reason of $\frac{d^2\tau^*}{d\bar{B}^2} > 0$.

**Remark 2.** $\frac{2-\alpha}{3-\alpha}(\rho+n) < \frac{2}{3}(\rho+n)$, which reflects the fact that an increase in $\bar{B}$ brings the higher interest rate and the higher cost of holding money.

Differently from models with full employment (e.g., Blanchard 1985 and Weil 1989), debt policy during recessions is Pareto improving by creating more employment if there is high population growth and the amount of outstanding bonds is relatively small. However, as public debt accumulates, the demand-stimulating effect declines whereas the cost through taxes swells. Consequently, when the outstanding issue is large, future generations are made worse off by heavy taxes. The result is also different from Rankin (1986) where debt policy during recessions is always beneficial to future generations.

# IV. Capital Accumulation

Modigliani's (1961) original concern is the crowding-out effect of public debt: capital accumulation is discouraged by raising the interest rate and welfare of future generations falls. The contribution of Rankin (1986) challenges this negative scenario by showing that debt issue interferes with investment but totally increases the steady-state output and welfare. For purpose of comparison, this section pays attention also to the effect on capital accumulation.

The optimal conditions (13) and (14) generate

$$r(t) = \beta\theta\left[\frac{\bar{W}}{(1-\beta)\theta P(t)}\right]^{\frac{\beta-1}{\beta}}. \tag{23}$$

Since the equilibrium condition of asset markets ensures $a(t) = k(t) + \frac{\bar{B}}{P(t)} + \frac{\bar{M}}{P(t)}$, the dynamic equation for $c(t)$ becomes

$$\frac{dc(t)}{dt} = [r(t)-\rho]c(t) - \rho n(1-\alpha)\left[k(t) + \frac{\bar{B}}{P(t)} + \frac{\bar{M}}{P(t)}\right]. \tag{24}$$

Keeping $y(t) = \frac{r(t)k(t)}{\beta}$ in mind, the equilibrium condition of the commodity market is

$$\frac{dk(t)}{dt} = \frac{r(t)k(t)}{\beta} - c(t) - nk(t). \tag{25}$$

The Fisher equation and (11) yield

$$\frac{dP(t)}{dt} = \left[\frac{\alpha}{1-\alpha}\frac{c(t)P(t)}{\bar{M}} - r(t)\right]P(t). \tag{26}$$

Equations (23)-(26) constitute the autonomous dynamic system.

Appendix II-A proves that the steady state is uniquely determined if it exists, and is dynamically stable. The analysis assumes the existence of the steady state. See Appendix II-B for the mathematical proof of the following argument.

To explain the effect on the steady-state values $(c^*, k^*, P^*)$, I propose three channels displayed in Figure 4. By the same logic as Proposition 1, Channel 1 signifies that taking $(r^*, k^*, P^*)$ as given in (24), debt policy directly raises $c^*$. Starting this point, there are two secondary effects denoted by Channel 2 and 3. From (23) and (26), I find that an increase in $c^*$ raises $P^*$.[15] In turn, $r^*$ rises from (23) and $\frac{\bar{B}}{P^*} + \frac{\bar{M}}{P^*}$ falls. Equation (24) implies that these secondary effects put downward pressure on $c^*$ (Channel 2). Equation (25) shows that there are conflicting effects on $k^*$: the demand-stimulating effect of Channel 1 directly raises $k^*$ but indirectly reduces it through raising $r^*$. I can prove that $k^*$ totally falls. Equation (24) implies that this secondary effect puts downward pressure on $c^*$ (Channel 3). Dynamic stability (Det$<0$) ensures that $c^*$ totally rises through Channel 1–3. I conclude that debt policy increases $c^*$ and $P^*$ and decreases $k^*$.

Responding to this result, the total effect on the steady-state output, $y^* = c^* + nk^*$, is ambiguous. When the prevailing interest rate is relatively low (i.e., $r^* < \frac{n\beta(3\beta-2)}{2\beta-1}$),

[15] This result depends on $\beta \in \left(0, \frac{1}{2}\right)$. If $\beta$ exceeds $\frac{1}{2}$, an increase in $c^*$ reduces $P^*$. Then, the total effects are non-positive on $c^*$, ambiguous on $k^*$, and non-negative on $P^*$.

the marginal crowding-out effect is large and debt policy is harmful to aggregate output. Modigliani's (1961) scenario holds true in this case: debt policy reduces future aggregate output through crowding out investment. However, the total effect including a change in taxes is much more important. Assuming that the economy is always in steady state, I obtain the welfare implication.

**Proposition 2.** *If* $\max(\rho, n\beta) \leq r^* < \frac{2-3\beta-\alpha(1-\beta)}{3-4\beta-\alpha(1-\beta)}(\rho+n)$*, debt policy is beneficial to all generations. If* $\max\left(\rho, n\beta, \frac{2-3\beta-\alpha(1-\beta)}{3-4\beta-\alpha(1-\beta)}(\rho+n)\right) \leq r^* \leq \rho+n$*, debt policy improves welfare of the initial generation and deteriorates that of future generations.*

*Proof.* See Appendix II-B.

**Remark 3.** $\frac{2-3\beta-\alpha(1-\beta)}{3-4\beta-\alpha(1-\beta)}(\rho+n) < \frac{2-\alpha}{3-\alpha}(\rho+n)$, which reflects the fact that an increase in $\bar{B}$ accompanies the negative effect through crowding out investment.

Basically, the implication of Theorem 1 is inherited even if capital accumulation is incorporated. Although there is the possibility that aggregate output drops, the economy is in dynamic-inefficiency at low levels of the interest rate. Therefore, if there is high population growth and the amount of outstanding bonds is small, debt finance improves welfare of all generations whether aggregate output drops or not.

# V. Conclusion

The present paper develops an overlapping-generations model with nominal wage rigidities and examines the welfare effects of debt policy during recessions. Further issues of public debt stimulate aggregate consumption demand and create employment. In so doing, future generations face both this beneficial employment effect and higher taxes. As

public debt accumulates, the beneficial effect gradually declines whereas the harmful effect steadily swells. If the outstanding issue is already large, heavy taxes lower the welfare of future generations. In contrast, if the outstanding issue is relatively small and there is high population growth, the beneficial effect dominates the harmful effect and debt policy is Pareto improving. These welfare implications are different from the results obtained in both the Blanchard-Weil model with full employment and the Diamond model with unemployment.

# Appendix I: Proof of Theorem 1

Differentiating (20) totally yields

$$\begin{aligned}\frac{dU(0,0)}{d\bar{B}} &= \frac{1}{\rho[r^*a(0,0)+c^*-\tau^*]}\frac{d[r^*a(0,0)+c^*-\tau^*]}{d\bar{B}} - \frac{\alpha}{\rho r^*}\frac{dr^*}{d\bar{B}},\\ \frac{dU(s,s)}{d\bar{B}} &= \frac{1}{\rho(c^*-\tau^*)}\frac{d(c^*-\tau^*)}{d\bar{B}} - \frac{\alpha}{\rho r^*}\frac{dr^*}{d\bar{B}} \quad \text{for } \forall s>0.\end{aligned}$$

Rewrite $r^*a(s,s)+c^*-\tau^*$ as a function of $r^*$ and $\bar{B}$ by using (11) and (18). Eliminate $\bar{B}$ from the result and Lemma 2 by use of (17). Utilizing these results, Lemma 1, and 3, the welfare effect of a change in $\bar{B}$ is calculated as

$$\begin{aligned}\frac{dU(0,0)}{d\bar{B}} &= \frac{n(2-\alpha)}{\rho[a(0,0)+h(0,0)](2r^*-\rho)\bar{P}} > 0,\\ \frac{dU(s,s)}{d\bar{B}} &= \frac{-(3-\alpha)\left[r^*-\frac{2-\alpha}{3-\alpha}(\rho+n)\right]}{\rho h(s,s)(2r^*-\rho)\bar{P}} \quad \text{for } \forall s>0.\end{aligned}$$

The former equation is strictly positive. The latter equation is positive if $0<r^*<\frac{2-\alpha}{3-\alpha}(\rho+n)$ and negative if $\frac{2-\alpha}{3-\alpha}(\rho+n)<r^*$. Taking account of Condition 1 completes the proof. ■

# Appendix II

## A. Steady State and Dynamic System

From (23)-(26), the steady state is, after tedious manipulations, characterized by

$$\Phi(r^*) \equiv r^{*3} - (\rho + n\beta)r^{*2} - \rho n\alpha \frac{\bar{B} + (1-\beta)\bar{M}}{\bar{M}} r^* + \rho n^2 \alpha\beta \frac{\bar{B} + \bar{M}}{\bar{M}} = 0.$$

$r^*$ must satisfy the following conditions: (i) $r^* \geq \rho$, which is required for $c^* \geq 0$ from (24); (ii) $r^* \geq n\beta$, which is required for $k^* \geq 0$ from (25); (iii) $r^* \leq \rho + n$, which is the non-negativity constraint of individual consumption. These conditions are summarized as

**Condition 2.** $\max(\rho, n\beta) \leq r^* \leq \rho + n$.

Since $\Phi(0) = \rho n^2 \alpha\beta \frac{\bar{B}+\bar{M}}{\bar{M}} > 0$ and $\Phi(n\beta) = -\rho n^2(1-\alpha)\beta^2 < 0$, $r^*$ is uniquely determined if it exists.

The dynamic system linearized around the steady state is

$$\begin{pmatrix} \frac{dc(t)}{dt} \\ \frac{dk(t)}{dt} \\ \frac{dP(t)}{dt} \end{pmatrix} = \begin{pmatrix} r^* - \rho & -\rho n(1-\alpha) & \frac{1-\alpha}{\alpha}\frac{\bar{M}}{P^{*2}}\left[\frac{1-\beta}{\beta} r^{*2} + \rho n\alpha \frac{\bar{B}+\bar{M}}{\bar{M}}\right] \\ -1 & \frac{r^*-n\beta}{\beta} & \frac{1-\alpha}{\alpha}\frac{\bar{M}}{P^{*2}}\frac{1-\beta}{\beta}\frac{r^{*2}}{r^*-n\beta} \\ \frac{\alpha}{1-\alpha}\frac{P^{*2}}{\bar{M}} & 0 & \frac{2\beta-1}{\beta} r^* \end{pmatrix} \begin{pmatrix} c(t) - c^* \\ k(t) - k^* \\ P(t) - P^* \end{pmatrix}.$$

For this coefficient matrix,

$$\begin{aligned} \text{Trace} &= 3r^* - (\rho + n), \\ \text{Det} &= -\frac{1-\beta}{\beta^2}\frac{r^*}{r^* - n\beta}\psi(r^*) < 0 \end{aligned}$$

in which $\bar{B}$ is eliminated by use of the above steady-state condition. The function $\psi(r^*)$ is given by

$$\psi(r^*) \equiv 2r^{*3} - (\rho + 4n\beta)r^{*2} + 2n\beta(\rho + n\beta)r^* - \rho n^2 \alpha\beta^2.$$

Det$<0$ is proved as follows. The first derivative of $\psi(r^*)$ is $\frac{d\psi(r^*)}{dr^*} = 2(r^* - n\beta)[3r^* - (\rho + n\beta)]$. First, consider the case where $\frac{\rho+n\beta}{3} < n\beta$. Because I have $\psi(n\beta) = \rho n^2(1-\alpha)\beta^2 > 0$, $\psi(r^*) > 0$ is satisfied within Condition 2. Next, consider the case where $\frac{\rho+n\beta}{3} \geq n\beta$. Since $\frac{\rho+n\beta}{3} < \rho$ is satisfied, it is sufficient to show $\psi(r^*) > 0$ for $r^* \geq \rho$. I have $\psi(\rho) = \rho(\rho - n\beta)^2 + \rho n^2\beta^2(1-\alpha) > 0$. Resultantly, $\psi(r^*)$ is always positive within Condition 2 and hence I get Det$<0$.

As $k(0)$ is predetermined and $c(0)$ and $P(0)$ are not, the steady state is dynamically stable: the dynamic path is either saddle-point stable or indeterminacy. In the case of saddle-point stability, divergent paths are inconsistent with the optimality of individual consumers.

## B. Comparative Statics

Differentiating the steady-state conditions totally, the steady-state effect of debt policy is obtained as

$$\begin{aligned}
\frac{dc^*}{d\bar{B}} &= \frac{\rho n(1-\alpha)(2\beta-1)r^*(r^*-n\beta)}{\beta^2 P^* \mathrm{Det}} \geq 0, \\
\frac{dk^*}{d\bar{B}} &= \frac{\beta^2[r^* - n(2\beta-1)]}{(2\beta-1)(r^*-n\beta)^2}\frac{dc^*}{d\bar{B}} \leq 0, \\
\frac{dP^*}{d\bar{B}} &= -\frac{\beta}{(2\beta-1)r^*}\frac{\alpha}{1-\alpha}\frac{P^{*2}}{\bar{M}}\frac{dc^*}{d\bar{B}} \geq 0.
\end{aligned}$$

As explained in the text, Det$<0$ and the restriction on $\beta$ produce the above signs. From these results, I get the effect on the steady-state output, $y^* = c^* + nk^*$:

$$\frac{dy^*}{d\bar{B}} = \frac{r^*}{(r^*-n\beta)^2}\left[r^* - \frac{n\beta(3\beta-2)}{2\beta-1}\right]\frac{dc^*}{d\bar{B}} \geq 0 \quad \text{if} \quad r^* \geq \frac{n\beta(3\beta-2)}{2\beta-1}.$$

By use of $w^*x^* = (1-\beta)y^*$, the restriction on $\beta$, and (15), the effect on the steady-state net wage income $w^*x^* - \tau^*$ is derived as

$$\frac{d(w^*x^* - \tau^*)}{d\bar{B}} = \frac{(3-4\beta)r^*\left[r^* - \frac{2-3\beta}{3-4\beta}(\rho+n)\right]}{\rho n(1-\alpha)(2\beta-1)}\frac{dc^*}{d\bar{B}} \geq 0 \quad \text{if} \quad r^* \leq \frac{2-3\beta}{3-4\beta}(\rho+n).$$

Assume that the economy is always in steady state. Following the similar procedure to Theorem 1, I obtain the welfare effect:

$$\begin{aligned}
\frac{dU(0,0)}{d\bar{B}} &= \frac{1}{\rho[a(0,0)+h(0,0)]}\frac{-[2-3\beta-\alpha(1-\beta)]}{\rho(1-\alpha)(2\beta-1)}\frac{dc^*}{d\bar{B}} > 0, \\
\frac{dU(s,s)}{d\bar{B}} &= \frac{1}{\rho h(s,s)}\frac{[3-4\beta-\alpha(1-\beta)]\left[r^* - \frac{2-3\beta-\alpha(1-\beta)}{3-4\beta-\alpha(1-\beta)}(\rho+n)\right]}{\rho n(1-\alpha)(2\beta-1)}\frac{dc^*}{d\bar{B}} \quad \text{for } \forall s > 0.
\end{aligned}$$

Notice that $3-4\beta > 2-3\beta > \alpha(1-\beta)$. $\frac{dU(s,s)}{d\bar{B}}$ is positive if $r^* \leq \frac{2-3\beta-\alpha(1-\beta)}{3-4\beta-\alpha(1-\beta)}(\rho+n)$ and negative otherwise. Taking into account Condition 2, the proof of Proposition 2 is completed.

# References


Blanchard, Olivier J., "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, XCIII (1985), 223-247.

__ and Nobuhiro Kiyotaki, "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review*, LXXVII (1987), 647-666.

Bowen, William G., Richard G. Davis, and David H. Kopf, "The Public Debt: A Burden on Future Generations?," *American Economic Review*, L (1960), 701-706.

Buiter, Willem H., "Death, Birth, Productivity Growth and Debt Neutrality," *Economic Journal*, XCVIII (1988), 279-293.

Diamond, Peter A., "National Debt in a Neoclassical Growth Model," *American Economic Review*, LV (1965), 1126-1150.

Iwamoto, Yasushi, "On the Role of Fiscal Policy in Stabilizing Business Fluctuations (in Japanese)," *Financial Review*, LXIII (2002), 8-28.

King, Robert G. and Sergio T. Rebelo, "Resuscitating Real Business Cycles," in M. Woodford and J. Taylor, eds., *Handbook of Macroeconomics*, Vol. 1B, (Amsterdam: North-Holland, 1999).

Lerner, Abba P., "The Burden of the National Debt," in L. A. Metzler, ed., *Income, Employment and Public Policy: Essays in Honor of Alvin H. Hansen*, (New York: W. W. Norton & Company, 1948).

Mankiw, N. Gregory and David Romer, eds., *New Keynesian Economics*, Vols. 1 and 2, (Cambridge, MA: MIT Press, 1991).

Mino, Kazuo and Akihisa Shibata, "Growth and Welfare Effects of Monetary Expansion in an Overlapping-Generations Economy," *Japanese Economic Review*, LI (2000), 407-430.

Modigliani, Franco, "Long-Run Implications of Alternative Fiscal Policies and the Burden of the National Debt," *Economic Journal*, LXX (1961), 730-750.

Ono, Yoshiyasu, *Gokai darake no Kouzou Kaikaku* (in Japanese), (Tokyo, Japan: Nihon Keizai Shinbun Sha, 2001).

Rankin, Neil, "Debt Policy under Fixed and Flexible Prices," *Oxford Economic Papers*, XXXVIII (1986), 481-500.

Saint-Paul, Gilles, "Fiscal Policy in an Endogenous Growth Model," *Quarterly Journal of Economics*, CVII (1992), 1243-1259.

Sen, Partha, "Welfare-Improving Debt Policy under Monopolistic Competition," *Journal of Economic Dynamics & Control*, XXVII (2002), 143-156.

Tobin, James, *Asset Accumulation and Economic Activity*, (Oxford: Basil Blackwell, 1980).

Weil, Philippe, "Permanent Budget Deficits and Inflation," *Journal of Monetary Economics*, XX (1987), 393-410.

___ , "Overlapping Families of Infinitely-Lived Agents," *Journal of Public Economics*, XXXVIII (1989), 183-198.

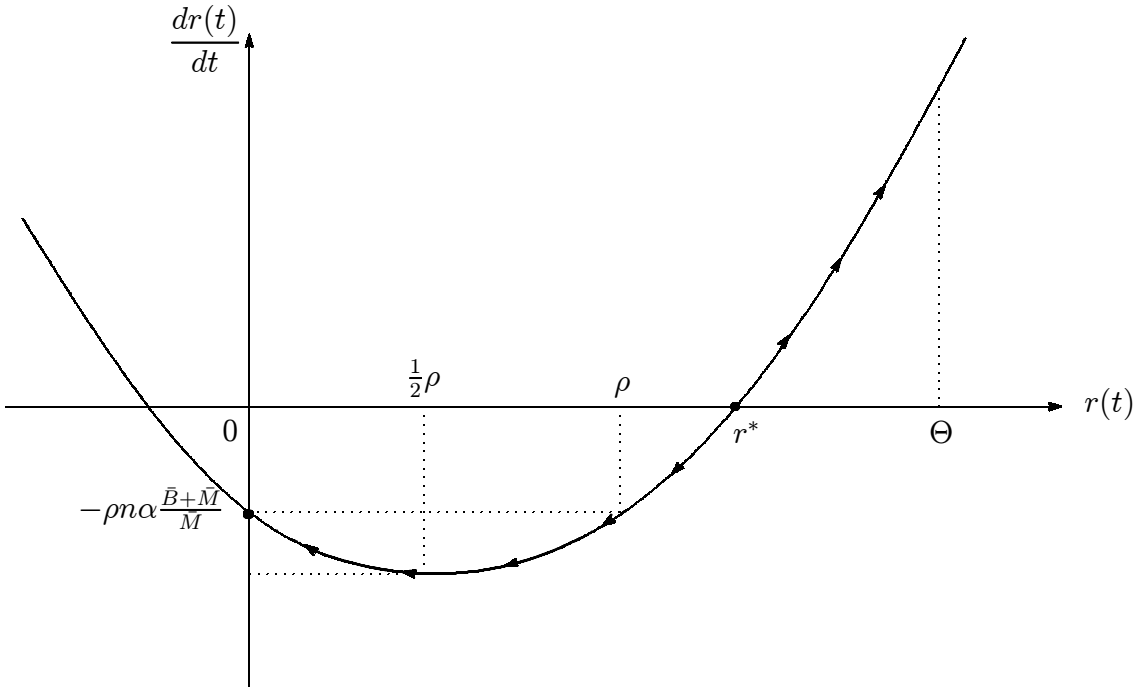


Figure 1: Aggregate Dynamics

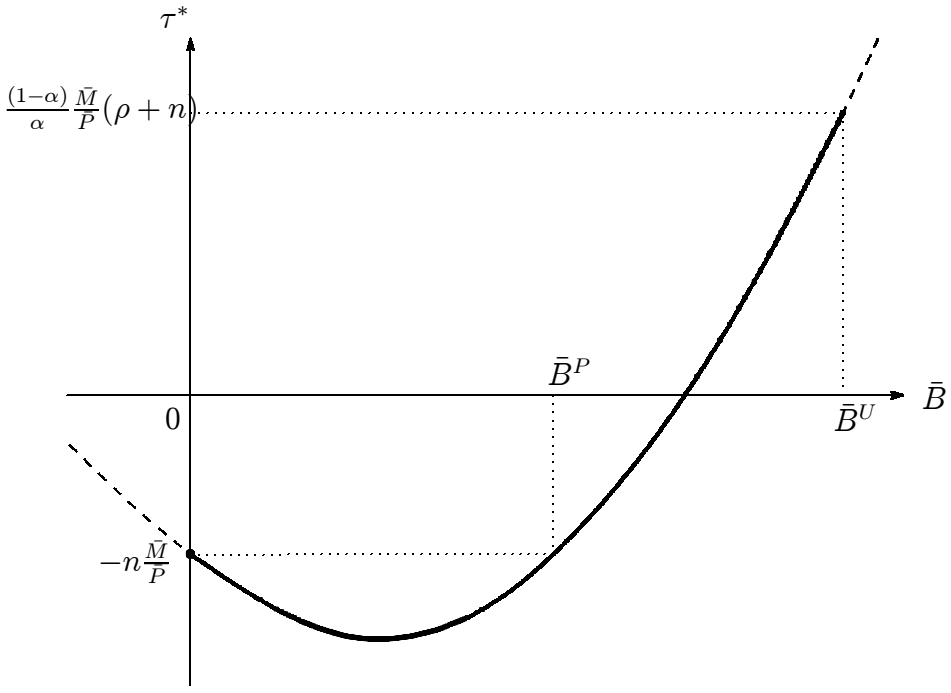


Figure 2(a): $\bar{B}^P > 0$

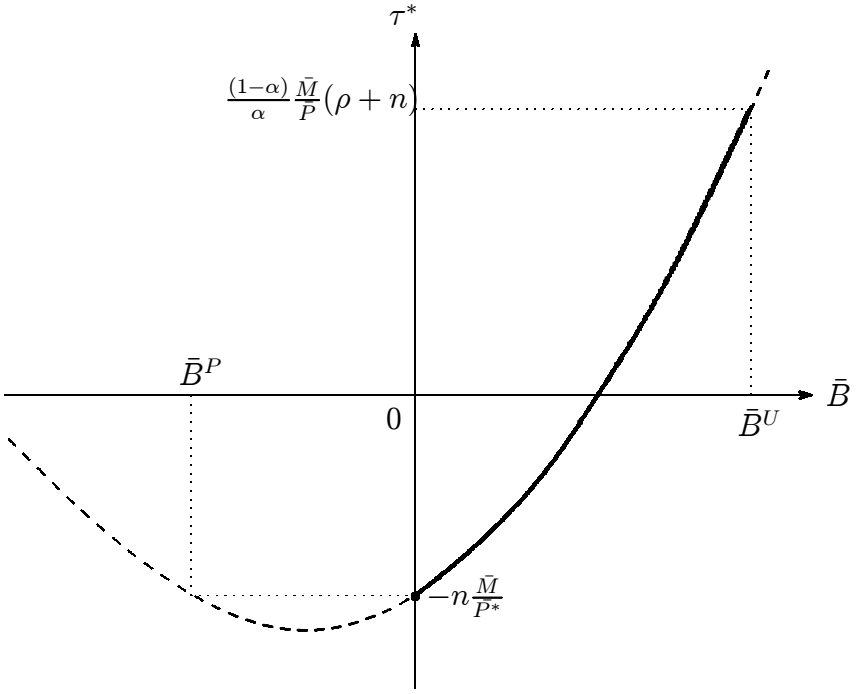


Figure 2(b): $\bar{B}^P \leq 0$

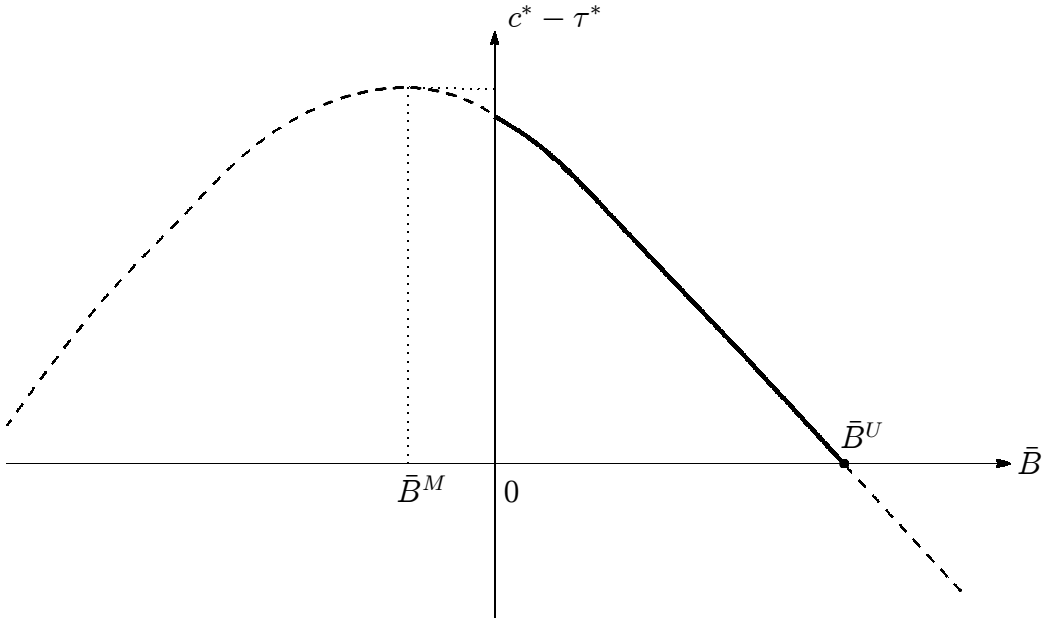


Figure 3(a): $\bar{B}^M \leq 0$

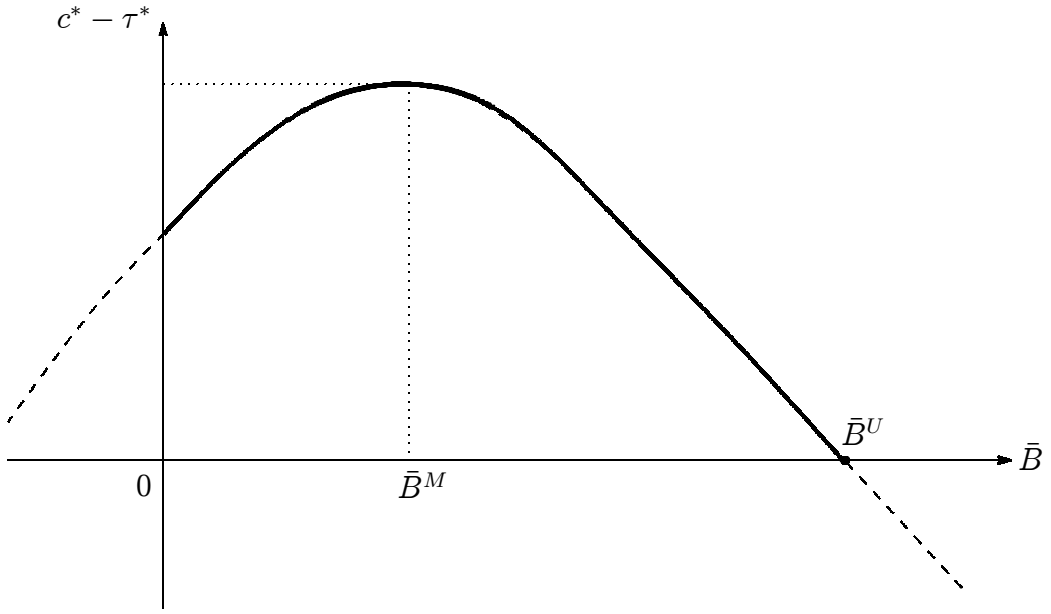


Figure 3(b): $\bar{B}^M > 0$

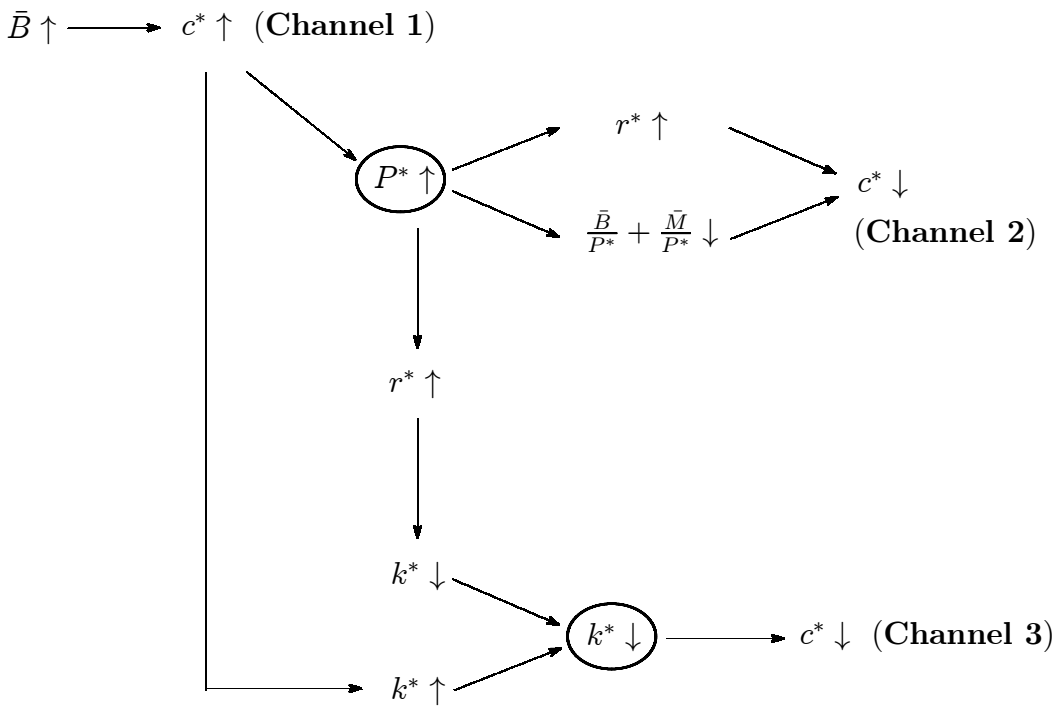


Figure 4: Channels