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Abstract

This paper constructs a two-region endogenous growth model with productive government expenditure to analyze the relationship between regional redistribution of public input and the welfare of residents in each region. This paper shows that the redistribution policy may be Pareto improving if the distribution rate of a more populous region is increased because it raises the equilibrium growth rate. Furthermore, the higher the inequalities between the labor populations are, the greater the possibility of a Pareto improving policy.

Key words: Endogenous growth; Government expenditure; Regional distribution; Welfare; Pareto improving policy
JEL classification: H53; O41; R58

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1 Introduction

Why does an increase in regional disparities cause some serious problems? Some insist that regional inequalities hurt immobile residents in the poorer regions, others state that an agglomeration of efficient regions has positive externalities. Thus, although issues about regional inequalities have been controversial for a long time, there have only been a few theoretical studies. Furthermore, despite limited theoretical understanding, in many countries inter-regional redistribution policies that include tied and untied subsidies are believed to be justified, as well as income redistribution policies between individuals. Therefore, it is important to construct a theoretical model in terms of regional inequalities and resident welfare.

The purpose of this paper is to investigate the relationship between the inter-regional redistribution of government expenditure and the welfare of residents in each region. Barro (1990) first formulated an endogenous growth model that incorporated public services as a productive input into private production. This paper extends Barro to two regions and examines a redistribution policy between the regions. Since Barro, although there are many endogenous growth models with productive public expenditure,[1] there has been hardly any extension to two-region or multi-region models.

This paper is also related to the literature on international economics that has examined effects of transfers on welfare. They have examined three effects when the transfer is done. The first is the normal case, in which a donor becomes worse off and a recipient becomes better off. The second is that a donor becomes better off and a recipient becomes worse off, called the *transfer paradox*. The third is that both of them are better (worse) off, called *Pareto improving (worsening)*. Although these studies are closely related to our model, they are almost all static[2] and has supposed that the transfers have no effect on productivity.[3]

The structure of our model is similar to the models that analyze the relationship between the composition of public expenditure and economic growth in a one-region endogenous growth model, such as Devarajan et al.

[1]See Barro and Sala-i-Martin (1992), Futagami et al. (1993), Glomm and Ravikumar (1994), Turnovsky (1996), and others.

[2]In international economics, there are many studies that analyze the relationship between forms of aid and welfare, for example Bhagwati et al. (1982). Ihori (1996) introduces public goods that are voluntarily provided to these frameworks.

[3]By way of exception, Galor and Polemarchakis (1987) analyze the existence of a transfer paradox in an overlapping generations model, and Yanagihara (1998) extends Galor and Polemarchakis to introduce public goods that have externalities only on domestic private production. However, those studies are not endogenous growth models in which there is a possibility that the transfer affects an equilibrium growth rate.

(1996) and Davoodi et al. (1998, 1999). They have shown that a change in the composition of public expenditure leads to an increase in the equilibrium growth rate. Thus, it is possible to say that our model is a two-region and multi-region expansion of their model.

However, our model is distinct from others in the respect that it incorporates an inter-regional redistribution policy and a simple endogenous growth model. Moreover, this is the first paper to investigate Pareto improving public policy in a two-region Barro's framework.[4]

The main results in this paper are as follows. A redistribution policy may be Pareto improving if the distribution rate of the more populated region is raised. On the contrary, it may be Pareto worsening if the distribution rate of the less populated region is raised. Furthermore, the higher the inequalities in labor population between the regions are, the possibility of a Pareto improving policy is more enhanced. This is because a change in the distribution rate has effects not only on initial consumption, but also on the economy-wide growth rate, which has a positive correlation with a change in the distribution rate as long as that of the more populous region is raised. The reasons why the economy-wide growth rate increases are because of scale effects in endogenous growth models[5] and spillover effects in government expenditure. Moreover, we extend this model to a multi-regional case. Similar to the two-region case, we show the possibility for Pareto improvement if the distribution rate of regions with higher than average population is raised.

The remainder of this paper is organized as follows. The basic model is presented in Section 2. Section 3 examines comparative statics with respect to policy variables of governments. Section 4 analyses the welfare effect of changes in the distribution rate and shows the possibility of a Pareto improving policy. Section 5 briefly considers the case of a lump-sum transfer. Section 6 extends the model to a multi-region model. Section 7 discusses the implications and concludes this paper.

2 The model

We first present the basic structure of the model. There are two regions, regions 1 and 2, in a country. The country has a central government that collects taxes from individuals in each region and distributes them to local governments, which supply local public goods that enhance productivity as

[4]Martin (1999) and Martin and Ottaviano (1999) have a similar idea to ours, but the structure of their models are more complicated since they consider industrial geography and they have analyzed public policies that lower intra- or inter-regional transaction costs.

[5]See Barro and Sala-i-Martin (1995) for more detail.

an externality in each region. The regions produce a homogeneous good and they are identical except for their initial assets and labor populations. We assume that capital is perfectly mobile between regions, but labor migration is prohibited.

2.1 Firms

Following Barro(1990), the production sector in each region has a linearly homogeneous production function, that is

$$Y_i \quad = \quad F(K_i, G_i L_i), \quad i = 1, 2, \tag{1}$$

where Y_i is output in region i, K_i is physical capital devoted to the production sector, G_i is local government expenditure in each region, and L_i denotes labor, which is identified as the size of the population, and is constant over time.

By taking the price of the good as numeraire, let r_i, w_i represent the rent on capital and the wage in region i, respectively. Since the production function in each region is linearly homogeneous, we can express the profit maximization problem of competitive firms as follows:

$$r_i \quad = \quad f'(k_i), \tag{2}$$

$$w_i \quad = \quad \left[\frac{f(k_i)}{k_i} - f'(k_i)\right]\frac{K_i}{L_i} = \omega(k_i)\frac{K_i}{L_i}. \tag{3}$$

In the above equations, $k_i \equiv K_i/L_iG_i$ is the quantity of capital per unit of effective labor, $\omega(k_i) = f(k_i)/k_i - f'(k_i)$ and $f(k_i) = F(K_i/L_iG_i, 1)$. We assume that $f(k_i)$ satisfies the standard neoclassical properties and the Inada conditions:

$$f'(k_i) > 0, \ f''(k_i) < 0, \ \lim_{k_i \to 0} f'(k_i) = \infty, \ \lim_{k_i \to \infty} f'(k_i) = 0.$$

where f' and f'' denote the first derivative and the second derivative with respect to the argument of the function, respectively.

2.2 Households

We assume that many identical households live in two regions and that each household provides its assets in both regions and inelastically one unit of labor in the region where they live. The objective function of the representative household in region i is

$$\int_0^\infty u(c_i)e^{-\rho t}dt, \tag{4}$$

where c_i is the per capita consumption of the household in region i and ρ is the constant rate of time preference. The felicity function is assumed to take the constant relative risk aversion form:

$$\begin{aligned} u(c_i) &= \frac{c_i^{1-\sigma}}{1-\sigma}, \quad \text{for} \ \ \sigma > 0, \quad \sigma \neq 1, && (5) \\ &= \ln c_i, \qquad \text{for} \ \ \sigma = 1, && (6) \end{aligned}$$

where σ is the degree of relative risk aversion (the reciprocal of the intertemporal elasticity of substitution), which is assumed to be a positive constant.

Due to the perfect capital mobility between the regions, a no-arbitrage condition $r_1 = r_2 = r$ must always hold without the case of the corner solution. Thus, the flow budget constraint of the household in region i is

$$\dot{a}_i = (1-\tau)(ra_i + w_i) - c_i, \tag{7}$$

where a_i, τ denotes the total asset per capita in region i, the rate of flat-rate income tax, respectively.[6] Given the amount of initial holdings $a_i(0) > 0$, the household maximizes (4) subject to (7). It is assumed that the household is endowed with perfect foresight, and it accurately anticipates the whole sequences of real rents $\{r(t)\}_{t=0}^\infty$ and the wage rate $\{w_i(t)\}_{t=0}^\infty$. As a result of the utility maximization problem of the household endowed with perfect foresight, the optimal consumption path and the transversality condition are given by

$$\begin{aligned} \frac{\dot{c}_i}{c_i} &= \frac{1}{\sigma}\left[(1-\tau)r^* - \rho\right], && (8) \\ \lim_{t\to\infty} \lambda_i(t)a_i(t)e^{-\rho t} &= 0, && (9) \end{aligned}$$

where λ represents the co-state variable of a_i.

[6] A dot above a variable represents taking the time-derivative.

2.3 Governments

There are three governments in the economy. The central government levies flat-rate income taxes in each region.[7] The budget constraints of each government read

$$G = \tau(Y_1 + Y_2), \tag{10}$$
$$G_1 = \beta G, \quad G_2 = (1-\beta)G, \quad \beta \in (0,1), \tag{11}$$

where G is the central government's revenue (total tax revenue in the economy[8]), G_i is the local government expenditure in each region, and β is the distribution rate for region 1. For simplicity, we assume that τ and β are time-invariant and that distribution from the central government is tied to government expenditure.[9] Moreover, note that equations (10) and (11) mean that each government runs a balanced budget.

2.4 Market equilibrium

There are two integrated markets, asset and goods,[10] and two disintegrated markets, labor in each region, in the economy. Each market equilibrium is described by

$$A_1 + A_2 = K_1 + K_2, \tag{12}$$
$$Y_1 + Y_2 = C_1 + C_2 + \dot{K}_1 + \dot{K}_2 + G_1 + G_2, \tag{13}$$

where A_1 and A_2 respectively denote the aggregate assets in regions 1 and 2.

Let us next define the new variable x which represents the GDP ratio (Y_2/Y_1), then from (1), (11) the quantity of capital per unit of effective labor in each region is as follows:

$$k_1 = f^{-1}\left(\frac{1}{\beta L_1 \tau(1+x)}\right), \tag{14}$$
$$k_2 = f^{-1}\left(\frac{1}{(1-\beta) L_2 \tau(1+x^{-1})}\right). \tag{15}$$

[7]The details of other taxation methods are given in Atkinson and Stiglitz (1980)

[8]For the sake of an exhaustion of product, $Y_i = rK_i + w_i L_i$ holds in each region. Using the flow budget constraint of the household and asset market clearing condition, we derive (10).

[9]If the distribution includes tied and untied forms, the result in this paper would still hold.

[10]Equities are the only financial asset in the economy.

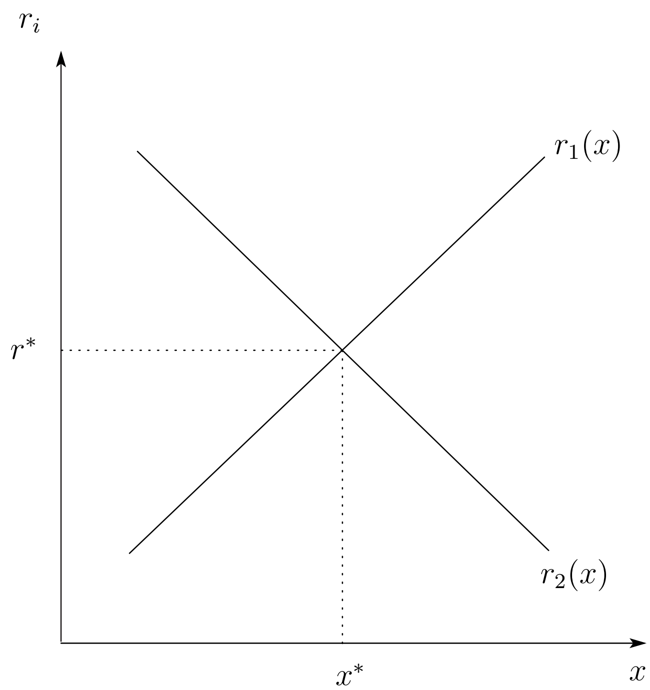

Figure 1: $r_i - x$ plane

Although the rent on physical capital in Barro's model, which is for one region with no distribution policy, could be derived in the form of simple AK technology, in our model this depends on the GDP ratio x. Figure 1 illustrates the rent on physical capital in each region, and it indicates that r_1 is monotonically increasing in x and r_2 is monotonically decreasing in x.[11]

As mentioned above, because the no-arbitrage condition always holds, $r_1 = r_2 \Leftrightarrow k_1 = k_2$ holds in equilibrium, and then, the equilibrium ratio x can be determined as follows.

$$x^* = (1-\beta)L_2/\beta L_1, \tag{16}$$

$$k^* = f^{-1}\left(\frac{1}{\tau[\beta L_1 + (1-\beta)L_2]}\right), \tag{17}$$

$$r^* = f'(k^*), \tag{18}$$

$$\omega^* = f(k^*)/k^* - f'(k^*), \tag{19}$$

where the asterisk denotes an equilibrium value. From (16) through (19), we show that the equilibrium value r^* and ω^* are time-invariant since x^* and k^* are time-invariant.

[11] The second order condition, the sign of $\partial^2 r_i/\partial x^2$ depends on the functional form.

2.5 Steady-growth equilibrium

We let γ^* be a steady-growth rate and in a steady-growth equilibrium, the economy is characterized by

$$\gamma^* \equiv \frac{\dot{C}_i}{C_i} = \frac{\dot{K}_i}{K_i} = \frac{\dot{A}}{A} = \frac{\dot{Y}_i}{Y_i} = \frac{\dot{G}_i}{G_i},$$

where all variables are at the aggregate level, and $A \equiv A_1 + A_2$ denotes the economy-wide assets. Let us examine the existence and stability of the steady-growth equilibrium. From (10), (12), (13) and recalling that $Y = (r + \omega)A$, we obtain

$$\dot{A} \;=\; (1-\tau)(r^* + \omega^*)A - C, \tag{20}$$

where $C \equiv C_1 + C_2$ represents the economy-wide consumption level. Define a new variable z as C/A, then from (8) and (20) we have the dynamics with respect to z:

$$\begin{aligned} \dot{z} &= \chi(z) \\ &\equiv z\left(\frac{1}{\sigma}((1-\tau)r^* - \rho) - (1-\tau)(r^* + \omega^*) + z\right). \end{aligned}$$

We can easily verify that $\chi(z) = 0$ has a unique positive solution, denoted by z^*, if and only if

$$\frac{\rho}{\sigma} + (1-\tau)\left(\frac{f(k^*)}{k^*} - \frac{f'(k^*)}{\sigma}\right) > 0. \tag{21}$$

Figure 2 depicts the graph of χ when (21) holds. In this graph, it is shown that z^* is globally unstable and hence the steady-growth equilibrium in this model uniquely exists. This implies that the economy has no transitional dynamics and is always in a steady-growth equilibrium.

3 Comparative statics

In this section, we consider the impacts of an unanticipated permanent increase in β and τ. The effects of changes in β and τ are summarized as follows:

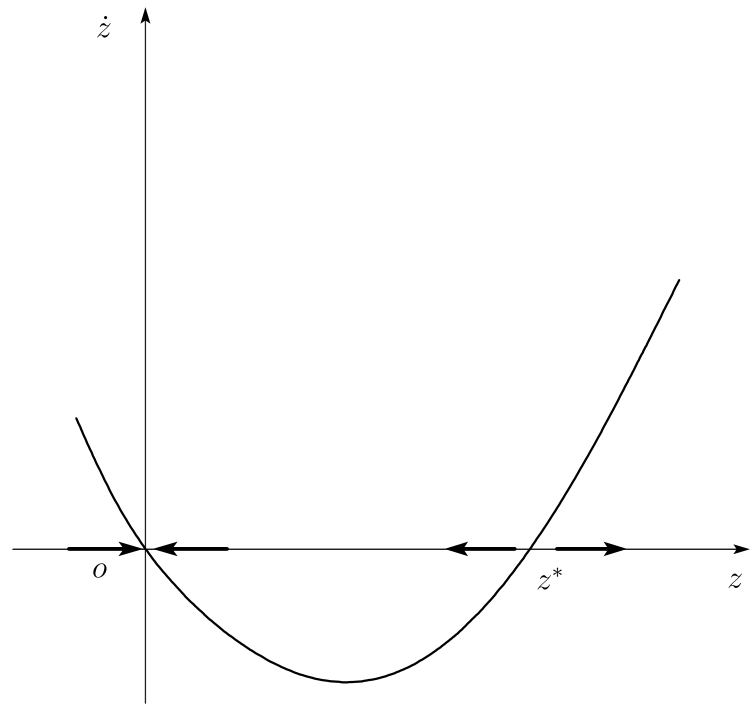

Figure 2: Graph of χ

$$\frac{d\gamma^*}{d\beta} = -\frac{1}{\sigma}(1-\tau)\frac{f''(k^*)}{f'(k^*)}\frac{L_1 - L_2}{\tau[\beta L_1 + (1-\beta)L_2]^2}, \tag{22}$$

$$\frac{dx^*}{d\beta} = -\frac{L_2}{\beta^2 L_1} < 0, \tag{23}$$

$$\frac{d\gamma^*}{d\tau} = \frac{1}{\sigma}\left[-(1-\tau)\frac{f''(k^*)}{f'(k^*)}\frac{1}{\tau^2[\beta L_1 + (1-\beta)L_2]} - r^*\right], \tag{24}$$

$$\frac{dx^*}{d\tau} = 0. \tag{25}$$

Proposition 1. *It is true that*

a. by increasing the distribution rate to the more populated region, the steady-growth rate can be enhanced.

b. since the central government levies the same tax rate on both regions, regional inequalities cannot be reduced by a tax policy.

Proof. See the equations from (22) to (25).

□

4 Welfare analysis (Pareto improving policy)

In this section, we examine the welfare effect in changing the distribution rate. The redistribution policy between regions would be justified if it enhanced the welfare of residents in less populated regions. We first predict the level of the consumption at time t in each region.

The budget constraint of households in the aggregate in region i is

$$\dot{A}_i(t) = (1-\tau)(r^* A_i(t) + w_i(t)L_i) - c_i(t)L_i.$$

By solving the first order differential equation with respect to $A_i(t)$, the initial consumption in region i is given by[12]

$$c_i(0) = -\phi \left[a_i(0) + (1-\tau) \int_0^{\infty} w_i(t) e^{-(1-\tau)r^* t} dt \right], \tag{26}$$

where $\phi \equiv (1/\sigma - 1)(1-\tau)r^* - \rho/\sigma < 0$.

Substituting (3), (12) and (19) into (26), we obtain:[13]

$$c_1(0) = -\phi a_1(0) + \frac{1}{1+x^*}\frac{1}{L_1}(1-\tau)\omega^* A(0), \tag{27}$$

$$c_2(0) = -\phi a_2(0) + \frac{x^*}{1+x^*}\frac{1}{L_2}(1-\tau)\omega^* A(0). \tag{28}$$

Thus the initial consumption depends on the initial asset (the first term in (27) and (28)) and the lifetime wage (the second term in (27) and (28)). In other words, the greater the initial asset or the lifetime wage is, the more the initial consumption is. Furthermore, the lifetime wage depends on GDP ratio (capital ratio) since it increases with capital invested where they live in an endogenous growth model.[14]

For the sake of simplicity, suppose that the rate of the relative risk aversion $\sigma = 1$, that is, the utility function is logarithmic. Further suppose that the production technology of the firms takes a Cobb-Douglas form, hence

$$Y_i = K_i^{\alpha}(G_i L_i)^{1-\alpha}, \quad \alpha \in (0,1),$$

[12] See appendix in detail.

[13] The derivation of (27) and (28) is in the appendix.

[14] In equilibrium, GDP ratio equals to capital ratio completely.

where α represents the distribution rate of capital.[15] Let us now define the welfare function in each region. The welfare over an infinite planning horizon of the household in region i (W_i) can be written as:

$$\begin{aligned} W_i &= \int_0^\infty \ln c_i(t) e^{-\rho t} dt, \\ &= \frac{1}{\rho} \ln c_i(0) + \frac{1}{\rho^2}\gamma^*. \end{aligned} \tag{29}$$

Let us now differentiate (29) with respect to β, in other words, we consider the welfare effects when the distribution rate to region 1 rises. For this operation the welfare effects on rising β are given by

$$\frac{dW_i}{d\beta} = \frac{1}{\rho}\frac{1}{c_i(0)}\frac{dc_i(0)}{d\beta} + \frac{1}{\rho^2}\frac{d\gamma^*}{d\beta}. \tag{30}$$

Equation (30) reveals that the welfare effect of a rise in β includes two distinct components. One, the first term on the right-hand side, is *the initial consumption effect*. It reflects whether the initial consumption of the residents in each region increases or not, by raising the distribution rate to region 1. The other, the second term on the right-hand side, is *the economy-wide growth effect*. It represents the effect of a rise in β on the steady-growth rate. As mentioned in proposition 1, this effect is always positive if and only if the labor population in region 1 is greater than that of region 2. Therefore, we have only to focus on the initial consumption effect. It can be derived as follows:

$$\frac{dc_1(0)}{d\beta} = \frac{1-\alpha}{\alpha} A(0)\psi \left[(1-\alpha)\beta(L_1 - L_2) + \alpha L_2\right], \tag{31}$$

$$\frac{dc_2(0)}{d\beta} = \frac{1-\alpha}{\alpha} A(0)\psi \left[(1-\alpha)(1-\beta)(L_1 - L_2) - \alpha L_1\right], \tag{32}$$

where $\psi \equiv (1-\tau)\tau^{\frac{1-\alpha}{\alpha}} B^{\frac{1-3\alpha}{\alpha}}$, $B \equiv \beta L_1 + (1-\beta)L_2$, respectively. Furthermore, in the case of the log-linear utility function, the initial consumption effect is divided into two components, both of which are linked to the lifetime wage. One, the first term in the square bracket on the right-hand side of equations (31) and (32), is *the wage growth effect*. It captures the effect of the wage increases with an economy-wide growth rate. The other, the second term in

[15] In the Cobb-Douglas case, the variables in equilibrium are given in appendix.

the bracket on the right-hand side, is *the productivity effect*. It means that a rise in the distribution rate (a rise in β) makes region 1 more attractive for investment since it enhances the productivity of capital in region 1. The wage growth effect is positive in both regions because we suppose $L_1 > L_2$. However, the productivity effect is positive in region 1 but negative in region 2. From the above argument we have the following proposition.

Proposition 2. *In the case that region 1 is more populous than region 2, a policy which increases the distribution rate in region 1 (region 2) is Pareto improving (Pareto worsening) if and only if* $L_2/L_1 \leq \Phi(L_2/L_1)$.

Proof. We first consider the effect of a steady-growth rate. By assuming region 1 is more populous, a rise in β enhances the steady-growth rate, that is $d\gamma^*/d\beta > 0$. Next, the two effects on the initial consumption in region 1 have the same sign (both positive), hence $dW_1/d\beta > 0$ from (31). On the other hand, those in region 2 have the opposite sign; that is, the wage growth effect is positive and the productivity effect is negative. Hence, the sign of $dW_2/d\beta$ is ambiguous because of this trade-off. The necessary and sufficient condition for a Pareto improving policy ($dW_2/d\beta \geq 0$) is as follows:

$$\frac{L_2}{L_1} \leq 1 - \frac{\alpha}{(1-\alpha)(1-\beta)\left[1+\frac{1}{\rho}(1-\tau)r^*\right] + \frac{a_2(0)}{A(0)}\alpha B} \equiv \Phi\left(\frac{L_2}{L_1}\right). \tag{33}$$

In the equation (33), they are variables $r^*, A(0), B$ that depends on L_2/L_1, and they are monotonically increasing in L_2/L_1. Therefore, the sign of $d\Phi(\cdot)/d(L_2/L_1)$ is ambiguous, but as a result of the numerical analysis, unless the parameters are extraordinary, Φ is increasing in L_2/L_1.[16]

Then it is useful for us to investigate the sufficient condition for the Pareto improving policy, which is $dc_2(0)/d\beta \geq 0$ if and only if[17]

$$\frac{L_2}{L_1} \leq 1 - \frac{\alpha}{(1-\alpha)(1-\beta)}. \tag{34}$$

□

From the equation (33), we derive the following proposition.

Proposition 3. *The possibility of a Pareto improving policy is greater in the following cases:*

[16] See appendix.

[17] Figure 4 shows the region for the Pareto improving policy.

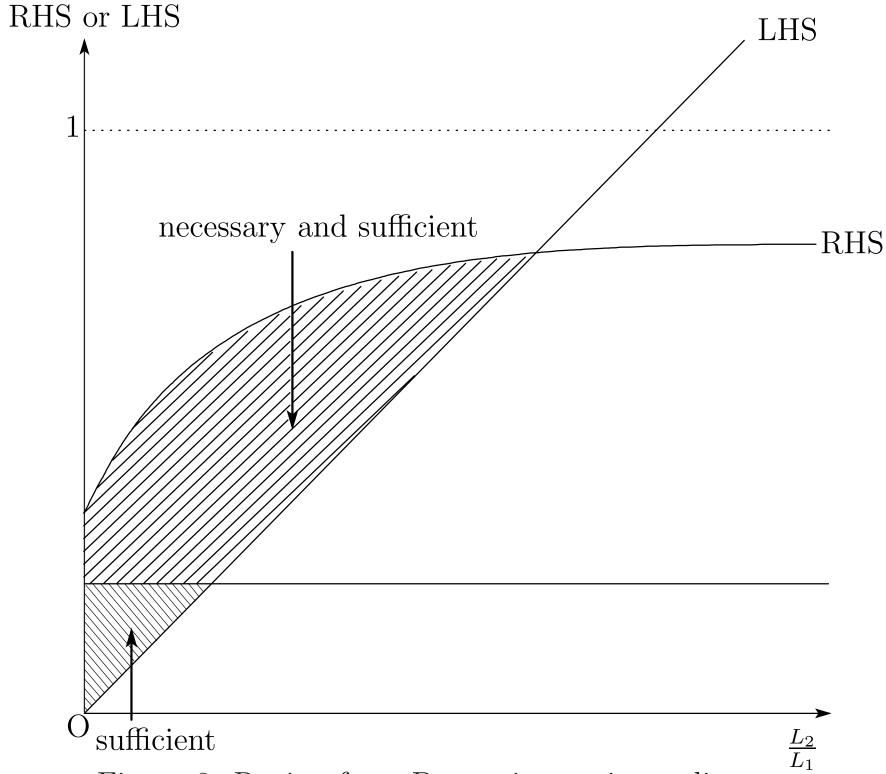

Figure 3: Region for a Pareto improving policy

a. *when the disparity of the labor population between regions (L_1/L_2) is greater.*

b. *when the tax rate is closer to the growth-maximizing tax rate.*

Proof. The proof of item a. is shown in Figure 3, as a result of the numerical analysis.[18] The proof of item b. is that the growth-maximizing tax rate is given by $\partial\gamma^*/\partial\tau = 0 \Leftrightarrow \partial(1-\tau)r^*/\partial\tau = 0$. In other words, when $\tau = \tau^*$, the right hand side of the equation (33) is the maximum, where τ^* is the growth-maximizing tax rate. $\square$

Let us next consider a more generalized case, $\sigma \neq 1$, in which the utility function takes the CRRA form. The welfare functions in each region are given by:

[18] We assumed that $L_1 > L_2$. Hence higher disparity of a labor population corresponds to a lower L_2/L_1.

$$
\begin{aligned}
W_i &= \int_0^\infty \frac{c_i(t)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt, \\
&= \frac{c_i(0)^{1-\sigma}}{(1-\sigma)[\rho - (1-\sigma)\gamma^*]}. \qquad (35)
\end{aligned}
$$

Partially differentiating (35) with respect to β, we obtain

$$
\begin{aligned}
\frac{dc_1(0)}{d\beta} &= \psi \left\{ \left(1 - \frac{1}{\sigma}\right) a_1(0)(L_1 - L_2)B + \frac{1-\alpha}{\alpha} A(0) \left[(1-\alpha)\beta(L_1 - L_2) + \alpha L_2\right] \right\}, \\
\frac{dc_2(0)}{d\beta} &= \psi \left\{ \left(1 - \frac{1}{\sigma}\right) a_2(0)(L_1 - L_2)B + \frac{1-\alpha}{\alpha} A(0) \left[(1-\alpha)(1-\beta)(L_1 - L_2) - \alpha L_1\right] \right\}.
\end{aligned}
$$

There is a slight difference from the log-linear utility function case in that the other effect, the first term on the right hand side, exists. We call this effect *the consumption propensity effect* which has the same sign as the economy-wide growth effect. Hence, the possibility of a Pareto improving policy is greater than in the log-linear utility function case.

5 The model with lump-sum transfers

In this section, preserving the basic structure of the model, we consider a simpler distribution form, which is a lump-sum income transfer. Suppose that the transfer T of the aggregate amount is done from region 2 to region 1. As is well known, the subjective equilibrium condition is not affected by the lump-sum transfers. The aggregate budget constraints of the household in each region change as follows:

$$
\begin{aligned}
\dot{A}_1(t) &= (1-\tau)(r^* A_1(t) + w_1(t)L_1) - c_1(t)L_1 + T, \\
\dot{A}_2(t) &= (1-\tau)(r^* A_2(t) + w_2(t)L_2) - c_2(t)L_2 - T.
\end{aligned}
$$

If we solve the ordinary differential equation for $c_i(0)$, we can derive

$$
\begin{aligned}
c_1(0) &= -\phi \left[a_1(0) + \frac{T}{(1-\tau)r^* L_1} \right] + \frac{1-\alpha}{\alpha} \frac{1}{1+x^*} \frac{1}{L_1} (1-\tau) r^* A(0), \\
c_2(0) &= -\phi \left[a_2(0) - \frac{T}{(1-\tau)r^* L_2} \right] + \frac{1-\alpha}{\alpha} \frac{x^*}{1+x^*} \frac{1}{L_2} (1-\tau) r^* A(0).
\end{aligned}
$$

The initial consumption in each region depends on initial assets, the amount of the lump-sum transfer, and the lifetime wage. Then, we examine the effects on welfare of changes in the lump-sum transfer. For simplicity, assume that $\sigma = 1$, and the following equation is obtained:

$$\frac{dW_i}{dT} = \frac{1}{\rho}\frac{1}{c_i(0)}\frac{dc_i(0)}{dT} + \frac{1}{\rho^2}\frac{d\gamma^*}{dT}. \tag{36}$$

From the equation (36) we can state the following proposition.

Proposition 4. *There is no possibility of a Pareto improving policy in the case of a lump-sum income transfer.*

Proof. The effects on welfare are of two types, which are *the initial consumption effect* and *the economy-wide growth effect*. The latter effect is zero, hence $d\gamma^*/dT = 0$, since the effect of the lump-sum transfer on the steady-growth rate is neutral. Let us next examine its effect on the composition of initial consumption. Although it is easily seen that there is no wage growth effect and no productivity effect, a new effect of the lump-sum transfer emerges. Therefore, we have only to contemplate this effect. The equation (36) changes into

$$\frac{dW_1(0)}{dT} = -\frac{\phi}{(1-\tau)r^* L_1} > 0, \tag{37}$$

$$\frac{dW_2(0)}{dT} = \frac{\phi}{(1-\tau)r^* L_2} < 0. \tag{38}$$

From equations (37) and (38), we have shown proposition. □

6 Multi-regional extension

This model can easily be extended to a multi-region model. Consider an economy with n regions: $i = 1, \cdots, n$. We assume that β_i denotes a share of government expenditure to region i, then the budget constraints of each government are changed as follows:

$$G = \tau \sum_{i=1}^{n} Y_i, \tag{39}$$

$$G_i = \beta_i G, \tag{40}$$

where $\sum_{i=1}^n \beta_i = 1$. Equations (39) and (40) respectively represent the budget constraints of the central government and the local government in region i. The profit maximization of firms and the utility maximization of households remain basically unchanged, and each market equilibrium condition is given by

$$\sum_{i=1}^n A_i = \sum_{i=1}^n K_i, \tag{41}$$

$$\sum_{i=1}^n Y_i = \sum_{i=1}^n C_i + \sum_{i=1}^n \dot{K}_i + G, \tag{42}$$

where (41) and (42) denote the integrated asset market and goods market, respectively.

Let us next define the new variable s_i which represents the GDP share $(Y_i/\sum_{k=1}^n Y_k)$ in region i, then from (40) the quantity of capital per unit of effective labor in region i is as follows:

$$k_i = f^{-1}\left(\frac{s_i}{\tau\beta_i L_i}\right).$$

From the no-arbitrage condition, $k_i = k_j$ holds for all $i,\ j,\ i \neq j$. Hence, the variables in equilibrium are given by

$$\begin{aligned} s_i^* &= \frac{\beta_i L_i}{\sum_{k=1}^n \beta_k L_k}, \\ k^* &= f^{-1}\left(\frac{1}{\tau\sum_{k=1}^n \beta_k L_k}\right), \\ r^* &= f'(k^*), \\ \omega^* &= f(k^*)/k^* - f'(k^*). \end{aligned}$$

As in the two-region model, we turn to examine the comparative statics. But in advance, we must determine the rule of the redistribution policy.

Assumption 1. *The increase in the share of region i on a minute scale equals the decrease in that of other regions, that is, $\Delta\beta_i = -\sum_{j\neq i}^n \Delta\beta_j$. For simplicity, we assume that the decrease in the share is the same amount in all regions except for region i, hence $\Delta\beta_j = \Delta\bar{\beta}$ for all $j \neq i$.*

Owing to the above assumption, we can derive the results on the comparative statics with respect to β_i as follows:[19]

$$\frac{d\gamma^*}{d\beta_i} = -\frac{1}{\sigma}(1-\tau)\frac{f''(k^*)}{f'(k^*)}\frac{1}{\tau(\sum_{k=1}^{n}\beta_k L_k)^2}\left(L_i - \frac{1}{n-1}\sum_{j\neq i}^{n} L_j\right), \quad (43)$$

$$\frac{ds_i^*}{d\beta_i} = \frac{1}{\sum_{k=1}^{n}\beta_k L_k}\left((1-s_i^*)L_i + \frac{s_i}{n-1}\sum_{j\neq i}^{n} L_j\right) > 0, \quad (44)$$

$$\frac{ds_j^*}{d\beta_i} = \frac{1}{\sum_{k=1}^{n}\beta_k L_k}\left(-\frac{L_j}{n-1} - s_j^*\left(L_i - \frac{1}{n-1}\sum_{j\neq i}^{n} L_j\right)\right) < 0. \quad (45)$$

Thus, we have the following proposition.

Proposition 5. *The steady-growth rate can be enhanced if and only if the share of government expenditure to the region which is more populous than the average for the remaining regions is increased.*

Proof. According to the equation (43), $d\gamma^*/d\beta_i > 0 \Leftrightarrow L_i > \sum_{i\neq j} L_j/(n-1)$. □

The equation (43) represents a multi-region case of the equation (22), and the equation (44) and (45) mean that an increase in the distribution rate of the region i leads to an increase (decrease) in the GDP share of the region i (region j).

As in the two-region model, we specify the production function of firms as the Cobb-Douglas.[20] Let us next derive the initial consumption by using the asset market clearing condition. The same operation in the two-region model leads to the following equation.

$$c_i(0) = -\phi a_i(0) + \frac{1-\alpha}{\alpha}\frac{1}{L_i}(1-\tau)r^* s_i^* A(0), \quad (46)$$

which holds for all i. Thereafter we differentiate (46) with respect to β_i, in other words, we examine the effects of an increase in the share of region i on the welfare within the region and in the other regions. Suppose again that $\sigma = 1$, then we obtain

$$\frac{dW_i}{d\beta_i} = \frac{1}{\rho}\frac{1}{c_i(0)}\frac{dc_i(0)}{d\beta_i} + \frac{1}{\rho^2}\frac{d\gamma^*}{d\beta_i}. \quad (47)$$

[19]The derivation of equations (43), (44) and (45) is in the appendix.
[20]Each variable in equilibrium is given in the appendix.

We have already investigated the economy-wide growth effect ($d\gamma^*/d\beta_i$) in the above comparative statics. Therefore we have only to consider the two initial consumption effects. ($dc_i(0)/d\beta_i$ and $dc_j(0)/d\beta_i$, $j \neq i$) They can be rewritten as:

$$
\begin{aligned}
\frac{dc_i(0)}{d\beta_i} &= \frac{1-\alpha}{\alpha}\Psi\beta_i\zeta^{\frac{1-3\alpha}{\alpha}}\left(\alpha\left[\left(\frac{1-s_i^*}{s_i^*}\right)L_i+\bar{L}\right]+(1-\alpha)(L_i-\bar{L})\right) \\
\frac{dc_j(0)}{d\beta_i} &= \frac{1-\alpha}{\alpha}\Psi\beta_j\zeta^{\frac{1-3\alpha}{\alpha}}\left(-\alpha\left[(L_i-\bar{L})+\frac{1}{n-1}\frac{\zeta}{\beta_j}\right]+(1-\alpha)(L_i-\bar{L})\right)
\end{aligned}
$$

where $\Psi \equiv (1-\tau)\tau^{\frac{1-\alpha}{\alpha}}A(0)$, $\zeta \equiv \sum_{k=1}^{n}\beta_k L_k$, $\bar{L} \equiv \frac{1}{n-1}\sum_{j\neq i}^{n} L_j$, respectively. Thus we can find the following proposition similar to the two-region model.

Proposition 6. *In the multi-region model, the policy that increases the share of government expenditure to region i may be Pareto improving if region i is more populous than the average of the other regions.*

Proof. We follow almost the same procedure as in the two-region model.[21] By assuming that region i is more populous than the average, a rise in β_i enhances the economy-wide growth rate, that is $d\gamma^*/d\beta_i > 0$. Next, the two effects on the initial consumption in region i have the same sign (both positive), hence $dW_i/d\beta_i > 0$. On the other hand, those in region j have the opposite sign, that is, the wage growth effect is positive and the productivity effect is negative. Hence, the sign of $dW_j/d\beta_i$ is ambiguous because of this trade-off. The necessary and sufficient condition for the Pareto improving policy ($dW_j/d\beta_i \geq 0$) is as follows:

$$
\frac{\bar{L}}{L_i} \leq 1 - \frac{\alpha}{[(n-1)(1-2\alpha)+D]}\frac{L_j}{s_j},
$$

where

$$
D \equiv \frac{a_j(0)}{A(0)}\frac{L_j}{s_j} + \frac{1}{\rho}\frac{1-\alpha}{\alpha}(1-\tau)r^*.
$$

Then it is useful for us to investigate the sufficient condition for the Pareto improving policy, that is, $dc_j(0)/d\beta_i \geq 0$ if and only if

[21] We pay attention to analyzing the Pareto improving policy in the multi-region model because we must check welfare in all regions. However, supposing that all regions are symmetric except for the labor population and GDP share, it is a sufficient condition of the possibility for the Pareto improving policy to assume that the region j is the least populous and has the biggest GDP share, which is $L_j = \min\{L_1, L_2, \cdots, L_n\}$ and $s_j = \max\{s_1, s_2, \cdots, s_n\}$.

$$\frac{\bar{L}}{L_i} \leq 1 - \frac{\alpha}{(n-1)(1-2\alpha)} \frac{1}{s_j} \frac{L_j}{L_i}.$$

$\square$

7 Concluding remarks

In this paper, we have constructed a two-region endogenous growth model with government expenditure, and theoretically explored the relationship between the regional redistribution policy of public input and the welfare of the residents in each region. We have shown that there is a possibility of Pareto improvement if the policy is for redistribution from the less populous region to the more populous region because it raises the equilibrium growth rate. Furthermore, this paper has extended a two-region model to a multi-region one, and has been successful in providing an analytical solution. This extension is highly fruitful and rewarding when we make policy recommendations.

In spite of our contributions, the possibility of further extension remains for future research. First, we have assumed no congestion costs of government expenditure. If these are taken into consideration, our results may be modified. Second, we have taken the regional share of government expenditure as given. It is much more fruitful to incorporate a framework of political decisions into our model.

A Appendix

A.1 Derivation of (26)

We solve the ordinary differential equation with respect to $A_i(t)$.

$$e^{-(1-\tau)r^*t}[\dot{A}_i(t) - (1-\tau)r^*A_i(t)] = e^{-(1-\tau)r^*t}[(1-\tau)w_i(t)L_i - c_i(t)L_i] \tag{A.1}$$

Integrating both sides within $[0, \infty)$, we derive

$$\left[e^{-(1-\tau)r^*t}A_i(t)\right]_0^\infty = \int_0^\infty e^{-(1-\tau)r^*t}[(1-\tau)w_i(t)L_i - c_i(t)L_i]dt \tag{A.2}$$

$$\lim_{t\to\infty} e^{-(1-\tau)r^*t}A_i(t) - A_i(0) = \int_0^\infty e^{-(1-\tau)r^*t}[(1-\tau)w_i(t)L_i - c_i(t)L_i]dt \tag{A.3}$$

The first term on the left hand side is 0 from the transversality condition. Then, using the Euler equation, we find $c_i(t) = c_i(0)e^{\gamma^* t}$. Substituting this into (A.3), we can get

$$c_i(0)\int_0^\infty e^{[\gamma^*-(1-\tau)r^*]t}dt = a_i(0) + (1-\tau)\int_0^\infty e^{-(1-\tau)r^* t}w_i(t)dt \tag{A.4}$$

Let us now define the new variable ϕ as follows:

$$\begin{aligned}
\phi &\equiv \gamma^* - (1-\tau)r^* && \text{(A.5)}\\
&= \frac{1}{\sigma}\left[(1-\tau)r^* - \rho\right] - (1-\tau)r^* && \text{(A.6)}\\
&= \left(\frac{1}{\sigma} - 1\right)(1-\tau)r^* - \frac{\rho}{\sigma} < 0 && \text{(A.7)}
\end{aligned}$$

Then $c_i(0)$ can be rewritten by

$$c_i(0) = -\phi\left[a_i(0) + (1-\tau)\int_0^\infty w_i(t)e^{-(1-\tau)r^* t}dt\right] \tag{A.8}$$

A.2 Derivation of (27) and (28)

Because of no transitional dynamics, the second term in the right hand side ($w_i(t)$) of equation (26) is

$$w_i(t) = \omega^* \frac{K_i(t)}{L_i}. \tag{A.9}$$

Using the asset market equilibrium condition (12), the equation (A.9) can be rewritten as

$$K_1(t) \;=\; \frac{1}{1+x^*}A(0)e^{\gamma^* t}.$$

By the same operation, the condition in region 2 is given by

$$K_2(t) \;=\; \frac{1+x^*}{x^*}A(0)e^{\gamma^* t}.$$

Thus, we can derive the equilibrium wage as follows:

$$w_1(t) = \frac{1}{1+x^*}\frac{1}{L_1}\omega^* A(0)e^{\gamma^* t}, \tag{A.10}$$

$$w_2(t) = \frac{x^*}{1+x^*}\frac{1}{L_2}\omega^* A(0)e^{\gamma^* t}. \tag{A.11}$$

Substituting equations (A.10) and (A.11) into the initial consumption (26), we obtain (27) and (28).

A.3 Cobb-Douglas specification (two-region model)

$$\begin{aligned}
x^* &= \frac{(1-\beta)L_2}{\beta L_1} \\
r^* &= \alpha\tau^{\frac{1-\alpha}{\alpha}}\left[\beta L_1 + (1-\beta)L_2\right]^{\frac{1-\alpha}{\alpha}} \\
\gamma^* &= \frac{1}{\sigma}\left((1-\tau)\alpha\tau^{\frac{1-\alpha}{\alpha}}\left[\beta L_1 + (1-\beta)L_2\right]^{\frac{1-\alpha}{\alpha}} - \rho\right) \\
w_i(t) &= \frac{1-\alpha}{\alpha}\frac{K_i(t)}{L_i}r^* \\
c_1(0) &= \left(\frac{\rho}{\sigma} + \left(1-\frac{1}{\sigma}\right)(1-\tau)r^*\right)a_1(0) + \frac{1-\alpha}{\alpha}\frac{1}{1+x^*}\frac{1}{L_1}(1-\tau)r^* A(0) \\
c_2(0) &= \left(\frac{\rho}{\sigma} + \left(1-\frac{1}{\sigma}\right)(1-\tau)r^*\right)a_2(0) + \frac{1-\alpha}{\alpha}\frac{x^*}{1+x^*}\frac{1}{L_2}(1-\tau)r^* A(0)
\end{aligned}$$

A.4 Cobb-Douglas specification (multi-region model)

$$\begin{aligned}
s_i^* &= \frac{\beta_i L_i}{\sum_{k=1}^n \beta_k L_k} \\
r^* &= \alpha\left(\tau\sum_{k=1}^n \beta_k L_k\right)^{\frac{1-\alpha}{\alpha}} \\
\gamma^* &= \frac{1}{\sigma}\left((1-\tau)\alpha\left(\tau\sum_{k=1}^n \beta_k L_k\right)^{\frac{1-\alpha}{\alpha}} - \rho\right) \\
w_i(t) &= \frac{1-\alpha}{\alpha}\frac{K_i(t)}{L_i}r^* \\
c_i(0) &= \left[\frac{\rho}{\sigma} + \left(1-\frac{1}{\sigma}\right)(1-\tau)r^*\right]a_i(0) + \frac{1-\alpha}{\alpha}\frac{1}{L_i}(1-\tau)r^* s_i^* A(0)
\end{aligned}$$

A.5 Derivation of (43), (44) and (45)

Because of the Euler equation the steady-growth rate is given by $\gamma^* = 1/\sigma((1-\tau)r^* - \rho)$. Then the comparative statics with respect to β_i is

$$\frac{d\gamma^*}{d\beta_i} = \frac{1}{\sigma}(1-\tau)\frac{dr^*}{d\beta_i}. \tag{A.12}$$

According to the equations (43) and (43), the equation (A.12) changes into

$$\frac{d\gamma^*}{d\beta_i} = -\frac{1}{\sigma}(1-\tau)\frac{f''(k^*)}{f'(k^*)}\frac{\tau}{(\tau\sum_{k=1}^{n}\beta_k L_k)^2}\frac{d(\sum_{k=1}^{n}\beta_k L_k)}{d\beta_i}. \tag{A.13}$$

By using assumption 1, $d(\sum_{k=1}^{n}\beta_k L_k)/d\beta_i$ in the equation (A.13) is rewritten as:

$$\begin{aligned} d\left(\sum_{k=1}^{n}\beta_k L_k\right) &= L_i d\beta_i + \sum_{j\neq i} L_j d\bar{\beta}, \\ &= \left(L_i - \frac{1}{n-1}\sum_{j\neq i} L_j\right) d\beta_i. \end{aligned} \tag{A.14}$$

Hence we can derive the equation (43) by substituting (A.14) into (A.13). [22]

Let us next derive the equation (44). Totally differentiating the equation (43), we obtain:

$$\frac{ds_i^*}{d\beta_i} = \frac{1}{\sum_{k=1}^{n}\beta_k L_k}\left(L_i - s_i^*\frac{d(\sum_{k=1}^{n}\beta_k L_k)}{d\beta_i}\right) \tag{A.15}$$

Substituting (A.14) into (A.15), we then derive the equation (44). We can obtain the equation (45) in a similar way.

[22]To derive (A.14) we use the condition $\sum_k \beta_k = 1 \Leftrightarrow \sum_k d\beta_k = 0$.

A.6 Numerical examples

This appendix shows the relationship between the possibility of a Pareto improving policy and the labor population ratio L_2/L_1 in a two-region model with numerical calculations. That is to say, we investigate the equation (33). As mentioned above, on the right hand side, although $d\Phi(L_2/L_1)/d(L_2/L_1)$ is ambiguous, Φ almost always increases in L_2/L_1 .

The specification of parameters is as follows:

$$\alpha = 0.4,\ \rho = 0.01,\ a_1(0) = 2,\ a_2(0) = 1,\ \tau = 0.5 \ \text{ in figure 4-6}$$
$$\alpha = 0.4,\ \rho = 0.01,\ a_1(0) = 2,\ a_2(0) = 1,\ \beta = 0.5 \ \text{ in figure 7-9}$$

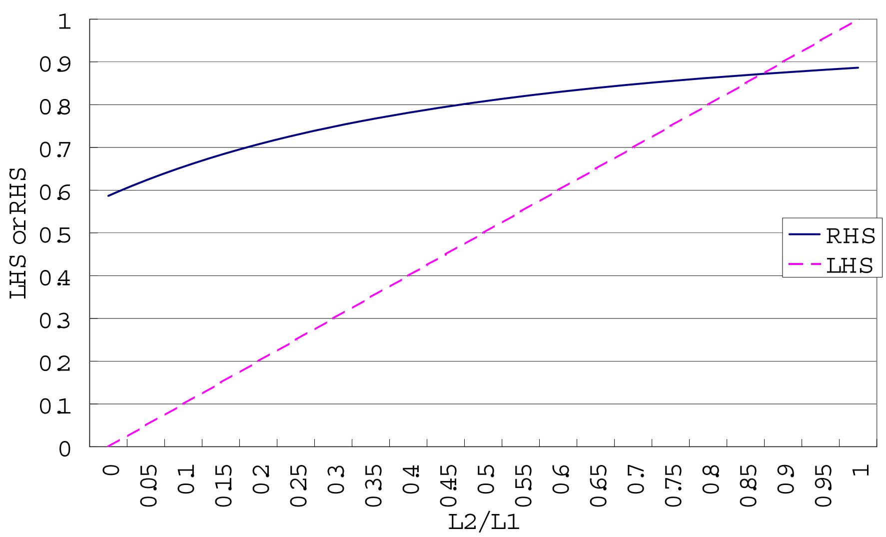

Figure 4: $\beta = 0.3$

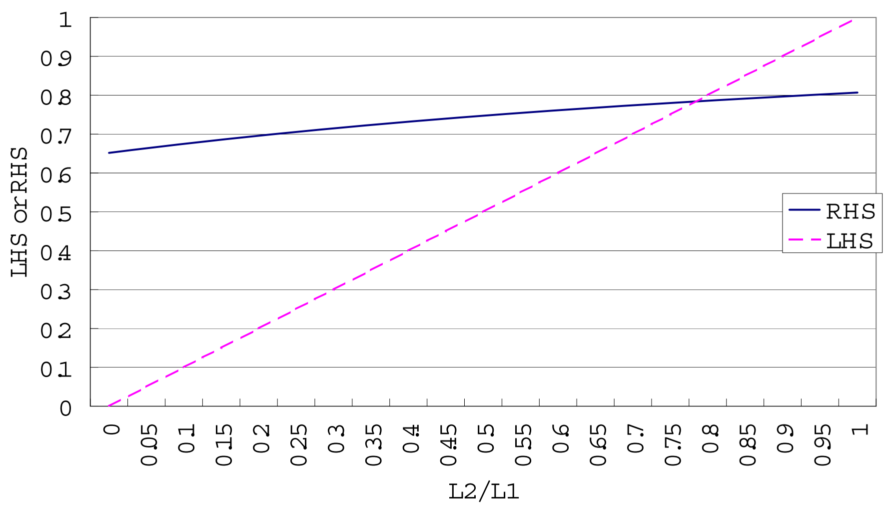

Figure 5: $\beta = 0.6$

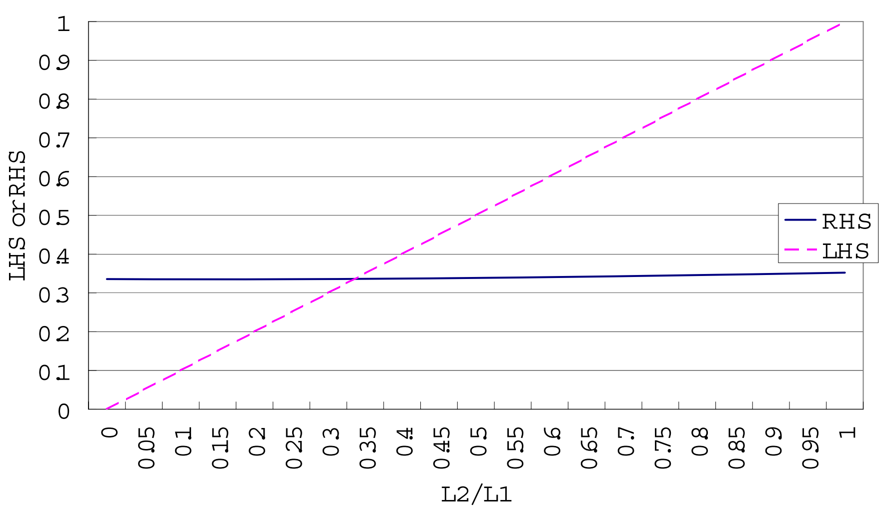

Figure 6: $\beta = 0.9$

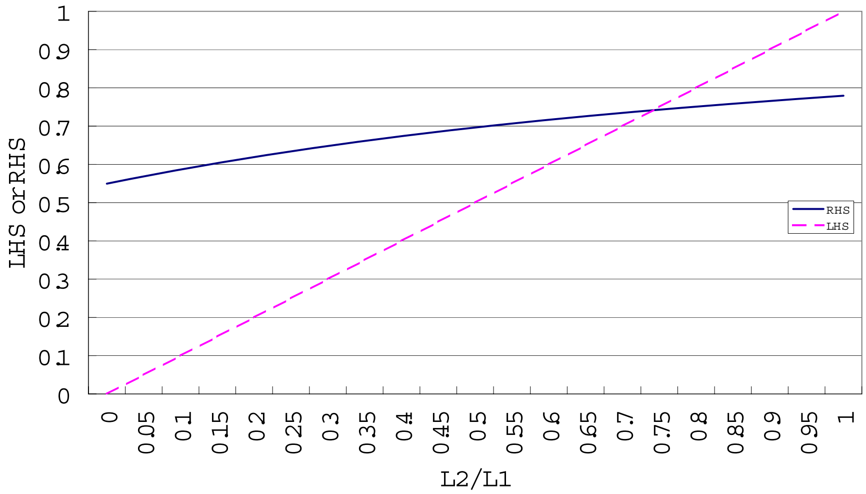

Figure 7: $\tau = 0.3$

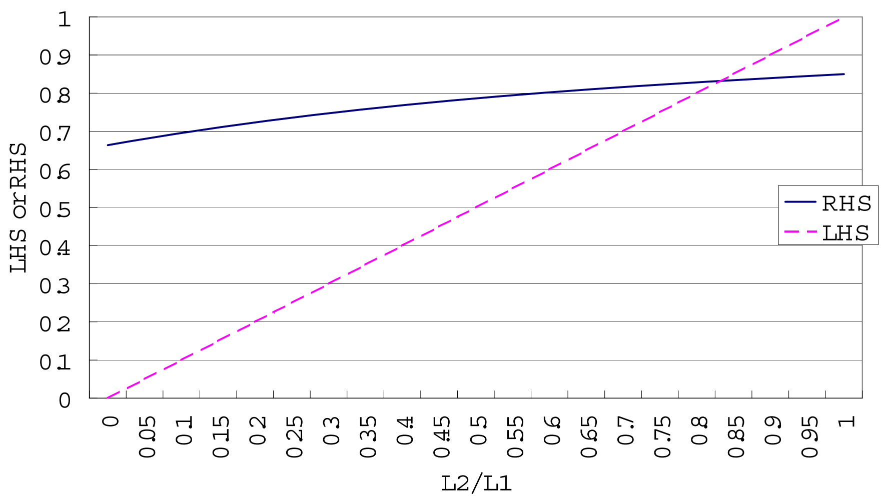

Figure 8: $\tau = 0.6$

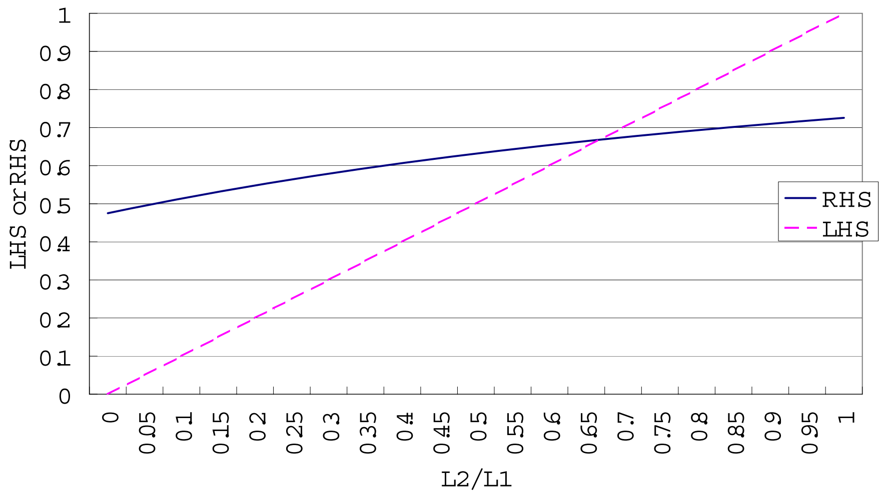

Figure 9: $\tau = 0.9$

References

[1] Atkinson and Stiglitz (1980) *Lectures on Public Economics*, McGraw Hill.

[2] Bhagwati, J.N., Brecher R.A. and Hatta, T. (1983) "The generalized theory of transfer and welfare: Bilateral transfers in a multilateral world", *American Economic Review, 83(3), 606-618.*

[3] Barro, R.J. (1990) "Government spending in a simple model of endogenous growth", *Journal of Political Economy 98, 103-125.*

[4] Barro, R.J. and X. Sala-i-Martin (1992) "Public Finance in Models of Economic Growth", *Review of Economic Studies 59, 645-661*

[5] Barro, R.J. and X. Sala-i-Martin (1995) *Economic Growth*, McGraw-Hill, New York.

[6] Davoodi, H. and H. Zou (1998) "Fiscal decentralization and economic growth: A cross-country study", *Journal of Urban Economics 43, 224-257.*

[7] Davoodi, H., D. Xie and H. Zou (1999) "Fiscal decentralization and economic growth in the United States", *Journal of Urban Economics 45, 228-239.*

[8] Devarajan, S., V. Swaroop and H. Zou (1996) "The composition of public expenditure and economic growth", *Journal of Monetary Economics 37, 313-344.*

[9] Futagami, K., Y. Morita and A. Shibata (1993) "Dynamic analysis of an endogenous growth model with public capital",*Scandinavian Journal of Economics 95, 607-625.*

[10] Glomm, G. and B. Ravikumar (1994) "Public Investment in Infrastructure in a Simple Growth Model",*Journal of Economics Dynamics and Control 18, 1173-1187.*

[11] Galor, O. and H. Ploemarchakis (1987) "Intertemporal equilibrium and transfer paradox", *Review of Economic Studies 54, 147-156.*

[12] Ihori, T. (1996) "International public goods and contribution productivity differentials", *Journal of Public Economics 61, 139-154.*

[13] Lejour, A.M. and H.A.A. Verbon (1997) "Tax competition and redistribution in a two-country endogenous-growth model", *Journal of Tax and Public Finance 4, 485-497.*

[14] Martin, P. (1999) "Public policies, regional inequalities and growth", *Journal of Public Economics 73, 85-105.*

[15] Martin, P. and G.I.P. Ottaviano (1999) "Growing locations: Industry location in a model of endogenous growth", *European Economic Review 43, 281-302.*

[16] Turnovsky, S.J. (1996) "Fiscal policy, adjustment costs, and endogenous growth", *Oxford Economic Papers 48, 361-381.*

[17] Yanagihara, M. (1998) "Public goods and the transfer paradox in an overlapping generations model", *Journal of International Trade and Economic Development 7:2, 175-205.*