# Discussion Papers In Economics And Business

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Discussion Paper 05-09

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# Wealth Heterogeneity and Escape from the Poverty-Environment Trap*

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**Summary.** A mutual link between poverty and environmental degradation is examined in an overlapping generations model with environmental externality, human capital, and credit constraints. Environmental quality affects labor productivity and thus wealth dynamics, whereas wealth distribution determines the degree to which agents rely upon natural resources and therefore the evolution of environmental quality. This interaction creates a 'poverty-environment trap,' where a deteriorated environment lowers income, which in turn accelerates environmental degradation. We show that greater wealth heterogeneity is the key to escaping the poverty-environment trap, although it has negative effects both on the environment and output when not in the trap.

**Keywords and Phrases:** Poverty trap, Environmental degradation, Wealth distribution, Human capital.

**JEL Classification Numbers:** O11, O13, O15.

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# 1 Introduction

During the last decade, a substantial number of both empirical and case studies have pointed out the mutual link between poverty and environmental degradation. Poverty forces people to rely heavily on natural resources for agriculture, fisheries, and timber production.[1] However, excessive use of natural resources, such as overgrazing, excessive fuelwood harvesting and unsustainable farming practice, is a primary cause of environmental degradation. As pointed out by Barbier (1997), poor rural households with limited access to capital tend to extract short-term rents from existing agricultural land, and such practices often result in land degradation, abandonment of existing land, and the expansion of agricultural activity into frontier forests.[2] Partly as a consequence of such exploitation, low-income countries lost about 73 million hectares (i.e., about 8 percent) of their forest in the 1990s while high-income countries reforested about 8 million hectares of forest in the same period (World Bank, 2004).

Environmental degradation, in turn, results in low productivity and makes poor households poorer. Deforestation is a major source of human-induced soil degradation in developing regions, accounting for around 40 percent of erosion in Asia and South America (Oldeman *et al.* 1990).[3] With reduction in land productivity, agricultural income of the poor falls and the risk of undernourishment rises. Moreover, poor households living in a degraded environment are confronted with a high possibility of water-related diseases from drinking untreated water (World Bank, 2004). In this way, a degraded environment lowers agricultural and other incomes of the

[1] Cavendish (2000) empirically analyzes the poverty-environment relationship in Zimbabwe and shows that environmental resources make a significant contribution to average rural incomes, and that poorer households are more resource dependent than are the rich.

[2] Barbier and Burgess (1997) show that the demand for forest conversion is negatively correlated with income per capita. Fuelwood harvesting by the poor is also responsible for deforestation in Peru (Swinton and Quiroz, 2003).

[3] Deforestation also makes fuelwood collection by the poor more time-consuming (Dasgupta, 1998).

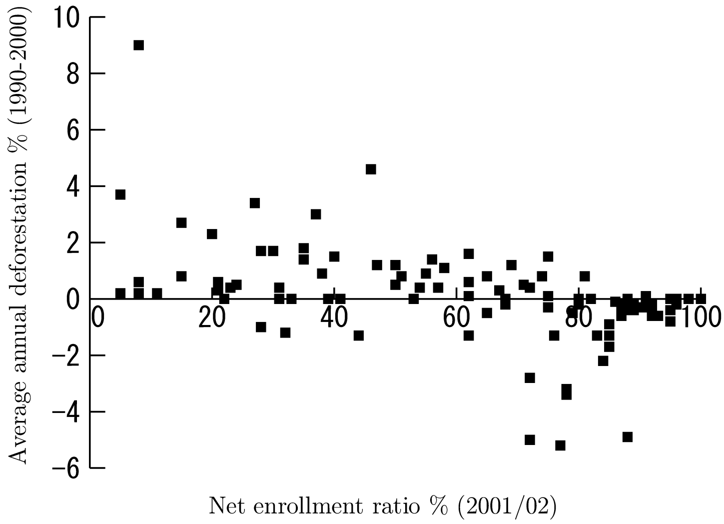


Figure 1: Education and Deforestation. The horizontal axis shows the average annual deforestation rate between 1990 and 2000; the vertical axis shows the net enrollment ratio of primary school in the 2001/02 period. *Data source: World Bank (2004).*

poor, which forces them to rely on natural resources more than ever, furthering the process of environmental degradation and increasing poverty.[4] This mutual link between poverty and environmental degradation creates a 'poverty-environment' trap, from which the poor find it hard to emerge.

This paper presents a model that formalizes this mutual link in an overlapping generations setting with environmental externality, human capital accumulation, and credit constraints. While the poverty-environment trap has been described largely in verbal terms in the literature, our model clarifies its properties by showing the existence of two stable steady states. In particular, one steady state is characterized by a combination of a high level of wealth, a high level of human capital investment and a good environment, whereas the other is characterized by poverty, little or no human capital investment, and a degraded environment. Once the economy settles into the latter steady state, it cannot escape from the poverty-environment trap.

A key specification of our model is that technologies that intensively use human

[4]This 'feedback loop' has been pointed out by Duraiappah (1998) and Borghesi and Vercelli (2003).

capital are less reliant on natural resources and therefore have smaller impacts on the environment than primitive technologies. A number of pieces of evidence are consistent with this specification. First, an international comparison among 126 developed and developing countries shows a clear negative relationship between the net enrollment ratio of primary school and the annual deforestation rate (see Figure 1). Second, Swinton and Quiroz (2003) show that human capital significantly promotes the use of sustainable agricultural practices (e.g., fallowing) and reduces the likelihood of tree felling. Torras and Boyce (1998) also report that, in low-income countries, higher literacy rates improve environmental quality. Since only wealthy households can afford education when credit markets are imperfect, these environmental effects of human capital imply that higher average wealth improves the environment, or, equivalently, that poverty causes environmental degradation.[5]

To complete the mechanics of the poverty-environment trap, the model also incorporates the fact that agents' capacity to work depends on environmental quality. Combined with the environmental effects of human capital, the productivity effect of the environment creates intergenerational feedback loops. With a degraded environment, agents' low productivity enables them to leave only small amounts of transfer for their children, who therefore have to rely on technologies that deteriorate the environment even more, falling into the poverty-environment trap. Conversely, when the environment is favorable, agents receive sufficient amounts of assets from their parents to invest in human capital out of received assets, improving the environment for future generations. We show that the fate of an economy depends on both the initial level of environmental quality and the initial distribution of wealth.

Poverty and the quality of the environment are often considered to be associated with inequality. Using the model explained above, we examine the dynamic consequences of distributional policies in both of the two steady states. Following Loury

[5]Bahamondes (2003) finds this causation in a panel data study examining how asset levels affect the choice of agricultural practices and how those practices affected natural resource status in arid central Chile. He concludes that the impressive recovery of a fragile natural resource is based on rising incomes.

(1981) and Owen and Weil (1998), we introduce heterogeneity in individuals' ability so that the distribution of wealth evolves over successive generations. It is shown that the effect of inequality on environment and development is completely opposite for the two steady states. If wealth inequality widens in the poverty trap, some wealthy households start or increase investment in human capital whereas poor households do not invest at all *ab initio*. The aggregate amount of human capital investment thus increases, improving the quality of the environment in the economy. The improved environment raises everyone's productivity and income, and the wealth of households over generations. It encourages poorer households to invest in human capital, further improving the quality of the environment.[6] Under a certain condition, this virtuous spiral allows the economy to escape from the poverty trap to a better steady state. It can be viewed as a particular case of the more general and abstract model of Galor and Tsiddon (1997), who demonstrate that inequality of human capital distribution may serve as a vehicle in the development of less-developed economies.

In the better steady state, conversely, our model shows that an appropriate level of redistribution is beneficial both for aggregate output and for the environment: a narrower wealth distribution lets poor households rely less on natural resources whereas the rich are always fully investing in human capital. Those contrasting results in the two steady states are consistent with an empirical finding by Barro (2000) that the relationship between inequality and growth is positive for low-income countries whereas it is negative for high-income countries.

In the literature of poverty traps, a number of theoretical studies examine the role of wealth distribution on economic development. In those studies, multiple steady states occur because of imperfections of credit markets combined with some nonconvexities in the model. Nonconvexities are usually introduced in terms of a discrete choice being faced by agents between being uneducated or educated (Galor and Zeira, 1993; Owen and Weil, 1998); a discrete choice between exerting a fixed amount of

[6]This process is similar to the trickle-down mechanism of Aghion and Bolton (1997). They consider the trickle-down process in terms of physical capital and interest rate, whereas ours is driven by human capital and the quality of the environment.

effort or not (Piketty, 1997); or a discrete choice between occupations (Banerjee and Newman, 1993). A high-yielding choice typically requires agents to possess at least a certain threshold level of wealth. Thus, agents below the threshold cannot escape from the poverty trap where a low-yielding choice is reinforced in subsequent periods. Our model differs from these in allowing agents to choose arbitrary amounts of investment in human capital. Rather, the obvious fact that the amount of human capital must be nonnegative generates a convex correspondence between wealth and investment.[7] Agents with assets lower than a certain threshold level choose not to invest at all since consumption is their urgent priority, while those with larger assets invest according to their wealth levels. Thus, from Jensen's inequality, a mean-preserving spread in the distribution of wealth clearly increases the average level of investment and hence promotes economic development, whereas the growth effect of wealth heterogeneity is less obvious in other models where nonconvexities arise from a discrete choice.[8]

The remainder of this paper is organized as follows. Section 2 presents the model and derives the interrelated evolutions of the distribution of wealth and environmental quality. As a benchmark, Section 3 considers a representative agent economy and demonstrates the existence of multiple steady states, one of which is characterized as a poverty-environment trap. Heterogeneity across agents is introduced in Section 4, where the effects of a redistributive policy on the environment and wealth distribution are analyzed. Section 5 concludes.

# 2 The Model

We consider a discrete time model of an overlapping generations economy, where each individual lives for two consecutive periods. Agents in their first and second periods

[7]Similarly, Moav (2002) obtains a convex bequest function by assuming that agents cannot leave negative bequests.

[8]In fact, Galor and Zeira (1993) and Banerjee and Newman (1993) conclude that equality in the initial distribution of wealth is important for economic growth.

are called young and adult individuals, respectively, and each adult is assumed to bear a single child (a young agent), implying that the total population is stationary. The size of each generation is normalized to unity, with each individual of generation $t$ being indexed by $j \in [0, 1]$ and $t$.

**Evolution of individuals' wealth over generations**

Individuals are endowed with a certain amount of labor for both periods of their lives. The amount of labor, or their ability to learn and work, depends on the quality of the environment at the time of their birth.[9] Let $Q_t \geq 0$ denote the quality of the environment in the economy, whose evolution is explained later. The labor endowment for each of an individual's periods is specified by[10]

$$\ell_{j,t} = Q_t \varepsilon_{j,t}. \tag{1}$$

In (1), $\varepsilon_j$ is a random variable with mean 1, distributed independently, identically and uniformly within $[1 - \sigma, 1 + \sigma]$, where $\sigma$ is a constant representing the degree of variations in ability. Inclusion of random variable $\varepsilon_j$ enables us to capture the heterogeneity of agents within an economy, as in Loury (1981).

From parents, young agent $j$ inherits $b_{j,t}$ units of goods, which are subject to an inheritance tax of rate $\tau \in [0, 1]$. The government revenue from the inheritance tax is distributed equally among young agents, so that an agent's total wealth at birth is

$$w_{j,t} = (1 - \tau) b_{j,t} + \int_0^1 \tau b_{j',t} dj'. \tag{2}$$

Those goods are divisible and nonstorable. A young individual uses a part of his/her wealth, $e_{j,t} \geq 0$, as an input for human capital investment. It is combined with

[9]As mentioned in the introduction, the risk of disease and malnutrition in developing countries increases with environmental degradation. Also in industrial countries, polluted air impairs the labor productivity of those with health problems such as asthma and bronchitis.

[10]We assume that an individual is endowed with the same amount of labor for each period, implying that his/her ability to work in adulthood depends on the quality of the environment in his/her youth. This assumption is not critical since the quality of the environment, $Q_t$, changes only gradually.

his/her endowed labor $\ell_{j,t}$ to produce

$$h_{j,t+1} = \gamma e_{j,t} \ell_{j,t} \tag{3}$$

units of human capital, where $\gamma > 0$ is a productivity parameter. As its subscript shows, this human capital $h_{j,t+1}$ becomes available in adulthood. We assume that credit markets are imperfect so that an individual can neither borrow nor lend.[11] It means that the amount of human capital investment is restricted by $e_{j,t} \in [0, w_{j,t}]$ and that the remainder is used for an agent's young period consumption

$$c^y_{j,t} = w_{j,t} - e_{j,t} \geq 0. \tag{4}$$

In the second period of life, the agent produces goods employing two kinds of technology. One is the primitive technology that produces goods from labor and natural resources according to

$$y^p_{j,t+1} = a^p \min\{r_{j,t+1}, \ell^p_{j,t+1}\}, \tag{5}$$

where constant $a^p > 0$ denotes the marginal productivity of labor and natural resources, $\ell^p_{j,t+1} \in [0, \ell_{j,t}]$ is the amount of labor used for the primitive technology out of his/her second period endowment $\ell_{j,t}$, and $r_{j,t+1} \geq 0$ is the amount of natural resources that he/she exploits without private costs.[12] In equilibrium, an individual uses exactly the same amount of $r_{j,t+1}$ as $\ell^p_{j,t+1}$ since the additional marginal product of natural resources is zero. Although each agent does not incur private costs, the aggregate usage of natural resources deteriorates the quality of the environment, as will be specified later.

The other production technology, which we call the sustainable technology, produces goods from labor and human capital according to

$$y^s_{j,t+1} = a^s \min\{h_{j,t+1}, \ell^s_{j,t+1}\}, \tag{6}$$

[11]The assumption that agents cannot save is made only for simplicity. It does not affect the equilibrium outcomes other than the consumption profile of individuals.

[12]The cost of exploitation can be interpreted as being included in $\ell^p_{j,t+1}$.

where constant $a^s > a^p$ denotes the marginal productivity of labor and human capital, $\ell^s_{j,t+1} \in [0, \ell_{j,t}]$ is the amount of labor used for the sustainable technology, and $h_{j,t+1}$ is the amount of human capital accumulated through (3). This production technology does not require natural resources, and in that sense is good for the environment.

Subject to the time constraint, $\ell^p_{j,t+1} + \ell^s_{j,t+1} = \ell_{j,t}$, each adult agent allocates labor so as to maximize the total output $y_{j,t+1} = y^p_{j,t+1} + y^s_{j,t+1}$. The latter can be written as $y_{j,t+1} = a^p(\ell_{j,t} - \ell^s_{j,t+1}) + a^s \min\{h_{j,t+1}, \ell^s_{j,t+1}\}$, since substituting $r_{j,t+1} = \ell^p_{j,t+1}$ into (5) shows that the output from the primitive technology is $a^p \ell^p_{j,t+1}$. Note that the assumption $a^s > a^p$ implies that the marginal product of labor is higher in the sustainable technology as long as there is a sufficient amount of human capital to be used. Thus, it is optimal to set $\ell^s_{j,t+1} = \min\{h_{j,t+1}, \ell_{j,t}\}$. Consequently, an agent's output is described as a function of human capital investment and labor endowment:

$$y(h_{j,t+1}, \ell_{j,t}) \equiv (a^s - a^p)\min\{h_{j,t+1}, \ell_{j,t}\} + a^p \ell_{j,t}. \tag{7}$$

An individual's objective throughout his/her life is given by a lifetime utility function,

$$U_{j,t} = \ln c^y_{j,t} + \alpha \ln c^a_{j,t+1} + \beta \ln b_{j,t+1}, \tag{8}$$

where $c^y_{j,t}$ and $c^a_{j,t+1}$ denote the amount of consumption in youth and adulthood, respectively, while $b_{j,t+1}$ represents the amount of transfer to offspring out of altruism.[13] An agent chooses the amounts of human capital investments $e_{j,t}$, transfer to his/her child $b_{j,t+1}$, and the agent's consumption schedule $c^y_{j,t}$, $c^a_{j,t+1}$ so as to maximize (8). The maximization is done subject to human capital production (3), the nonnegativity constraint on investment $e_{j,t} \geq 0$,[14] the first-period budget constraint (4), and

[13]The third term in (8), $\beta \ln b_{j,t+1}$, shows that we employ the joy-of-giving formulation of altruism. Alternatively, we can replace it with $\beta \ln(1-\tau) b_{j,t+1}$ so that the parents receive utility only from the amount of transfer net of the inheritance tax. This reformulation does not affect equilibrium in any way because $\beta \ln(1-\tau)b_{j,t+1} = \beta \ln b_{j,t+1} + \text{constant}$.

[14]Since the utility function satisfies the Inada conditions, the nonnegativity constraints on $c^y_{j,t}$ and $c^a_{j,t+1}$ never bind in the optimal solution and therefore can be omitted. We also omit the inequality constraint $e_{j,t} \leq w_{j,t}$ since it is equivalent to $c^y_{j,t} \geq 0$ from (4).

that of the second-period $c^a_{j,t+1} + b_{j,t+1} = y(h_{j,t+1}, \ell_{j,t})$, where function $y(h_{j,t+1}, \ell_{j,t})$ is defined by (7).

This problem can be solved using the Kuhn-Tucker method as detailed in Appendix A. The optimal amount of investments in human capital is found as a function of $w_{j,t}$.

$$e_{j,t} = e(w_{j,t}) \equiv \begin{cases} 0 & \text{if } w_{j,t} \leq \underline{w}, \\ (\alpha+\beta)(1+\alpha+\beta)^{-1}(w_{j,t} - \underline{w}) & \text{if } w_{j,t} \in (\underline{w}, \overline{w}), \\ 1/\gamma & \text{if } w_{j,t} \geq \overline{w}, \end{cases} \tag{9}$$

where $\underline{w} \equiv a^p((\alpha+\beta)\gamma(a^s - a^p))^{-1}$ and $\overline{w} \equiv a^s((\alpha+\beta)\gamma(a^s - a^p))^{-1} + 1/\gamma$. Investments in human capital increase with wealth because agents with insufficient wealth are liquidity constrained. Specifically, when $w_{j,t} \leq \underline{w}$ agents cannot afford to invest in human capital since consumption in youth is quite low and therefore is their first priority. Agents with $w_{j,t} \in (\underline{w}, \overline{w})$ are also liquidity constrained, but they can afford to invest in human capital, although less than the desirable amount $1/\gamma$.

The procedure in Appendix A also finds the optimal amount of transfer that is left for his/her offspring as a function of $w_{j,t}$ and $\ell_{j,t}$. In particular, it can be written as $b_{j,t+1} = b(w_{j,t})\ell_{j,t}$, where

$$b(w_{j,t}) \equiv \begin{cases} \beta(\alpha+\beta)^{-1}a^p & \text{if } w_{j,t} \leq \underline{w}, \\ \beta(1+\alpha+\beta)^{-1}\left(a^p + \gamma(a^s - a^p)w_{j,t}\right) & \text{if } w_{j,t} \in (\underline{w}, \overline{w}), \\ \beta(\alpha+\beta)^{-1}a^s & \text{if } w_{j,t} \geq \overline{w}. \end{cases} \tag{10}$$

The amount of transfer $b(w_{j,t})\ell_{j,t}$ depends on $w_{j,t}$ because initial wealth determines the amount of human capital investment in youth and hence an agent's income in adulthood. This creates an intergenerational linkage in wealth. The dynamics of wealth over generations are obtained by substituting this result into (2) and eliminating $\ell_{j,t}$ by (1):

$$w_{j,t+1} = Q_t \left( (1-\tau) b(w_{j,t})\varepsilon_{j,t} + \tau \int_0^1 b(w_{j',t}) dj' \right). \tag{11}$$

### Evolution of the quality of the environment

Both nature and the activity of individuals drive the evolution of the quality of the environment $Q_t$. The environment gradually deteriorates when people exploit natural resources to use as an input for their primitive technology. However, natural resources have the ability to recover their pristine abundance if exploitation ceases.[15] Specifically, the change in the quality of the environment over one generation is given by

$$\Delta Q_{t+1} \equiv Q_{t+1} - Q_t = \delta(\widetilde{Q} - Q_t) - \eta \int_0^1 r_{j,t+1} dj. \tag{12}$$

Parameter $\delta \in (0, 1)$ measures the speed with which the environment recovers towards pristine quality $\widetilde{Q}$. It is natural to assume that the initial level of the environment does not exceed the pristine level; i.e., $Q_0 \in (0, \widetilde{Q}]$. The last term of (12), $\eta \int_0^1 r_{j,t+1} dj$, shows that the quality of the environment deteriorates in proportion to the aggregate amount of exploitation, where parameter $\eta \in (0, 1)$ measures the magnitude of this effect.

Individual usage of natural resources $r_{j,t+1}$ can be found from preceding arguments. From the Leontief property of primitive technology (5), $r_{j,t+1}$ coincides with labor input $\ell^p_{j,t+1}$, which is from the time constraint equal to labor endowment $\ell_{j,t}$ less the amount used for the sustainable technology $\ell^s_{j,t+1}$. The labor input for the sustainable technology $\ell^s_{j,t+1}$ in turn coincides with the amount of human capital $h_{j,t}$, again from the Leontief property of (6). Human capital is produced through $h_{j,t} = \gamma e_{j,t} \ell_{j,t}$ as in (3), where $e_{j,t}$ and $\ell_{j,t}$ are respectively given by (1) and (9). In sum, we obtain

$$r_{j,t+1} = (1 - \gamma e(w_{j,t}))\, Q_t \varepsilon_{j,t}. \tag{13}$$

Recall that human capital investment function $e(w_{j,t})$ is increasing with the agent's initial wealth $w_{j,t}$, as shown by (9), due to credit market imperfections. Therefore, (13) describes the tendency of poorer households to rely more on natural resources. In particular, those with initial wealth $w_{j,t}$ below $\underline{w}$ do not invest in hu-

[15] This assumption is appropriate when we consider renewable kinds of resources, such as soil, forests, water and atmosphere.

man capital at all and produce goods entirely through the primitive technology that exploits natural resources, whereas those with $w_{j,t}$ above $\overline{w}$ fully invest in human capital and make use of the sustainable technology without harming the environment. Agents with $w_{j,t} \in (\underline{w}, \overline{w})$ also exploit some amount of natural resources, but not as much as poorer agents do. Interestingly, (13) also implies that, *ceteris paribus*, agents utilize natural resources more when $Q_t$ is larger since better environments make agents more active.

Aggregating (13) over agents and substituting the result into (12) gives the evolution of $Q_t$ over generations:

$$Q_{t+1} = \delta \widetilde{Q} + \left( 1 - \delta - \eta + \gamma \eta \int_0^1 e(w_{j,t}) dj \right) Q_t. \tag{14}$$

Equation (14) shows that the evolution of an environmental state is determined by the average level of human capital investments, which is derived from the distribution of wealth through the investment function (9). In the other direction, the evolution of the distribution of wealth (11) is dependent on environmental quality since the latter determines the ability of workers. Distribution of wealth and environmental quality are thus interdependent so that dynamics are simultaneously determined. The following section demonstrates that this interdependency gives rise to multiple steady states.

# 3 Homogeneous Economy

As a benchmark, this section examines the working of an economy when all individuals are identical in their abilities to work and in their initial wealth. Specifically, we assume that variation in ability, defined by $\sigma$, is zero so that agents' labor endowments are given by $\ell_{j,t} = Q_t$ for all $j$ (see equation 1). If both ability and wealth are the same across agents, they leave the same amount of wealth for their offspring. Thus, given that agents in the initial generation start from the same amount of wealth, $w_{j,t}$'s are the same among individuals for all subsequent periods, and therefore can simply be written as $w_t$. Since any redistributive policy (e.g., the inheritance

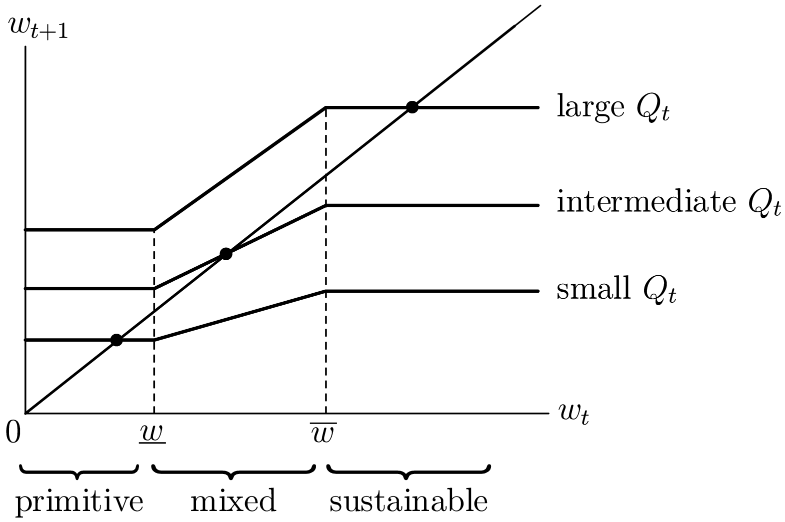


Figure 2: Dynamics of wealth of the representative individual. Parameters: $a^s = 0.5$, $a^p = 0.3$, $\alpha = 0.4$, $\beta = 0.6$, $\gamma = 6$, $\delta = 0.3$, $\eta = 0.9$, $\sigma = 0$. The values for $Q_t$ are 1 (small), 1.7 (intermediate), and 2.5 (large).

tax) has no effect in this case, the dynamics of wealth (11) simplifies to

$$w_{t+1} = b(w_t)Q_t. \tag{15}$$

Figure 2 depicts the evolution of the representative agent's wealth $w_t$ for given levels of $Q_t$'s. As shown by the figure, there is an intersection between (15) and the $w_{t+1} = w_t$ line. The point of intersection represents the fixed point of the dynamics, toward which $w_t$ converges over generations. For each $Q_t$, the fixed point is unique since the elasticity of $b(w_t)Q_t$ with respect to $w_t$ is always less than unity (see equation 10). A straightforward calculation gives the fixed point in terms of $Q_t$,

$$w^*(Q_t) \equiv \begin{cases} \beta(\alpha+\beta)^{-1}a^pQ_t \leq \underline{w} & \text{if } Q_t \leq \underline{Q}, \\ \beta(\alpha+\beta+1-Q_t/\underline{Q})^{-1}a^pQ_t \in (\underline{w}, \overline{w}) & \text{if } Q_t \in (\underline{Q}, \overline{Q}), \\ \beta(\alpha+\beta)^{-1}a^sQ_t \geq \overline{w} & \text{if } Q_t \geq \overline{Q}, \end{cases} \tag{16}$$

where $\underline{Q} \equiv (\beta\gamma(a^s - a^p))^{-1}$ and $\overline{Q} \equiv \underline{Q} + (\alpha+\beta)(\beta\gamma a^s)^{-1}$. A larger $Q_t$ shifts the whole schedule upward, and thus also raises $w^*(Q_t)$.

Even before analyzing the dynamics of $Q_t$, Figure 2 and equation (16) are already suggestive of the existence of multiple steady states. If the environment is sufficiently good ($Q_t > \overline{Q}$), the wealth of representative agent $w_t$ converges to a high level above $\overline{w}$ so that everyone uses the sustainable technology, maintaining a good environment. With a bad environment ($Q_t < \underline{Q}$), conversely, $w_t$ converges to a low level below $\underline{w}$ so that everyone uses the primitive technology, further deteriorating their environment. The latter situation corresponds to what we call the poverty-environment trap.

To obtain the precise nature of multiple steady states, we now examine the simultaneous movements of $w_t$ and $Q_t$. From equation (14), it is straightforward to show that $Q_{t+1} \gtreqless Q_t$ if and only if

$$Q_t \lesseqgtr Q^*(E_t) \equiv \frac{\delta \widetilde{Q}}{\delta + \eta(1-\gamma E_t)}, \tag{17}$$

where $E_t \equiv \int_0^1 e(w_{j,t})dj$ represents the aggregate amount of human capital investment. Since $E_t = e(w_t)$ in the representative agent setting, substituting (9) into (17) gives

$$Q^*(e(w_t)) = \begin{cases} \delta\widetilde{Q}/(\delta+\eta) & \text{if } w_t \leq \underline{w}, \\ \delta\widetilde{Q}/\left(\delta + \eta\big(\overline{w}-\underline{w}\big)^{-1}(\overline{w}-w)\right) & \text{if } w_t \in (\underline{w}, \overline{w}), \\ \widetilde{Q} & \text{if } w_t \geq \overline{w}. \end{cases} \tag{18}$$

Using (16) and (18), the evolution of wealth and environmental quality is described by the phase diagram in Figure 3, drawn in $(w, Q)$ space. The $w_{t+1} = w_t$ locus is given by $w_t = w^*(Q_t)$, whereas the $Q_{t+1} = Q_t$ locus is from $Q_t = Q^*(e(w_t))$. The figure is drawn by assuming that the pristine quality of the environment $\widetilde{Q}$ is within

$$\widetilde{Q} \in \Big(\overline{Q}, (1+\eta/\delta)\underline{Q}\Big), \tag{19}$$

for which case there exist three steady states.[16] Intuitively, multiple steady states emerge since the $Q_{t+1} = Q_t$ locus has an S-shape while the $w_{t+1} = w_t$ locus has an

[16]The interval in the RHS of (19) is nonempty when $\eta/\delta$ is large. Recall that parameters $\eta$ and $\delta$ represent the speed of the environment's recovery and that of deterioration due to exploitation, respectively. Therefore, multiple steady states occur only when the impacts of exploitation on the environment are considerably stronger than its ability to recover. We consider this case throughout the paper.

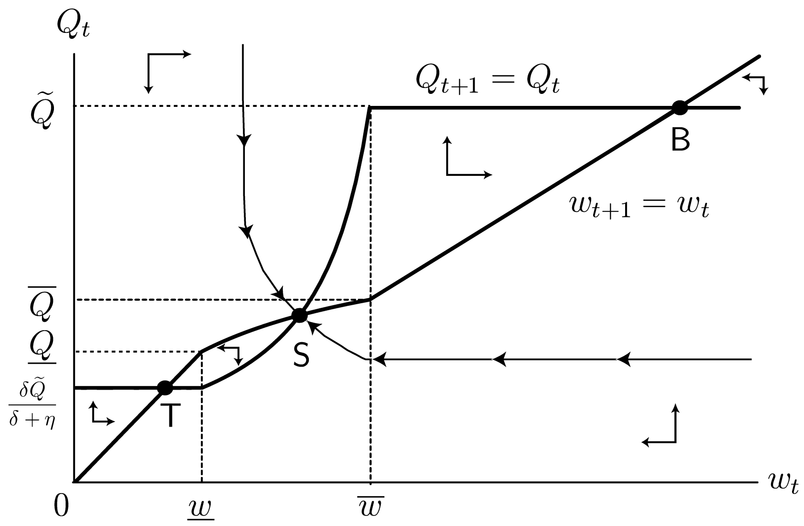


Figure 3: Dynamics of wealth and environmental quality. The parameters are the same as in Figure 2. There exist three steady states since assumption (19) implies $\delta\widetilde{Q}/(\delta+\eta) < \underline{Q}$ and $\overline{Q} < \widetilde{Q}$.

inverted S-shape. These properties derive from the upper and lower bounds of the investments in human capital. On one hand, the $Q_{t+1} = Q_t$ locus is upward sloping if an increase in wealth raises the amount of human capital investment, which occurs only when $w_t \in (\underline{w}, \overline{w})$. On the other hand, the slope of the $w_{t+1} = w_t$ locus is flatter when $w_t$ is within $(\underline{w}, \overline{w})$ than otherwise since only in this range a change in the environmental quality affects not only the ability of workers but also their (descendants') human capital investment behavior, resulting in greater variation in the steady state wealth.

The phase diagram has two stable steady states, denoted by B (one with $Q_t = \widetilde{Q}$ and $w_t \geq \overline{w}$) and T (where $Q_t < \underline{Q}$ and $w_t \leq \underline{w}$), and one saddle point S (where $Q_t \in (\underline{Q}, \overline{Q})$ and $w_t \in (\underline{w}, \overline{w})$). There is a downward sloping saddle path, i.e., the set of $(w_0, Q_0)$ from which $(w_t, Q_t)$ converges to saddle point S. However, convergence to S is unlikely since the initial $(w_0, Q_0)$ pair is given historically and therefore it is an event of measure zero that the pair happens to be exactly on the saddle path.

The economy converges to a better steady state B if it starts from the upper right side of the saddle path. Steady state B is better in that both wealth of the representative agent and the quality of the environment are higher than the other stable steady state $\mathsf{T}$. In this steady state, the productivity of every agent is high thanks to good environments. High productivity enables agents to leave large transfers to their offspring, who use the received wealth to obtain human capital. High levels of investment in human capital in turn contribute to maintaining good environments because human capital enables people to adopt the sustainable technology. This virtuous cycle makes both high wealth and good environment sustainable over generations.

When the initial pair of $(w_t, Q_t)$ is on the other side of the saddle path, the economy converges to steady state $\mathsf{T}$. In contrast to steady state $\mathsf{B}$, here a deteriorated environment reduces agents' ability to work, and also the amount of wealth that can be left to their offspring. Agents of subsequent generations cannot afford to invest in human capital, and have no other choice than to use the primitive technology that exploits natural resources, and as a consequence degrade the environment more and more. Once the economy falls into this poverty-environment trap, no one changes their practices of environmental degradation since they are equally poor.

The last point suggests that the existence of the poverty-environment trap depends on the assumption of homogeneity among agents. If we allow heterogeneity in their ability and wealth, some relatively rich agents may be able to afford to invest in human capital even though most agents are trapped in the vicious cycle of poverty and bad environment. The next section examines this possibility and shows that wealth heterogeneity is in fact the key to escaping the poverty-environment trap.

# 4 Economy with Heterogeneous Individuals

This section introduces heterogeneous individuals differing in their ability (i.e., in the amount of labor endowment) and also in the initial wealth inherited from their parents. Our first task is to establish a way to find the steady states in which both the level of environmental quality and the distribution of wealth are stationary. Then we examine the effect of the redistribution of wealth in the two stable steady states.

Let $F_t(w)$ denote the cumulative distribution function of wealth among generation $t$. Given $F_t(w)$ and environmental quality $Q_t$, equation (11) generates the distribution of wealth among generation $t+1$. Let this process be denoted by

$$F_{t+1}(\cdot) = T_{Q_t} F_t(\cdot), \tag{20}$$

where $T_{Q_t}$ is an operator that maps a wealth distribution to that of the next period under a given level of $Q_t$. In Appendix B, we show that $Q_t$ evolves within $(0, \widetilde{Q}]$ for all $t$ and that for any $Q \in (0, \widetilde{Q}]$ mapping $T_Q$ has an invariant distribution. Let us denote by $F^*(\cdot; Q)$ the cumulative distribution function of wealth that solves the functional equation $T_Q F^*(\cdot; Q) = F^*(\cdot; Q)$. Then the condition that the distribution of wealth among agents does not change over generations is written as $F_t(\cdot) = F^*(\cdot; Q_t)$.

In steady states, the quality of the environment must also be stationary. Recall from condition (17) that this is the case if and only if $Q_t = Q^*(E_t)$, where $E_t$ is the aggregate amount of human capital investments. The aggregate amount of human capital investment, $E_t$, is in turn determined by the distribution of wealth and the individual investment function. Specifically, this relationship can be written as $E_t = \int e(w) dF_t(w) \equiv \overline{E}[F_t(\cdot)]$, where $\overline{E}[\cdot]$ is a functional that maps a wealth distribution to the aggregate human capital investments.

In sum, the triple $\{F_t(\cdot), Q_t, E_t\}$ constitute a steady state when the following simultaneous equations are satisfied:

$$F_t(\cdot) = F^*(\cdot; Q_t), \quad Q_t = Q^*(E_t), \quad E_t = \overline{E}[F_t(\cdot)]. \tag{21}$$

Although equations in (21) well describe the mutual relationship among wealth distribution, environmental quality and human capital, we need to eliminate one of them to obtain a two-dimensional figure of the steady state. Let us eliminate $F_t(w)$ by substituting the first equation into the RHS of the third and define $E^*(Q) \equiv \overline{E}[F^*(\cdot; Q_t)]$. Function $E^*(Q)$ gives the amount of human capital investment when wealth distribution is stationary under environment $Q_t$. Using this, (21) can be simplified to

$$E_t = E^*(Q), \quad Q_t = Q^*(E_t). \tag{22}$$

The pair $\{E_t, Q_t\}$ constitutes a steady state if and only if simultaneous equations (22) are satisfied. Once such a pair is found, the stationary wealth distribution $F_t(\cdot)$ can be calculated by the first equation in (21).

The advantage of the representation of steady states by (22) is that both $E_t$ and $Q_t$ are scalar variables and therefore their relationships can be drawn as two curves in $(E_t, Q_t)$ space. We numerically calculated the curves for the conditions $E_t = E^*(Q)$ and $Q_t = Q^*(E_t)$, and the results are shown in Figure 4. To examine the effects of redistribution, calculations are performed for four different rates of inheritance tax while keeping other parameters fixed.

The upper left panel of Figure 4 shows the extreme case of $\tau = 1$, which serves as a benchmark. In this case all transfers from parents are expropriated by the authority and then equally distributed among all young agents. That is, there is no heterogeneity in wealth at birth, and, from (9), educational investment is also the same among individuals. Thus, the steady states should coincide with ones derived in Section 3. In fact, we see that there are three steady states; again denoted as $\mathsf{B}$, $\mathsf{S}$ and $\mathsf{T}$. Note that the aggregate levels of human capital investment are exactly $1/\gamma$ (the maximum) and zero at steady states $\mathsf{B}$ and $\mathsf{T}$, respectively, because the $E_t = E^*(Q_t)$ curve is vertical at these two values. That is, when $Q \leq \underline{Q}$ no agent invests in human capital and therefore the $E_t = E^*(Q_t)$ curve is vertical at $E_t = 0$. The curve is also vertical at $E_t = 1/\gamma$ since all agents fully invest in human capital whenever $Q_t \geq \overline{Q}$.

When the magnitude of redistribution is not so large ($\tau < 1$), the initial wealth of agents differs, depending on their parents' ability and wealth. In this case, the $E_t = E^*(Q_t)$ locus no longer has vertical segments because the choice of technology among agents is not uniform. As the tax rate $\tau$ decreases, wealth distribution becomes more heterogeneous. Then the choice of technology becomes more mixed and the $E_t = E^*(Q_t)$ curve gets smoother. Given that the upward sloping $Q_t = Q^*(E_t)$ curve does not depend on the degree of heterogeneity, this change implies that both $E_t$ and $Q_t$ decreases at steady state $\mathsf{B}$ whereas the opposite holds at steady state $\mathsf{T}$. Although heterogeneity among individuals deteriorates the environment and reduces

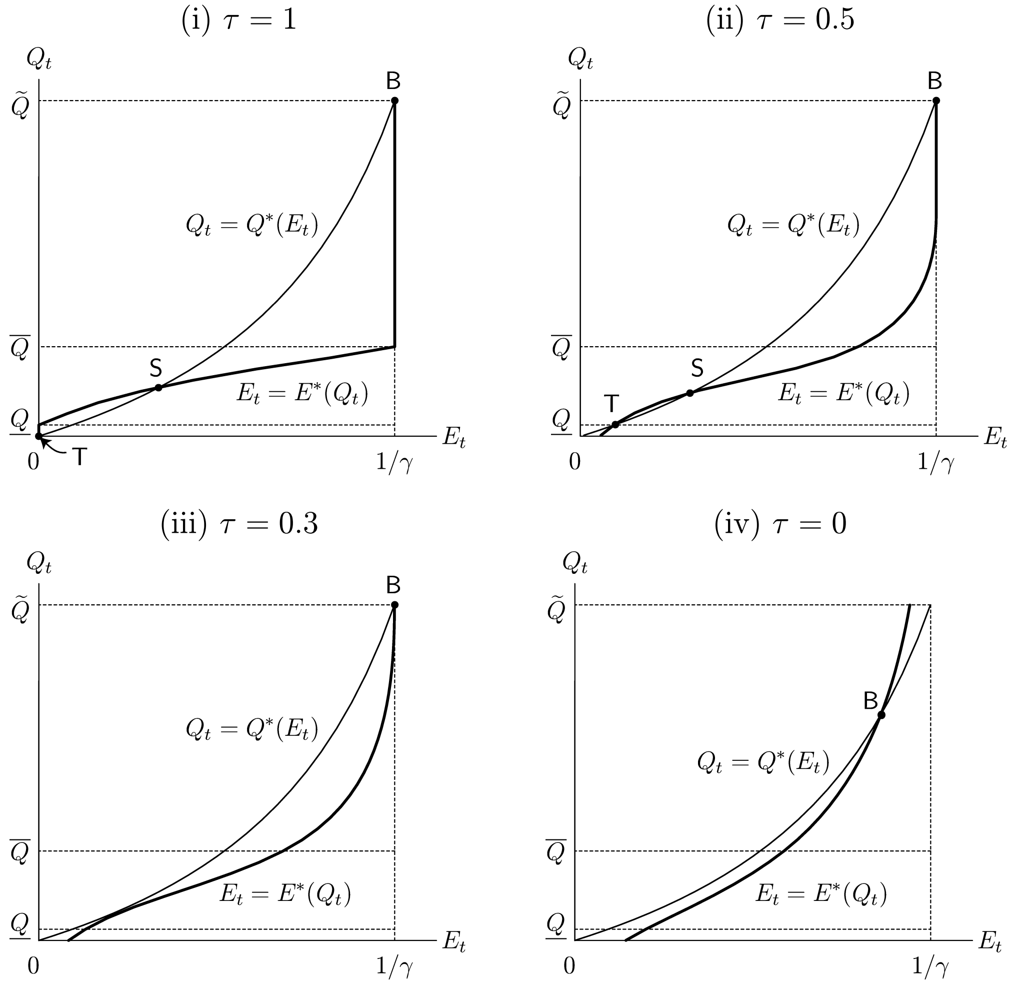


Figure 4: Steady States with Heterogeneity. Parameters: $a^s = 0.5$, $a^p = 0.3$, $\alpha = 0.4$, $\beta = 0.6$, $\gamma = 5.8$, $\eta/\delta = 2$, $\sigma = 0.7$, $\widetilde{Q} = 4$. The poverty-environment trap disappears when $\tau \leq 0.3$.

output when the economy is in the better steady state, it mitigates environmental degradation and poverty when the economy is in the poverty-environment trap.

Moreover, our numerical example shows that the poverty-environment trap disappears when the rate of inheritance tax is below a certain threshold.[17] Figure 5 illustrates the relationship between the level of $\tau$ and the steady state values of $Q_t$.

[17]The threshold level of $\tau$ varies considerably depending on parameters. Under different sets of parameters, it is possible that reductions in $\tau$ cause the better steady state to disappear before the poverty-environment trap vanishes.

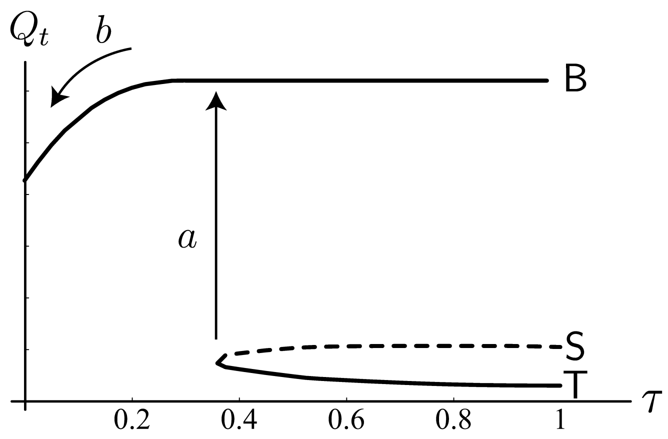


Figure 5: Rate of inheritance tax and the steady state levels of environmental quality. Parameters are the same as figure 4.

Thick curves represent stable steady states while the dotted curve represents a saddle point. Arrow $a$ shows that when the degree of wealth heterogeneity is increased by reductions in $\tau$ to a certain extent, an economy initially in the poverty-environment trap escapes from the trap and shifts to the better steady state. However, a further heterogeneity in wealth in the better steady state causes environmental deterioration, as shown by arrow $b$.

The result obtained in this section can be interpreted in terms of an intergenerational linkage in wealth and a trickle-down effect. Consider an economy trapped in the poverty-environment trap and suppose that the heterogeneity of wealth among agents is increased by a reduction in the rate of inheritance tax. Then, even when the environment is bad, agents with relatively high ability may be able to leave sufficient wealth for their children to afford to invest in human capital. Those receiving sufficient wealth to invest in human capital employ the sustainable technology, which is more productive than the primitive technology. Thus, they can leave more wealth, giving their offspring an even greater chance to invest in human capital. In this way, the intergenerational linkage in wealth enables fortunate households to get out of the poverty trap. In addition, the wealth of rich households trickles down to the poor through their adoption of the sustainable technology. As rich households switch to the sustainable technology, the aggregate environmental load of production decreases. The quality of the environment gradually improves, and the ability of every individual increases. When redistribution is reduced considerably, even the poorest may eventually obtain enough wealth to invest in human capital. In that case, the shrinkage in redistribution lets the economy escape from the poverty-environment trap to the better steady state.

However, this result is reversed if the economy is already in the better steady state, where a majority of individuals employ the sustainable technology and the quality of the environment is relatively good. In this case, greater inequality lets some individuals use the primitive technology and deplete natural resources through their production activities. These practices result in environmental degradation, which gradually erodes the ability and wealth of every individual. Thus, redistribution of income or wealth is desirable in this case because it not only reduces inequality but also improves the environment and the productivity of all individuals.

# 5 Conclusion

We model an overlapping generation economy with heterogeneous agents, whose ability depends on environmental quality. The relationship between wealth heterogeneity, the environment, and economic development are examined in that setting. The dynamics of wealth distribution are affected by environmental quality through the ability of agents, whereas wealth distribution affects the evolution of environmental quality through the use of primitive technology that exploits natural resources. This interaction creates a "poverty-environment trap", where environmental degradation lowers the productivity of the poor and reduces the amount of wealth left for their offspring, which, in turn, deprives them of the opportunity to invest in human capital and thereby accelerates deterioration in environmental quality.

We show that increased heterogeneity in the amount of wealth at birth, resulting from a reduction in the rate of inheritance tax, has a positive effect on both the environment and aggregate output in a trapped economy. When relatively capable individuals can leave a large amount of wealth, their children can employ the sustainable technology and leave more wealth to their offspring. In addition, the wealth of fortunate households trickles down to the poor since these households' adoption of the sustainable technology improves environmental quality and therefore the productivity of every individual. Accumulation of those effects may eventually enable the economy to escape from the poverty-environment trap to a better steady state. However, it should be noted that wide heterogeneity has a negative effect on both the environment and aggregate output when the economy is already in the better steady state.

# Appendix

### Appendix A

Using (3), we eliminate $h_{j,t+1}$ from (7) to obtain

$$y(\gamma e_{j,t}\ell_{j,t}, \ell_{j,t}) = \begin{cases} \big((a^s - a^p)\gamma e_{j,t} + a^p\big)\ell_{j,t} & \text{for } e_{j,t} \in [0, 1/\gamma], \\ a^h \ell_{j,t} & \text{for } e_{j,t} > 1/\gamma. \end{cases} \tag{23}$$

Equation (23) shows that any investment in human capital above $1/\gamma$ has no effect on the output in the second period. Since the marginal utility of consumption in the first period is always positive, it cannot be optimal to invest more than $1/\gamma$.

Considering this fact, the current problem is restated as follows: to maximize (8) with respect to $c^y_{j,t}, c^a_{j,t+1}, e_{j,t}$ and $b_{j,t+1}$ under

$$c^y_{j,t} + e_{j,t} = w_{j,t} \tag{24}$$

$$c^a_{j,t+1} + b_{j,t+1} = \big((a^s - a^p)\gamma e_{j,t} + a^p\big)\ell_{j,t}, \tag{25}$$

$$e_{j,t} \geq 0,\ \ e_{j,t} \leq 1/\gamma. \tag{26}$$

The Lagrangian for this problem is

$$\begin{aligned} \mathscr{L} =& \ln c^y_{j,t} + \alpha \ln c^a_{j,t+1} + \beta \ln b_{j,t+1} + \lambda_1 \left[w_{j,t} - c^c_{j,t} - e_{j,t}\right] \\ &+ \lambda_2 \left[\big((a^s - a^p)\gamma e_{j,t} + a^p\big)\ell_{j,t} - c^a_{j,t+1} - b_{j,t+1}\right] + \lambda_3 e_{j,t} + \lambda_4 \left[1/\gamma - e_{j,t}\right]. \end{aligned}$$

The first-order conditions with respect to $c^y_{j,t}, c^a_{j,t+1}, e_{j,t}$ and $b_{j,t+1}$ are respectively given by

$$\lambda_1 = 1/c^y_{j,t}, \tag{27}$$

$$\lambda_2 = \alpha/c^a_{j,t+1} \tag{28}$$

$$\lambda_2 = \beta/b_{j,t+1} \tag{29}$$

$$-\lambda_1 + \lambda_2(a^s - a^p)\gamma\ell_{j,t} + \lambda_3 - \lambda_4 = 0. \tag{30}$$

Since (26) is a pair of inequality constraints, complementary slackness conditions are also required:

$$\lambda_3 e_{j,t} = 0, \quad \lambda_3 \geq 0, \tag{31}$$

$$\lambda_4[1/\gamma - e_{j,t}] = 0, \quad \lambda_4 \geq 0. \tag{32}$$

The optimal solution is characterized by $\{c^y_{j,t}, c^a_{j,t+1}, e_{j,t}, b_{j,t+1}, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ that solve (24)-(32), the latter consisting of eight equality and four inequality conditions.

The solution is found by a guess-and-verify method. The first guess is that $e_{j,t} = 0$. Then from (24) and (27) $\lambda_1 = 1/c^y_{j,t} = 1/w_{j,t}$. From (25), (28) and (29) $\lambda_2 = \alpha/c^a_{j,t+1} = \beta/b_{j,t+1} = (\alpha+\beta)/(a^p l_{j,t})$. From (32), $\lambda_4 = 0$. Substituting $\lambda_1$, $\lambda_2$ and $\lambda_4$ into (30) gives $\lambda_3 = 1/w_{j,t} - (\alpha+\beta)\gamma(a^s - a^p)/a^p$. If $\lambda_3 \geq 0$, the remaining condition (31) is also satisfied, implying that the guess $e_{j,t} = 0$ is correct. This last condition is met if and only if $w_{j,t} \leq a^p((\alpha+\beta)\gamma(a^s - a^p))^{-1} \equiv \underline{w}$.

The next guess is that $e_{j,t} \in (0, 1/\gamma)$. Then (31) and (32) show $\lambda_3 = \lambda_4 = 0$. From (24) and (27), $\lambda_1 = 1/c^y_{j,t} = (w_{j,t} - e_{j,t})^{-1}$. From (25), (28) and (29), $\lambda_2 = \alpha/c^a_{j,t+1} = \beta/b_{j,t+1} = (\alpha+\beta)/\big[((a^s - a^p)\gamma e_{j,t} + a^p)\ell_{j,t}\big]$. Substituting $\lambda_1, \lambda_2, \lambda_3$ and $\lambda_4$ into the remaining condition (30) gives $e_{j,t} = (\alpha+\beta)(1+\alpha+\beta)^{-1}(w_{j,t} - \underline{w})$. If the latter value is within $(0, 1/\gamma)$ then our guess is correct. This is the case if and only if $\underline{w} < w_{j,t} < \underline{w}_{j,t} + (1+\alpha+\beta)(\alpha+\beta)^{-1}\gamma^{-1} \equiv \overline{w}$.

Finally, we guess that $e_{j,t} = 1/\gamma$. Then from (24) and (27) $\lambda_1 = 1/c^y_{j,t} = 1/(w_{j,t} - 1/\gamma)$. From (25), (28) and (29) $\lambda_2 = \alpha/c^a_{j,t+1} = \beta/b_{j,t+1} = (\alpha+\beta)/(a^s l_{j,t})$. From (31), $\lambda_3 = 0$. Substituting $\lambda_1$, $\lambda_2$ and $\lambda_3$ into (30) gives $\lambda_4 = -1/(w_{j,t} - 1/\gamma) + (\alpha+\beta)\gamma(a^s - a^p)/a^s$. If $\lambda_4 \geq 0$, the remaining condition (32) is also satisfied, implying that the guess $e_{j,t} = 1/\gamma$ is correct. This last condition is met if and only if $w_{j,t} \geq a^s((\alpha+\beta)\gamma(a^s - a^p))^{-1} + 1/\gamma \equiv \overline{w}$.

It can be confirmed that an agent with any arbitrary $w_{j,t} \geq 0$ fits (exactly) into one of the above three cases.

**Appendix B**

We first prove that $Q_t \in (0, \widetilde{Q}]$ for all $t \geq 0$. Since $e(w_{j,t}) \in [0, 1/\gamma]$ from (9), equation (14) implies that

$$Q_{t+1} \leq \quad \widetilde{Q} - (1-\delta)(\widetilde{Q} - Q_t), \tag{33}$$

$$Q_{t+1} \geq \quad \delta(\widetilde{Q} - Q_t) + (1-\eta)Q_t, \tag{34}$$

where $\delta, \eta \in (0,1)$. Note that if $Q_t \in (0, \widetilde{Q}]$, then (33) and (34) gives $Q_{t+1} \in (0, \widetilde{Q}]$. Since the initial level of $Q_t$ is assumed to be within $Q_0 \in (0, \widetilde{Q}]$, by induction we obtain $Q_t \in (0, \widetilde{Q}]$ for all $t \geq 0$.

We next show that $w_{j,t}$ is within a bounded interval for all $j$ and $t \geq 1$. Note that equation (10) shows $b(w_{j,t}) \in [(\alpha+\beta)^{-1}\beta a^p, (\alpha+\beta)^{-1}\beta a^s]$. Also recall that $\varepsilon_{j,t} \in [1-\sigma, 1+\sigma]$ and $\tau \in [0,1]$. Substituting these and the property $Q_t \in (0, \widetilde{Q}]$ as proved above into equation (11) gives $w_{j,t} \in (0, W]$ for all $j$ and $t \geq 1$, where $W \equiv (\alpha+\beta)^{-1}\beta(1+\sigma)a^s\widetilde{Q}$.

Therefore, to represent the wealth distribution for $t \geq 1$, it is sufficient to consider the family of distribution on a closed and bounded (i.e., compact) interval $[0, W]$. In addition, since the number of agents in each generation is 1 for all $t$, we can further restrict our attention to distributions such that $F(W) = 1$. Let us denote by $\Lambda$ the family of such distributions. Then, whenever $Q \in (0, \widetilde{Q}]$, operator $T_Q$ defined by (20) maps $\Lambda$ into itself, i.e., $T_Q : \Lambda \to \Lambda$. For later use, we order elements in $\Lambda$ by first-order dominance and denote the ordering by operator $\succcurlyeq$. That is, $F_a \succcurlyeq F_b$ means $F_a(w) \leq F_b(w)$ for all $w \in [0, W]$.

The aim of this appendix is to show that mapping $T_Q : \Lambda \to \Lambda$ has a fixed point. The proof is based on Theorem 1 of Hopenhayn and Prescott (1992).[18]

[18] Aghion and Bolton (1997) and Owen and Weil (1998) rely on Theorem 2 of Hopenhayn and

**Theorem 1 (Hopenhayn-Prescott)** *Let $\Lambda$ be a compact subset of $\mathscr{M}(S)$ and $T : \Lambda \to \Lambda$ an increasing map. Then $T$ has a fixed point if and only if there exists a measure $\mu_a$ in $\Lambda$ such that $T\mu_a \succcurlyeq \mu_a$.*

In this theorem, $\mathscr{M}(S)$ refers to the space of finite measures on $(S, \mathscr{S})$, where $\mathscr{S}$ is the Borel $\sigma$-algebra of subsets of $S$. In our context, $S = [0, W]$ and $\Lambda$ is a closed set of uniformly bounded measures, satisfying the requirements in Theorem 1.[19] Let $\delta_0$ represent the distribution where all the mass is concentrated at 0 (i.e., $\delta(w) = 1$ for all $w \in [0, W]$.) Then $\delta_0 \in \Lambda$ is the minimum element in $\Lambda$ in that $F \succcurlyeq \delta_0$ for all $F \in \Lambda$ and therefore $T_Q\delta_0 \succcurlyeq \delta_0$.

To apply Theorem 1 for $T_Q$, then, we only need to prove that $T_Q$ is an increasing map. Consider two distributions of wealth $F_a, F_b \in \Lambda$ and assume that $F_a$ first-order dominates $F_b$; i.e. $F_a \succcurlyeq F_b$. By definition, $T_Q$ is an increasing map if $T_QF_a \succcurlyeq T_QF_b$ for any such $F_a$ and $F_b$. To show this, we use the fact that the wealth of the next generation, given by (11), is the sum of individual transfer, $Q_t(1-\tau)b(w_{j,t})\varepsilon_{j,t}$, and the amount of redistribution, $Q_t\tau \int_0^1 b(w_{j',t})dj'$. Note that function $b(w_{j,t})$ is increasing in the previous generation's wealth $w_{j,t}$, as shown by (10), and $\varepsilon_{j,t}$ is distributed independently of $w_{j,t}$. These imply that the distribution of individual transfer, when the distribution of the previous generation's wealth is given by $F_a$, first-order dominates that when the distribution of the previous generation's wealth is $F_b$. In addition, the amount of redistribution is proportional to the average amount of individual transfer, which is again higher when the distribution of the previous generation's wealth is $F_a$ than when it is $F_b$. Thus, the distribution of wealth of the next generation under $F_a$ first-order dominates that under $F_b$; i.e., $T_QF_a \succcurlyeq T_QF_b$.

We have shown that Theorem 1 can be applied for mapping $T_Q : \Lambda \to \Lambda$, which

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Prescott (1992), which is simpler but requires the mapping $T$ to be linear. Specifically, the latter theorem is applicable when individual wealth follows an independent Markov Process since in that case $T(F_a + F_b) = TF_a + TF_b$. This is not the case for our model, since the amount of redistribution depends on the entire wealth distribution in the economy and therefore the evolution of individual wealth is interrelated.

[19] Set $\Lambda$ conforms to example (c) of Hopenhayn and Prescott (1992, p. 1390).

assures the existence of an invariant distribution.

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