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in Multicriteria Outranking Analysis

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Discussion Paper 05-22

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Abstract

The outranking analysis has been frequently used to deal with the complex decisions involving qualitative criteria and imprecise data. So far, various versions of ELECTRE have been proposed for ranking alternatives in the outranking analysis. Among others, ELECTRE III has been widely used. A distillation procedure using a qualification index is proposed to rank alternatives from the valued outranking relation. A weakness of ELECTRE III, however, is to involve the arbitrariness in the selection of the discrimination threshold function for the distillation procedure.

On the other hand, various variants of PROMETHEE are also proposed for the outranking analysis. PROMETHEE intends to be simple and easy to understand. A deficiency of PROMETHEE is that it does not take into account the preference intensity of alternatives in the in-preference flow and out-preference flow for each alternative.

We propose a new preference ranking procedure based on eigenvector using the "weighted" in- and out-preference flows of each alternative in the outranking analysis. The basic idea of the procedure proposed here is that it should be better to outrank a "strong" alternative than a "weak" one and, conversely, it is less serious to be outranked by a "strong" alternative than by "weak" one in a PROMETHEE context. It has a completely different interpretation with the AHP (Analytic Hierarchy Process) since the components of the valued outranking relation matrix are neither ratios nor reciprocal as in the AHP.

JEL Classification: C44, C61, C63

Keywords: Multiple criteria analysis; PROMETHEE; ELECTRE; Valued outranking relations

1 Introduction

The outranking analysis has been frequently used to deal with the complex decisions involving qualitative criteria and imprecise data (see, Bana e Costa, 1990, Roy,1996; Roy and Vanderpooten,1997; Roy and Vincke,1984; Vincke,1992;Larichev and Olson,2001). So far, various versions of ELECTRE (ELimination Et Chix Traduisant la REalité) have been proposed for ranking alternatives in the outranking analysis. Among others, ELECTRE III is very familiar and has been widely used (see, Roy,1996; Rogers, Bruen and Maystre, 2000; Pomerol and Romero, 2000). A distillation procedure using a qualification index is proposed to rank alternatives from the valued outranking relation. A weakness of ELECTRE III, however, is to involve the arbitrariness in the selection of the discrimination threshold function for the distillation procedure.

On the other hand, various variants of and PROMETHEE (Preference Ranking Organization METHod for Enririching Evaluations) have also been widely used for the outranking analysis (Brans and Vincke, 1985; Brans and Mareschal,1992; Brans,Vincke and Mareschal, 1986; Pomerol and Romero, 2000; Albadvi,2004). PROMETHEE intends to be simple and easy to understand. PROMETHEE is based on the positive (out-) and negative (in-) preference flows for each alternative in the valued outranking relation to derive the ranking of alternatives. The positive flow is expressing how much an alternative is outranking the other ones, and the negative flow how much it is outranked by the other ones. Based on the preference flows, PROMETHEE I provides a partial preorder. PROMETHEE II is also introduced to obtain a complete preorder by using a net flow, though it loses much information of preference relations (Brans, Vincke and Mareschal, 1986). A deficiency of PROMETHEE is that it does not take into account the preference intensity of alternatives in the in-preference flow and out-preference flow for each alternative.

We propose a new preference ranking procedure based on eigenvector using the “weighted” in- and out-preference flows of each alternative in the outranking analysis. The basic idea of the procedure proposed here is that it should be better to outrank a “strong” alternative than a “weak” one and, conversely, it is less serious to be outranked by a “strong” alternative than by “weak” one in a PROMETHEE context. It has a completely different interpretation with the AHP (Analytic Hierarchy Process) since the components of the outranking relation matrix are neither ratios nor reciprocal as in the AHP (see Saaty,1990). Macharis et al (2004) discussed the strengths and weaknesses of PROMETHEE and AHP. And recommendations are formulated to integrate into PROMETHEE a number of useful AHP features, especially a tree-like structure similar to the one found in AHP and the determination of weights. They, however, didn’t suggest the preference ranking based on the eigenvector in a PROMETHEE context. Thus, this new procedure differs from the AHP and the approach adopted by Macharis.

The rest of this paper is organized as follows: In the next section, we shall briefly review the PROMETHEE analysis with the simple preference flows. In section 3, we shall generalize the simple preference flows and introduce the weighted preference flows in a PROMETHEE context. It is shown that the preference ranking procedure based on the weighted preference flows yields the eigenvalue problem. A rationale of the eigenvalue approach is provided in the theorem. Concluding remarks are given in the final section.

2 Preference Flows in a PROMETHEE Context

Let us consider the set A of n alternatives:

$$A = \{\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_n\}.$$

Let $g_1, g_2, \ldots, g_m$ be m-criteria. Thus, each alternatives $\boldsymbol{a}_i$ is characterized by a multiattribute outcome denoted by a vector

$$(g_1(\boldsymbol{a}_i), g_2(\boldsymbol{a}_i), \ldots, g_m(\boldsymbol{a}_i)).$$

The valued outranking relation is constructed from the notions of quasi-criterion and pseudo-criterion. In particular, PROMETHEE constructs it using a preference function which represents the decision maker's preference for an alternative $\boldsymbol{a}_i$ with regard to $\boldsymbol{a}_j$. Several types of preference functions are considered for the criteria such as usual criterion, quasi-criterion, criterion with linear preference, level criterion, pseudo-criterion with linear preference and indifference area, and Gaussian criterion (see, Brans and Vincke, 1985 and Brans, Vincke and Mareschal, 1986). To be more precise, let

$$P_k(\boldsymbol{a}_i, \boldsymbol{a}_j) = f\left[g_k(\boldsymbol{a}_i) - g_k(\boldsymbol{a}_j)\right]$$

be the preference function associated with the criterion $g_k(\cdot)$. As $f(\cdot)$, six types of functions are proposed to cover most of the cases in practical applications. Then, the valued outranking relation $\pi(\boldsymbol{a}_i, \boldsymbol{a}_j)$ of $\boldsymbol{a}_i$ over $\boldsymbol{a}_j$ is defined as the weighted sum of the preference functions P_k:

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_j) = \sum_k P_k(\boldsymbol{a}_i, \boldsymbol{a}_j) w_k,$$

where w_k is a weight for criterion k. Thus, $\pi(\boldsymbol{a}_i, \boldsymbol{a}_j)$ represents the intensity of the preference of $\boldsymbol{a}_i$ over $\boldsymbol{a}_j$ for all the criteria: the closer to 1, the greater the preference. From a valued outranking relation, a valued outranking graph with nodes signifying alternatives and arcs $(\boldsymbol{a}_i, \boldsymbol{a}_j)$ having values $\pi(\boldsymbol{a}_i, \boldsymbol{a}_j)$ is depicted.

Then, the preference out-flow and preference in-flow of each node $\boldsymbol{a}_i$ are respectively defined by

$$\phi^+(\boldsymbol{a}_i) = \sum_j \pi(\boldsymbol{a}_i, \boldsymbol{a}_j), \tag{1}$$

$$\phi^-(\boldsymbol{a}_i) = \sum_j \pi(\boldsymbol{a}_j, \boldsymbol{a}_i). \tag{2}$$

The higher the preference out-flow and the lower the preference in-flow, the better alternative.

The out-flow and in-flow induce respectively the following complete preorders,

$$\begin{aligned} &\boldsymbol{a}_i P^+ \boldsymbol{a}_j \quad \text{if and only if } \phi^+(\boldsymbol{a}_i) > \phi^+(\boldsymbol{a}_j), \\ &\boldsymbol{a}_i I^+ \boldsymbol{a}_j \quad \text{if and only if } \phi^+(\boldsymbol{a}_i) = \phi^+(\boldsymbol{a}_j). \end{aligned}$$

$$\begin{aligned} &\boldsymbol{a}_i P^- \boldsymbol{a}_j \quad \text{if and only if } \phi^-(\boldsymbol{a}_i) < \phi^-(\boldsymbol{a}_j), \\ &\boldsymbol{a}_i I^- \boldsymbol{a}_j \quad \text{if and only if } \phi^-(\boldsymbol{a}_i) = \phi^-(\boldsymbol{a}_j). \end{aligned}$$

PROMETHEE I provides a partial preorder by considering the intersection of these two complete preorders:

(a) $\boldsymbol{a}_i P_I \boldsymbol{a}_j$ is defined by

1. $\phi^+(\boldsymbol{a}_i) > \phi^+(\boldsymbol{a}_j)$ and $\phi^-(\boldsymbol{a}_i) < \phi^-(\boldsymbol{a}_j)$ or
2. $\phi^+(\boldsymbol{a}_i) > \phi^+(\boldsymbol{a}_j)$ and $\phi^-(\boldsymbol{a}_i) = \phi^-(\boldsymbol{a}_j)$ or
3. $\phi^+(\boldsymbol{a}_i) = \phi^+(\boldsymbol{a}_j)$ and $\phi^-(\boldsymbol{a}_i) < \phi^-(\boldsymbol{a}_j)$

(b) $\boldsymbol{a}_i I_I \boldsymbol{a}_j$ is defined by

1. $\phi^+(\boldsymbol{a}_i) = \phi^+(\boldsymbol{a}_j)$ and $\phi^-(\boldsymbol{a}_i) = \phi^-(\boldsymbol{a}_j)$

(c) $\boldsymbol{a}_i$ and $\boldsymbol{a}_j$ are incomparable, otherwise.

Example 1. Consider the following outranking relation matrix.

$$\Pi = \begin{array}{c} \\ \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \\ \boldsymbol{a}_4 \end{array} \begin{array}{c} \begin{array}{cccc} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 & \boldsymbol{a}_4 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

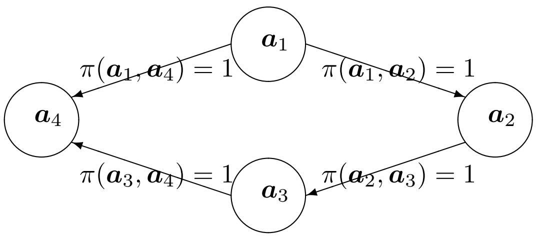


Figure 1. Outranking graph

In general, it is reasonable to assume the ranking:

$$\boldsymbol{a}_1 \longrightarrow \boldsymbol{a}_2 \longrightarrow \boldsymbol{a}_3 \longrightarrow \boldsymbol{a}_4.$$

In fact, ELECTRE III derives the same ranking as above. Employing PROMETHEE I, however, we have the following ranking:

$$\boldsymbol{a}_1 \longrightarrow \{\boldsymbol{a}_2, \boldsymbol{a}_3\} \longrightarrow \boldsymbol{a}_4$$

We will revisit this example later.

3 Weighted Preference Flows in a PROMETHEE context

We now introduce the "weighted" preference (out-) flows.

$$\lambda\psi^+(\boldsymbol{a}_i) = \sum_{j=1}^{n} \pi(\boldsymbol{a}_i, \boldsymbol{a}_j)\psi^+(\boldsymbol{a}_j), \qquad i = 1, 2, \ldots, n, \tag{3}$$

where λ is a constant and $\psi^+(\boldsymbol{a}_j)$ is the strength of preference of $\boldsymbol{a}_j$.

This implies that it should be better to outrank a "strong" alternative than a "weak" one.

Similarly, we define the weighted preference (in-) flows

$$\lambda\psi^-(\boldsymbol{a}_i) = \sum_{j=1}^{n} \pi(\boldsymbol{a}_j, \boldsymbol{a}_i)\psi^-(\boldsymbol{a}_j), \qquad i = 1, 2, \ldots, n. \tag{4}$$

which implies that it is less serious to be outranked by a "strong" alternative than by "weak" one.

In what follows, with no loss of generality, let $\psi^+(\boldsymbol{a}_i)$ $(\psi^+(\boldsymbol{a}_i))$ be normalized such that

$$\max_j \psi^+(\boldsymbol{a}_j) = 1 \qquad \left(\max_j \psi^-(\boldsymbol{a}_j) = 1\right).$$

Thus, using the weighted preference flows yields the eigenvalue problem:

$$\Pi\boldsymbol{\psi}^+ = \lambda_{\max}\boldsymbol{\psi}^+ \qquad \left(\boldsymbol{\psi}^-\Pi = \lambda_{\max}\boldsymbol{\psi}^-\right), \tag{5}$$

where $\Pi = (\pi(\boldsymbol{a}_i, \boldsymbol{a}_j))$ and $\boldsymbol{\psi}^+ = (\psi^+(\boldsymbol{a}_i))$, $(\boldsymbol{\psi}^- = (\psi^-(\boldsymbol{a}_i)))$, is the right (left) eigenvector associated with the maximum eigenvector $\lambda_{\max}$ of Π.

We have a well-known (see, for instance, Saaty, 1990, p. 170)

Lemma (Perron). *Let Π be any positive square matrix. Then*

1. *Π has a real positive simple eigenvalue λ_{max} whose modulus exceeds the moduli of all other eigenvalues.*
2. *The eigenvector of Π corresponding to λ_{max} has positive components and is essentially (to within multiplication by a constant) unique.*

In what follows, to assure that the eigenvectors $\boldsymbol{\psi}^+ = (\psi^+(\boldsymbol{a}_i))$ and $\boldsymbol{\psi}^- = (\psi^-(\boldsymbol{a}_i))$ corresponding to the maximum eigenvalue of any valued outranking relation Π have positive components, for the sake of calculation, we replace $\pi(\boldsymbol{a}_i, \boldsymbol{a}_j) = 0$ by $\pi(\boldsymbol{a}_i, \boldsymbol{a}_j) = \varepsilon$ where ε is a sufficiently small positive number.

In the example 1, we have

$$\boldsymbol{\psi}^+ = \left(\psi^+(\boldsymbol{a}_i)\right) = \begin{pmatrix} 1 & 0.06 & 0.003 & 0.0002 \end{pmatrix}$$

and

$$\boldsymbol{\psi}^- = \left(\psi^-(\boldsymbol{a}_i)\right) = \begin{pmatrix} 0.0002 & 0.003 & 0.06 & 1 \end{pmatrix},$$

where $\varepsilon = 0.0001$.

The ranking is

$$\boldsymbol{a}_1 \longrightarrow \boldsymbol{a}_2 \longrightarrow \boldsymbol{a}_3 \longrightarrow \boldsymbol{a}_4$$

Consider a special outranking relation Π with $\pi(\boldsymbol{a}_i, \boldsymbol{a}_j) = 1$ or ε for any $\boldsymbol{a}_i, \boldsymbol{a}_j$. Let us define, for any $\boldsymbol{a}_i, \boldsymbol{a}_j$,

$$\boldsymbol{a}_i P \boldsymbol{a}_j \quad \text{if and only if } \pi(\boldsymbol{a}_i, \boldsymbol{a}_j) = 1 \quad \text{and} \quad \pi(\boldsymbol{a}_j, \boldsymbol{a}_i) = \varepsilon,$$

$$\boldsymbol{a}_i I \boldsymbol{a}_j \quad \text{if and only if } \pi(\boldsymbol{a}_i, \boldsymbol{a}_j) = 1 \quad \text{and} \quad \pi(\boldsymbol{a}_j, \boldsymbol{a}_i) = 1.$$

Then Π is referred to as a complete preorder if

(a) for any $\boldsymbol{a}_i, \boldsymbol{a}_j$, one and only one of the following relations holds: $\boldsymbol{a}_i P \boldsymbol{a}_j$, $\boldsymbol{a}_j P \boldsymbol{a}_i$, or $\boldsymbol{a}_i \boldsymbol{I} \boldsymbol{a}_j$.

(b) P is asymmetric and transitive

(c) I is an equivalent relation(i.e., reflexive,symmetric and transitive).

As is well-known, conditions (a),(b) and (c) hold if and only if there exists a value function $v(\boldsymbol{a}_j)$ on a finite set A which represents the decision maker's preferences, that is, for any $\boldsymbol{a}_i, \boldsymbol{a}_j$,

$$\boldsymbol{a}_i P \boldsymbol{a}_j \quad \text{if and only if } v(\boldsymbol{a}_i) > v(\boldsymbol{a}_j),$$

$$\boldsymbol{a}_i I \boldsymbol{a}_j \quad \text{if and only if } v(\boldsymbol{a}_i) = v(\boldsymbol{a}_j).$$

As a rationale of the eigenvector approach, we have

Theorem. *If the outranking relation $\Pi = (\pi(\boldsymbol{a}_i, \boldsymbol{a}_j))$ is a complete preorder, $\psi^+(\cdot)$ and $-[\psi^-(\cdot)]$ are the value functions, that is,*

$$\boldsymbol{a}_i P \boldsymbol{a}_j \quad \text{if and only if } \psi^+(\boldsymbol{a}_i) > \psi^+(\boldsymbol{a}_j), \qquad \left(\psi^-(\boldsymbol{a}_i) < \psi^-(\boldsymbol{a}_j)\right),$$

$$\boldsymbol{a}_i I \boldsymbol{a}_j \quad \text{if and only if } \psi^+(\boldsymbol{a}_i) = \psi^+(\boldsymbol{a}_j), \qquad \left(\psi^-(\boldsymbol{a}_i) = \psi^-(\boldsymbol{a}_j)\right).$$

Proof: See Appendix.

Example 2. Consider the following special valued outranking relation matrix.

$$\begin{array}{c} \\ \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \\ \vdots \\ \boldsymbol{a}_n \end{array} \begin{array}{c} \begin{array}{ccccc} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 & \cdots & \boldsymbol{a}_n \end{array} \\ \begin{pmatrix} 1 & \theta & \theta & \cdots & \theta \\ (1-\theta) & 1 & \theta & \cdots & \theta \\ (1-\theta) & (1-\theta) & 1 & \cdots & \theta \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (1-\theta) & (1-\theta) & (1-\theta) & \cdots & 1 \end{pmatrix} \end{array}$$

$$\begin{aligned}
\boldsymbol{\psi}^+ &= \left(\psi^+(\boldsymbol{a}_i)\right) = \left(1, \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{n}}, \left(\frac{1-\theta}{\theta}\right)^{\frac{2}{n}}, \ldots, \left(\frac{1-\theta}{\theta}\right)^{\frac{n-1}{n}}\right) \\
\boldsymbol{\psi}^- &= \left(\psi^-(\boldsymbol{a}_i)\right) = \left(\left(\frac{1-\theta}{\theta}\right)^{\frac{n-1}{n}}, \left(\frac{1-\theta}{\theta}\right)^{\frac{n-2}{n}}, \ldots, \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{n}}, 1\right).
\end{aligned}$$

$$\lambda_{\max} = 1 + \theta \left(\tfrac{1-\theta}{\theta}\right)^{\frac{1}{n}} + \theta \left(\tfrac{1-\theta}{\theta}\right)^{\frac{2}{n}} + \cdots + \theta \left(\tfrac{1-\theta}{\theta}\right)^{\frac{n-1}{n}}$$

From this, the ranking is:

$$\begin{aligned}
&\boldsymbol{a}_1 \to \boldsymbol{a}_2 \to \cdots \to \boldsymbol{a}_n, && \text{if } \frac{1}{2} < \theta < 1, \\
&\{\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_n\}, && \text{if } \theta = \frac{1}{2}.
\end{aligned}$$

Example 3. (A valued outranking relation without discordance) (Table 3 in Brans et al. 1986).

	$\boldsymbol{a}_1$	$\boldsymbol{a}_2$	$\boldsymbol{a}_3$	$\boldsymbol{a}_4$	$\boldsymbol{a}_5$	$\boldsymbol{a}_6$
$\boldsymbol{a}_1$	1	0.296	0.250	0.268	0.100	0.185
$\boldsymbol{a}_2$	0.462	1	0.389	0.333	0.296	0.500
$\boldsymbol{a}_3$	0.236	0.180	1	0.333	0.056	0.429
$\boldsymbol{a}_4$	0.399	0.505	0.305	1	0.223	0.212
$\boldsymbol{a}_5$	0.444	0.515	0.487	0.380	1	0.448
$\boldsymbol{a}_6$	0.286	0.399	0.250	0.432	0.133	1

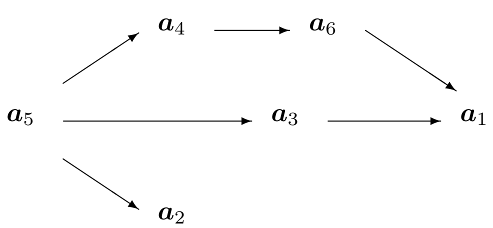
We get

$$\boldsymbol{\psi}^+ = \left(\psi^+(\boldsymbol{a}_i)\right) = (\ 0.533 \quad 0.873 \quad 0.593 \quad 0.763 \quad 1.000 \quad 0.704\ ),$$

and

$$\boldsymbol{\psi}^- = \left(\psi^-(\boldsymbol{a}_i)\right) = (\ 0.984 \quad 1.000 \quad 0.894 \quad 0.953 \quad 0.498 \quad 0.978\ ).$$

From this, the partial preorder is given by:



This is the same result as PROMETHEE I.

Though, when constructing the outranking relations, PROMETHEE does not take discordance into account (see Keyser and Peeters, 1996), since the concept of discordance plays the important role in the outranking analysis, let us consider the outranking relation involving the discordance index.

Example 4. (A valued outranking relation with discordance)

criteria		C_1	C_2	C_3	C_4
weight (w_i)		0.3	0.3	0.3	0.1
alternatives	$\boldsymbol{a}_1$	15	25	40	20
	$\boldsymbol{a}_2$	10	20	30	70
	$\boldsymbol{a}_3$	8	15	25	130
	$\boldsymbol{a}_4$	5	10	20	90

Let threshold values p_i (preference), q_i (indifference) and v_i (veto) of each criterion C_i be:

	C_1	C_2	C_3	C_4
p_i	1	1	1	1
q_i	0	0	0	0
v_i	100	100	100	100

Then a valued outranking relation is:

$$\begin{array}{c} \\ \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \\ \boldsymbol{a}_4 \end{array} \begin{array}{c} \begin{array}{cccc} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 & \boldsymbol{a}_4 \end{array} \\ \begin{pmatrix} 1 & 0.9 & \varepsilon & 0.9 \\ 0.1 & 1 & 0.9 & 0.9 \\ 0.095 & 0.1 & 1 & 1.0 \\ 0.086 & 0.1 & \varepsilon & 1 \end{pmatrix} \end{array}$$

We have

$$\boldsymbol{\psi}^+ = \left(\psi^+(\boldsymbol{a}_i)\right) = \left(\ 1.000 \quad 0.760 \quad 0.422 \quad 0.189\ \right)$$

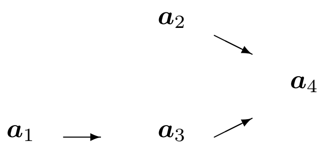
and

$$\boldsymbol{\psi}^- = \left(\psi^-(\boldsymbol{a}_i)\right) = \left(\ 0.183 \quad 0.353 \quad 0.372 \quad 1.000\ \right)$$

Thus, the ranking by the eigenvector procedure is:

$$\boldsymbol{a}_1 \longrightarrow \boldsymbol{a}_2 \longrightarrow \boldsymbol{a}_3 \longrightarrow \boldsymbol{a}_4$$

On the other hand, the ranking by PROMETHEE I is:



We employed the distillation method in ELECTRE III in which a discrimination threshold function is set at the following:

$$s(\lambda) = 0.3 - 0.15\,\lambda.$$

Then, we have

$$\boldsymbol{a}_1 \longrightarrow \boldsymbol{a}_2 \longrightarrow \boldsymbol{a}_3 \longrightarrow \boldsymbol{a}_4$$

which is the same as the eigenvector procedure.

4 Concluding Remarks

We have proposed a new eigenvector procedure based on the weighted preference in-flows and out-flows in the outranking analysis. The eigenvector procedure is easy to understand and calculate. Various numerical examples suggest that PROMETHEE I, ELECTRE III and the eigenvector procedure have the same or almost the same results in valued outranking relations *without discordance* and that the eigenvector procedure gives results that are closer to ELECTRE III than PROMETHEE in valued outranking relations *with discordance*. However, further properties specifying the strengths and weaknesses of the eigenvector procedure remain to be explored.

Appendix

Proof of Theorem:

From (3), we have for any $\boldsymbol{a}_i$, $\boldsymbol{a}_j$,

$$\lambda_{max}\left(\psi^+(\boldsymbol{a}_i) - \psi^+(\boldsymbol{a}_j)\right) = \sum_{k=1}^{n} \left(\pi(\boldsymbol{a}_i, \boldsymbol{a}_k) - \pi(\boldsymbol{a}_j, \boldsymbol{a}_k)\right) \psi^+(\boldsymbol{a}_k) \tag{6}$$

Note that, for any $\boldsymbol{a}_i$, $\boldsymbol{a}_j$, one and only one of (i) $\boldsymbol{a}_i P \boldsymbol{a}_j$ or (ii) $\boldsymbol{a}_j P \boldsymbol{a}_i$ or (iii) $\boldsymbol{a}_i I \boldsymbol{a}_j$ holds.

We shall prove, if $\boldsymbol{a}_i P \boldsymbol{a}_j$, then $\psi^+(\boldsymbol{a}_i) > \psi^+(\boldsymbol{a}_j)$.

From $\boldsymbol{a}_i P \boldsymbol{a}_j$, we get

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_j) = 1 \text{ and } \pi(\boldsymbol{a}_j, \boldsymbol{a}_i) = \varepsilon \tag{7}$$

It follows from the reflexivity of I that

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_i) = 1 \text{ and } \pi(\boldsymbol{a}_j, \boldsymbol{a}_j) = 1. \tag{8}$$

From (7) and (8), we have

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_j) = \pi(\boldsymbol{a}_j, \boldsymbol{a}_j), \tag{9}$$

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_i) > \pi(\boldsymbol{a}_j, \boldsymbol{a}_i). \tag{10}$$

For any $\boldsymbol{a}_k (k \neq i, j; k = 1, 2, \ldots, n)$, we have either $\boldsymbol{a}_j P \boldsymbol{a}_k$ or $\boldsymbol{a}_k P \boldsymbol{a}_j$ or $\boldsymbol{a}_j I \boldsymbol{a}_k$ exclusively.

(a) If $\boldsymbol{a}_j P \boldsymbol{a}_k$, then $\boldsymbol{a}_i P \boldsymbol{a}_k$ as P is transitive. That is,

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_k) = \pi(\boldsymbol{a}_j, \boldsymbol{a}_k) = 1. \tag{11}$$

(b) If $\boldsymbol{a}_k P \boldsymbol{a}_j$,then

$$\pi(\boldsymbol{a}_j, \boldsymbol{a}_k) = \varepsilon. \tag{12}$$

Since $\pi(\boldsymbol{a}_i, \boldsymbol{a}_k) \geq \varepsilon$, we have from (12)

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_k) \geq \pi(\boldsymbol{a}_j, \boldsymbol{a}_k) = \varepsilon. \tag{13}$$

(c) If $\boldsymbol{a}_j I \boldsymbol{a}_k$, then $\boldsymbol{a}_i P \boldsymbol{a}_k$. Thus, we have

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_k) = \pi(\boldsymbol{a}_j, \boldsymbol{a}_k) = 1. \tag{14}$$

From (9), (10), (11), (13) and (14), we have

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_k) \geq \pi(\boldsymbol{a}_j, \boldsymbol{a}_k), \qquad k = 1, 2, \ldots, n, \tag{15}$$

and

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_i) > \pi(\boldsymbol{a}_j, \boldsymbol{a}_i). \tag{16}$$

Since, by Lemma, $\lambda_{max} > 0$ and $\psi^+(\boldsymbol{a}_k) > 0$, $k = 1, 2, \ldots, n$, it follows from (6),(15) and (16) that

$$\psi^+(\boldsymbol{a}_i) > \psi^+(\boldsymbol{a}_j).$$

Thus, it is shown that

$$\text{if } \boldsymbol{a}_i P \boldsymbol{a}_j, \text{ then } \psi^+(\boldsymbol{a}_i) > \psi^+(\boldsymbol{a}_j) \tag{17}$$

Similarly, we have

$$\text{if } \boldsymbol{a}_j P \boldsymbol{a}_i, \text{ then } \psi^+(\boldsymbol{a}_j) > \psi^+(\boldsymbol{a}_i) \tag{18}$$

Finally, let us assume (iii) $\boldsymbol{a}_i I \boldsymbol{a}_j$. It follows that

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_j) = \pi(\boldsymbol{a}_j, \boldsymbol{a}_j) = 1, \tag{19}$$

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_i) = \pi(\boldsymbol{a}_j, \boldsymbol{a}_i) = 1. \tag{20}$$

For any $\boldsymbol{a}_k$, $(k \neq i, j; k = 1, 2, \ldots, n)$, we have either $\boldsymbol{a}_j P \boldsymbol{a}_k$ or $\boldsymbol{a}_k P \boldsymbol{a}_j$ or $\boldsymbol{a}_j I \boldsymbol{a}_k$ exclusively.

(a) If $\boldsymbol{a}_j P \boldsymbol{a}_k$ then $\boldsymbol{a}_i P \boldsymbol{a}_k$. Therefore,

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_k) = \pi(\boldsymbol{a}_j, \boldsymbol{a}_k) = 1. \tag{21}$$

(b) If $\boldsymbol{a}_k P \boldsymbol{a}_j$, then $\boldsymbol{a}_k P \boldsymbol{a}_i$. Thus

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_k) = \pi(\boldsymbol{a}_j, \boldsymbol{a}_k) = \varepsilon. \tag{22}$$

(c) If $\boldsymbol{a}_j \boldsymbol{I} \boldsymbol{a}_k$, then $\boldsymbol{a}_i \boldsymbol{I} \boldsymbol{a}_k$ by the transitivity of I. Thus we have

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_k) = \pi(\boldsymbol{a}_j, \boldsymbol{a}_k) = 1. \tag{23}$$

From (19) through (23) we have

$$\pi(\boldsymbol{a}_i, \boldsymbol{a}_k) = \pi(\boldsymbol{a}_j, \boldsymbol{a}_k), \qquad k = 1, 2, \ldots, n.$$

It follows from (6) that

$$\text{if } \boldsymbol{a}_i \boldsymbol{I} \boldsymbol{a}_j, \text{ then } \psi^+(\boldsymbol{a}_i) = \psi^+(\boldsymbol{a}_j). \tag{24}$$

Since, for any $\boldsymbol{a}_i$, $\boldsymbol{a}_j$, one and only one of the following three cases; $\boldsymbol{a}_i \boldsymbol{P} \boldsymbol{a}_j$ or $\boldsymbol{a}_j \boldsymbol{P} \boldsymbol{a}_i$ or $\boldsymbol{a}_i \boldsymbol{I} \boldsymbol{a}_j$ holds, we have

$$\boldsymbol{a}_i P \boldsymbol{a}_j \quad \text{if and only if } \psi^+(\boldsymbol{a}_i) > \psi^+(\boldsymbol{a}_j),$$
$$\boldsymbol{a}_i \boldsymbol{I} \boldsymbol{a}_j \quad \text{if and only if } \psi^+(\boldsymbol{a}_i) = \psi^+(\boldsymbol{a}_j).$$

In a similarly way, we have

$$\boldsymbol{a}_i P \boldsymbol{a}_j \quad \text{if and only if } \psi^-(\boldsymbol{a}_i) < \psi^-(\boldsymbol{a}_j),$$
$$\boldsymbol{a}_i \boldsymbol{I} \boldsymbol{a}_j \quad \text{if and only if } \psi^-(\boldsymbol{a}_i) = \psi^-(\boldsymbol{a}_j).$$

which ends the proof.

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