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Abstract

This paper studies sustainability of economic growth considering the risk of natural disasters caused by pollution in an endogenous growth model with physical and human capital accumulation. We consider an environmental tax policy, and show that economic growth is sustainable only if the tax rate on the polluting input is increased over time and that the long-term rate of economic growth follows an inverted V-shaped curve relative to the growth rate of the environmental tax. The social welfare is maximized under a positive steady-state growth in which faster accumulation of human capital compensates the productivity loss due to declining use of the polluting input.

Keywords: natural disasters, human capital, endogenous depreciation, economic growth.

JEL Classification Numbers: O41, O13, E22.

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1 Introduction

Natural disasters have a large impact on economic activity primarily through destruction of capital stock. For example, National Oceanic and Atmospheric Administration (NOAA, 2005) estimates that Hurricane Katrina, occurred on August 2005 in the Gulf of Mexico, caused over 100 billion dollars' worth of damage mainly on physical capital. This magnitude of destruction might seem unusual, but the losses caused by landfalling hurricanes in the United States in the previous year, 2004, were also considerably large—approximately 45 billion dollars, as reported by NOAA. This magnitude of damage is not negligible even in comparison to the whole size of the U.S. physical capital stock.

Despite various forms of preventive efforts, the occurrence of natural disasters is not declining but in a growing trend.[1] The long-term consequence of the increased possibility of such disasters critically hinge on whether their occurrences are purely exogenous phenomena to the economic system, or they are caused for some part by economic activity. That is, if the latter is true, economic growth itself creates a threat to economic activity, making the sustainability of economic growth questionable.

Recent meteorological research shows that, unfortunately, the latter argument is likely to be true. For example, it is clearly stated in the Intergovernmental Panel on Climate Change third assessment report (IPCC, 2001) that "Emissions of greenhouse gas and aerosols due to human activities have a great impact on global climate change." Global warming, or more specifically, increasing sea surface temperature,[2]

[1]In Hoyois et al. (2005), the global number of reported disasters is 1.55 times higher in 2000-2004 period than in 1995-1999 period, and the number of reported extreme temperature disasters, floods, and people affected by disasters are almost 2 times, 1.74 times, and 1.33 times higher, respectively. A disaster in the database must be fulfilled at least one of the following criteria: 10 or more people reported killed; 100 people reported affected; declaration of a state of emergency; call for international assistance.

[2]The tropical sea surface temperature in the North Atlantic shows a large upswing in the last decade. Emamuel (2005) argues such a upswing is related to the El Niño and reflects the effect of global warming.

is in turn suspected to increase hurricane frequency and intensity (Emamuel, 2005; Webster et al., 2005).

Those observations suggest that there is a two-way causality between economic activities and the occurrence of natural disasters. This paper investigates the sustainability of economic growth in the presence of this two-way causality, by introducing the endogenous risk of natural disasters into a Uzawa-Lucas type of endogenous growth model. Following the literature (e.g., Copeland and Taylor, 1994; Bovenberg and Smulders, 1995; Stokey, 1998), we assume that polluting inputs such as fossil fuels are necessary for economic activities and those inputs are subject to an environmental tax.[3] Differently from earlier studies, however, this paper examines the case in which the use of polluting inputs raises the probability that capital stocks are destroyed by natural disasters. Agents make saving decisions taking into account the possibilities of loss of asset due to natural disasters.

Using the model, we show that the economic growth is, in fact, not sustainable if the (per-unit) tax rate on polluting inputs is kept constant. Intuitively, under the constant tax rate, firms are willing to use increasing amounts of polluting inputs as the economy grows. However, as increased use of polluting inputs raises the risk of natural disasters, it reduce incentive agents to invest in capital stock since they face a higher possibilities of asset loss. Thus, even in the long run, capital stock cannot exceed a constant threshold under a constant tax rate.

To overcome this limitation, we next consider a time-varying tax on polluting inputs. If the per-unit tax rate is raised over time, private firms owing a Cobb-Douglas production technology will increase the use of other inputs, including human capital, relative to polluting inputs. As a result, the use of polluting inputs can be is bounded above, making the economic growth sustainable. It is also shown that the growth rate of environmental tax has both positive and negative effects on economic growth. The faster the rate at which the environmental tax is increased,

[3]Copeland and Taylor (1994) and Stokey (1994) assumed that there are a continuum of technology generating a different level of pollution. However, the technology can be written with capital and the total level of pollution as inputs.

the lower is the asymptotic amount of pollution and therefore the lower is probability of disasters. This gives households more incentive to save, which promotes growth. However, the increased cost of using polluting input faced by private firms reduces their productivity at each date, which has a negative effect on growth. Due to those opposite effects, the rate of economic growth rate is shown to follow an inverted V-shaped curve relative to the growth rate of environmental tax.

Having shown that the sustained growth is feasible, this paper then examine whether it is desirable or not. This question may seem trivial, but in a AK-growth model with pollution Stokey (1988) shows that, even when production technology allows sustained growth, it is theoretically possible that agents prefer a no-growth state with a good environment.[4] Contrary to Stokey's analysis, we show that the social welfare is maximized on a steady-state growth path, where the environmental tax is raised at a positive rate, although this does not coincide with the growth maximizing path.

The difference of our result from Stokey's stems not from our assumption that the use of polluting input only affects the risk of natural disasters without directly affecting consumer's utility.[5] Rather, it comes from the our two-sector specification that the growth is driven both by physical and human capital accumulation. This paper's analysis shows that sustained growth with a Cobb-Douglas technology is feasible under a limited use of polluting input because human capital stock is accumulated much faster than the rate at which output is increased. In fact, provided that human capital stock has lower degree of vulnerability to disasters, as suggested by Skidmore

[4]Stokey (1998) assumed additive separable preference for consumption and pollution in which the marginal utility from consumption is declining whereas the marginal disutility from pollution is increasing. Then, as consumption increases due to capital accumulation, reducing pollution becomes more important than increasing consumption. She shows that further growth is not optimal at a high level of capital stock.

[5]In section 4.1, we examine the case in which pollution directly causes disutility to consumers, in addition to raising the risk of disasters. It is shown that a positive steady-state growth is compatible to welfare maximization even in this case.

and Toya (2002), the risk raises incentives for more investment in human capital stock relative to physical capital stock.

To our best knowledge, this study is a first attempt to examine the consequences of natural disasters in an endogenous growth model. This does not mean, of course, that our study is independent from the previous literature. In fact, great attention has been paid to the sustainability of economic growth in the literature of growth theory, by noticing that the finite nature of natural environment may potentially restrict sustainability of economic growth. With regard to the finiteness of natural resources, Aghion and Howitt (1998), Scholz and Ziemes (1999), Schou (2000), Grimaud and Rougé (2003), and Agnai, Gutiérrez and Iza (2005) examined sustainability of economic growth in endogenous growth models with non-renewable resources.[6]

Complementary to those studies, Stokey (1998) and Uzawa (2003) examined sustainability focusing on emission of pollutants. Since the global atmosphere is finite, the negative effects from pollutants will become unacceptably serious when the usage of polluting input increases without bound. Therefore, both Stokey (1998) and Uzawa (2003) concludes that, without exogenous technological change, the economy should converge to a no-growth steady state. Our study is related to their studies in that it is focusing on pollutants, but is more closely related to Bovenberg and Smulders (1995) who explicitly considers accumulation of knowledge. When accumulation of knowledge improves the productivity of other inputs (including polluting inputs), the economy can grow using constant or even declining amounts of polluting inputs. Their prediction that sustained growth is possible under an appropriate policy also holds in our model.

The rest of the paper is organized as follows. Section 2 presents the model and proves that growth cannot be sustained under a constant tax rate on polluting inputs.

[6]For example, Grimaud and Rougé (2003) analyzed sustainability of economic growth introducing a non-renewable natural resource into a Schumpeterian endogenous growth model. They showed that whether both optimal and equilibrium growth is positive at the steady-state depends on the value of the subjective discount rate relative to the productivity of R&D.

The steady-state growth with an increasing environmental tax is analyzed in Section 3. The social planners's problem is examined in Section 4 so as to investigate the desirability of sustained growth. Section 5 concludes.

2 The Model

This section presents a model of natural disasters and economic growth. In the first subsection, the risk of natural disasters is introduced into a two-sector growth model based on Uzawa (1965) and Lucas (1998). In the second subsection, the behavior of households and firms is examined. After presenting equilibrium conditions, the final subsection proves that sustained growth is not possible when the environmental tax rate is kept constant.

2.1 The risk of natural disasters

In the model, output is produced by a constant-returns-to-scale production technology using physical capital K_t, human capital H_t, and polluting input P_t such as fossil fuels that emits pollutants or greenhouse gases. The production function is given by:

$$Y_t = F(K_t, u_t H_t, P_t) = AK_t^{\alpha}(u_t H_t)^{1-\alpha-\beta} P_t^{\beta},$$

where u_t is the time share devoted to production of goods, α is a constant share of physical capital, and β is that of the polluting input. Output is either consumed or added to physical capital stock. Since we focus on the risk of natural disasters, we ignore the extraction cost and/or production cost of polluting inputs and finite nature of natural resources. The risk of natural disasters on capital stock is assumed to be inevitable and to depend on the amount of pollution.

Specifically, suppose that economy consists of continuum of local areas. Let q_t be the arrival rate of natural disasters per unit of time at each area:

$$q_t = \overline{q} + \widehat{q} P_t, \tag{1}$$

where $\overline{q}$ and $\widehat{q}$ are positive constants. Equation (1) says that the arrival rate raises as the amount of aggregate polluting inputs increases, as in the case of hurricanes and fossil fuels. When a natural disaster occurs at an area, it causes damage to physical capital. For example, if natural disasters occur at an area where the existing aggregate physical capital stock is $\widetilde{K}_t$, the expected loss of physical capital is $\overline{\phi}\widetilde{K}_t$, where $\overline{\phi} > 0$ is the average damage to physical capital stock. Note that, for various reasons, a natural disaster harms human capital stock as well.[7] Similarly to $\overline{\phi}$, define $\overline{\psi} > 0$ as the average damage to human capital stock. The damage on human capital measured in relative to the existing stock is, however, smaller than the damage on physical capital stock, and therefore it is reasonable to assume $\overline{\phi} > \overline{\psi} > 0$.

For simplicity, each area is assumed to be small enough and the occurrence of natural disasters in one area is assumed not to be correlated to others.[8] By the law of large numbers with (1), the aggregate damage to physical capital stock and human capital stock are respectively:

$$q_t \cdot \overline{\phi} K_t = (\overline{\phi}\overline{q} + \overline{\phi}\widehat{q}P_t)K_t, \tag{2}$$

$$q_t \cdot \overline{\psi} H_t = (\overline{\psi}\overline{q} + \overline{\psi}\widehat{q}P_t)H_t. \tag{3}$$

Let $\overline{\delta}_K$ and $\overline{\delta}_H$ be the constant rates of depreciation of physical capital stock and human capital stock, respectively. Then, similarly to Lucas (1988), the resource constraints for physical and human capital stocks are written as:

$$\dot{K}_t = F(K_t, u_t H_t, P_t) - C_t - (\delta_K + \phi P_t)K_t, \tag{4}$$

$$\dot{H}_t = B(1 - u_t)H_t - (\delta_H + \psi P_t)H_t, \tag{5}$$

where $\delta_K \equiv \overline{\delta}_K + \overline{\phi}\overline{q}$, $\phi \equiv \overline{\phi}\widehat{q}$, $\delta_H \equiv \overline{\delta}_H + \overline{\psi}\overline{q}$, $\psi \equiv \overline{\psi}\widehat{q}$, and C_t, B, and $1 - u_t$ is the aggregate consumption, the constant productivity of human capital accumulation,

[7]The death toll in Katrina rose to over 1000 (NOAA, 2005) and the number of the injured was much more. In addition, many education institutions are forced to remain closed for extended periods of time and a large number of data and documents storing valuable knowledge are lost after the disasters pass.

[8]This assumption is not realistic when considering large scale disasters. Without it, the law of large numbers does not apply, and disasters cause short-term fluctuations. However, since we are focusing on long-term behavior of the economy, analysis of such fluctuations are out of the scope of this paper.

and a fraction of time devoted to production of human capital, respectively. Equations (4) and (5) shows that the risk of natural disasters effectively augments the depreciation rates of physical and human capital stocks, in proportion to the use of polluting input. We assume that $\alpha + \beta > \phi$ so that the sensitivity of the augmented depreciation rate to the polluting input is not too high.

Observe that, unlike standard endogenous growth models, the right hand sides of equations (4) and (5) are not homogenous of degree one in terms of quantities. This implies that balanced growth that presents a homothetic expansion is not feasible, which reflects the finiteness of natural environment.

2.2 The market economy

Since firms do not take into consideration the externality such that the use of polluting inputs increases the risk of natural disasters, the market equilibrium does not correspond with the solution of the social planner's problem. The followings consider explicitly the market economy where per-unit tax τ_t, in terms of final goods, is levied on the use of polluting inputs. Since there is no uncertainty, consumers predicts $\{\tau_t\}_{t=0}^{\infty}$ at the initial time under perfect foresight.[9] The government balances its budget at each moment and equally distribute the tax revenue $T_t = \tau_t P_t$ among households in a lump-sum fashion.

Households

The economy is populated by a unit mass of infinitely lived homogeneous households. Each household owns physical capital stock, k_t, and human capital stock, h_t. However, due to natural disasters, they are faced with the risk of damages to both types of capital stock. The insurance market is assumed to be complete. Under this assumption, it is optimal for households to take out insurance that cover the all losses associated with natural disasters. Since their expected damages to physical capital stock and human capital stock are $q_t \overline{p} hik_t$ and $q_t \overline{p} sih_t$, respectively, the

[9]This paper does not consider the issue of dynamic inconsistencies.

budget constraint of households can be written as:[10]

$$\dot{k}_t = r_t k_t + w_t u_t h_t - (\delta_K + \phi P_t)k_t - c_t + T_t, \quad (6)$$

$$\dot{h}_t = B(1-u_t)h_t - (\delta_H + \psi P_t)h_t, \quad (7)$$

where r_t, w_t, and c_t denote the real interest rate, the real wage rate, and the amount of consumption, respectively. Note that the cost associated with depreciation and insurance is paid by the owner of the capital.

The utility function of the representative household is given by:

$$\int_0^\infty \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \quad (8)$$

where $\theta > 1$ is the inverse of the elasticity of intertemporal substitution and ρ is the rate of time preference. We assume $B > \rho$ so that households have enough incentive to investment in human capital. Each household maximizes (8) subject to the constraints (6) and (7). From the first-order condition for maximization problem, we obtain the Keynes-Ramsey Rule:

$$-\theta \frac{\dot{c}_t}{c_t} = \rho + \phi P_t + \delta_K - r_t. \quad (9)$$

This condition is similar to that obtained in the original Uzawa-Lucas model, except that it depends on the risk of natural disasters, ϕP_t.

The shadow price of human capital relative to that of physical capital is w_t/B, which equals to the market price of human capital measured by physical capital stock. Hence, the arbitrage condition between human capital investment and physical capital investment is given by:

$$\frac{\dot{w}_t}{w_t} = r_t - (\phi - \psi)P_t - (\delta_K - \delta_H) - B. \quad (10)$$

[10]Equations (6)-(7) implicitly assume that damaged human capital is compensated in the form of human capital. Obviously, a more realistic setting is that this compensation is done in the form of goods. Nonetheless, as long as the amount of compensation in terms of goods is calculated using the appropriate price of human capital, w_t/B, the equilibrium outcomes do not change in the aggregate level.

In (10), the left hand side implies the rate of change in the relative shadow price of human and physical capital while the right hand side implies the difference between the marginal return to investment in physical capital and human capital. Note that this condition must be satisfied in the long run. If it is not, the solution would be either $u_t = 0$ or $u_t = 1$ for all agents, and therefore one of the two kinds of aggregate capital stock approaches zero due to depreciation, clearly inconsistent with the Cobb-Douglas production function. The transversality conditions are $\lim_{t\to\infty} k_t \nu_t e^{-\rho t} = 0$ for physical capital stock and $\lim_{t\to\infty} h_t \mu_t e^{-\rho t} = 0$ for human capital stock, where $\nu_t = c_t^{-\theta}$ and $\mu_t = (w_t/B) c_t^{-\theta}$.

Firms

There is a continuum of firms producing final goods in competitive market. We consider a representative firm maximizing its profit. The firm pays the wages for labor input, the rental rate for physical capital input, and the environmental tax as well. Given factor prices r_t, w_t and τ_t, the profit maximization problem is:

$$\max_{K_t, N_t, P_t} F(K_t, N_t, P_t) - r_t K_t - w_t N_t - \tau_t P_t,$$

where $N_t \equiv u_t H_t$ is the amount of human capital employed by the firm. The first-order conditions for this problem are $r_t = \alpha Y_t / K_t$, $w_t = (1 - \alpha - \beta) Y_t / N_t$, and $\tau_t = \beta Y_t / P_t$. Substituting the profit maximizing polluting input, $P_t = \beta Y_t / \tau_t$, into the production function, the output can be written as:

$$Y_t = \left(\widetilde{A} \tau^{-\frac{\beta}{1-\beta}} \right) K_t^{\widehat{\alpha}} N_t^{1-\widehat{\alpha}}, \tag{11}$$

where $\widetilde{A} \equiv \beta^{\beta/(1-\beta)} A^{1/(1-\beta)}$ and $\widehat{\alpha} \equiv \alpha/(1-\beta)$. When written in the form of (11), it becomes clear that the environmental tax lowers the effective total factor productivity, $\widetilde{A} \tau^{-\beta/(1-\beta)}$. Using this notation, the first order conditions are expressed as

$$r_t = \alpha \widetilde{A} \tau_t^{-\frac{\beta}{1-\beta}} \left(\frac{K_t}{N_t} \right)^{\widehat{\alpha}-1}, \tag{12}$$

$$w_t = (1 - \alpha - \beta) \widetilde{A} \tau_t^{-\frac{\beta}{1-\beta}} \left(\frac{K_t}{N_t} \right)^{\widehat{\alpha}}. \tag{13}$$

2.3 Equilibrium conditions

Since the population is homogenous and normalized to unity, $K_t = k_t$, $H_t = h_t$, $C_t = c_t$, and $u_t = H_t/N_t$ hold in equilibrium. From the budget constraint of households, the evolution of physical capital stock is given by:

$$\frac{\dot{K}_t}{K_t} = \frac{Y_t}{K_t} - \frac{C_t}{K_t} - (\delta_K + \phi P_t), \tag{14}$$

From the production function of human capital stock, its evolution is given by:

$$\frac{\dot{H}_t}{H_t} = B(1 - u_t) - (\delta_H + \psi P_t). \tag{15}$$

From (10) and (13), the evolution of labor supply u_t must satisfy:

$$-\frac{\beta}{1-\beta} \cdot \frac{\dot{\tau}_t}{\tau_t} + \widehat{\alpha}\Big(\frac{\dot{K}_t}{K_t} - \frac{\dot{H}_t}{H_t} - \frac{\dot{u}_t}{u_t}\Big) = \alpha\frac{Y_t}{K_t} + (\psi - \phi)P_t - (\delta_K - \delta_H) - B. \tag{16}$$

The dynamics of consumption is given by the Keynes-Ramsey Rule:

$$-\theta\frac{\dot{C}_t}{C_t} = \rho - \alpha\frac{Y_t}{K_t} + \delta_K + \phi P_t. \tag{17}$$

Finally, from the firm's f.o.c., the amount of polluting input is determined by

$$P_t = \beta Y_t/\tau_t. \tag{18}$$

A necessary condition for the transversality condition associated with physical capital stock is $\dot{K}_t/K_t + \dot{\nu}_t/\nu_t - \rho < 0$ as $t \to \infty$. We will show in Section 3 that $\dot{K}_t/K_t = \dot{C}_t/C_t$ in the long run. Thus, from $\rho > 0$ and $\nu_t = c_t^{-\theta}$, we obtain $\dot{K}_t/K_t + \dot{\nu}_t/\nu_t = (1-\theta)\dot{Y}_t/Y_t$ so that the transversality condition associated with physical capital stock is satisfied so long as $\dot{Y}_t/Y_t \geq 0$ (i.e., unless the economy is shrinking toward zero). Likewise, the transversality condition associated with human capital stock is satisfied whenever $\dot{H}_t/H_t + \dot{\mu}_t/\mu_t - \rho < 0$. Differentiating logarithmically with respect to time in $\mu_t = (w_t/B)c_t^{-\theta}$ and using (9), (10), and (15), we obtain $\dot{H}_t/H_t + \dot{\mu}_t/\mu_t - \rho = -Bu_t$. Because $B > 0$, the transversality condition associated with human capital stock is satisfied if $u_t > 0$.

2.4 Sustainability under a constant tax rate

Observe, from (18), that pollution increases in proportion to output Y_t if the government do not change the environmental tax rate. Since the increasing usage of polluting inputs makes natural disasters more and more frequent, it seems that economic growth is not sustainable under such a static environmental policy. This subsection formally proves that this insight is correct.

The proof goes via *reductio ad absurdum*. When the government sets a constant environmental tax rate (i.e., $\tau_t = \tau_0$ for all t), the Keynes-Ramsey Rule (17) can be rewritten, from (18), as:

$$-\theta\frac{\dot{C}_t}{C_t} = \rho + \delta_K - \left(\alpha - \frac{\phi\beta}{\tau_0}K_t\right)\frac{Y_t}{K_t}.$$

This equation states that, if consumption grow in the long-run (i.e., $C_t \to \infty$ as $t \to \infty$), the sign of the value in the parentheses must be positive. Hence, K_t must be bounded above by a constant value $\tau_0\alpha/\phi\beta$ (i.e., $\lim_{t\to\infty} K_t < \tau_0\alpha/\phi\beta$). To interpret this result, observe that that, from (11) and (12), the rental price of physical capital is $r_t = \alpha Y_t/K_t$ and therefore the last term of represents marginal rate of return of holding capital net of insurance cost $\phi P_t = \phi\beta Y_t/\tau_0$. As physical capital accumulates, the insurance cost increases in relative to interest rate due to increased risk of natural disasters. Since this lowers the incentive to save, the stock of physical capital should not become too large in order to maintain sustained growth.

This raises another question, however, of maintaining output growth under a limited size of physical capital. From (11), the positive growth rate of output requires the positive growth rate of human capital stock devoted to production due to the supremum of K_t. That is, $\lim_{t\to\infty} \dot{N}_t \geq 0$ must hold in order to support increasing consumption. Under a constant environmental tax rate equation, (16) can be rewritten as:

$$\widehat{\alpha}\Big(\frac{\dot{K}_t}{K_t} - \frac{\dot{N}_t}{N_t}\Big) = \alpha\frac{Y_t}{K_t} + (\psi - \phi)P_t - (\delta_K - \delta_H) - B.$$

Consider the behavior of both hands of the above equation in the long-run. The left hand side implies the growth rate of wage, which eventually becomes negative value,

$-\widehat{\alpha}\dot{N}_t/N_t$. Conversely, the right hand is given by:

$$\left(\alpha - \frac{\phi\beta}{\tau_0}K_t\right)\frac{Y_t}{K_t} - \delta_K + \psi P_t - B + \delta_H,$$

which is the difference between the marginal rate of return on both types of capital stock. From the condition for the positive growth rate of consumption derived above, the sign of the value in the parentheses must be positive, and thus, the value of the right hand side goes to infinity as $Y_t \to \infty$. These results imply that the equality in (16) fails to hold, and therefore C_t and Y_t cannot grow in the long run. To summarize,

Proposition 1 *If the environmental tax rate is constant, economic growth is not sustainable.*

3 Steady-state growth

Given Proposition 1, this section considers a time-varying tax policy. In order to focus on the long-term behavior of the economy, the following considers a policy that has the constant growth rate of environmental tax:

$$\tau_t = \tau_0 \exp(g_\tau t), \quad \tau_g : \text{positive constant.}$$

The main task of this section is to examine the dependence of long-term rate of economic growth, denoted by

$$g^* \equiv \lim_{t\to\infty} \frac{\dot{Y}_t}{Y_t},$$

on the growth rate of environmental tax g_τ. Note that, for any g_τ chosen by the government, the corresponding equilibrium value of g^* cannot be larger than g^τ since it means the usage of polluting input, $P_t = \beta Y_t/\tau_t$, become infinite in the long run, and therefore is impossible for the same reason as explained in subsection 2.4. Hence, $g^* \leq g_\tau$, which leads to $\lim_{t\to\infty} \dot{P}_t/P_t \leq 0$ from (18). Since the amount of polluting input is nonnegative, this means that P_t converges to a constant value, denoted by P^*, in the long-run.

To examine the long-term property of the equilibrium dynamics, we consider the steady-state growth path in which the various quantities grow at constant rates:

$\dot{K}_t/K_t = g_K$, $\dot{C}_t/C_t = g_C$, $\dot{H}_t/H_t = g_H$, and $\dot{u}_t/u_t = g_u$. In this steady state, (15) and (17) implies that the value of u_t and Y_t/K_t must be constant. In addition, from (14), these properties in turn imply C_t/K_t must also be constant. Thus we denote those constant values by $u_t = u$, $Y_t/K_t = z$, $C_t/K_t = \chi$.

Since $g_K = g_C = g^*$ and $g_u = 0$ are already known, the following focuses on the determination of g^* and g_H. Differentiating logarithmically with respect to time in (11) and using $g_K = g^*$ yield the growth rate of human capital stock:

$$g_H = g^* + \frac{\beta}{1-\alpha-\beta} g_\tau. \tag{19}$$

Equation (19) shows that, in the long run, the use of human capital grows faster than physical capital (and output). It compensates for the decreasing use of polluting input due to the increasing environmental tax. Constancy of u leads to $\dot{N}/N = \dot{H}/H$. Hence, from (13), the rate of change in wage is given by:

$$\frac{\dot{w}}{w} = -\frac{\beta}{1-\alpha-\beta} g_\tau. \tag{20}$$

Recall that the price of a unit of human capital stock measured in goods is w_t/B. Equation (20) shows that it decreases as human capital stock increases.

Applying the above discussion, the equilibrium conditions (14)-(18) can be simplified along the steady-state growth path, as follows.

$$\text{evolution of } K_t\text{: } g^* = z - \chi - (\delta_K + \phi P^*), \tag{21}$$

$$\text{evolution of } H_t\text{: } g^* + \frac{\beta}{1-\alpha-\beta} g_\tau = B(1-u) - (\delta_H + \psi P^*), \tag{22}$$

$$\text{arbitrage condition: } -\frac{\beta}{1-\alpha-\beta} g_\tau = \alpha z - B + (\psi - \phi) P^* - (\delta_K - \delta_H), \tag{23}$$

$$\text{Keynes-Ramsey rule: } -\theta g^* = \rho - \alpha z + (\delta_K + \phi P^*), \tag{24}$$

$$\text{asymptotic value of } P_t\text{: } P^* \begin{cases} \geq 0 \text{ if } g^* = g_\tau & \text{(case 1)}, \\ = 0 \text{ if } g^* < g_\tau & \text{(case 2)}. \end{cases} \tag{25}$$

Those five conditions, (21)-(25), determine five unknowns, g^*, $z \equiv Y_t/K_t$, $\chi \equiv K_t/C_t$, u, and P^*, in the steady state growth path. Note that, however, they include a complementary slackness condition (25), and we cannot know whether $P^* = 0$ or

$g^* = g^\tau$ holds in advance. Thus we need to examine two possible cases in turn, and then to determine which case actually occurs in equilibrium under a particular set of exogenous parameters.

Let us first derive the steady-state growth path assuming that Case 1 actually occurs (whether this assumption is appropriate will be checked immediately). Substituting $g^* = g_\tau$ into (24) and (23), we obtain the steady-state value of polluting input:

$$P^* = \frac{1}{\psi}\Big[B - \rho - \Big(\theta + \frac{\beta}{1-\alpha-\beta}\Big)g_\tau - \delta_H\Big]. \tag{26}$$

In (26), we can see that the condition, $P^* \geq 0$ is satisfied if g_τ is within the following region:

$$g_\tau \leq \frac{B - \rho - \delta_H}{\theta + \frac{\beta}{1-\alpha-\beta}} \equiv g^{\max}.$$

Hence, Case 1 is possible only if $g_\tau \leq g^{\max}$. From (21) to (24), we obtain the value of other variable as follows:

$$z = \frac{1}{\alpha}\left(\theta g_\tau + \delta_K + \phi P^* + \rho\right), \tag{27}$$

$$\chi = \frac{1}{\alpha}\Big((\theta-\alpha)g_\tau + (1-\alpha)(\delta_K + \phi P^*) + \rho\Big), \tag{28}$$

$$u = \frac{1}{B}\Big((\theta-1)g_\tau + \rho\Big). \tag{29}$$

Next, we examine the possibility of steady-state growth in Case 2. Suppose that the amount of polluting input is zero at the equilibrium. The same procedure as the above yields the steady-state growth rate:

$$g^* = \frac{1}{\theta}\Big(B - \delta_H - \frac{\beta}{1-\alpha-\beta}g_\tau - \rho\Big), \tag{30}$$

we can see that the condition $g^* < g_\tau$ is satisfied if $g_\tau > g^{\max}$. The steady-state value of other variables are:

$$z = \frac{1}{\alpha}\Big(B + \delta_K - \delta_H - \frac{\beta}{1-\alpha-\beta}g_\tau\Big), \tag{31}$$

$$\chi = \Big(\frac{1}{\alpha} - \frac{1}{\theta}\Big)\Big(B - \delta_H - \frac{\beta}{1-\alpha-\beta}g_\tau\Big) + \frac{1-\alpha}{\alpha}\delta_K + \frac{\rho}{\theta}, \tag{32}$$

$$u = \frac{1}{B\theta}\Big[(\theta-1)\Big(B - \delta_H - \frac{\beta}{1-\alpha-\beta}g_\tau\Big) + \rho\Big]. \tag{33}$$

It can be confirmed that all z, χ and u in (31)-(33) are positive if g_τ is lower than $g^{\lim} \equiv (1-\alpha-\beta)\beta^{-1}(B+\delta_K-\delta_H)$.

Note that the possibilities of Case 1 and Case 2 are mutually exclusive—that is, under a given set of parameters, only one case is possible. Therefore, the steady-state path is always unique, and it is characterized by $g^* = g_\tau$ and (26)-(29) if $g_\tau \in [0, g^{\max}]$, and by $P^* = 0$ and (30)-(33) if $g_\tau \in (g^{\max}, g^{\lim})$. In Appendix, we show that the steady state is saddle stable in both cases, given that either δ_K is not too large or g_τ is below a certain level:

$$\widetilde{g}_\tau \equiv \frac{(\gamma+\theta)(1-\alpha-\beta)(B-\delta_H)+\rho(1-\alpha-\beta)}{(\theta-1)\beta}.$$

Since $g^{\lim}$ and $\widetilde{g}_\tau$ are considerably high under reasonable parameter values, we restrict our attention to $g_\tau < \min\{g^{\lim}, \widetilde{g}_\tau\}$.

Figure 1 illustrates the relationship between the growth rate of environmental tax and the long-run growth rate of human capital stock in (22), physical capital stock and output in (21), and the growth rate of polluting inputs or pollution which is given by $\dot{P}/P = g^* - g_\tau$. The asymptotic amount of pollution, P^*, is largest when the environmental tax rate is constant (i.e., $g_\tau = 0$), which results in the highest probability of natural disasters and zero growth. When g_τ in increased within the range of $[0, g^{\max}]$, the long-run amount of pollution, P^* decreases while the growth rates of Y and K increase in parallel with g_τ. The growth rate of human capital g_H also increases, at a faster pace. This enables sustained growth under (asymptotically) constant use of polluting input. When the environmental tax rate is raised so rapidly that g_τ exceeds $g^{\max}$, the use of polluting inputs P_t is continually reduced, converging asymptotically to zero level. In this case, the growth rates of Y and K can no longer increase in parallel with g_τ, but decreasing in g_τ. Nonetheless, provided that $g_\tau < g^{\lim}$, the economy can grow at a positive rate growth by accumulating g_H much faster than output grows. The following proposition summarizes the obtained results.

Proposition 2 *If the environmental tax rate is raised at constant rate $g_\tau \in (0, \widetilde{g}_\tau)$, there exists a unique and saddle-stable steady-state growth path with positive rate of*

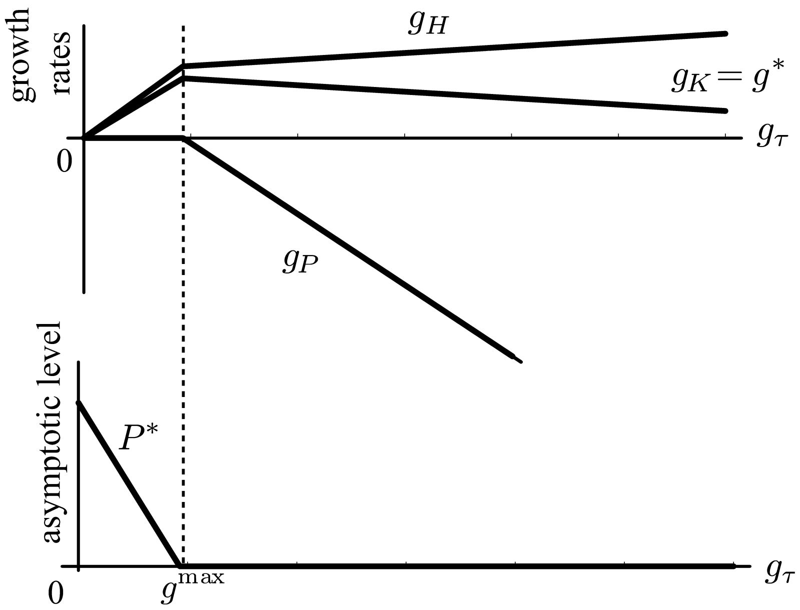

Figure 1: Growth rate of environmental tax and the steady state growth. The upper panel shows the relationship between the growth rate of environmental tax (g_τ) and that of human capita (g_H), phisical capital (g_K), output (g^*), and pollution (g_P). The lower panel shows the level to which the level of pollution converges to in the long run ($P_t \to P^*$). Parameters: $\alpha = .4$, $\beta = .1$, $\theta = .2$, $\rho = .98$, $B = 2$, $\psi = .005$, $\phi = .01$.

growth. The long-term rate of growth follows an inverted-V shape against the growth rate of the environmental tax rate, and it is maximized when $g_\tau = g^{\max}$.

It is possible to interpret Proposition 2 in terms of the Keynes-Ramsey rule. Note that, from the arbitrage condition between human capital investment and physical capital investment, the rates of return to both of capital net of insurance payments are equal. Therefore, using the arbitrage condition (10) and the Keynes-Ramsey Rule (9) and the fact that $g_C = g^*$, the growth rate at the steady state can be rewritten as:

$$g^* = \frac{1}{\theta}\left(B - \psi P_t - \frac{\beta}{1-\alpha-\beta} g_\tau - \rho - \delta_H\right). \tag{34}$$

The second term in the parenthesis of (34) expresses the expected loss of human capital due to natural disasters. If the growth rate of environmental tax is accelerated, the effective productivity of private firms, $\widetilde{A}$ falls, the value of human capital is reduced, and individuals invest less in human capital. Given P^*, thus, the growth rate of economy decreases. As a result, the growth rate of pollution, $\dot{P}_t/P_t = g^* - g_\tau$ decreases. However, when P^* is positive (Case 1), the risk of damage to human capital stock decreases due to a reduction in P^*, which allows the growth rate of the economy to increase, as shown in (34). This boost continues until g^* catches up with g_τ. However, when P_t is already approaching $P^* = 0$ (Case 2), the reduction in $\dot{P}/P$ accelerates the speed at which P_t converges to $P^* = 0$, but it does not change the asymptotic value P^*. Thus, the steady-state growth rate remains low.

4 Welfare

The previous section established that sustained growth is feasible by raising the rate of environmental tax rate over time. It is yet to shown, however, such an environmental policy is desirable in terms of welfare. This section investigates the social planner's problem so as to derive the welfare-maximizing environmental policy. The planner maximizes (8) subject to the following constraints:

$$\dot{K}_t = F(K_t, u_t H_t, P_t) - C_t - (\delta_K + \phi P_t K_t), \tag{35}$$

$$\dot{H}_t = B(1 - u_t)H_t - (\delta_H + \psi P_t H_t). \tag{36}$$

From the first-order conditions for optimality, the dynamics of K_t, H_t, and C_t, and the arbitrage condition between two types of capital stocks are the exactly same as (14)-(17), which are parts of market equilibrium conditions. Since the social planner takes into consideration the externality of polluting emissions, that is, the risk of natural disasters, it chooses the amount of polluting input according to the following rule:

$$\frac{\beta Y_t}{P_t} = \phi K_t + \psi H_t \cdot \frac{(1 - \alpha - \beta)Y_t}{B u_t H_t}. \tag{37}$$

Equation (37) says that P_t is determined such that the marginal benefit is equal to the marginal cost of using polluting inputs. The left hand side of (37) is the marginal productivity of polluting input whereas the right hand side is the sum of increases in damage to physical capital stock and in damage to human capital stock, both measured in terms of final goods.

Recall that P_t is determined according to (18) in the market equilibrium. Substituting (18) into (37) reveals that the optimal growth path can be realized in equilibrium by setting the optimal environmental tax rate:

$$\tau_t^{\text{opt}} = \phi K_t + \psi \frac{(1-\alpha-\beta)Y_t}{Bu_t}. \tag{38}$$

In order to implement such a policy, it is necessary for the government to know equilibrium paths of K_t, Y_t, and u_t, which appear in the RHS of (38). However, K_t, Y_t, and u_t are endogenously determined depending the future path of τ_t that is expected by consumers.

Although it seems excessively difficult to solve this dynamic fixed point problem, the long-term property of the optimal policy can be conveniently analyzed by focusing on the usage of polluting inputs, P_t, which approaches a constant steady state value, rather than in terms of the time-varying tax rate imposed on it. Note that optimality condition (37) can be stated as

$$P_t^{\text{opt}} = \beta \frac{Y_t}{\tau_t^{\text{opt}}} = \beta \left(\phi \frac{K_t}{Y_t} + \frac{\psi(1-\alpha-\beta)}{Bu_t} \right)^{-1}. \tag{39}$$

In the previous section, we have shown that Y_t/K_t and u_t appearing in the RHS of (39) become constant in the steady-state growth path, and their values are derived explicitly in terms of g^τ (see equations 26-28 and 31-32). Therefore, the welfare-maximizing amount of polluting input P^{opt} in the long run can be calculated as a function of g_τ. As depicted in Figure 2, P^{opt} is positive and continuous in g_τ for all $g_\tau \in [0, g^{\text{lim}})$.

The actual amount of polluting inputs used in equilibrium is, however, determined not by (39) but by (26) for $g_\tau < g^{\max}$ and $P^* = 0$ for $g^* \geq g^{\max}$, which is also depicted in Figure 2. Unless consumers heavily discount the utility in the future, it can be shown that the intercept of P^{opt} curve is smaller than that of P^*. Therefore, there exists a level of $g_\tau \in (0, g^{\max})$, denoted as g_τ^{opt}, such that $P^* = P_t^{\text{opt}}$ in the long run. This means that the optimal growth path can be (asymptotically) implemented by raising the environmental tax rate at the rate of g_τ^{opt}.

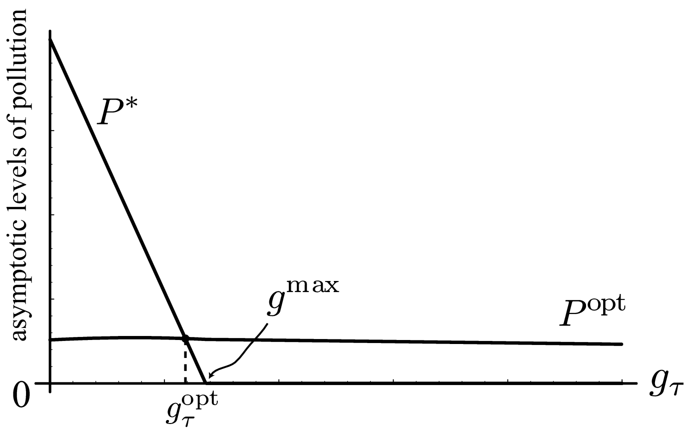

Figure 2: Determination of the optimal growth rate of environmental tax. The optimal growth rate of environmental tax, g_τ^{opt}, is given by the intersection of P^* and P^{opt}, and is lower than the growth maximizing rate, $g^{\max}$. The parameters are the same as in Figure 1.

Note that, since $g_\tau^{\text{opt}} \in (0, g^{\max})$, the long-term rate of economic growth g^* coincides with g_τ^{opt}, and therefore it is positive. Thus the above analysis shows that the sustained growth implemented by raising the environmental tax rate is not only feasible but also desirable. It is also notable, however, that the optimal policy does not coincide with the growth maximizing policy since $g_\tau^{\text{opt}} < g^{\max}$. That is, if the government care about welfare it should tighten the environmental policy more slowly than when growth is its first priority. This result may seem at odds with the usual growth vs. environment arguments, but its reasoning is similar to the modified golden rule argument familiar to economists. Although an aggressive environmental policy that aims to eliminate the emission of pollutants in the long run (i.e., $P^* = 0$) may maximize the economic growth rate in the very long run, the cost in the form of reduced effective productivity that must be incurred in the transition can overwhelm the benefit that cannot be reaped in far future.

4.1 Disutility of pollution

Our result that the sustained growth is both feasible and desirable is in contrast to the previous literature. Notably, Stokey (1998) has shown that even when sustained growth is feasible, it is not desirable when production of goods emits pollutants that harm the utility of consumers. The difference of the results of course comes from the setting of models. More specifically, our model significantly differs from Stokey (1998) in two aspects; (1) we are considering human capital accumulation differently from Stokey's AK model; (2) so far pollutants are assumed to cause disasters but do not directly give disutility.

In this subsection, we clarify that the critical reason behind the difference in the results is not (ii) but (i). To this end, we present an extended model in which agents suffer from not only damages to capital stocks caused by natural disasters but also disutility of pollution. Suppose that consumers has an utility function of

$$\int_0^\infty \left(\frac{c_t^{1-\theta}-1}{1-\theta} - \frac{P_t^{1+\gamma}}{1+\gamma} \right) e^{-\rho t} dt, \quad \gamma > 0. \tag{40}$$

Since function (40) is separable with respect to c_t and P_t, behavior of all agents, who takes P_t as given, does not change. That is, the equilibrium outcome is exactly the same as in analyzed in sections 2 and 3.

Let us examine how the planner's problem is affected. Under resource constraints (35) and (36), the planner maximizes function (40). Then, the optimality condition with respect to polluting input becomes

$$\frac{\beta Y_t}{P_t} = \phi K_t + \psi H_t \cdot \frac{(1-\alpha-\beta) Y_t}{B u_t H_t} + \frac{P_t^\gamma}{C^{-\theta}}. \tag{41}$$

When compared to (37), there is an additional marginal cost of polluting input in terms of utility. Multiplying both sides of (41) by P_t/Y_t yields the condition

$$\beta = \left(\phi \frac{K_t}{Y_t} + \psi \frac{(1-\alpha-\beta)}{B u_t} \right) P_t + \left(\frac{C_t}{Y_t} \right)^\theta Y_t^{\theta-1} P_t^{1+\gamma}, \tag{42}$$

which is convenient since the LHS is constant.

Our goal is to find the environmental tax policy such that (42) holds in the long run. Recall that on any steady-state growth path, K_t/Y_t, u_t, and C_t/Y_t in (42) do not change. Therefore, for the optimality condition to hold, $Y_t^{\theta-1} P_t^{1+\gamma}$ must be constant. Since $\dot{P}_t/P_t = g^* - g_\tau$ from (18), this condition requires $(\theta-1)g^* + (1+\gamma)(g^* - g_\tau) = 0$, or equivalently

$$g^* = \frac{1+\gamma}{\theta+\gamma} g_\tau. \tag{43}$$

Observe that, since $\theta > 1$, condition (43) requires that the economy should grow slower than the rate at which the environmental tax is increased.[11] As we see in section 3, this occurs in equilibrium only when $g_\tau > g^{\max}$. In this case, the actual value of g^* is determined by (30), which equals to (43) if and only if

$$\widehat{g}_\tau^{\text{opt}} = \frac{(\gamma+\theta)(1-\alpha-\beta)(B-\delta_H-\rho)}{\theta(1+\gamma)(1-\alpha-\beta)+\beta(\gamma+\theta)}.$$

This expression gives welfare-maximizing growth rate of environmental tax rate: Since $\widehat{g}_\tau^{\text{opt}}$ is positive and smaller than $\widetilde{g}_\tau$ and $g^{\lim}$, the optimal rate of growth is positive and the dynamics is saddle stable.

To summarize, the main result that sustained growth is desirable does not change even when disutility of pollution is introduced into the model. However, the desirable speed at which the environmental tax is increased is now higher than the growth maximizing speed, $g^{\max}$. This implies that, if pollution affects the utility of agents directly, the emission of pollutant should be eliminated in the long run even at the cost of accepting a slower (although positive) rate of economic growth.

5 Concluding Remarks

The sustainability of economic growth has been analyzed in a two-sector model of endogenous growth, taking into account the risk of natural disasters. Polluting inputs are necessary for production, but they intensify the risk of natural disasters. In this setting, we obtained following results.

First, the long-run economic growth can not be sustained if the private cost of using the polluting input is kept constant.[12] Since, for simplicity, we do not consider the cost associated with extracting resources or the finiteness of those inputs,

[11] Strictly speaking, condition (43) does not rule out $g^* = g_\tau = 0$. However, similarly to the previous discussion, if the tax rate is kept constant, P^* is higher than the optimal amount of pollutants (this should be even lower due to the disutility of pollution) unless the consumers discount the future very heavily. Therefore, the optimal rate at which the tax rate is increased must be positive.

[12] When the damage to physical capital stock is much larger than that to human capital stock

this result implies that the environmental tax rate should be increased over time. However, it should be noted that if the private cost changes for some ignored reasons, the environmental tax rate must be adjusted to absorb those changes. More substantially, a next step in the research agenda would be to integrate the analysis of natural disasters with the studies of finiteness of natural resources, although it is beyond the scope of this first endeavor.

Second, the rate of the economic growth rate follows the inverted V-shaped curve relative to the growth rate of the environmental tax. When the rate of environmental tax is initially slow growing, its acceleration will reduce the long-run level of emission and the risk of natural disasters, which enhances the incentive to save and hence promotes economic growth. When the rate of environmental tax is already fast growing, the amount of polluting input at the steady state is fairly small so that further acceleration of environmental tax excessively impair the productivity of private firms, which works against economic growth. Therefore, the economic growth can be maximized with choice of the most gradual increase in environmental tax rate that minimizes the amount of pollution in the long-run.

Third, the sustained growth, realized by ever increasing tax rate on polluting inputs, is not only feasible but also desirable. Although economic growth *ceteris paribus* induces private firms to use more polluting input, an appropriate environmental policy can lead firms to use more of human capital (e.g., by investing in alternative technology), which decreases their reliance on polluting inputs. The optimal speed at which the environmental tax rate is increased is lower than the growth maximizing speed if pollution only causes disasters, while it is higher when direct disutility from pollution is accounted for.

(i.e., $\phi >> \psi$), the steady-state value of P^* is rather large (see Equation 26). In this case, the speed of convergence to the steady state is slow, the economic growth declines gradually, and amount of pollution increases during the transition to the steady state.

Appendix

Case 1

First, we examine the transitional dynamics and the stability of the steady state in Case 1. From (14), (15), and (16), we obtain the dynamics of u as follows:

$$\frac{\dot{u}}{u} = B(u-u^*) - (\chi-\chi^*) + \beta(z-z^*) + \frac{(1-\alpha-\beta)(\phi-\psi)}{\alpha}(P-P^*), \tag{44}$$

where z^*, χ^*, u^*, and P^* are the steady state value of z, χ, u, and P, respectively. From (14) and (17), the dynamics of χ is given by:

$$\frac{\dot{\chi}}{\chi} = (\chi-\chi^*) - \frac{\theta-\alpha}{\theta}(z-z^*) + \frac{(\theta-1)\phi}{\theta}(P-P^*). \tag{45}$$

From (11) and (44), the dynamics of z and of P are given by:

$$\frac{\dot{z}}{z} = -(1-\alpha-\beta)(z-z^*) + \frac{(1-\alpha-\beta)(\psi-\phi)}{\alpha}(P-P^*), \tag{46}$$

$$\frac{\dot{P}}{P} = -(\chi-\chi^*) + \frac{\alpha+\beta(1-\alpha-\beta)}{1-\beta}(z-z^*) + \frac{(1-2\alpha-\beta)\phi-(1-\alpha-\beta)\psi}{\alpha}(P-P^*). \tag{47}$$

The Jacobi matrix of (44) - (47) is written as:

$$J^1 = \begin{bmatrix} B & -1 & \beta & \Lambda \\ 0 & 1 & -\frac{\theta-\alpha}{\theta} & \frac{(\theta-1)\phi}{\theta} \\ 0 & 0 & -(1-\alpha-\beta) & \Lambda \\ 0 & -1 & \frac{\alpha+\beta(1-\alpha-\beta)}{1-\beta} & \Omega \end{bmatrix},$$

where

$$\Lambda \equiv \frac{(1-\alpha-\beta)(\phi-\psi)}{\alpha}, \quad \text{and} \quad \Omega \equiv \frac{(1-2\alpha-\beta)\phi-(1-\alpha-\beta)\psi}{\alpha}.$$

Define a matrix as follows:

$$J^* = \begin{bmatrix} 1 & -\frac{\theta-\alpha}{\theta} & \frac{(\theta-1)\phi}{\theta} \\ 0 & -(1-\alpha-\beta) & \Lambda \\ -1 & \frac{\alpha+\beta(1-\alpha-\beta)}{1-\beta} & \Omega \end{bmatrix}.$$

The eigenvalues of J^* are the solutions of its characteristic equation:

$$-\lambda^3 + \mathrm{Tr}J^*\lambda^2 - \mathrm{B}J^*\lambda + \mathrm{Det}J^* = 0, \tag{48}$$

where

$$\begin{aligned}
\mathrm{Tr}J^* &= \Lambda + (\alpha + \beta - \phi) > 0, \qquad \text{if} \quad \alpha + \beta > \phi \\
\mathrm{B}J^* &= \begin{vmatrix} 1 & -\frac{\theta-\alpha}{\theta} \\ 0 & -(1-\alpha-\beta) \end{vmatrix} + \begin{vmatrix} -(1-\alpha-\beta) & \Lambda \\ \frac{\alpha+\beta(1-\alpha-\beta)}{1-\beta} & \Omega \end{vmatrix} + \begin{vmatrix} 1 & \frac{(\theta-1)\phi}{\theta} \\ -1 & \Omega \end{vmatrix}, \\
&= \frac{\theta(1-\alpha-\beta)(\phi-1) - \phi}{\theta} < 0, \\
\mathrm{Det}J^* &= \frac{\psi(1-\alpha-\beta)}{\theta} > 0.
\end{aligned}$$

We determine the sign of the real parts of the roots of (48) based on Theorem 1 of Benhabib and Perli (1994).

Theorem 1 (Benhabib-Perli) *The number of roots of the polynomial in (48) with positive real parts is equal to the number of variations of sign in the scheme*

$$-1 \qquad TrJ^* \qquad -BJ^* + \frac{DetJ^*}{TrJ^*} \qquad DetJ^*.$$

Since we have $\mathrm{Tr}J^* > 0$, $-\mathrm{B}J^* + \frac{\mathrm{Det}J^*}{\mathrm{Tr}J^*} > 0$, and $\mathrm{Det}J^* > 0$, there can only be one positive real parts in the matrix J^* under the assumption $\alpha + \beta > \phi$. Hence, there can be two positive eigenvalues and two eigenvalues with negative real parts in the matrix J^1 and the steady state is saddle path stable.

Case 2

Turning to Case 2, the transitional dynamics of z, χ, u, and P are given by:

$$\frac{\dot{u}}{u} = Bu - \chi + \beta z + \Lambda P + \frac{1-\alpha-\beta}{\alpha}(B + \delta_K - \delta_H) - \frac{\beta}{\alpha}g_\tau, \tag{49}$$

$$\frac{\dot{\chi}}{\chi} = \chi - \frac{\theta-\alpha}{\theta}z + \frac{\theta-1}{\theta}\phi P - \frac{\rho}{\theta} + \frac{\theta-1}{\theta}\delta_K, \tag{50}$$

$$\frac{\dot{z}}{z} = -(1-\alpha-\beta)z + \Lambda P + \frac{1-\alpha-\beta}{\alpha}(B + \delta_K - \delta_H) - \frac{\beta}{\alpha}g_\tau, \tag{51}$$

$$\begin{aligned}
\frac{\dot{P}}{P} &= -\chi + \frac{\alpha + (1-\alpha-\beta)\beta}{1-\beta}z + \Omega P + \frac{1-\alpha-\beta}{\alpha}B - \frac{\alpha+\beta}{\alpha}g_\tau \\
&\quad + \frac{(1-2\alpha-\beta)\delta_K - (1-\alpha-\beta)\delta_H}{\alpha}.
\end{aligned} \tag{52}$$

Linearizing from (49) to (52) around the steady state and substituting $P^* = 0$ and other steady state values into the linearization, we obtain:

$$J = \begin{bmatrix} \dot{u} \\ \dot{\chi} \\ \dot{z} \\ \dot{P} \end{bmatrix} \simeq \begin{bmatrix} J_{11} & * & * & * \\ 0 & J_{22} & * & * \\ 0 & 0 & J_{33} & * \\ 0 & 0 & 0 & J_{44} \end{bmatrix} \begin{bmatrix} u - u^* \\ \chi - \chi^* \\ z - z^* \\ P - P^* \end{bmatrix},$$

where

$$\begin{aligned} J_{11} &= \frac{\theta - 1}{\theta}(B - \delta_H) - \frac{\theta - 1}{\theta(1 - \alpha - \beta)}\beta g_\tau + \frac{\rho}{\theta}, \\ J_{22} &= \frac{\theta - \alpha}{\alpha\theta}(B - \delta_H) - \frac{(\theta - \alpha)\beta}{\alpha\theta(1 - \alpha - \beta)}g_\tau + \frac{\rho}{\theta}, \\ J_{33} &= -\frac{1 - \alpha - \beta}{\alpha}(B - \delta_H) + \frac{\beta}{\alpha}g_\tau + \frac{1 - \alpha}{\alpha}\delta_K, \\ J_{44} &= \frac{1}{\theta}(B - \delta_H) - \frac{\rho}{\theta} - \frac{\theta(1 - \alpha - \beta) + \beta}{\theta(1 - \alpha - \beta)}g_\tau, \end{aligned}$$

Substituting $g^{\max}$ into g_τ of the above eigenvalues, we obtain:

$$J_{11} > 0, \qquad J_{22} > 0, \qquad J_{33} < 0, \qquad J_{44} = 0.$$

Moreover, substituting $g^{\lim}$ into g_τ, the above eigenvalues are:

$$J_{11} > 0, \qquad J_{22} > 0, \qquad J_{33} = 0, \qquad J_{44} < 0.$$

Thus, for $g_\tau \in (g^{\max}, g^{\lim})$, there are two positive eigenvalues and two eigenvalues with negative real parts in the matrix J and the steady state is saddle path stable.

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