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Does the Diversity of Human Capital Increase GDP? A Comparison of Education Systems[*]

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Abstract

This paper examines how different education systems affect GDP by influencing the diversity of human capital. We construct an overlapping generation model in which agents are heterogeneous in income and innate ability, and the final goods are produced with differentiated intermediate goods. We analyze an economy in which an income distribution converges to a stationary distribution. It is shown that the diversity of human capital induced by income inequality always lowers the GDP of the next period, while the diversity of human capital induced by heterogeneous ability can increase GDP, if the produced intermediate goods are sufficiently substitutable and firms have a large span of control. Hence, as public education equalizes education resources across households, it mitigates the negative effect of income inequality on GDP, while the effects of ability tracking crucially depend on the production structure of the economy.

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1 Introduction

Economists typically consider that education can improve the human capital of workers and raise GDP. Several researchers estimate the *level* of human capital from education attainment and examine the impact of human capital on GDP or economic growth [e.g., Mankiw, Romer and Weil, 1992]. On the other hand, relatively little is known about the effect of the *diversity* of human capital on GDP.

Apparently, the diversity of human capital differs across countries. Several recent international surveys reveal this variation. Although different surveys compare different abilities at different ages, some common tendencies can be found in the surveys.[1] Brown, Micklewright, Schnepf and Waldmann (2005) find that among 18 OECD countries, results from three surveys (Trends in International Mathematics and Science Study, Programme for International Student Assessment, and International Adult Literacy Survey) consistently indicate that Finland and the Netherlands have relatively small inequalities of achievements; the United Kingdom, New Zealand and the USA have relatively large inequalities of achievements.[2]

How does this diversity of human capital influence GDP? The impor-

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[1] Trends in International Mathematics and Science Study (TIMSS) covers achievement in mathematics and science for early or middle teens, the Programme for International Student Assessment (PISA) covers achievement in reading, mathematics and science for early or middle teens, the International Adult Literacy Survey (IALS) examines 'document', 'prose' and 'quantitative' literacy for all people of working age, and the Progress in International Reading Literacy Study (PIRLS) examines the reading skills of young children.

[2] Brown, Micklewright, Schnepf and Waldmann (2005) compare the difference between the 95th percentile and 5th percentile of achievement distributions for 18 OECD countries, converted from the data in TIMSS, PISA, and the late teens and early 20s in IALS. The 18 countries include Finland, Denmark, Netherlands, Canada, Czech Republic, Sweden, Australia, Portugal, Norway, Germany, Ireland, Italy, Belgium, Hungary, Switzerland, UK, New Zealand and USA. The countries are ordered from the most equal achievement to the least equal achievement using the average ranking of the three surveys.

tance of this question can be understood when we recognize that one of the central aims of an education policy is to provide students with equal education resources. For example, several reforms have been conducted to achieve equity in education outcomes in the United States. The 1971 landmark decision in Serrano v. Priest transformed the public education system in California, and other states (e.g., Michigan in 1994 and Washington in 1979) have also centralized their education systems in order to achieve equity in education resources. More recently, the "No Child Left Behind Act" by the George W. Bush administration aims to achieve equity in and a high quality of education by raising the performance of the lowest achieving students. Hence, the previous question leads us to ask a more important question: can an egalitarian education policy raise GDP?

The answer is not obvious. On the one hand, if a government fails to provide everybody with enough literacy skills, it would be difficult for workers to communicate and cooperate with each other. On the other hand, as top managers' decisions are influential in a company, we want them to understand the varieties of opinions and to make sound decisions. Hence, some may insist that an education policy should target the bottom of ability distribution; others may advocate the importance of education for the elite.

In order to evaluate the impact of an egalitarian education policy on GDP, we need a model to link education reform, the diversity of human capital and GDP in a unified framework. This paper aims to accomplish this task. It constructs an overlapping generation model in which education systems influence GDP by changing the variance in human capital and compares alternative education systems by their effects on GDP.

This model is distinguished from the previous literature in two aspects. First, we tractably parameterize the structure of industries and firms and

examine how an education system and the production structure of an economy have an interactive effect on GDP. In particular, this paper pays special attention to the span of control in a firm and the complementarity of goods in an industry. A large span of control gives an individual the authority to reallocate large amounts of resources. Without authority, an able person cannot fully utilize his/her unusual talents. Hence, a high level of control favors an education system that produces a few highly educated workers. If complementarity of goods exists, the value of a firm's product depends on other firms' product, and a good produced by incompetent persons may reduce the value of other firms' product. Hence, high complementarity of goods demands an education system that produces many reasonably well-trained workers.[3]

Secondly, different from the previous literature that analyzes education policies in a dynamic general equilibrium model, education systems are characterized not only by their financing systems, but also by their ability-tracking programs. Hence, education systems change the way the heterogeneities of both income and ability influence the diversity of human capital. A private education system yields more diverse human capital than a public education system because the rich spend more on education than the poor.[4]

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[3]For example, in a financial market, fund managers are allowed to allocate a large amount of resources to buy different stocks that are highly substitutable. It is likely to demand unusual talent. On the other hand, firms in the car industry need to combine a number of complementary intermediate goods (e.g., the quality of tires is likely to influence the value of brakes). This might demand many well-trained workers. The level of control in the car industry would be influenced by the structure in the firm. If intermediate good sectors are less vertically integrated, or bottom-up decision making is common, the level of control would be low. Again, it must demand reasonably well-trained workers. It is interesting to note that the structure of industries and firms in the U.S., which is regarded as having relatively heterogeneous human capital, seems to relatively favor unusual talent, while that in Japan, which is seen as having a relatively homogeneous human capital, relatively favors reasonably well-trained workers.

[4]A private education system is defined as an education system in which individuals finance the costs of education, and a public system is defined as an education system in

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On the other hand, ability tracking students into separate groups according to their ability restricts those with whom they can interact as schoolmates or classmates. Since advantaged students interact with advantaged students, ability-tracking benefits advantaged students more than disadvantaged students through the peer effects. Hence, a streamed program yields more diverse human capital than an untracked program by amplifying the benefits from high innate ability.

This paper analyzes an aggregate economy in which an income distribution converges to a stationary distribution, and shows that a public system yields higher GDP than a private system regardless of industry and firm structure, while the effect of an ability-tracking program on GDP depends on the production structure. As far as the variance of log income converging to a finite value, this paper shows that, given a current GDP and an ability distribution, a larger income difference reduces GDP at the next period. Since the public system always lowers income inequality more than the private system through the redistribution of income, it always attains a higher GDP than the private system.

A similar mechanism is emphasized in the previous literature when the human capital accumulation function is concave in expenditure on education and the production function is linear in human capital (e.g., Loury, 1981, and Glomm and Ravikumar, 1992). Our result shows that this still holds even if the diversity of human capital increases GDP on the production side.

However, when the diversity of human capital is enhanced by ability tracking, the structure of the production side becomes important. Different from the dynamics of income distribution, the distribution of ability is exogenously given and the variance of ability is always finite. It is shown

which revenues from labor income tax are assumed to finance the costs.

that a rise in inequality in ability can increase GDP at the next period. It is also shown that an ability-tracking program can attain higher GDP if goods in an industry are fairly substitutable and if the span of control in a firm is sufficiently large. Hence, the private education system may increase GDP if a private school has more incentives or advantages to screen students through its entrance examination.[5] This result highlights a distinctive role of ability tracking in macroeconomics.

This paper is based on the literature that compares the performance of different education systems in a dynamic general equilibrium model (e.g., Glomm and Ravikumar, 1992, Bénabou, 1996 and Fernández and Rogerson, 1998). These papers compare different financing methods for education. In particular, Bénabou (1996) examines the effect of diversity of human capital on economic growth when the human capitals of individual agents have interactive effects on GDP. His main focus is to examine the role of complementarity of human capital at the community level and at the production level. By contrast, we do not consider a local interaction at the community level and pay more attention to the interaction at a production level. In particular, we explicitly examine the role of complementarity of products and a manager's span of control to compare education systems. We also explicitly analyze the effect of ability sorting on GDP.

Ability tracking has been examined by Epple, Newlon and Romano (2002) and Brunello, Giannini and Ariga (2004). Although these papers examine the benefits and costs of ability tracking, they do not examine the role of the production structure. The interaction between an education system and the structure of an industry and firm is the main focus of our

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[5]A model in Epple and Romano (1998) predicts that a private school attracts more able students than a public school.

paper.

The rest of this paper is organized as follows. Section 2 describes the model, and Section 3 shows some results about the relationship between income inequality and GDP. Section 4 compares the two education systems, the public and the private. Section 5 considers the case with ability tracking in public schools. Section 6 discusses some extensions and concludes. All proofs are in the Appendix.

2   The Model

In this section, we construct an overlapping generation model with human capital accumulation. The model features complementarities among intermediate goods in the final good production and the span of control in the production of intermediate goods. Individuals are required to participate in both tax-financed (public) schools and self-financed (private) schools in their life. In our benchmark model, private schools are allowed to sort students based on their ability, while public schools are not. This assumption is relaxed later. Using this model, we derive the dynamics of individual income and examine its aggregate behavior in the next section.

2.1   Technology

Consider an economy in which there is only one final (numeraire) good. The final good is produced by combining intermediate goods $\{x^i\}$ where i is the index for ith intermediate good. The production function of the final good is given by:

$$Y_t = \left[\int (x_t^i)^\rho di \right]^{1/\rho}, 0 < \rho < 1,$$

where x_t^i is the amount of the intermediate good i at date t.[6]

The final good producers sell their products in a competitive market. Taking the prices of the intermediate goods $\{p_t^i\}$ as given, a (representative) final good producer chooses $\{x_t^i\}$ to maximize profits:

$$\max \left[\int (x_t^i)^\rho di \right]^{1/\rho} - \int p_t^i x_t^i di.$$

The first order necessary conditions for profit maximization imply that the demand function for ith good at date t is:

$$p_t^i = Y_t^{1-\rho}(x_t^i)^{\rho-1}. \tag{1}$$

Intermediate goods are produced by monopolistically competitive producers. Specifically, the production function of the intermediate good i is given by:

$$x_t^i = h_t^i (k_t^i)^\lambda, 0 < \lambda < 1, \tag{2}$$

where h_t^i is the human capital of the producer i at date t and k_t^i is the physical capital employed by the producer i at date t. This production function captures the idea of the span of control a la Lucas (1978) in the sense that the marginal product of capital declines as capital accumulates. Hence, it is not productive for one manager to manage all capital. Note that the larger is λ, the larger is the marginal product of capital. That is, a larger λ allows the manager to productively operate more capital. Hence, λ measures the degree of the span of control.

For a given (gross) interest rate r_t and his/her own human capital h_t^i, the intermediate good producer i chooses $\{k_t^i\}$ to maximize his/her profits:

$$\max p_t^i x_t^i - r_t k_t^i,$$

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[6]The elasticity of substitution of intermediate goods is given by $1/(1-\rho) > 1$.

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subject to the demand function (1) and the production function (2). From the first order necessary conditions, we have the demand for physical capital by the producer i:

$$k_t^i = \left[\frac{\lambda \rho}{r_t} Y_t^{1-\rho} (h_t^i)^{\rho} \right]^{\frac{1}{1-\lambda\rho}},$$

and thus the profit function of the producer i is given by:

$$\pi_t^i = (1 - \lambda\rho) \left(\frac{\lambda\rho}{r_t} \right)^{\frac{\lambda\rho}{1-\lambda\rho}} Y_t^{\frac{1-\rho}{1-\lambda\rho}} (h_t^i)^{\frac{\rho}{1-\lambda\rho}}. \tag{3}$$

Note that for a given interest rate and total output, the producer i produces the intermediate good i by the following amount:

$$x_t^i = \left[\frac{\lambda\rho}{r_t} Y_t^{1-\rho} \right]^{\frac{\lambda}{1-\lambda\rho}} (h_t^i)^{\frac{1}{1-\lambda\rho}},$$

and thus the total output of the economy at date t is given by:

$$Y_t = \left(\frac{\lambda\rho}{r_t} \right)^{\frac{\lambda}{1-\lambda}} H_t^{\frac{1}{1-\lambda}}, \tag{4}$$

where $H_t = \left[\int (h_t^i)^{\frac{\rho}{1-\lambda\rho}} di \right]^{\frac{1-\lambda\rho}{\rho}}$ is the aggregate level of human capital.

Note that if the elasticity of substitution in intermediate goods in the final good production and/or the degree of the span of control are so large that $\frac{\rho}{1-\lambda\rho} > 1$ (i.e., $\rho > \frac{1}{1+\lambda}$), H_t is *increasing* in the variance of human capital. One of the immediate implications of this observation is that the heterogeneity in human capital across intermediate good producers is a source of gains from society's point of view if the intermediate goods are less complementary in the production of the final good, and/or the degree of the span of control is large enough.

Let Π_t denote the total profit of the intermediate producers, that is, $\Pi_t \equiv \int \pi_t^i di$. The relationship between total output and total profit is $\Pi_t = (1 - \lambda\rho)Y_t$. Hence, the share of physical capital is $1 - \lambda\rho$, and the share of intermediate good producers is $\lambda\rho$.

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2.2 Preferences and Human Capital Formation

There is a continuum of overlapping-generation families i of unit measure. In each period, each family consists of one child, one young adult and one aged person. Each individual lives for three periods. In the first period, children obtain an education while they are still supported by their parents. In the second period, young individuals work, spend part of their income for current consumption and the education of their children, and save the rest. In the third period, old individuals consume their savings.

Human capital is formed with individual ability ξ_{t+1}^i (i.i.d. with mean 1), and public and private schooling. It is assumed that a child must spend $1 - \theta$ fraction of his/her time at a public school and θ fraction of his/her time at a private school. The production function of human capital is given by:

$$h_{t+1}^i = (\xi_{t+1}^i)^\phi \left[(R_t^i)^\theta (U_t)^{1-\theta} \right]^\alpha, \quad \phi > 0, \quad 0 \le \theta \le 1, \quad 0 < \alpha < 1,$$

where U_t and R_t^i are the quality of public and private education, respectively. The quality of schooling consists of education expenditure *and* the average ability (that is, a measure of the quality of a peer group) in each type of school:

$$U_t = (\bar{\xi}_{t+1}^{Pub})^\nu g_t, \quad R_t^i = (\bar{\xi}_{t+1}^{Pri,i})^\nu e_t^i, \quad \nu \ge 0,$$

where $\bar{\xi}_{t+1}^{Pub}$ is the average ability of students in public school, g_t is the public education expenditure per pupil, $\bar{\xi}_{t+1}^{Pri,i}$ is the average ability of students in the private school the child of household i attends, and e_t^i is the tuition cost of this private school. A similar specification of human capital formation can be found in, for example, Bénabou (1996), Epple and Romano (1998), Nechyba (2000, 2003), Epple, Newlon and Romano (2002), and Brunello,

Giannini and Ariga (2004).

The quality of a peer group in each type of school crucially depends on the feasibility of sorting based on student ability. As a benchmark, we consider the case in which private schools are allowed to choose students based on their ability (and thus students are perfectly sorted in school by ability), but public schools are not. This is a standard case studied by, for example, Epple and Romano (1998), and we think that this is a plausible case because private schools have more incentives and advantages to screen students through their entrance exams than public schools. However, we will discuss how our results change if public schools are allowed to sort students based on ability in a later section.

The quality of a peer group in each type of school is determined as follows. On the one hand, since students are randomly assigned to a public school, the average ability in each public school is the same as the population mean (that is, $\bar{\xi}_{t+1}^{Pub} = 1$). On the other hand, since students are assumed to be sorted in the private school by ability, the average ability of students in a private school for students with ability ξ_{t+1}^i is given by $\bar{\xi}_{t+1}^{Pri,i} = \xi_{t+1}^i$. Hence, the reduced form of the production function of human capital is given by:

$$h_{t+1}^i = (\xi_{t+1}^i)^{\phi+\alpha\theta\nu} \left((e_t^i)^\theta (g_t)^{1-\theta}\right)^\alpha. \tag{5}$$

The utility maximization problem of the young individual in household i is characterized as follows. The young individual of household i chooses consumption when young $c_t^{y,i}$, his/her child's private education expenditure e_t^i and consumption when old $c_{t+1}^{o,i}$ to maximize his/her utility:

$$\max \ln c_t^{y,i} + \beta \ln h_{t+1}^i + \gamma \ln c_{t+1}^{o,i},$$

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subject to the budget constraint:

$$c_t^{y,i} + e_t^i + s_t^i = (1 - \tau_t)\pi_t^i, \quad c_{t+1}^{o,i} = r_{t+1}s_t^i,$$

and the production function of human capital (5), where $\tau_t \in [0,1]$ is a tax rate for public education and $g_t = \tau_t \int \pi_t^i di$ is per capita public education expenditure.

From this utility maximization problem, we have the optimal private education and saving for the young individual i at date t:

$$e_t^i = \frac{\alpha\beta\theta}{1 + \gamma + \alpha\beta\theta}(1 - \tau_t)\pi_t^i, \quad s_t^i = \frac{\gamma}{1 + \gamma + \alpha\beta\theta}(1 - \tau_t)\pi_t^i.$$

Hence, each household allocates constant fractions of his/her own disposable income to each current and future consumption and the private education of his/her child.

2.3 Physical Capital Market Clearing

The total supply of physical capital at date $t + 1$ (call it K_{t+1}) is given by the total saving at date t:

$$K_{t+1} \equiv \int s_t^i di = \int \frac{\gamma(1 - \tau_t)}{1 + \gamma + \alpha\beta\theta}\pi_t^i di = \frac{\gamma(1 - \tau_t)}{1 + \gamma + \alpha\beta\theta}\Pi_t. \qquad (6)$$

On the other hand, the total demand for physical capital at date $t + 1$ is given by:

$$\int k_{t+1}^i di = \int \left[\frac{\lambda\rho}{r_{t+1}}Y_{t+1}^{1-\rho}(h_{t+1}^i)^\rho\right]^{\frac{1}{1-\lambda\rho}} di = \frac{\lambda\rho}{r_{t+1}}Y_{t+1}.$$

Hence, the market-clearing interest rate at date $t + 1$ is given by:

$$r_{t+1} = \lambda\rho\frac{Y_{t+1}}{K_{t+1}} = \lambda\rho Y_{t+1}\frac{1 + \gamma + \alpha\beta\theta}{\gamma(1 - \tau_t)}\Pi_t^{-1}. \qquad (7)$$

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2.4 Choice of Tax Rate

Each young individual votes for a tax rate that maximizes his/her lifetime utility. We assume that when voters choose their favorite tax rate, they take the future interest rate as given.[7] Hence, they vote for a tax rate that resolves the trade-off between the benefit from and the tax burden for public education.

The indirect lifetime utility of the young individual in the family i is given by:

$$v(\tau_t; \pi_t^i, \xi_{t+1}^i, \Pi_t, r_{t+1}) = A(\pi_t^i, \xi_{t+1}^i, \Pi_t, r_{t+1}) + (1+\alpha\beta\theta+\gamma)\ln(1-\tau_t) + (1-\theta)\alpha\beta\ln\tau_t,$$

where $A(\pi_t^i, \xi_{t+1}^i, \Pi_t, r_{t+1}) = (1+\alpha\beta\theta+\gamma)\ln\pi_t^i + \beta\phi\ln\xi_{t+1}^i + \alpha\beta(1-\theta)\ln\Pi_t + \gamma\ln r_{t+1} + \ln\left[(\alpha\beta\theta)^{\alpha\beta\theta}\gamma^\gamma/(1+\alpha\beta\theta+\gamma)^{1+\alpha\beta\theta+\gamma}\right]$.

The favorite tax rate of the household with income y_i is given by:

$$\tau_t^* = \frac{\alpha\beta(1-\theta)}{1+\alpha\beta+\gamma}. \tag{8}$$

Note that the favorite tax rate is independent of household income (and any other household characteristics). Although we assume that the tax rate is chosen by majority voting, equation (8) implies that $\tau_t = \tau_t^*$ is chosen by unanimity no matter what the income distribution. That is, equation (8) shows that the tax rate chosen in this paper is constant over time.[8]

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[7]In principle, the choice of tax rate affects the next period interest rate through the aggregate human capital in the next period. Here we assume that voters ignore this effect of tax choice.

[8]If only the young individuals have the voting right, then τ_t^* is chosen by unanimity. Even if both the young and the old individuals are franchised, $\tau_t = \tau_t^*$ is chosen by a majority because τ_t^* is always supported by at least half of the population (that is, all the young individuals always support $\tau_t = \tau_t^*$).

3 Evolution of Income Distribution

In this section, we study the evolution of income distribution using the model described in the previous section. In particular, we show that the heterogeneity in household income reduces the GDP of the next period, but the effects of the heterogeneity in ability can be positive or negative depending on the production structure. These results are crucial to understanding the comparison of education systems in the next section.

3.1 Distribution Dynamics

To study the evolution of an income distribution, we track the dynamics of the distribution of (pretax) profit income of young individuals (π_t^i), because this corresponds to the present value of lifetime (pretax) income in our model (hereafter, we call it simply "income").

For the given total output in period $t+1$, the tax rate and the total (profit) income in period t, and the income of the young individual i in period t, the income of his/her child (and thus the income of the young individual i in the next period) π_{t+1}^i is given by:

$$\pi_{t+1}^i = (\xi_{t+1}^i)^{\frac{(\phi+\alpha\theta\nu)\rho}{1-\lambda\rho}} (\pi_t^i)^{\frac{\alpha\theta\rho}{1-\lambda\rho}} Y_{t+1}^{\frac{1-\rho(1+\lambda)}{1-\lambda\rho}} \Pi_t^{\frac{\rho[\lambda+\alpha(1-\theta)]}{1-\lambda\rho}} \bar{B}, \qquad (9)$$

where $\bar{B} \equiv (1-\lambda\rho) \left(\frac{\gamma^\lambda (\alpha\beta\theta^\theta(1-\theta)^{1-\theta})^\alpha}{(1+\gamma+\alpha\beta)^{\alpha+\lambda}} \right)^{\frac{\rho}{1-\lambda\rho}}$.

In order to characterize the dynamics of income distribution, we assume that income and ability are distributed across individuals by the following log-normal distributions: $\ln \pi_t^i \sim N(m_t, \Delta_t^2)$ and $\ln \xi_{t+1}^i \sim N(-\sigma^2/2, \sigma^2)$. Under these assumptions, income of the next period is also distributed by the log-normal distribution $\ln \pi_{t+1}^i \sim N(m_{t+1}, \Delta_{t+1}^2)$. The mean and the

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variance of log income in the next period are given by:

$$
\begin{aligned}
m_{t+1} = & (\alpha + \lambda)m_t + \left[\frac{1-\rho(1+\lambda)}{\rho}\frac{(\alpha\theta\rho)^2}{(1-\lambda\rho)^2} + \lambda + \alpha(1-\theta)\right]\frac{\Delta_t^2}{2} \\
& + (\phi + \alpha\theta\nu)\left[\frac{1-\rho(1+\lambda)}{\rho}\frac{\rho^2(\phi+\alpha\theta\nu)}{(1-\lambda\rho)^2} - 1\right]\frac{\sigma^2}{2} + \tilde{B},
\end{aligned}
\tag{10}
$$

$$
\Delta_{t+1}^2 = \frac{\rho^2(\phi + \alpha\theta\nu)^2}{(1-\lambda\rho)^2}\sigma^2 + \frac{(\alpha\theta\rho)^2}{(1-\lambda\rho)^2}\Delta_t^2,
\tag{11}
$$

where $\tilde{B} \equiv (1-\lambda\rho)/\rho \ln \bar{B} + [\rho(1+\lambda) - 1]/\rho \ln(1-\lambda\rho)$.

The evolution of income distribution is fully described by equations (10) and (11). However, in tracking aggregate growth, we shall focus on GDP, Y_t. For this purpose, we would like to transform equation (10) to the dynamics of $\ln Y_t$. As $\ln Y_t = m_t + \Delta_t^2/2 - \ln(1-\lambda\rho)$, we can rewrite the equation (10) as follows:

$$
\ln Y_{t+1} = R \ln Y_t + \alpha\theta\left[\frac{\alpha\theta\rho}{1-\lambda\rho} - 1\right]\frac{\Delta_t^2}{2} + (\phi+\alpha\theta\nu)\left[\frac{(\phi+\alpha\theta\nu)\rho}{1-\lambda\rho} - 1\right]\frac{\sigma^2}{2} + B,
\tag{12}
$$

where $B \equiv \tilde{B} + (R-1)\ln(1-\lambda\rho)$ and $R \equiv \alpha + \lambda$. In the rest of this paper, we mostly analyze the property of equation (11) and (12).

3.2   Transition

We first analyze the effect of diversity of human capital on GDP during the transition process, and later discuss the same effect when the aggregate economy reaches a steady state. Transition dynamics of GDP are characterized by equation (12). Results from direct investigations on (12) are summarized by Proposition 1.

Proposition 1: *For given Y_t,*

1. *Y_{t+1} is strictly increasing in σ^2 if and only if $\rho > 1/(\phi + \alpha\theta\nu + \lambda)$.*

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2. Y_{t+1} is strictly decreasing in Δ_t^2 if and only if $\frac{\alpha\theta\rho}{1-\lambda\rho} < 1$.

The first part of Proposition 1 says that heterogeneity in ability may have *positive* effects on the next period's GDP. The variance of ability can affect the next period's GDP through the following two channels. The first channel is ability tracking. If ability tracking has substantial peer group effects, the overall variance of ability in the economy affects the average level of the next period's human capital. In particular, if the elasticity of peer group effects in the private school ν is so high that the total elasticity of human capital to ability $(\phi + \alpha\theta\nu)$ is larger than unity, the human capital accumulation function is convex in ability and therefore the next period's average human capital is *increasing* in the variance of ability.

The second channel is the span of control. Remember that a large span of control gives an individual the authority to reallocate a large amount of resources, and thus a large span of control favors workers with very high ability. If the degree of the span of control is large (and complementarity among produced goods is relatively small), the diversity of ability increases GDP. This effect is reflected in equation (4). It shows that if $\frac{\rho}{1-\lambda\rho} > 1$, a large variance in human capital increases GDP.

Consequently, if the "total" elasticity of income to ability $((\phi+\alpha\theta\nu)\rho/(1-\lambda\rho))$ is larger than unity, the variance of ability has positive effects of the next period's GDP.[9]

The second part of Proposition 1 shows that the effect of inequality in current income on GDP at the next period depends on the parameters $\frac{\alpha\theta\rho}{1-\lambda\rho}$. The mechanical reason can be understood by equation (9). Note that the

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[9]Note that the variance in *ability* raises the next period's GDP even if GDP is decreasing in the variance of *human capital* when $\rho \in (1/(\phi + \alpha\theta\nu + \lambda), 1/(1 + \lambda))$. In this case, the first (positive) effect dominates the second (negative) effect.

left-hand side of the equation (9) is strictly concave in π_t^i if and only if $\alpha\theta\rho/(1-\lambda\rho) < 1$. Hence, the high variance of π_t^i reduces the expected value of π_{t+1}^i, and, therefore, GDP, if and only if $\alpha\theta\rho/(1-\lambda\rho) < 1$.

On the other hand, equation (11) shows that the variance of log income converges to a finite value if and only if $\alpha\theta\rho/(1-\lambda\rho) < 1$. Hence, the following corollary is the immediate result.

Corollary 1: *Suppose that the variance of log income does not diverge. An increase in current income inequality lowers GDP at the next period.*

This negative effect of the variance of income on GDP is interpreted as the result of the concavity of the production function of human capital with respect to household income, as in the previous literature. For example, Loury (1981) and Fernandez and Rogerson (1998) derive the same results when the production function of human capital is concave in current income and the production function of consumption goods is linear in human capital. In their setting, the mapping from current income to next-period income is concave and, therefore, their income dynamics converge to a stationary process.

Different from the previous literature, our production function in the consumption goods sector can be strictly convex in human capital if the span of control in a firm is large enough. It provides a possibility that the income in the next period can be a strictly convex function of current income. Nonetheless, Corollary 1 implies that the concavity of the production function for human capital must outweigh the convexity of the production function for consumption goods when the variance of log income converges to a finite value.

An intuitive reason can be understood as follows. Whenever an increase

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in inequality in current income always raises the average income at the next period, the mapping from current income to next income must exhibit convexity. However, the convexity of the mapping implies that the current rich are likely to receive disproportionately high income at the next period. This means that inequality at the next period is larger than that in the current period. Hence, the variance of log income is infinite in the long run. In other words, insofar as we restrict our attention to the income dynamics that have a finite variance in the long run, the mapping from the current income to the future income must exhibit concavity for some income level.[10]

3.3   Steady State

Next, we examine the relationship between the steady-state variance in income and steady-state GDP. For this purpose, we must know when two aggregate state variables, $\ln Y_t$ and Δ_t^2, converge to steady states. Equation (11) shows that the variance of log income converges to a stationary point if and only if $\frac{\alpha\theta\rho}{1-\lambda\rho} < 1$. Provided that the variance of log income converges to a stationary point, equation (12) shows that the logarithm of GDP converges to a stationary point if and only if $R < 1$. Note that $R < 1$ guarantees $\frac{\alpha\theta\rho}{1-\lambda\rho} < 1$. This reasoning proves the following lemma.

Lemma 1: An aggregate economy converges to a stationary distribution if and only if $R < 1$.

Since we would like to analyze the relationships among income inequality,

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[10]Technically, we obtain Corollary 1 because local concavity (around a steady state) implies global concavity in equation (9). This is a standard assumption in this literature and we can still apply Corollary 1 when a deviation from the assumption is small. However, if a mapping from current income to future income exhibits a more general form, we may need additional assumptions on the variance of ability to guarantee the negative relationship between current inequality and future GDP.

the heterogeneity in innate ability, and GDP in the long run, we assume $R < 1$ in the rest of this paper.[11] As the variance of log income has a finite value on the stationary distribution, an increase in inequality in current income lowers GDP at the next period during the transition process. We want to show how the result changes in the long run.

Using equations (11) and (12), the logarithm of GDP in the steady state is given by:

$$\ln Y_\infty = \frac{1}{1-R}\left[\frac{1}{2\rho^2(\phi+\alpha\theta\nu)}D(\rho;\alpha,\lambda,\theta,\nu,\phi)\Delta_\infty^2 + \ln\tilde{B}\right],$$

where:

$$D(\rho;\alpha,\lambda,\theta,\nu,\phi) = \rho^2(\alpha\theta(\alpha\theta-\phi-\alpha\theta\nu)-\lambda(\phi+\alpha\theta\nu+\lambda))+\rho(\phi+\alpha\theta\nu+2\lambda)-1,$$
$$(13)$$

and

$$\Delta_\infty^2 = \frac{\rho^2(\phi+\alpha\theta\nu)^2}{(1-\lambda\rho)^2-(\alpha\theta\rho)^2}\sigma^2.$$

If $D > 0$, then the greater is the variance of income, the higher the long-run GDP (and vice versa). The effects of income inequality on steady-state GDP is summarized as Proposition 2.

Proposition 2: *(1) If $\lambda < 1-\phi+(1-\nu)\alpha\theta$, the higher is the steady-state variance of income, the lower is the steady-state GDP. (2) If $\lambda > 1-\phi+(1-\nu)\alpha\theta$, the higher is the steady-state variance of income, the higher is the steady-state GDP if and only if $\rho \in (\underline{\rho},1)$ where $\underline{\rho} = 1/(\phi+\alpha\theta(\nu-1)+\lambda)$.*

This proposition tells us that when the production of the final good is sufficiently substitutable and the degree of the span of control is large

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[11]Even if we allow endogenous growth (that is, $R = 1$), the results regarding transition dynamics in the rest of the paper remain unchanged. Almost all results from the steady-state analysis also can be applied to the analysis of the long run growth if we relabel steady-state GDP to "the long-run growth rate".

enough, the relationship between income inequality and GDP is positive in the steady state. Note that, similar to the case of transition, the variance of income affects the next period's GDP negatively. However, since the steady-state variance of income is determined by the variance of ability, which can have a positive impact on GDP, the total association between income variance (and hence the variance of ability) and GDP in the steady state depends on the combination of these two effects. When the final good production is sufficiently substitutable and the degree of the span of control is large enough, the positive effect of the high variance in ability dominates the negative effect of the variance of income.

In summary, the (causal) effect of income inequality on GDP is negative in our stationary economy, but heterogeneity in ability can have positive effects on GDP due to ability tracking and the span of control. Moreover, the long-run association between income inequality and GDP can be positive or negative depending on the production structure of the economy.

4 Public vs Private Education System

In the benchmark model, we allow both public and private education investment in human capital accumulation (when $\theta \in [0, 1]$). However, comparison between public and private education systems may be of interest because such comparison is useful to identify the advantages and disadvantages of each education system. In the literature of education macroeconomics, many researchers study the costs and benefits of each education system through such comparison (e.g., Glomm and Ravikumar 1992, Bénabou 1996, Fernandez and Rogerson 1998, and so on). In this section, we also study the comparison between the public and the private education systems in the

short run and in the long run.

4.1 Short Run

Imagine that the economy with the distribution of household income $\ln \pi_t^i \sim N(m_t, \Delta_t^2)$ is about to choose between the private ($\theta = 1$) or the public ($\theta = 0$) education system. Starting from this income distribution, the next period's GDP is given by the equation (12). The difference of the next period's GDP under the two systems is given by:

$$\ln Y_{t+1}^{Pub} - \ln Y_{t+1}^{Pri} = -\alpha \left(\frac{\alpha \rho}{1 - \lambda \rho} - 1 \right) \frac{\Delta_t^2}{2} - \alpha \nu \left[\frac{\rho(2\phi + \alpha \nu)}{1 - \lambda \rho} - 1 \right] \frac{\sigma^2}{2}, \ (14)$$

where Y_{t+1}^{Pub} and Y_{t+1}^{Pri} are the next period's GDP under the public (with $\theta = 0$) and the private system (with $\theta = 1$), respectively.

Under the assumption that the aggregate economy converges to a steady state in the long run, the first term of this equation is always positive. This is because income inequality reduces average human capital in the next period in the private system, but this effect is absent in the public system as expenditure on education does not depend on households' income in the public system.

The total comparison between the public and the private systems crucially depends on the sign of the second term and the relative size of the variance in household income and ability. This comparison is summarized as Proposition 3.

Proposition 3: *(1) If $\rho \leq 1/(2\phi + \alpha \nu + \lambda)$, the public system generates higher GDP in the next period than the private system. (2) If $\rho > 1/(2\phi + \alpha \nu + \lambda)$, the private system generates higher GDP in the next period than the public system if and only if $\Delta_t^2 < \frac{\nu[\rho(2\phi + \alpha \nu + \lambda) - 1]}{1 - \rho(\alpha + \lambda)} \sigma^2$.*

Observe that the second term of the equation (14) is positive if $\rho <$ $1/(2\phi + \alpha\nu + \lambda)$. In this case, the public system is always better than the private system. Intuitively, if the production of the final good is complementary enough in terms of human capital, public education is better because it generates low variance of human capital.

However, the second term of the equation (14) is negative if $\rho > 1/(2\phi + \alpha\nu + \lambda)$. In this case, the private system has an advantage because the variance of human capital is a source of gains. As the private system generates a higher variation in human capital than the public system, it generates a higher output.

The total comparison of these two systems crucially depends on the variance of household income when the production of the final good has less complementarity. If the variance of household income is small enough relative to the variance of ability, the private system is better than the public system because the positive effect in the second term dominates the negative effect in the first term.

4.2   Long Run

Next, we compare steady-state GDP under the two systems. In the short run, in the comparison of education systems the initial variance of household income is taken as given and common to the two systems. However, in the long run, the degree of income inequality depends on which education system the economy adopts. Hence, we need to compare steady-state GDP and income inequality under these two systems.

The difference in the steady-state GDP under the two systems is given

by:

$$
\ln Y_\infty^{Pub} - \ln Y_\infty^{Pri} = \frac{1}{1-R}\left[-\alpha\left(\frac{\alpha\rho}{1-\lambda\rho}-1\right)\frac{\Delta_\infty^{2,Pri}}{2} - \alpha\nu\left(\frac{\rho(2\phi+\alpha\nu)}{1-\lambda\rho}-1\right)\frac{\sigma^2}{2}\right],
$$

(15)

where $\Delta_\infty^{2,Pri} = \{\rho^2(\phi+\alpha\nu)^2\sigma^2\}/\{(1-\lambda\rho)^2-(\alpha\rho)^2\}$. Similar to the case of transition, the first term is positive, and the sign of the second depends on the production structure of the economy. We state the results of the long-run comparison between the public and the private system in Proposition 4.

Proposition 4: *(1) When $\rho \leq 1/(2\phi+\alpha\nu+\lambda)$, the public system generates a higher steady-state GDP than the private system. (2) When $\rho > 1/(2\phi+\alpha\nu+\lambda)$, the public system generates a higher steady-state GDP than the private system if and only if $\Delta_\infty^{2,Pri} > \frac{\nu[\rho(2\phi+\alpha\nu+\lambda)-1]}{1-\rho(\alpha+\lambda)}\sigma^2$. Moreover, for each $(\rho,\alpha,\lambda,\phi)$ satisfying the assumptions $R < 1$ and $\rho > 1/(2\phi+\alpha\nu+\lambda)$, there exists a unique $\nu^*(\rho,\alpha,\lambda,\phi)$ such that the private system generates higher steady-state GDP than the public system if and only if $\nu > \nu^*(\rho,\alpha,\lambda,\phi)$.*

The first part of this proposition states that the public system generates higher GDP when the production of the final good is complementary enough even in the steady state.

More interesting, the second part of this proposition tells us that when the final good production is less of a complement the long-run comparison of GDP under the two systems depends on the long-run variance under the private system. In this case, the private system generates higher steady-state GDP than the public if the elasticity of human capital production to the quality of a peer group is large enough. Intuitively, when the elasticity of

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human capital production to the quality of a peer group is large, the production function of human capital becomes more *convex* in individual ability, and thereby sorting by ability in private schools contributes to generating a large amount of human capital.

5   Ability Tracking in Public Schools

So far, we assume that the ability sorting occurs only in private schools. Recently, however, ability tracking in public schools is one of the main issues in the debate on education reforms (e.g., Lefgren 2004). In this section, we first examine how our results on the system comparison change if we allow for public schools choosing students based on their ability. Next, we compare the public system with ability tracking to that without tracking in order to evaluate the effect of ability tracking on GDP.

5.1   Public vs Private System

When the public school is allowed to select students based on ability, the average ability of students in a public school for students with ability ξ_{t+1}^i is given by $\bar{\xi}_{t+1}^{Pub,i} = \xi_{t+1}^i$. The reduced form of the production function of human capital is given by: $h_{t+1}^i = (\xi_{t+1}^i)^{\phi+\alpha\nu} \left((e_t^i)^\theta (g_t)^{1-\theta} \right)^\alpha$. In this case, the equations (14) and (15) become:

$$\ln Y_{t+1}^{Pub} - \ln Y_{t+1}^{Pri} = -\alpha \left(\frac{\alpha\rho}{1-\lambda\rho} - 1 \right) \frac{\Delta_t^2}{2} > 0,$$

$$\ln Y_\infty^{Pub} - \ln Y_\infty^{Pri} = \frac{1}{1-R} \left[-\alpha \left(\frac{\alpha\rho}{1-\lambda\rho} - 1 \right) \frac{\Delta_\infty^{2,Pri}}{2} \right] > 0,$$

where $\Delta_\infty^{2,Pri} = \{\rho^2(\phi+\alpha\nu)^2\sigma^2\}/\{(1-\lambda\rho)^2 - (\alpha\rho)^2\}$. Therefore, the public system generates higher GDP than the private system both in transition and in the steady state. In this case, the only difference between the public

and the private system is their source of finance. As a reduction in the current variance of income increases GDP at the next period, the public system generates a higher GDP than does the private.[12]

5.2 Tracking vs Mixing in Public Schools

Can ability tracking in public schools increase GDP? To answer this question, we compare GDP under the public system with ability tracking to that under the public system without tracking.

The difference of the logarithm of GDP under these two education systems during transition is given by:

$$\ln Y_{t+1}^{Pub,S} - \ln Y_{t+1}^{Pub,M} = \alpha\nu \left(\frac{\rho(2\phi + \alpha\nu)}{1 - \lambda\rho} - 1 \right) \frac{\sigma^2}{2},$$

$$\ln Y_{\infty}^{Pub,S} - \ln Y_{\infty}^{Pub,M} = \frac{1}{1 - R} \left[\alpha\nu \left(\frac{\rho(2\phi + \alpha\nu)}{1 - \lambda\rho} - 1 \right) \frac{\sigma^2}{2} \right],$$

where $Y_t^{Pub,S}$ and $Y_t^{Pub,M}$ are GDP under the public system with and without ability tracking, respectively. From these equations, we have the following result:

Proposition 5: *Ability tracking in public schools generates higher GDP in both transition and steady state if and only if $\rho > 1/(2\phi + \alpha\nu + \lambda)$.*

From Proposition 5, we may conclude that ability tracking in public schools increases GDP when the production of the final good is a sufficiently substitutable (large ρ), the degree of the span of control (λ) is large enough, the production function of human capital is less concave (large α), and/or the elasticity of human capital to the peer group effect (ν) is large enough.

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[12]In the case where neither public nor private schools are allowed to select students based on ability, the public system generates higher GDP than the private system both in transition and in the steady state.

In summary, when public schools are allowed to select students based on their ability, the public system generates a higher GDP than the private both in transition and in the steady state. However, whether the ability tracking in public schools increases GDP crucially depends on the interaction between the production structure of the final and the intermediate goods and the technology of human capital accumulation.

6   Concluding Remarks

In this paper, we examined how different education systems change GDP through their influence on the diversity of human capital. We analyze an economy in which an income distribution converges to a stationary distribution. We show that the diversity of human capital due to income inequality always lowers GDP, while diverse human capital induced by heterogeneous ability can increase GDP if produced goods are sufficiently substitutable and firms have a large span of control. Hence, we may conclude that a public education system always yields higher GDP than a private education system, though the effect of ability tracking on GDP depends on the structure of industries and firms.

It is worth mentioning some cautions to the interpretation of our results. We focus on the effect of different education systems on GDP. But obviously, GDP is not the only variable that governments should be concerned. In our model, both a private system and ability tracking increase the variance of log income and lower social mobility. Hence, our model indicates that equalizing school resources can also promote efficiency, but that ability tracking may induce a serious trade-off between efficiency and equity in some societies. One of messages from this paper is that if a government wants to know

how serious the trade-off is, it must carefully investigate the structure of industries and firms.

Although we examine the effect of ability tracking on GDP in this paper, we can extend our analysis to other policies that can change a mapping from ability to human capital. More specifically, we can extend our analysis to policies that influence the human capital accumulation function through the following path, $h_{t+1}^i = (\xi_{t+1}^i)^{\phi(\theta,p)} \left((e_t^i)^\theta (g_t)^{1-\theta} \right)^\alpha$, where p is a policy parameter. For example, when policy allows individuals more flexibility in choosing how many credits they can take in a year, it is likely that talented students take more courses than ordinary students do.[13] In this case, more able students accumulate a larger human capital not only because they are talented, but also because they study harder. Hence, the effect of ability on human capital is more than proportional and $\phi(\theta, p)$ becomes larger by this policy. According to our analysis, the impact of this policy on GDP depends on the structure of production. In this way, our analysis suggests that we cannot evaluate the economic consequences of a liberal education policy without the knowledge of how industries and firms are organized.

As indicated by this example, there will be more unexplored issues about the relationship between education policy and the structure of the production sector. We hope that this paper helps researchers in unraveling part of the complicated relationships.

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[13] The Japanese government substantially reduced the number of subjects that students must study in elementary school, junior high school and high school during 1990s. According to Kariya (2001), as a result of this policy change, students who previously received poorer grades reallocate their time more to leisure than those who previously received a better grade.

A Proofs

A.1 Proposition 1

Proof. Obvious from the equation (12). Q.E.D.

A.2 Proposition 2

Proof. We first show (2) of the proposition. The second part of the proposition is true if $D > 0$ when $\rho \in (\underline{\rho}, 1)$ and $D < 0$ when $\rho \in (0, \underline{\rho})$. Solving equation $D(\rho; \alpha, \lambda, \theta, \nu, \phi) = 0$ for ρ, we have $\underline{\rho} = 1/(\phi + \alpha\theta(\nu - 1) + \lambda), \bar{\rho} = 1/(\lambda + \alpha\theta)$. Since $\alpha\theta(\alpha\theta - \phi - \alpha\theta\nu) - \lambda(\phi + \alpha\theta\nu + \lambda) < 0$, $\phi + \alpha\theta\nu + 2\lambda > 0$ and $D(0; \alpha, \lambda, \theta, \nu, \phi) = -1 < 0$, $D > 0$ if and only if $\underline{\rho} < \rho < \bar{\rho}$. Under the assumption $\lambda > 1 - \phi + (1 - \nu)\alpha\theta$, $\underline{\rho} < 1 < \bar{\rho}$. Therefore, $D > 0$ when $\rho \in (\underline{\rho}, 1)$ and $D < 0$ when $\rho \in (0, \underline{\rho})$.

Next we show (1) in the proposition. For the first part, note that $\underline{\rho} > 1$ if $\lambda < 1 - \phi + (1 - \nu)\alpha\theta$. This implies that $D(\rho; \alpha, \lambda, \theta, \nu, \phi) < 0$ for any $\rho \in (0, 1)$. Therefore, the association between GDP and income variance in the steady state is negative when $\lambda < 1 - \phi + (1 - \nu)\alpha\theta$. Q.E.D.

A.3 Proposition 3

Proof. (1) When $\rho \leq 1/(2\phi + \alpha\nu + \lambda)$, $\ln Y_{t+1}^{Pub} - \ln Y_{t+1}^{Pri} > 0$.

(2) When $\rho > 1/(2\phi + \alpha\nu + \lambda)$, $\ln Y_{t+1}^{Pub} - \ln Y_{t+1}^{Pri} < 0$ if and only if $\Delta_t^2 < \frac{\nu[\rho(2\phi + \alpha\nu + \lambda) - 1]}{1 - \rho(\alpha + \lambda)}\sigma^2$. Q.E.D.

A.4 Proposition 4

Proof. For the first part, observe that $\left(\frac{\rho(2 + \alpha\nu)}{1 - \lambda\rho} - 1\right) < 0$ when $\rho \leq 1/(2 + \alpha\nu + \lambda)$. Hence $\ln Y_{\infty}^{Pub} - \ln Y_{\infty}^{Pri} > 0$.

For the first half of the second part, note that $\ln Y_\infty^{Pub} - \ln Y_\infty^{Pri} > 0$ if and only if $\Delta_\infty^{2,Pri} > \frac{\nu[\rho(2+\alpha\nu+\lambda)-1]}{1-\rho(\alpha+\lambda)}\sigma^2$.

Finally, we prove the last half of the second part. The long-run variance of household income under the private system is given by:

$$\Delta_\infty^{2,Pri} = \frac{\rho^2(1+\alpha\nu)^2}{(1-\lambda\rho)^2 - (\alpha\rho)^2}\sigma^2.$$

Hence, $\ln Y_\infty^{Pub} - \ln Y_\infty^{Pri} > 0$ if and only if:

$$\frac{\rho^2(1+\alpha\nu)^2}{(1-\lambda\rho)^2 - (\alpha\rho)^2}\sigma^2 > \frac{\nu[\rho(2+\alpha\nu+\lambda)-1]}{1-\rho(\alpha+\lambda)}\sigma^2,$$

and this inequality is equivalent to:

$$\alpha\rho(1-\lambda\rho)\nu^2 + [(1-\lambda\rho)(\rho(2+\lambda)-1) + \alpha\rho(\rho\lambda-\alpha)]\nu - \rho^2 < 0.$$

When $\nu = 0$, the left-hand side (LHS) is negative and thus $\ln Y_\infty^{Pub} - \ln Y_\infty^{Pri} > 0$. When $\nu \to \infty$, LHS is infinity and thus $\ln Y_\infty^{Pub} - \ln Y_\infty^{Pri} < 0$. Since $\alpha\rho(1-\lambda\rho) > 0$ and LHS is negative when $\nu = 0$, there exists a unique $\nu^*(\rho,\alpha,\lambda) > 0$ such that LHS $= 0$ when $\nu = \nu^*(\rho,\alpha,\lambda)$ and $\ln Y_\infty^{Pub} < \ln Y_\infty^{Pri}$ if and only if $\nu > \nu^*(\rho,\alpha,\lambda)$. Q.E.D.

A.5 Proposition 5

Proof. $\left(\frac{\rho(2\phi+\alpha\nu)}{1-\lambda\rho} - 1\right) > 0$ (and thus $Y_\infty^{Pub,S} > Y_\infty^{Pub,M}$) if and only if $\rho > 1/(2\phi + \alpha\nu + \lambda)$. Q.E.D.

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