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Discussion Paper 06-27

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Abstract

This note develops a one-sector, two-period, overlapping generations model that incorporates endogenous labor–care choice. Care choice is modeled by allowing young agents to participate in the production of household health status. Using this model, we derive the steady-state equilibrium dynamics.

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1 Introduction

Along with rapidly aging populations, it has been argued that health care and long-term care are increasing in industrialized countries.[1] Thus, this note demonstrates an economy in which young agents supply their time for taking care of their parents and verifies the equilibrium dynamics of that economy.

Care supply is modeled by allowing young agents to participate in the production of household health status. In this note, we analyze labor–care choice in an overlapping generations model that incorporates lifetime and health status uncertainty. Our model has two key features. First, young agents are heterogeneous with respect to their parents' death–illness status (death, good health, and bad health), and exhibit one-sided altruism (from children to parents) that is derived from a "joy of giving" to their parents. Second, the health status of aged parents depends on the care supply from their children.

Under this analytical framework, we show that the equilibrium dynamics can feature multiple equilibria. More specifically, there exist multiple equilibria, comprising a unique saddle stable equilibrium and other equilibria, which have an infinite number of trajectory paths.

The remainder of this paper is organized as follows. Section 2 sets up the basic model. Section 3 derives the equilibrium dynamics. Section 4 concludes this paper.

2 The Model

In this paper, we use an overlapping generations model in which a continuum of agents $[0, 1]$ is born every period and each agent lives for a maximum of two periods. Each agent is endowed with one unit of time in young age. Young agents have a probability p of surviving throughout old age. In addition, an agent who is alive in old age has a probability $1 - \psi$ of being in bad health. Thus a fraction $p\psi$ of agents are of type g who have good health, a fraction $p(1 - \psi)$ of agents are of type b who have bad health. Type d agents, who constitute a fraction $1 - p$ of agents, die when old. We express the death–illness status of each agent's parents as index i.

[1] In OECD countries as a whole, the average density of practicing nurses per 1,000 persons was about 2.5 in 1960 and 8.1 in 2003 (OECD (2005)).

Each young agent of generation t allocates their unit of time between firms $l_{i,t}^{t}$ and the production of household health status $q_{i,t}^{t}$. He or she earns wage income $w_t l_{i,t}^{t}$, and saves all wage income, where w_t is the real wage rate. Old agents of generation t consume the proceeds of their savings, which we denote by $c_{i,t+1}^{t}$. We assume the existence of actuarially fair insurance in this paper (see Yaari (1965) and Blanchard (1985)). Thus the rate of return on the annuities is R_{t+1}/p if they are alive and 0 if they die at the end of period t, where R_{t+1} is the real rental rate. Let us assume that the health status of type g and type b old agents at time t is produced in the household using the following household health production function:

$$h_{g,t} = dq_{g,t}^{t}, \quad (1)$$

$$h_{b,t} = q_{b,t}^{t}, \quad (2)$$

where $d > 1$ is a productivity parameter. Thus the marginal productivity of care supply of the agents with index $i = g$ is higher than that of the agents with index $i = b$. Each young agent of generation t solves the following optimization problem at time t:

$$U_{i,t} = \beta \ln h_{i,t} + pc_{i,t+1}^{t} \qquad i = g, b, d$$

s.t.

$$\begin{aligned} c_{i,t+1}^{t} &= R_{t+1} w_t l_{i,t}^{t}, \\ q_{i,t}^{t} + l_{i,t}^{t} &= 1, \\ 0 \le q_{i,t}^{t} \le 1, \quad 0 &\le l_{i,t}^{t} \le 1, \\ (1), (2)&, \end{aligned} \quad (3)$$

where $\beta \in (0,1)$ measures the degree of altruism towards parents. The optimal care supply to the production of household health status is derived as follows:

$$q_{g,t}^{t} = q_{b,t}^{t} = \begin{cases} \dfrac{\beta}{R_{t+1} w_t} & \text{if } \beta \le R_{t+1} w_t, \\ 1 & \text{if } R_{t+1} w_t \le \beta. \end{cases} \quad (4)$$

Due to the quasi-linear utility function, when $R_{t+1} w_t$ is sufficiently large, the opportunity cost of care supply to the production of household health status is high, and each agent decreases his or her transfers of care supply.[2] If parents die, young agents do not derive any utility from household health status; thus we obtain:

$$q_{d,t}^{t} = 0. \tag{5}$$

Using (3), (4), and (5), we have the aggregate labor supply as follows:

$$L_t \equiv l_t N_t = \begin{cases} \left(1 - \dfrac{p\beta}{R_{t+1} w_t}\right) N_t & \text{if } \beta \leq R_{t+1} w_t, \\ (1-p) N_t & \text{if } R_{t+1} w_t \leq \beta. \end{cases} \tag{6}$$

Firms are perfectly competitive profit maximizers that produce output using a production function of the Cobb–Douglas form $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t is aggregate output, $A > 0$ is a productivity parameter and K_t is the aggregate capital stock. We assume that capital depreciates fully in the process of production. Thus profit maximization in the competitive market equates the marginal products of private labor and capital to the real wage and the real rental rate, respectively:

$$w_t = (1-\alpha) A k_t^{\alpha}, \quad R_t = \alpha A k_t^{\alpha - 1}, \tag{7}$$

where $k_t \equiv K_t / L_t$.

Capital market equilibrium in period t requires total savings in the previous period equal to:

$$K_{t+1} = s_t N_t = w_t l_t N_t. \tag{8}$$

3 Equilibrium Dynamics and Indeterminacy

Using (6) through (8), we obtain the following complete dynamic system:

<Regime I: $\beta \leq R_{t+1} w_t$ >

$$l_{t+1}^{1-\alpha} = \frac{p\beta l_t^{1-\alpha}}{\alpha A (1 - l_t)((1-\alpha) A k_t^{\alpha})^{\alpha}}, \tag{9}$$

$$k_{t+1}^{1-\alpha} = \frac{(1-\alpha) A k_t^{\alpha} \alpha A (1 - l_t)}{p\beta}. \tag{10}$$

[2]If a log-linear utility function is assumed, the care supply to aged parents becomes constant. This is not consistent with the actual data. When we consider that the care supplied is in order to take care of their parents, the data show a negative correlation between the rate of resident and income. (See U.N. (2005)).

<Regime II: $R_{t+1}w_t \leq \beta$ >

$$\begin{aligned} l_t &= 1-p, & (11)\\ k_{t+1} &= (1-\alpha)Ak_t^{\alpha}. & (12)\end{aligned}$$

Before stating the equilibrium, we consider the borderline between the regimes. Substituting (7), (8), and (9) into the borderline, we express the borderline as follows:

$$l_t = 1-p. \tag{13}$$

Thus, Regime I and Regime II is respectively feasible on the area $1-p \leq l_t \leq 1$ and $l_t \leq 1-p$.

The equations (9) through (12) characterize the economic equilibria that are represented sequences of $\{k_t, l_t\}_{t=1}^{\infty}$ with an initial condition $(k_1, l_1) \geq 0$. Now let us draw the phase diagram on the (k_t, l_t) plane. We refer to the loci representing $k_{t+1} = k_t$ as "KK" and that representing $l_{t+1} = l_t$ as "LL". We have the "KK" and "LL" loci from (9) through (12):

<Regime I: $1-p \leq l_t$ >

$$LL_1 \;:\; l_t = 1 - \frac{p\beta}{\alpha A((1-\alpha)Ak_t^{\alpha})^{\alpha}}, \tag{14}$$

$$KK_1 \;:\; k_t = \Big(\frac{(1-\alpha)A\alpha A(1-l_t)}{p\beta}\Big)^{\frac{1}{1-2\alpha}} \tag{15}$$

LL_1 and KK_1 loci respectively intersect the borderline $l_t = 1-p$ at the points A^{ll} and A^{kk}, where: $A^{ll} \equiv (k_t, l_t) = (((\frac{\beta}{\alpha A})^{\frac{1}{\alpha}}\frac{1}{(1-\alpha)A})^{\frac{1}{\alpha}},\ 1-p)$ and $A^{kk} \equiv (k_t, l_t) = ((\frac{(1-\alpha)A\alpha A}{\beta})^{\frac{1}{1-2\alpha}},\ 1-p)$.

<Regime II : $l_t \leq 1-p$ >

$$LL_2 \;:\; l_t = 1-p, \tag{16}$$

$$KK_2 \;:\; k_t = ((1-\alpha)A)^{\frac{1}{1-\alpha}}. \tag{17}$$

For simplicity of analysis, we assume the following:

Assumption 1 *(i)* $\alpha < \frac{1}{2}$, *and* *(ii)* $\Big(\frac{\beta}{\alpha}\Big)^{1-\alpha}\Big(\frac{1}{1-\alpha}\Big)^{\alpha} < A.$

Under Assumption 1(i), the KK locus is decreasing in l_t, and the KK and LL loci intersect in Regime I under Assumption 1(ii).[3]

The initial point at which the economy starts can be derived from:

$$l_1 = \frac{K_1}{k_1 N_1}. \tag{18}$$

The surface representing (18) can be drawn in (k_t, l_t) space as the initial per capita capital stock K_1/N_1 is given exogenously. Therefore, the economy must be initially on the line (18). As can be verified immediately from (18), the contour of (18) drawn in the (k_t, l_t) plane is downward. Thus, if an initial condition is given, the dynamics of this economy are determinable. The phase diagram of this economy is depicted in Figure 1.

Let us first consider the case in which the initial level of capital is sufficiently large. In this case, the initial contour is drawn in the upper-right corner in Figure 1, and the trajectory is drawn like J. Since any trajectory that is above J does not satisfy the physical condition, we can exclude these trajectories from the equilibrium. Thus contour J shows the boundary trajectory in this economy. Next, let us consider the case $l_1 = K_1/k_1N_1$ in Figure 1. If the economy initially happens to be on the line SS, it converges to E_2.[4] If the economy initially happens to be above (below) the line SS, it converges to E_1 (E_3). In this case, there exists an infinite number of trajectory paths that leads to the equilibria E_1 (E_3). Therefore, for a given level of the initial capital stock, the economy converges to one of the equilibria E_1, E_2, or (E_3) in the long run.

In order to obtain intuitive implications for this result, let us firstly consider the equilibrium E_1 in Figure 1. On the equilibrium path towards E_1, any trajectory that starts above the line "SS" at G in Figure 1, initially has a higher labor supply for firms and savings than other equilibrium paths. Higher savings are related to the larger capital stock and an increased labor supply in next period. Thus, the economy initially increases both labor supply and the capital–labor ratio, however, when it crosses the KK locus at G' in Figure 1, the increased labor supply decreases the

[3]Assumption 1(i): $\frac{\partial k}{\partial l} = -\frac{1}{1-2\alpha}\left(\frac{(1-\alpha)A\alpha A(1-l_t)}{p\beta}\right)^{\frac{2\alpha}{1-2\alpha}} < 0$, if $\alpha < \frac{1}{2}$.
Assumption 1(ii): We find the point k becomes $A^{ll} < ((1-\alpha)A)^{\frac{1}{1-\alpha}} < A^{kk}$, if $(\frac{\beta}{\alpha})^{1-\alpha}(\frac{1}{1-\alpha})^{\alpha} < A$.

[4]See Appendix A for the conditions for the stability of the steady state.

capital–labor ratio. A lower capital–labor ratio increases labor supply (see (9)), thus, any trajectory of this economy converges to E_1. For the equilibrium E_1, the capital–labor ratio converges to zero; that is, no production can be carried out. Thus, the economy is doomed on this equilibrium.

Next, let us consider the equilibrium E_3 in Figure 1. On the equilibrium path of E_3, any trajectory that starts below the line "SS" at I in Figure 1, initially has a lower labor supply for firms and savings than other equilibrium paths. The economy initially increases both labor supply and the capital–labor ratio, however, when it crosses the LL locus at I' in Figure 1, labor supply is restrained. A decreased labor supply increases the capital–labor ratio, making labor supply lower–bounded (at I'' in Figure 1). On the economy where labor supply becomes lower–bounded, the capital–labor ratio decreases and converges to E_3.

4 Conclusion

We have investigated the steady-state equilibrium dynamics in a model that incorporates lifetime and health status uncertainty; and endogenous labor–care choice. Care choice is modeled by allowing young agents to participate in the household health production. Using this model, we have shown that the equilibrium dynamics obtained from the model has multiple equilibria. More specifically, there exists a unique saddle stable equilibrium and other equilibria, which have an infinite number of trajectory paths.

Appendix

Appendix A

We examine the stability of the equilibrium E_2 in this Appendix. To examine the local dynamics of equilibrium E_2, we take a first-order Taylor expansion of the system around the steady state (k^*, l^*). With $\check{l}_t \equiv l_t - l^*$ and $\check{k}_t \equiv k_t - k^*$, the linearization is expressed as:

$$\begin{pmatrix} \check{k}_{t+1} \\ \check{l}_{t+1} \end{pmatrix} = \begin{pmatrix} \dfrac{\alpha}{(1-\alpha)} & -\dfrac{\alpha A}{p\beta} \cdot \dfrac{((1-\alpha)A)^{\frac{1+\alpha}{1-\alpha}}}{1-\alpha} \\ -\dfrac{l^* \alpha^2}{(1-\alpha)((1-\alpha)A)^{\frac{1}{1-\alpha}}} & 1 + \dfrac{((1-\alpha)A)^{\frac{\alpha}{1-\alpha}} l^* \alpha A}{(1-\alpha)p\beta} \end{pmatrix} \begin{pmatrix} \check{k}_t \\ \check{l}_t \end{pmatrix},$$

where $l^* \equiv 1 - \frac{p\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}}$. The characteristic polynomial becomes:

$$P(\kappa) = \kappa^2 - T\kappa + D,$$

$$\begin{aligned} T &= \frac{\alpha}{1-\alpha} + 1 + \frac{((1-\alpha)A)^{\frac{\alpha}{1-\alpha}} l^* \alpha A}{(1-\alpha)p\beta}, \\ D &= \frac{\alpha}{1-\alpha} + \frac{\alpha}{(1-\alpha)} \cdot \frac{((1-\alpha)A)^{\frac{\alpha}{1-\alpha}} l^* \alpha A}{p\beta}. \end{aligned}$$

Azariadis (1993) checks that the steady state is a saddle point, when $1 - T + D < 0$ holds. It is clear that:

$$1 - T + D = -\frac{((1-\alpha)A)^{\frac{\alpha}{1-\alpha}} l^* \alpha A}{p\beta} < 0.$$

Therefore, E_2 is a saddle point.

References

Azariadis, C. (1993): *Intertemporal Macroeconomics*. Basil Blackwell, Oxford.

Blanchard, O. J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93(2), 223–47.

OECD (2005): *OECD Health Data 2005: Statistics and Indicators for 30 Countries.* Organization for Economic.

U.N. (2005): *Living Arrangements of Older Persons Around the World.* United Nations Publications.

Yaari, M. E. (1965): "Uncertain lifetime, life insurance, and the theory of the consumer," *Review of Economic Studies*, 32(2), 137–150.

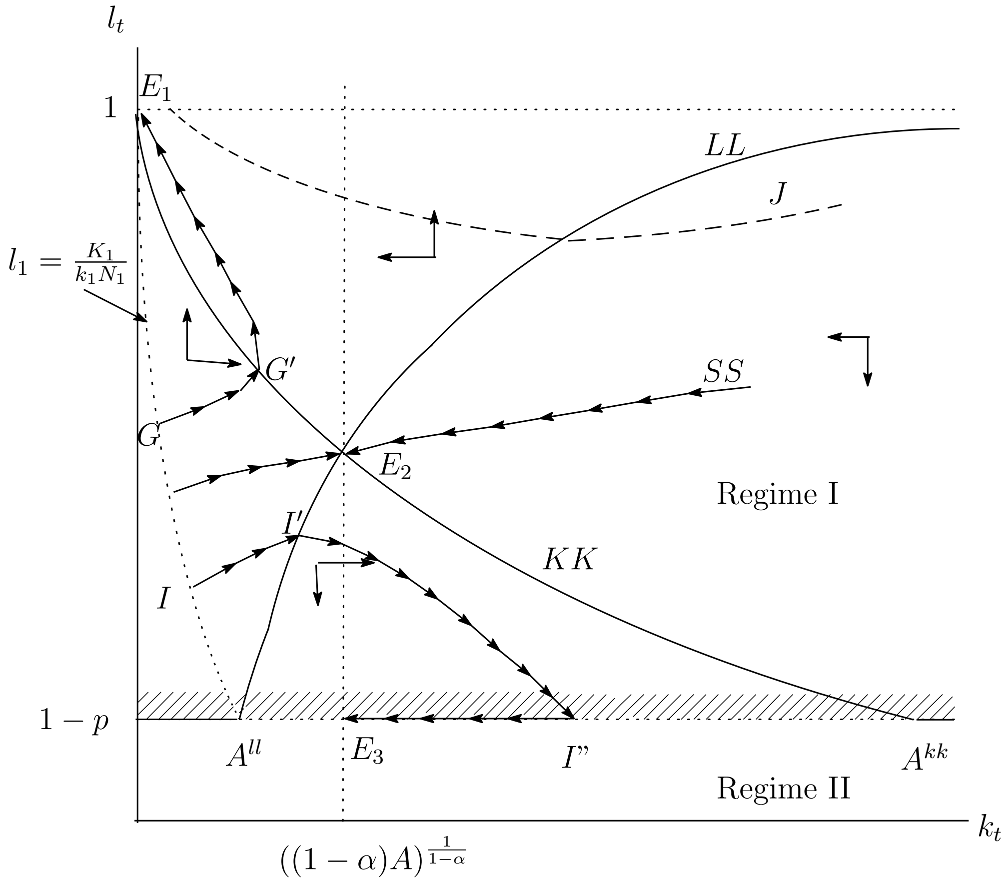

Figure 1: The phase diagram analysis