Discussion Papers In Economics And Business

Capacity Expansion in Markets with Intertemporal Consumption Externalities

Hiroshi Kitamura

Discussion Paper 07-11

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Capacity Expansion in Markets with Intertemporal Consumption Externalities

Hiroshi Kitamura

Discussion Paper 07-11

April 2007

この研究は「大学院経済学研究科・経済学部記念事業」
基金より援助を受けた、記して感謝する。

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Capacity Expansion in Markets with Intertemporal Consumption Externalities*

Hiroshi Kitamura†
Graduate School of Economics, Osaka University

April 2, 2007

Abstract

This paper analyzes market capacity expansion in the presence of intertemporal consumption externalities such as consumer learning, networks, or bandwagon effects. The externality leads to an endogenous shift of market demand that responds to past market capacity. Whereas market capacity grows in waves, its magnitude depends on the degree of market concentration. The competitive environment contributes to S-shaped time patterns of market capacity expansion that is slow from the social viewpoint. On the other hand, using an introductory price, a monopolist plans an initially larger, but eventually smaller, amount of market cultivation than a competitive market capacity expansion.

JEL Classifications Code: D11, L11, L14.
Keywords: Intertemporal consumption externalities; S-shaped diffusion; Market structure; Introductory price.

---

*In the process of this study, I talked to many people, and their knowledge and ideas contributed significantly to the analysis in this paper. I especially thank Katsuya Takii, Junichiro Ishida, and Ryo Horii for helpful discussions and comments. I also thank Tatsuhiko Nariu, Kazuo Mino, Tetsuya Shinkai, Keizo Mizuno, Tsuyoshi Toshimitsu, Masamichi Kawano, Noriyuki Doi, Toshihiro Okada, Kohei Daido, Yasutomo Murasawa, Eiichi Miyagawa, Tatsuhiro Shichijo, Misato Sato, and the seminar participants at the Japanese Economic Association, Osaka University, Kwansei Gakuin University, and Osaka Prefecture University for constructive comments. All errors are mine.

†Address:Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka, 560-0043, Japan. Email address:gge003kh@mail2.econ.osaka-u.ac.jp

1 Introduction

In a number of successful product or services markets, the level of market demand often increases over time. One of the significant features of this growth is that it is gradual rather than instantaneous. Although different explanations are possible for this gradual expansion, there is an important but relatively neglected reason in the literature: an intertemporal consumption externality where market demand is endogenously determined as an increasing function of past market capacity, defined as the number of consumers who buy the product. Several reasons exist for such an externality. First, there may be consumer learning. An increase in past market capacity leads to an accumulation of product information among consumers and to an updating of consumer preferences. Second, indirect network externalities may exist[1]. The variety of complementary products is an increasing function of the number of past product users (for example, computers and software). Another reason, which is purely psychological, is the bandwagon effect. Consumers often wish to consume a popular product. They may regard past market capacity as a sign of popularity[2].

When an intertemporal consumption externality exists, the market equilibrium contains dynamic aspects: market capacity increases in waves. More importantly, market capacity expansion may be highly dependent on the degree of market concentration. Intuitively, market concentration enables firms to efficiently expand the market by internalizing the externality, but also earn a large, socially inefficient level of profits. In contrast, the competitive environment restricts firm profits but makes the internalization of the externality difficult. It is apparently ambiguous how these differences affect the level, time pattern, and product price of the market capacity expansion. The main aim of this paper is to theoretically examine the relation between market concentration and the properties of the market capacity expansion in the presence of an intertemporal consumption externality.

In this paper, we develop a dynamic model of market capacity expansion with endogenous market demand in an infinite-horizon framework. The market demand increases with increases in the previous period's market capacity because of an intertemporal consumption externality. We examine two types of market capacity expansions: a competitive (decentralized) market capacity expansion and a monopoly

[1] See Katz and Shapiro (1994).

[2] This may be naive behavior, but is consistent with the existence of advertising and media reports containing information on past product sales. See Monterio and Gonzalez (1999) who analyze the role of advertising past sales.

(centralized) market capacity expansion. In the competitive market, small and identical firms enter the market responding to each period's demand growth given the equilibrium price determined by the zero profit condition. In contrast to competitive firms, a monopolist has the ability to control price. This ability enables the monopolist to control both the level of current market capacity and future demand growth because of the internalization of the externality. By increasing current market capacity, the monopolist generates a large amount of subsequent period demand. However, an increase in current market capacity reduces current profits. The main difference between the two market capacity expansions is whether the intertemporal trade off between current revenue and future demand growth exists or not.

The two market capacity expansions differ with respect to the levels, the time patterns, and the equilibrium prices, respectively. Assuming that the degree of demand growth (the benefit of an externality) decreases with increases in the previous period's market capacity, the analysis provides several interesting results. First, the competitive market capacity expansion is initially slower and smaller than the monopoly market capacity expansion. However, it gradually becomes faster and larger in the long run. Because of the externality, the competitive market capacity expansion is slow and inefficient from the social viewpoint. In contrast, the monopolist plans a profit structure in order to earn large profits from a large market demand by sacrificing early profits. The monopolist has an incentive for a large amount of early market cultivation by internalizing the externality. This leads to faster market capacity expansion than in the competitive market. However, the monopolist slows down the rate of market cultivation to yield large profits by restricting market capacity.

Second, the initially slow competitive market capacity expansion has an S-shaped time pattern that is reported by a number of researchers who investigate the time patterns of a number of firms[3] and products[4]. In contrast, monopoly market capacity expansion has difficulty in following the initial convex part of the S-shaped time patterns. The intuitive logic for the difference in the initial time patterns is as follows. In the competitive market, the attractiveness of a new entry is determined by the strength of its externality

[3]See, for example, Gort and Klepper (1982) who investigate the time patterns of a number of firms in 46 product markets. Note that S-shaped diffusion is not an isolated phenomenon. Empirical evidence shows that the time patterns of the intra-firm and inter-firm technology diffusion processes also tend to be S-shaped: see the seminal work of Griliches (1957), Mansfield (1968), and the survey of technological diffusion by Stoneman (2002). For a discussion of the theoretical mechanism of adoption of new technology, see Jensen (1982) and Rengnum (1981).

[4]S-shaped product diffusion is treated as a stylized fact in the marketing literature. It is observed in a number of markets such as color televisions (Karshenas and Stoneman (1992)), fax machines (Economides (1995)), and clothes dryers (Krishnan, Bass, and Jain (1999)). This phenomenon is also observed in the services market in the presence of network externalities such as the mobile telecommunications services market and the Digital Subscriber Line services market.

effect. A strong externality effect makes a current period entry more profitable than in the previous period and it raises the attractiveness of the new entry. This leads to the initial convex part of the S-shaped time pattern. On the other hand, a strong externality effect also raises the monopolist's benefit of internalizing the externality. This provides the monopolist with a strong incentive for a large amount of early market cultivation in order to generate a large amount of demand sooner. Therefore, the initial market capacity expansion by the monopolist is less likely to increase, but more likely to maintain, the concave time patterns.

Finally, the two market capacity expansions also differ with respect to the equilibrium prices. Whereas the competitive equilibrium price is constant over time, the monopolist has an incentive to initially offer a low introductory price[5]. The initial low introductory price contributes to the large amount of early market cultivation by the monopolist. The eventual high price induces the monopolist to generate large profits with the larger demand.

This paper is related to a number of literatures. First, this paper is most relevant to the industrial organization literature concerned with market capacity expansion in the presence of intertemporal externalities. The majority of previous studies on market capacity expansion are related to firm learning: learning by doing (Jovanovic and Lach (1989)), learning consumer demand (Rob (1991)), and two sided learning between consumers and firms (Bergemann and Välimäki (1997) and Vettas (1998)). S-shaped diffusion has been explained in these literatures. In contrast, this paper does not focus on the role of firm learning, but focuses on the role of the consumption externality.

In the literature on firm learning, Vettas (2000) analyzes market capacity expansion with an intertemporal consumption externality. He shows that the competitive diffusion path becomes S-shaped and that it is always slower than the optimal path by a planner or a monopolist. In his model, however, the benefits of market concentration are overestimated because the monopolist does not have an incentive to restrict output because of perfectly elastic demand: the demand curve is a horizontal straight line. In addition, the perfectly elastic demand induces his model to require firm learning for gradual market diffusion[6] and

[5]A number of studies state that introductory pricing is possible when network externalities exist (Rohlfs (1974), Katz and Shapiro (1985, 1986), and Cabral, Salant, and Woroch (1999)). In addition, an introductory price has been theoretically observed in the optimal pricing of experience goods (Schmalensee (1982), Shapiro (1983), and Bergemann and Välimäki (2006)) and to gradually raise prices. In the marketing literature, Krishnan, Bass, and Jain (1999) analyze the optimal pricing strategy for new products based on the Bass model and show the role of introductory pricing.

[6]If firm learning does not exist, market capacity expansion becomes instantaneous. The structure of firm learning in his model builds on Rob (1991).

develops an unnatural relation between market capacity and prices: while monopoly market capacity is larger than for the competitive case, monopoly price is always higher[7]. In contrast, the demand here has a more common structure: the demand curve is a downward sloping line and leads to gradual diffusion without firm learning, the inefficient properties of monopoly diffusion, and the natural relation between the price and the market capacity.

Furthermore, this paper is related to the literature concerned with consumption externalities. One of the established literatures concerned with intertemporal consumption externalities is rational addiction where a consumer's utility is positively related to the volume of own past consumption (see Becker and Murphy (1988)). Frank (1989) studies an intertemporal effect where consumers' own past experience affects present consumption from a perspective of relative consumption. While these literatures do not focus on the role of social learning on a consumer's preferences, social learning also leads to intertemporal consumption externalities (see Banerjee (1992), Bikhchandai, Hirsheifer, and Welch (1992), Ellison and Fundenberg (1993), and MacFadden and Train (1996)). Becker (1991) studies restraint pricing where consumer demand is positively related to market capacity, and Caminal and Vives (1996) analyze the importance of past market share as a signal of product quality[8]. The model in this paper builds on these points of view[9]. Assuming the existence of an intertemporal consumption externality, this paper explores how it affects market capacity expansion depending on market concentration.

The remainder of this paper is organized as follows. Section 2 sets up the model. Section 3 introduces the concept of competitive equilibrium and analyzes its properties: the existence of S-shaped diffusion. Section 4 sets up the social planner problem and shows that competitive market capacity expansion is slow from the social viewpoint. Section 5 sets up the monopoly problem and compares the competitive market capacity expansion with the monopoly case. Section 6 contains concluding remarks. The proofs of all results are provided in the Appendix.

---

[7]In contrast to our model, an increase in current market capacity does not lower market price, but raises future prices in the case of perfectly elastic demand. In addition, as stated by Rob (1991), monopoly market capacity is always larger than in the competitive case because of the informational externality generated by the firm's learning. Therefore, a larger monopoly capacity leads to higher market prices.

[8]See also Doganoglu (2003) who examines dynamic price competition in a horizontally differentiated duopoly market.

[9]In the marketing literature, the modeling of S-shaped product diffusion relies largely on so-called "epidemic" models established by Bass (1969). See Mahajan, Muller and Bass (1990, 1995). In such models, information of a new product is assumed to spread from users to nonusers by personal contact. This social interaction may provide an explanation of intertemporal consumption externalities in this analysis. Therefore, this paper may be regarded as a complementary economic analysis of this marketing literature.

2 Model

This section develops the model. We characterize the consumers' behavior in 2.1 and the firms' behavior in the competitive market in 2.2. We assume that time is discrete and the horizon is infinite. It is also assumed that the market in this paper is a perishable good market or a services market in which the service fee is charged in every period.

2.1 Consumers

There are a number of mass unit consumers for all periods. Each consumer has a different preference for a product. Let θ be the type of consumer, which is stationary for all periods and is uniformly distributed on the interval [0,1]. The market capacity, the number of consumers who purchase the product, at period t is denoted by q_t. The consumers' willingness to pay depends on the previous period's market capacity because of the intertemporal consumption externality. We assume the following reservation price for type θ consumer at periods $t = 1, 2, .., v_t(\theta)$:

Assumption 1.

$$v_t(\theta) = V(\theta, q_{t-1}) = \rho\theta + \sigma(q_{t-1}) \tag{1}$$

where $\rho > 0$, $\sigma'(q) > 0$, $\sigma''(q) < 0$, $\sigma(0) = 0$, $\lim_{q\to 0} \sigma'(q) = \infty$, *and* $\lim_{q\to\infty} \sigma'(q) = 0$, *and where* ρ *is a preference parameter.*

$\sigma(q_{t-1})$ represents the intertemporal consumption externality, which has two properties. First, $\sigma'(q_{t-1})$ > 0 implies that each consumer's reservation price at period t is strictly increasing in the previous period's market capacity. Second, $\sigma''(q_{t-1}) < 0$ implies that the increase in the reservation price, or equivalently the benefit of the externality, is strictly decreasing in the previous period's market capacity. Of course, there may exist a locally increasing part especially at smaller market capacities but it is unrealistic that the increasing part would be observed for larger market capacities. This assumption guarantees that there is a unique upper limit for market capacity for every market capacity expansion in this analysis[10].

A consumer of type θ pays p_t for a product and enjoys consumer surplus of $v_t(\theta) - p_t$. The consumer is assumed to purchase the product if and only if consumer surplus is nonnegative, i.e., $v_t(\theta) - p_t \geq 0$.

[10]If the benefit of the externality increases initially, but eventually decreases with increases in the previous period's market capacity, multiple steady states may exist.

Then, the inverse demand function at period t, $P(q_{t-1}, q_t)$, becomes:

$$P(q_{t-1}, q_t) = \begin{cases} \rho + \sigma(q_{t-1}) - \rho q_t & 0 \leq q_t \leq 1, \\ 0 & q_t > 1. \end{cases} \tag{2}$$

for all $t = 1, 2,..,$ and $0 \leq q_{t-1} \leq 1$. It is easy to see that the inverse demand function is strictly increasing in the previous period's market capacity, but strictly decreasing in the current period's market capacity.

2.2 Firms under Competitive Environment

Firms in the competitive market are identical, small and price takers. At the beginning of each period, the set of firms is composed of two subsets associated with potential entrants and incumbents. There is assumed to be no asymmetry of information between the two subsets and to be no demand uncertainty. At the beginning of each period, potential entrants decide whether to enter the market or not, and incumbents decide whether to exit the market or not. Potential entrants enter the market with entry cost $c > 0$, which is the initial investment in purchases such as machines. We assume that machines are durable and are operated for multiple periods. Machines can be operated at any level between zero to one unit for each period in an environment of constant return to scale. To simplify the analysis, we assume that the scrap value of machines is zero and the marginal cost is zero.

Let x_t be the number of incumbents at period t, y_t the number of new firms entering the market at the beginning of period t. Because the entry cost is not recoverable and the marginal cost is zero, incumbents do not have an incentive to exit the market. Therefore, $y_t = x_t - x_{t-1} \geq 0$ for all $t = 1, 2,...$ Assuming that $x_0 = 0$, we have $x_t = \sum_{\tau=1}^{t} y_\tau$. Let $i > 0$ be a constant interest rate and the discount factor is denoted by $\beta \equiv 1/(1+i)$. For each period, firms maximize the discounted sum of future operation profits, which is denoted by $R(x_{t-1}, y_t)$[11], i.e.:

$$R(x_{t-1}, y_t) = p_t + \beta R(x_t, y_{t+1}), \tag{3}$$

for all $t = 1, 2,,..,$ where p_t represents the direct operation profit (market price) at period t and $\beta R(x_t, y_{t+1})$ represents the discounted future operation profits. Potential entrants enter the market if and only if the present value of net profits is positive, i.e., $R(x_{t-1}, y_t) > c$. If the demand is initially low and the fixed cost or the discount factor is high, then entry may not occur in the first period. The following assumption guarantees first period entry.

[11] Because entrants and incumbents are symmetric and the horizon is finite, they have the same present value of their future revenue streams.

Assumption 2.

$$\rho > (1 - \beta)c \tag{4}$$

Assumption 2 implies that entering the market is profitable in the first period. If $\rho \leq (1-\beta)c$, then first period entry is not attractive and it does not occur. Because this condition holds for all following periods, the market capacity expansion never occurs.

3 Analysis

This section provides the characterization of competitive equilibrium and explores the existence of S-shaped market diffusion. We first characterize the competitive equilibrium in 3.1. Then, the existence of S-shaped market diffusion is examined.

3.1 Competitive Equilibrium

Each period's equilibrium condition is determined by the market clearing condition and the zero profit condition. Let r_t be the number of new consumers who purchase the product at period t. Now, we define the competitive equilibrium as follows:

Definition. *The competitive equilibrium consists of three sequences* $\{p_t^c, r_t^c, y_t^c\}$ *that simultaneously satisfy the following conditions:*

1. *The market clears for all* $t = 1, 2, ...$

$$r_t^c = y_t^c \quad (\Leftrightarrow q_t^c = x_t^c). \tag{5}$$

2. *The market price is determined by the inverse demand of consumers for all* $t = 1, 2, ...$

$$p_t^c = P(x_{t-1}^c, x_{t-1}^c + y_t^c). \tag{6}$$

3. *New entry occurs until excess profits become zero for all* $t = 1, 2, ...$

$$R(x_{t-1}^c, y_t^c) \leq c, \tag{7}$$

with equality if $y_t^c > 0$.

According to the above definition, the properties of the competitive market capacity expansion are identified. From the market clearing condition, the number of new consumers, r_t^c is nondecreasing. In addition, from the zero profit condition and equation (3), the equilibrium price in the competitive market becomes:

$$p_t^c = (1-\beta)c, \tag{8}$$

for all $t \geq 1$ such that $y_t^c > 0$. This implies that the equilibrium price is constant as long as the market is in the transition process and it does not depend on the externality. Given this equilibrium price, potential entrants enter the market in response to demand growth in each period. By putting the inverse demand function into equation (8), the competitive market capacity expansion is summarized as follows:

$$\rho + \sigma(x_{t-1}^c) - \rho[x_{t-1}^c + y_t^c] + \beta c = c \tag{9}$$

for all $t \geq 1$ such that $y_t^c > 0$. Equation (9) implies that the competitive market capacity expansion is represented by a first order difference equation with respect to x_t^c. Denote the steady state of equation (9) by x^c. Then, the dynamical system of equation (9) is summarized in Figure 1. It is easy to see that the sequences $\{x_t^c\}_0^\infty$ satisfy $x_t^c \in [0, x^c]$ for all $t = 1, 2, ..,$ and monotonicity, $x_0^c = 0$ and $x_t^c \to x^c$ as $t \to \infty$[12].

3.2 S-shaped Market Diffusion

From now on, we examine the time pattern of the competitive market capacity expansion and show that it becomes S-shaped (initially convex, but eventually concave) when the externality effect is initially strong enough. In terms of firm entry, S-shaped market diffusion implies that the level of new entry is initially increasing, but eventually decreasing. We first explore the determinants of the amount of new entry at each period. Then, the initial convexity and the eventual concavity of the market diffusion path are examined, respectively.

Let $\Delta y_t^c \equiv y_{t+1}^c - y_t^c$. The time path of the market capacity expansion becomes S-shaped if $\Delta y_t^c > 0$ initially, but $\Delta y_t^c < 0$ eventually. We also define the potential firms' profitability of entry at the beginning

[12]By rearranging the terms of equation (9), we have:

$$x_t^c = X(x_{t-1}^c) = \sigma(x_{t-1}^c)/\sigma + [\rho - (1-\beta)c]/\rho. \tag{9'}$$

From the properties of σ, we have $X(0) = [\rho - (1-\beta)c]/\rho > 0$, $X'(x_{t-1}^c) > 0$, $X''(x_{t-1}^c) < 0$, $\lim_{x_{t-1}^c \to 0} X'(x_{t-1}^c) = \infty$, and $\lim_{x_{t-1}^c \to \infty} X'(x_{t-1}^c) = 0$. Therefore, $X(x_{t-1}^c)$ crosses the $x_t^c = x_{t-1}^c$ line only once and there is a unique steady state, x^c.

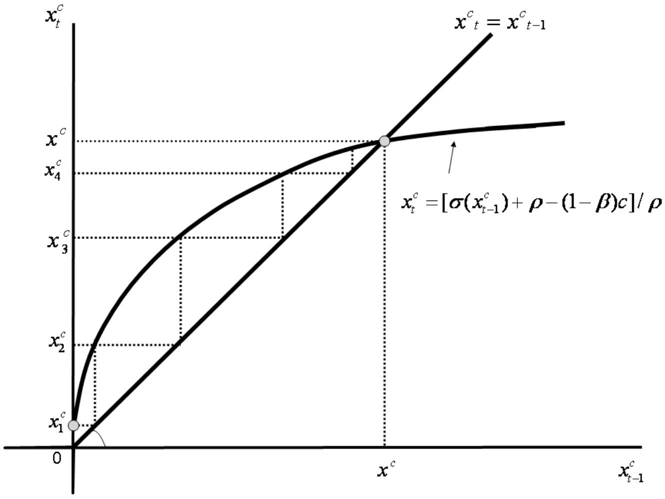

Figure 1: The dynamical system of competitive diffusion path

of each period as $\Pi(x^c_{t-1}) \equiv R(x^c_{t-1}, 0) - c$, i.e.:

$$\Pi(x^c_{t-1}) = \rho + \sigma(x^c_{t-1}) - \rho x^c_{t-1} - (1-\beta)c, \tag{10}$$

for all $t = 1, 2, ...$ Equation (10) implies that the net profit of new entry at the beginning of each period is a function of the previous market capacity. It does not depend on the current period's market capacity. Let $\Delta\pi^c_t \equiv \pi^c_{t+1} - \pi^c_t$. The following lemma shows the relation between the profitability of new entry and the level of new entry at each period.

Lemma 1. *Suppose that* $x^c_0 = 0$, *and* $\sigma(1) \leq (1-\beta)c$ *so that* $x^c \leq 1$. *Then, for all* $t = 1, 2, ..,$ $\Delta y^c_t \gtreqless 0$ *if and only is* $\Delta\pi^c_t \gtreqless 0$.

Proof. See Appendix. □

Lemma 1 implies that an increase or decrease in the amount of new entry is equivalent to an increase or decrease in the profitability of entering the market at the beginning of each period. If the profitability increases (decreases), then the amount of new entry increases (decreases). Therefore, we focus on the properties of $\Pi(x^c_{t-1})$ more precisely in order to explore the time pattern of competitive market capacity expansion. The following lemma summarizes the properties of the profitability of new entry:

Lemma 2. $\Pi(x^c_{t-1})$ *has a single peaked property; more precisely,*

1. *$\Pi(x)$ satisfies $\Pi(x) > 0$ for $0 \leq x < x^c$, $\Pi(0) = \rho - (1-\beta)c$, and $\Pi(x^c) = 0$.*

2. *There is a unique maximizer $\bar{x} = \sigma'^{-1}(\rho) \in (0, x^c)$ such that $\Pi(\bar{x}) = \bar{\pi} > \rho - (1-\beta)c$.*

Proof. See Appendix. □

The single peaked property of the profitability of new entry is summarized in Figure 2. This property follows from the features of the inverse demand function with $y_t^c = 0$, $P(x_{t-1}^c, x_{t-1}^c) = \rho + \sigma(x_{t-1}^c) - \rho x_{t-1}^c$. An increase in the previous period's market capacity leads to two independent effects associated with the effect of demand growth because of the externality (as captured by $\sigma(x)$) and with the effect of declining market price because of the downward sloping demand curve (as captured by ρx). First, market demand is increasing in the previous period's market capacity because of the intertemporal consumption externality. The effect of demand growth on the profitability of new entry is always positive. However, the downward sloping demand curve leads to a decline in the market price as the previous period's market capacity increases. This effect is always negative and it reduces the profitability of new entry. Therefore, the net effect on the profitability of new entry is dependant on the relative magnitude of the two effects.

From the property of $\sigma(\cdot)$, the effect of demand growth is initially stronger than that of a declining market price, and the profitability of new entry is increasing for all $0 \leq x_{t-1}^c < \bar{x}$. However, as the market capacity increases, the effect of demand growth becomes weaker and the profitability of new entry decreases for all $x_{t-1}^c > \bar{x}$. Its value becomes zero at the steady state, x^c. As a result, $\Pi(x_{t-1}^c)$ has the single peaked property.

The single peaked property of the profitability of new entry and lemma 1 imply that the amount of new entry increases for $0 < x_{t-1}^c \leq \bar{x}$ but decreases for $\bar{x} < x_{t-1}^c < x^c$. Because market capacity is strictly increasing for all periods, it is easy to see that the time path of competitive market capacity expansion eventually becomes concave. Therefore, it becomes S-shaped if $0 < x_1^c \leq \bar{x}$. This condition is equivalent to $\sigma'(x_1^c) > \rho$. This implies that the strong externality effect contributes to the initial convexity of the time path of competitive market capacity expansion. Moreover, the necessary and sufficient condition for the initial convexity is $\Pi(x_1^c) > \Pi(x_0^c)$, where $x_0^c = 0$. Let x^* be $x > 0$ such that $\Pi(x) = \Pi(x_0^c)$. From Figure 2, it is easy to see that $\Pi(x_1^c) > \Pi(x_0^c)$ if and only if $x_1^c < x^*$. More precisely, we have the following proposition:

Proposition 1. *Suppose that* $x_0^c = 0$. *Then, the time pattern of competitive market capacity expansion becomes S-shaped if and only if*

$$\sigma\left(\frac{\rho - (1-\beta)c}{\rho}\right) > \rho - (1-\beta)c. \tag{11}$$

Proof. See Appendix. □

One of the important properties of the competitive capacity expansion is that the first period entry, $(\rho - (1-\beta)c)/\rho$, does not depend on the externality, $\sigma(\cdot)$, but the subsequent period entry does. Therefore, the strong externality effect does not lead to a larger market capacity expansion in the first period, but it does in subsequent periods. From inequality (11) and the properties of $\sigma(\cdot)$, it is easy to see that S-shaped market diffusion is more likely to be observed under small values of initial profitability, $\Pi(x_0^c) = \rho - (1-\beta)c$, which follows from the low preference parameter, the low discount factor, and the high entry cost. Therefore, we conclude that the S-shaped time pattern of competitive market capacity expansion is observed for a strong externality effect and low initial profitability.

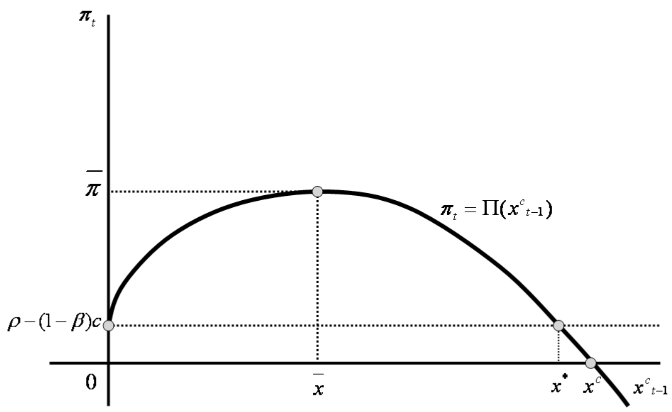

Figure 2: Property of $\Pi(\cdot)$

4 Welfare

In the previous section, we showed that the competitive market capacity expansion has an S-shaped time pattern. We examine its welfare properties in this section and compare it with the monopoly market capacity expansion in the following section. We first set up a social planner problem in 4.1 and then examine the properties of a socially optimal market capacity expansion in 4.2.

4.1 The Planner's Problem

A social planner expands market capacity with expansion cost c per unit and zero marginal operation cost. The planner maximizes the discounted sum of future welfare. Let x_t^o be the market capacity set by the planner at period t and y_t^o the amount of new capacity expansion at period t. Socially optimal planning satisfies the following Bellman Equation:

$$V(x_{t-1}^o) = \max_{y_t^o \geq 0} \left\{ \int_0^{x_{t-1}^o + y_t^o} [\rho + \sigma(x_{t-1}^o) - \rho w - c y_t^o] dw + \beta V(x_{t-1}^o + y_t^o) \right\}. \tag{12}$$

The interpretation of the above equation is that the present value of the sum of future welfare is current welfare plus the discounted next period value of the sum of future welfare. We now characterize the feature of the social optimum market capacity expansion as follows:

Proposition 2. *Let x^o be the market size in the steady state of the social planner problem and $\eta \equiv [\rho - \sigma''(x^o)x^o]/\beta\sigma'(x^o) > 0$. Suppose that $\sigma(1) + \beta\sigma'(1) \leq (1-\beta)c$ (so that the Euler equation does not jump), $\beta\eta^2 > 4$ (so that the eigenvalues are real numbers), and $\beta\eta > 1 + \beta$ (so that x^o is saddle). Then, there is a unique $x^o > 0$, and for all $t = 1, 2, ..$, there exists a unique optimal solution of the planner's problem $y_t^o > 0$, which satisfies the following conditions:*

1. *For all $t = 1, 2, ..$, the optimal solution y_t^o satisfies the following second order difference equation:*

$$R(x_{t-1}^o, y_t^o) + \beta\sigma'(x_{t-1}^o + y_t^o)[x_{t-1}^o + y_t^o + y_{t+1}^o] = c, \tag{13}$$

   where $R(x_{t-1}, y_t) = \rho + \sigma(x_{t-1}) - \rho[x_{t-1} + y_t] + \beta c$, and

2. *For all $t = 1, 2, ..$, $x_t^o \in [0, x^o]$ with $x_0^o = 0$ and $x_t^o \to x^o = 0$ as $t \to \infty$.*

Proof. See Appendix. □

Note that there is a second order difference equation, but one initial condition, $x_0^o = 0$. Therefore, another boundary condition is required. Proposition 2 shows that there exists a unique steady state x^o whose local property is saddle. It also shows that there exist unique sequences $\{x_t^o\}_0^\infty$ converging on

x^o, which is determined by two boundary conditions, and satisfies monotonicity and $x_t^o \in [0, x^o]$ for all $t = 1, 2, ...$

Equation (13) shows that the marginal expansion cost is equal to the marginal social benefit at the socially optimal plan. The right hand side is the marginal expansion cost. The left hand side is the marginal social benefit, which is composed of two elements. The first term represents the discounted sum of future revenue. The second term indicates the social benefit from the internalization of the externality; increased current sales increase the subsequent period's market demand.

4.2 Optimal Capacity Expansion

Note that the only difference between the competitive market capacity expansion and the socially optimal market capacity expansion is whether to internalize the social benefit from the externality or not. From the social viewpoint, it is more efficient to design a market capacity expansion taking into consideration an increase in the subsequent period's demand as a result of increased current sales. On this point, the competitive market capacity expansion is socially inefficient.

Let p_t^o be the shadow price of the product in the planner's problem. From equation (13), it is denoted by:

$$p_t^o = (1 - \beta)c - \beta\sigma'(x_t^o)x_{t+1}^o > 0, \tag{14}$$

for all $t \geq 1$[13]. Note that the equilibrium price in the competitive market does not depend on the strength of the externality effect and it is constant for all periods, i.e., $p_t^c = (1 - \beta)c$ for all $t = 1, 2, ...$ On the other hand, the shadow price in the planner's planning depends on the strength of the externality effect and changes responding to the strength of the externality effect. It is easy to see that it is lower than the equilibrium price in the competitive market. This property leads to the socially efficient market capacity expansion, which is faster than the competitive one:

Proposition 3. *Suppose that $x_0^i = 0$, for all $i = c, o$ and $\beta \in (0, 1]$. Then, for all $x_{t-1}^c \leq x_{t-1}^o$, we have $x_t^c < x_t^o$.*

Proof. See Appendix. □

[13]The last inequality follows from the assumption $\sigma(1) + \sigma'(1) \leq (1 - \beta)c$.

Moreover, the two market capacity expansions differ according to their time patterns. Whereas the externality does not affect the first period's market capacity expansion in the competitive market, it does under socially optimal planning. The strong externality effect makes the competitive market capacity expansion initially too slow from the social viewpoint and initially convex. On the other hand, it leads to an initially large market capacity under optimal planning and makes the S-shaped time pattern more difficult to obtain.

5 Comparing Competitive Diffusion and Monopoly Diffusion

In this section, we examine how the market capacity expansion differs depending on the degree of market concentration. We first set up the monopoly market capacity expansion in 5.1. Then, we compare it with the competitive one in 5.2.

5.1 The Monopolist's Planning

A monopolist is assumed to expand market capacity with expansion cost c per unit and the zero marginal operation costs. The monopolist maximizes the discounted sum of future profits. Let x_t^m be the market capacity by the monopolist at period t and y_t^m the amount of new capacity expansion at period t. The profit maximization problem is summarized by the following Bellman equation:

$$V(x_{t-1}^m) = \max_{y_t^m \geq 0} \left\{ \rho + \sigma(x_{t-1}^m) - \rho[x_{t-1}^m + y_t^m] - cy_t^m + \beta V(x_{t-1}^m + y_t^m) \right\}. \quad (15)$$

The interpretation of the above equation is that the present value of the sum of future profits is composed of the current profit and the discounted next period profit. The following proposition characterizes the monopolist's planning.

Proposition 4. *Let x^m be the market size in the steady state under the monopolist's plan and $\gamma = [2\rho - \sigma''(x^m)x^m]/\beta\sigma'(x^o) > 0$. Suppose that $\sigma(1) + \sigma'(1) \leq \rho + (1-\beta)c$, and $\beta\gamma^2 > 4$. Then, there exists a unique $x^m > 0$ and for all $t = 1, 2, ..,$ there exists a unique $y_t^m > 0$, which satisfies the following conditions:*

1. *For all $t = 1, 2, ..,$ the optimal solution $y_t^m > 0$ satisfies the following second order difference equation:*

$$R(x_{t-1}^m, y_t^m) + \beta\sigma'(x_{t-1}^m + y_t^m)[x_{t-1}^m + y_t^m + y_{t+1}^m] - \rho[x_{t-1}^m + y_t^m] = c. \quad (16)$$

2. *For all* $t = 1, 2, .., x_t^m \in [0, x^m]$ *with* $x_0^m = 0$ *and* $x_t^m \to x^m$ *as* $t \to \infty$.

Proof. See Appendix. □

Equation (16) implies that the marginal expansion cost for the monopolist is equal to the marginal benefit, which is composed of three elements. The first term on the right hand side of equation (16) is the present value of the sum of direct future revenues. The second term is the discounted value of the future benefit in which an increased current market capacity raises subsequent period demand. The last term represents a marginal loss of current revenue. This term is regarded as the current benefit, in which the decreased current market capacity raises the current revenue.

The last two terms represent the intertemporal trade off with respect to the market cultivation strategies of the monopolist. From the discounted future benefit, the monopolist has an incentive to lower the current market price and increase current market capacity. This incentive coincides with the social planner's incentive to maximize social welfare. However, the monopolist has the incentive to raise the current market price and to reduce output because of the current benefit. Therefore, the optimal planning by the monopolist is determined by the magnitude of both benefits.

5.2 Initial Efficiency and Eventual Inefficiency of Monopoly Diffusion

The comparison of both market capacity expansions starts from the initial market capacity expansions. Then, the eventual market capacity expansions are examined. Note that the only difference between both market capacity expansions is that the monopolist faces an intertemporal trade off.

Let p_t^m be the monopoly equilibrium price at period t. It is denoted by:

$$p_t^m = (1 - \beta)c - \beta\sigma'(x_t^m)x_{t+1}^m + \rho x_t^m, \tag{17}$$

for all $t = 1, 2, ...$ It is easy to see that the monopoly equilibrium price is endogenously determined by the intertemporal trade off between the discounted future benefit and the current benefit. By comparing (8) with (17), the monopoly equilibrium price at the current period is lower (higher) than the competitive one if the net benefit of increasing current output is positive (negative). Because the inverse demand is strictly decreasing in the current market capacity, the larger (smaller) monopoly capacity is observed as long as the previous monopoly market capacity is at least as large (small) as the previous competitive market capacity.

Proposition 5. *Suppose that* $x_0^i = 0$ *for all* $i = c, m$ *and* $\beta\sigma'(x_t^m)x_{t+1}^m \lesseqgtr \rho x_t^m$*. Then, for all* $x_{t-1}^m \lesseqgtr x_{t-1}^c$*, we have* $x_t^m \lesseqgtr x_t^c$*.*

Proof. See Appendix. □

Note that the higher value of the discounted future benefit arises in the environment of a strong externality effect, $\sigma'(x_t^m)$, the large new market capacity expansion in the subsequent period, y_{t+1}^m, and the low discount value, β. On the other hand, the degree of the current benefit, ρx_t^m is constant under the current market capacity. Therefore, we conclude that the monopoly market capacity is larger than the competitive one in the early periods because the externality effect is decreasing over time and y_{t+1}^m is initially large[14].

Next, we compare the eventual capacity expansions in the both markets. As the market capacity increases, the net benefit of increasing the current market capacity decreases and the monopoly equilibrium price increases. The following lemma shows that the net benefit of increasing the current market capacity becomes negative in the steady state.

Lemma 3. *In the steady state,* $\rho > \beta\sigma'(x^m)$*.*

Proof. See Appendix. □

From lemma 3, it is easy to see that the monopoly equilibrium price in the steady state is higher than the competitive one by comparing equations (8) and (17). This implies that the monopolist has an incentive to slow down the market cultivation and to eventually earn positive profits. The following proposition shows that the slow downed monopoly market capacity expansion leads to a smaller market size in the steady state than for the competitive market capacity expansion.

Proposition 6. *In the steady state,* $x^c > x^m$*.*

Proof. See Appendix. □

The characteristic monopoly equilibria[15] may explain the low introductory price. In the presence of the intertemporal consumption externality, it is optimal for the monopolist to reduce the initial profits

[14]Whereas the concavity of $\sigma(x)$ contributes this result, this result holds even if the externality effect is constant, i.e., $\sigma'(x) = \bar{\sigma} < \rho$. In this setting, the discounted sum of the future benefit in the first period is denoted by $\bar{\sigma}[x_1^m + y_2^m]$. Because y_2^m is positive, the monopoly market capacity expansion is initially faster than the competitive one if $\rho - \bar{\sigma}$ is close to zero.

[15]The results here differ from the results of Vettas (2000). The monopoly price in Vettas is dependent only on the previous period's market capacity because of perfectly elastic demand: an increase in the monopolist's output does not lower price. In the equilibrium, the large amount of market capacity in the previous period raises prices and revenues. In this environment, the

by using the introductory price, and to start to earn larger profits with higher demand and a higher price in the earlier periods. If the externality effect is initially strong enough, the monopolist earns negative profits in the initial periods, but soon obtains strong demand growth. It is interesting to recognize that the monopolist's incentive to cultivate the market initially leads to a socially more efficient output level than the competitive market under a strong externality effect.

Furthermore, the above results provide some implications about the time pattern of the monopoly market capacity expansion. As well as the socially optimal capacity expansion, the first period monopoly market capacity expansion is influenced by the externality. In addition, it eventually leads to a smaller market size than the competitive one. Therefore, the monopoly market capacity expansion in the first period is relatively larger than the competitive one. It tends to be concave and has the greatest difficulty of the three market capacity expansions in becoming S-shaped.

6 Conclusion

This paper presents a dynamic model of market capacity expansion where the market demand endogenously shifts in response to the previous period's market capacity because of an intertemporal consumption externality. We explore how the degree of market diffusion depends on the market structures with respect to its levels, time patterns, and prices. The major results reported here are summarized as follows. First, the competitive environment leads to a constant equilibrium price, which is not influenced by the externality. This contributes to the S-shaped time pattern of market capacity expansion. In addition, the competitive market capacity expansion is socially inefficient and initially too slow because it does not internalize the benefit of the externality through the market price. Therefore, subsidies for initial entrants can be an effective policy.

On the other hand, market concentration enables the firm to control the market price by internalizing the effect of the externality. The monopolist has an incentive to initially cultivate the market at a fast pace using a low introductory price and eventually earn large profits with higher levels of demand. Whereas this strategy leads to negative profits initially, it is optimal because the monopolist generates high levels of market demand sooner.

monopolist has no incentive to restrict the current market capacity, but has an incentive to increase it instead. In addition, the monopoly market capacity is efficient and larger than the competitive one because of firm learning. As a result, the monopoly market capacity is efficient and larger than the competitive one, while the monopoly price is higher.

Whereas the model has ignored the impact of heterogeneous firms, demand uncertainty, technological progress, and strategic behavior, which mainly impact the producer, these elements may be important issues for market diffusion. However, our concern here is to examine how market diffusion differs between market structures when an intertemporal consumption externality exists, and to provide an alternative explanation of S-shaped market diffusion and a low introductory price as simply as possible without these elements. Therefore, this paper is to be regarded as a complement to these issues.

There are the several issues requiring future work. First, the empirical importance of the intertemporal consumption externality and the relation between market capacity expansion and market concentration. In addition, this paper ignores the aspect of oligopolistic market diffusion. My conjecture is that an oligopolistic market capacity expansion is faster than the monopoly, but slower than the competitive market, and that its time pattern is likely to be S-shaped as the number of firms increase. Finally, there is concern over how general our results are. While the analysis here is couched in terms of a parametric example, the results may extend to more general settings. We hope this study helps researchers address these issues.

Appendix

Proof of Lemma 1

Note that for all $t = 1, 2, ..$, new entry $y_t^c > 0$ satisfies $\pi_t^c = \rho y_t^c$. It is straightforward that π_t^c is positively related to y_t^c. □

Proof of Lemma 2

By differentiating $\Pi(x)$ with respect to x, we have,

$$\Pi'(x) = \sigma'(x) - \rho, \tag{18}$$

and

$$\Pi''(x) = \sigma''(x). \tag{19}$$

By the definition of $\sigma(.)$, $\Pi(x)$ is a strictly concave function for $x \geq 0$ and has a unique maximizer $\bar{x} = \sigma'^{-1}(\rho)$. It is easy to see that $\Pi(\bar{x}) > \Pi(0) = \rho - (1 - \beta)c$ and $\Pi(x^c) = 0$. □.

Proof of Proposition 1

Because the competitive diffusion path eventually becomes concave, it becomes S-shaped if and only if $\pi_1^c > 0$. This condition is equivalent to:

$$\sigma(x_1^c) > \rho x_1^c. \tag{20}$$

Putting $x_1^c = [\rho + (1-\beta)c]$ into this inequality and rearranging terms, we have inequality (11). □

Proof of Proposition 2

Note that there is a second order difference equation and one initial condition, $x_0^o = 0$. By using a phase diagram, we show that there exist unique sequences, $\{x_t^o\}_0^\infty$, which satisfy (i) for all $t = 1, 2, .., x_t^o \in [0, x^o]$, and (ii) monotonicity, $x_0^o = 0$ and $x_t^o \to x^o$ as $t \to \infty$.

We first show that there exists a unique $x^o > 0$. On the steady state,

$$\rho x^o - \sigma(x^o) - \beta\sigma'(x^o)x^o = \rho - (1-\beta)c. \tag{21}$$

Let $T(x) = \rho x - \sigma(x) - \beta\sigma'(x)$. It is easy to see that $T(x)$ is strictly increasing in $x > 0$ such that $T(x) > 0$. Therefore, there exists a unique $x^o > 0$.

To analyze the property of equation (13) further, we transform equation (13) into an equivalent system in the (x_{t-1}^o, x_t^o) space. Solving (13) with respect to x_{t+1}^o, we have:

$$x_{t+1}^o = \frac{1}{\beta\sigma'(x_t^o)}\{\rho x_t^o - \sigma(x_{t-1}^o) - [\rho - (1-\beta)c]\}, \tag{22}$$

for all $t = 1, 2, ...$ Note that equation (22) is a second order difference equation with one variable, x^o. By defining $z_t^o \equiv x_{t-1}^o$, we translate equation (22) to a simultaneous equation with two variables, x^o and z^o. Let the right hand side of equation (20) be $\phi(x_t^o, x_{t-1}^o)$. Then, equation (20) becomes:

$$\begin{cases} x_{t+1}^o = \phi(x_t^o, z_t^o), \\ z_{t+1}^o = x_t^o. \end{cases} \tag{23}$$

for all $t = 1, 2, ...$ By using Taylor's formula, the linearized system around x^o is denoted by:

$$\begin{bmatrix} x_{t+1}^o - x^o \\ z_{t+1}^o - x^o \end{bmatrix} = \begin{bmatrix} \eta & -\frac{1}{\beta} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_t^o - x^o \\ z_t^o - x^o \end{bmatrix}. \tag{24}$$

for all $t = 1, 2, ...$ Then, the characteristic equation becomes:

$$(\lambda^o)^2 - \eta\lambda^o + \frac{1}{\beta} = 0. \tag{25}$$

Assuming $\beta\eta^2 > 4$ and $\beta\eta > 1 + \beta$, we have eigenvalues $\lambda_1^o, \lambda_2^o \in \mathbb{R}_+^2$ such that $0 < \lambda_1^o < 1 < \lambda_2^o$ and eigenvector corresponding to λ_1^o is:

$$(e_1^o, e_2^o) = (\lambda_1^o, 1). \tag{26}$$

Therefore, the local property of x^o is saddle.

We finally draw the phase diagram. From equation (23), $z_t^o = z_{t+1}^o$ locus is $z_t^o = x_t^o$, that is, the 45 degree line of the (z_t^o, x_t^o) plane. On the other hand, the $x_t^o = x_{t+1}^o$ locus is $x_t^o = \phi(x_t^o, z_t^o)$. By solving $\phi(x_t^o, z_t^o)$ with respect to z_t^o, we have:

$$z_t^o = \sigma^{-1}([\rho - \beta\sigma'(x_t^o)])x_t^o - [\rho - (1-\beta)c]), \tag{27}$$

for all $t = 1, 2, ...$ Because $[\rho - \beta\sigma'(x_t^o)]x_t^o$ is strictly increasing in x_t^o and convex for $[\rho - \beta\sigma'(x_t^o)]x_t^o > 0$, and $\sigma^{-1}(x)$ is strictly increasing and convex, the $x_t^o = x_{t+1}^o$ locus is strictly increasing and convex in x_t^o. We next examine whether z_t^o and x_t^o are increasing or decreasing above and below the phase-lines. It is easy to check that for the points above the $z_t^o = z_{t+1}^o$ line, we have $z_t^o > z_{t+1}^o$, and for the points below the $z_t^o = z_{t+1}^o$ line, we have $z_t^o < z_{t+1}^o$. Also, for the points above $x_t^o = x_{t+1}^o$ line, we have $x_t^o > x_{t+1}^o$, and for the points below $x_t^o = x_{t+1}^o$, we have $x_t^o < x_{t+1}^o$. The phase diagram is summarized in Figure 3 where the arrows show the direction of increase at each point.

We now characterize the optimal solution by using the above properties. It satisfies the following conditions: (i) the $x_t^o = x_{t+1}^o$ locus is increasing and (ii) $x_t^o \in [\overline{x}^o, x^o]$ for all $t = 1, 2, ..$, where:

$$\overline{x}^o = \{x \in \mathbb{R}_{++} : [\rho - \beta\sigma'(x)]x = \rho - (1-\beta)c\}. \tag{28}$$

Because $[\rho - \beta\sigma'(x)]x$ is strictly increasing in x for all $[\rho - \beta\sigma'(x)]x > 0$, there exists a unique $\overline{x}^o > 0$. This implies that the $x_t^o = x_{t+1}^o$ locus intersects the $x_{t-1}^o = 0$ axis once at $\overline{x}^o > 0$. Any $x_t^o \notin [\overline{x}^o, x^o]$ does not become the optimal solution. Therefore, we conclude that the optimal solution that satisfies conditions (i) and (ii) is restricted in the shaded portion. Any points outside the shaded portion diverge and do not become the optimal solution. For the points above the $z_t^o = z_{t+1}^o$ locus, x_t^o becomes negative in finite time but this violates a feasibility condition. On the other hand, for the points above $z_{t+1}^o > z_t^o$ but below $x_t^o = x_{t+1}^o$, x_t^o is increasing over time satisfying $z_{t+1}^o > z_t^o$ ($x_t^o > x_{t-1}^o$). However, for large x_t^o such that $x_t^o > x_{t-1}^o$, the inverse demand function becomes $P(x_{t-1}^o, x_t^o) = 0$. In these circumstances, Euler equation

(13) does not hold. It is easy to see that there exists a unique path that converges to x^o and the optimal solution consists of the points that belong to the stable manifold of x^o. □

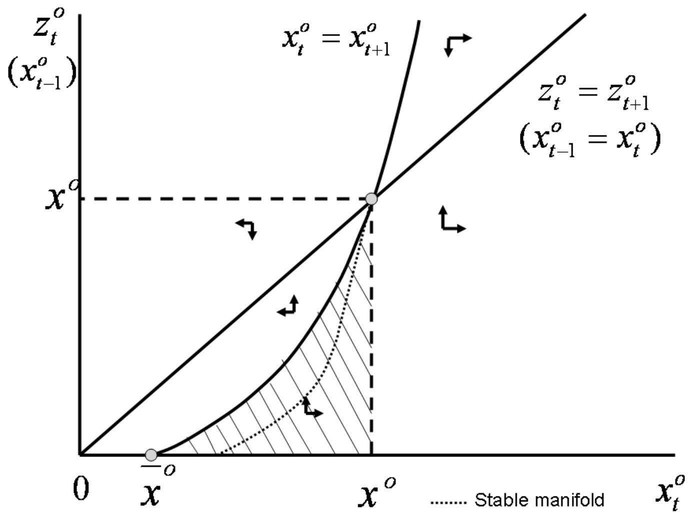

Figure 3: The dynamical system of socially optimal diffusion path

Proof of Proposition 3

Let $U(x_{t-1}^o, x_t^o) \equiv \rho + \sigma(x_{t-1}^o) - \rho x_t^o + \beta\sigma'(x_t^o)x_{t+1}^o$. Rearranging equation (9), the competitive diffusion path is denoted by:

$$U(x_{t-1}^c, x_t^c) - \beta\sigma'(x_t^c)x_{t+1}^c = c. \tag{29}$$

Let $x_{t-1}^c \leq x_{t-1}^o$. Suppose in negation that $x_t^c \geq x_t^c$. Then, by using the properties of $U(.,.)$, we have the following inequalities:

$$U(x_{t-1}^o, x_t^o) \geq U(x_{t-1}^o, x_t^c) \geq U(x_{t-1}^c, x_t^c), \tag{30}$$

for all $t = 1, 2, ..$, where the first inequality follows from $U_2(x_{t-1}, x_t) < 0$ and the second inequality follows from $U_1(x_{t-1}, x_t) > 0$. Because $U(x_t^o, x_t^o) = c$ in the equilibrium, we have $U(x_{t-1}^c, x_t^c) \leq c$ which contradicts equation (29) because $\beta\sigma'(x_t^c)x_{t+1}^c$ is positive. □

Proof of Proposition 4

Note that there is a second order difference equation and one initial condition, $x_0^m = 0$. By using a phase diagram, we show that there exist unique sequences, $\{x_t^m\}_0^\infty$, which satisfy (i) for all $t = 1, 2, ..,$ $x_t^m \in [0, x^m]$, and (ii) monotonicity, $x_0^m = 0$ and $x_t^m \to x^m$ as $t \to \infty$.

We first show that there exists a unique $x^m > 0$. In the steady state,

$$2\rho x^m - \sigma(x^m) - \beta\sigma'(x^m)x^m = \rho - (1-\beta)c. \tag{31}$$

Let $F(x) = 2\rho x - \sigma(x) - \beta\sigma'(x)$. It is easy to see that $F(x)$ is strictly increasing in $x > 0$ such that $F(x) > 0$. Therefore, there exists a unique $x^m > 0$.

To analyze the properties of equation (16) further, we transform equation (16) to an equivalent system in (x_{t-1}^m, x_t^m) space. Solving (16) with respect to x_{t+1}^m, we have:

$$x_{t+1}^m = \frac{1}{\beta\sigma'(x_t^m)}\{2\rho x_t^m - \sigma(x_{t-1}^m) - [\rho - (1-\beta)c]\}, \tag{32}$$

for all $t = 1, 2, ...$ Note that equation (32) is a second order difference equation with one variable, x^m. By defining $z_t^m \equiv x_{t-1}^m$, we translate equation (32) to a set of simultaneous equations with two variables, x^m and z^m. Let the right hand side of equation (32) be $\psi(x_t^m, x_{t-1}^m)$. Then, equation (32) becomes:

$$\begin{cases} x_{t+1}^m = \psi(x_t^m, z_t^m), \\ z_{t+1}^m = x_t^m. \end{cases} \tag{33}$$

for all $t = 1, 2, ...$ By using Taylor's formula, the linearized system around x^m is denoted by:

$$\begin{bmatrix} x_{t+1}^m - x^m \\ z_{t+1}^m - x^m \end{bmatrix} = \begin{bmatrix} \gamma & -\frac{1}{\beta} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_t^m - x^m \\ z_t^m - x^m \end{bmatrix}. \tag{34}$$

for all $t = 1, 2, ...$ Then, the characteristic equation becomes:

$$(\lambda^m)^2 - \gamma\lambda^m + \frac{1}{\beta} = 0. \tag{35}$$

Assuming $\beta\gamma^2 > 4$, it is easy to check that $\beta\gamma > 1 + \beta$ because $\rho > \beta\sigma'(x^m)$[16]. Therefore, we have eigenvalues $\lambda_1^m, \lambda_2^m \in \mathbb{R}_+^2$ such that $0 < \lambda_1^m < 1 < \lambda_2^m$ and an eigenvector corresponding to λ_1^m is:

$$(e_1^m, e_2^m) = (\lambda_1^m, 1). \tag{36}$$

[16]This property is proved in Lemma 3.

Therefore, the local property of x^m is saddle.

We finally draw the phase diagram. From equation (33), the $z_t^m = z_{t+1}^m$ locus is $z_t^m = x_t^m$, that is, the 45 degree line of the (z_t^m, x_t^m) plane. On the other hand, the $x_t^m = x_{t+1}^m$ locus is $x_t^m = \psi(x_t^m, z_t^m)$. By solving $\psi(x_t^m, z_t^m)$ with respect to z_t^m, we have:

$$z_t^m = \sigma^{-1}([2\rho - \beta\sigma'(x_t^m)])x_t^o - [2\rho - (1-\beta)c]), \tag{37}$$

for all $t = 1, 2, ...$ Because $[2\rho - \beta\sigma'(x_t^m)]x_t^m$ is strictly increasing in x_t^m and convex for $[2\rho - \beta\sigma'(x_t^m)]x_t^m > 0$, and $\sigma^{-1}(x)$ is strictly increasing and convex, the $x_t^m = x_{t+1}^m$ locus is strictly increasing and convex in x_t^m. We next examine whether z_t^m and x_t^m are increasing or decreasing above and below the phase-lines. It is easy to check that for the points above $z_t^m = z_{t+1}^m$ line, we have $z_t^m > z_{t+1}^m$, and for the points below $z_t^m = z_{t+1}^m$, we have $z_t^m < z_{t+1}^m$. Also, for the points above $x_t^m = x_{t+1}^m$ line, we have $x_t^m > x_{t+1}^m$, and for the points below $x_t^m = x_{t+1}^m$, we have $x_t^m < x_{t+1}^m$. The phase diagram is presented in Figure 4 where the arrows show the direction of increase at each point.

We now characterize the optimal monopoly solution by using the above properties. It satisfies the following conditions: (i) the $x_t^m = x_{t+1}^m$ locus is increasing and (ii) $x_t^m \in [\overline{x}^m, x^m]$ for all $t = 1, 2, ..$, where:

$$\overline{x}^m = \{x \in \mathbb{R}_{++} : [2\rho - \beta\sigma'(x)]x = \rho - (1-\beta)c\}. \tag{38}$$

Because $[2\rho - \beta\sigma'(x)]x$ is strictly increasing in x for all $[2\rho - \beta\sigma'(x)]x > 0$, there exists a unique $\overline{x}^m > 0$. This implies that the $x_t^m = x_{t+1}^m$ locus intersects the $x_{t-1}^m = 0$ axis at $\overline{x}^m > 0$. Any $x_t^m \notin [\overline{x}^m, x^m]$ does not become the optimal monopoly solution. Therefore, we conclude that the optimal monopoly solution that satisfies conditions (i) and (ii) is restricted to the shaded portion. Any points outside the shaded portion diverge and do not become the optimal monopoly solution. For the points above the $z_t^m = z_{t+1}^m$ locus, x_t^m becomes negative in finite time, but this violates a feasibility condition. On the other hand, for the points above $z_{t+1}^m > z_t^m$, but below $x_t^m = x_{t+1}^m$, x_t^m is increasing over time satisfying $z_{t+1}^m > z_t^m$ $(x_t^m > x_{t-1}^m)$. However, for large x_t^m such that $x_t^m > x_{t-1}^m$, the inverse demand function becomes $P(x_{t-1}^m, x_t^m) = 0$. In these circumstances, Euler equation (16) does not hold. It is easy to see that there exists a unique path that converges to x^m and the optimal monopoly solution consists of the points that belong to the stable manifold of x^m. □

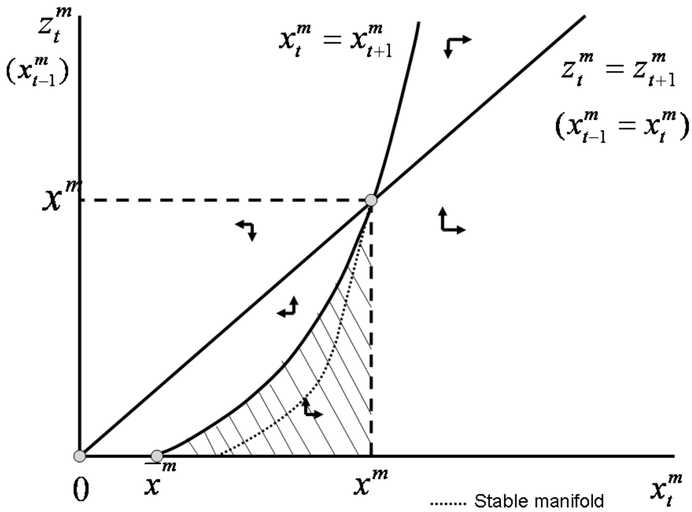

Figure 4: The dynamical system of monopoly diffusion path

Proof of Proposition 5

Let $L(x_{t-1}^c, x_t^c) \equiv R(x_{t-1}^c, y_t^c)$. We prove the first case. Let $\beta\sigma'(x_t^m)x_{t+1}^m < \rho x_t^m$ and $x_{t-1}^m \leq x_{t-1}^c$. Suppose in negation that $x_t^m \geq x_t^c$. Then, using the property of $L(.,.)$, we have the following inequalities:

$$L(x_{t-1}^c, x_t^c) \geq L(x_{t-1}^c, x_t^m) \geq L(x_{t-1}^m, x_t^m), \tag{39}$$

for all $t \geq 1$ such that $x_{t-1}^m \leq x_{t-1}^c$, where the first inequality follows from $L_2(x_{t-1}, x^t) < 0$ and $L_1(x_{t-1}, x_t) > 0$. Because $L(x_{t-1}^c, x_t^c) = c$ in the competitive equilibrium, we have $L(x_{t-1}^m, x_t^m) \leq c$. This is a contradiction to equation (16) because $\beta\sigma'(x_t^m)x_{t+1}^m < \rho x_t^m$. In the same way, we can prove the second case. □

Proof of Lemma 3

Suppose in negation that $\rho x^m \leq \beta\sigma'(x^m)$. Then, we have the following inequalities:

$$\rho x^m \leq \beta\sigma'(x^m)x^m < \sigma(x^m), \tag{40}$$

Where the last inequality follows from the property of $\sigma(.)$. These inequalities imply that:

$$2\rho x^m \leq \sigma(x^m) + \beta\sigma'(x^m)x^m. \tag{41}$$

This contradicts the steady state condition, $2\rho x^m > \sigma(x^m) + \beta\sigma'(x^m)x^m$. □

Proof of Proposition 6

Suppose in negation that $x^c \leq x^m$. This implies that $\Pi(x^m) \leq 0$. But this contradicts the steady state condition:

$$\Pi(x^m) = \rho x^m - \beta\sigma'(x^m)x^m > 0. \tag{42}$$

Therefore, $x^c > x^m$. □

References

[1] Banerjee, Abhijit. 1992, ‘A Simple Model of Herd Behavior’, *Quarterly Journal of Economics*. 107, pp797-817.

[2] Bass, Frank. 1969, ‘A New Product Growth Model for Consumer Durables’, *Management Science*, 15, pp215-227.

[3] Becker, Gary. 1991, ‘A Note on Restaurant Pricing and Other Examples of Social Influences on Price’, *Journal of Political Economy*. 99, pp1109-1116.

[4] Becker, Gary. and Murphy, Kevin. 1988, ‘A Theory of Rational Addiction’, *Journal of Political Economy*. 96, pp675-700.

[5] Bergemann, Dirk. and Välimäki, Juuso. 1997, ‘Market Diffusion with Two-sided Leaning’, *Rand Journal of Economics*. 28, pp773-795.

[6] Bergemann, Dirk. and Välimäki, Juuso. 2006, ‘Dynamic Pricing of New Experience Goods’, *Journal of Political Economy*. 114, pp713-743.

[7] Bikhchandai, Sushi., Hirsheifer, David. and Welch, Ivo. 1992, ‘A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades’, *Journal of Political Economy*, 100, pp992-1026.

[8] Cabral, Luis. Salant, David. and Woroch, Glenn. 1999, ‘Monopoly Pricing with Network Externalities’, *International Journal of Industrial Organization*, 17, pp199-214.

[9] Caminal, Ramon. and Vives, Xavier. 1996, ‘Why Market Shares matter: An Informational-Based Theory’, *Rand Journal of Economics*. 27, pp221-239.

[10] Caves, Richard. 1998, 'Industrial Organization and New Findings on the Turnover and Mobility of Firms', *Journal of Economic literature*. 36, pp1947-1982.

[11] Doganoglu, Toker. 2003, 'Dynamic Price Competition with Consumption Externalities', *Netomics*. 5, pp43-69.

[12] Economides, Nicholas. and Himmelberg, Charles. 1995,'Critical Mass and Network Size with Application to the US Fax Market', *Discussion Paper, Stern School of Business, NYU*, EC-95-11.

[13] Ellison, Glenn. and Fundenberg, Drew. 1993, 'Rules of Thumb for Social Learning', *Journal of Political Economy*. 101, pp612-643.

[14] Frank, Robert. 1989, 'Frames of Reference and the Quality of Life', *American Economic Review*. 79, pp80-85.

[15] Gort, Michael. and Klepper, Steven. 1982, 'Time Paths in the Diffusion of Product Innovations', *Economic Journal*. 92, pp630-652.

[16] Grliches, Zyi. 1957, 'Hybrid Corn: An Exploration in the Economics of Technological Change', *Econometrica*. 25, pp501-522.

[17] Jensen, Richard. 1982, 'Adoption and Diffusion of an Innovation of Uncertain Profitability', *Journal of Economic Theory*. 27, pp182-193.

[18] Jovanovic, Boyan. and Lach, Saul. 1989, 'Entry, Exit, and Diffusion with Learning by Doing', *American Economic Review*. 79, pp690-699.

[19] Jovanovic, Boyan. 1998, 'Michael Gort's Contribution to Economics', *Review of Economic Dynamics*. 1, pp327-337.

[20] Karshenas, Massoud. and Stoneman, Paul. 1992, 'Flexible Model of Technological Diffusion Incorporating Economic Factors with an Application to the Spread of Colour Television Ownership in the UK', Journal of Forecasting. 11, pp577-601.

[21] Katz, Michael. and Shapiro, Carl. 1985, 'Network Externalities, Competition, and Compatibility', *American Economic Review*, 75, pp424-440.

[22] Katz, Michael. and Shapiro, Carl. 1986, 'Technology Adoption in the Presence of Network Externalities', *Journal of Political Economy*, 94,pp822-841.

[23] Katz, Michael. and Shapiro, Carl. 1994, 'Systems Competition and Network Effects', *Journal of Economic Perspective*, 8, pp93-115.

[24] Klepper, Steven. 1996, 'Entry, Exit, Growth, and Innovation over the Product Life Cycle', *American Economic Review*, 86, pp562-583.

[25] Krishnan, Trichy., Bass, Frank., Jain, Dipack. 1999, 'Optimal Pricing Strategy for New Products', *Marketing Science*, 45, pp1650-1663.

[26] Mahajan, Vijay., Muller, Eitan., and Bass, Frank. 1990, 'New Product Diffusion Models in Marketing: A Review and Directions for Research', *Journal of Marketing*, 54, pp1-26.

[27] Mahajan, Vijay., Muller, Eitan., and Bass, Frank. 1995, 'Diffusion of New Products: Empirical Generalizations and Managerial Uses', *Marketing Science*, 14, pp79-88.

[28] Mansfield, Edwin. 1968, *'Industrial Research and Technological Innovation'*, New York: W.W. Norton, 1968.

[29] McFadden, Daniel. and Train, Kenneth. 1996, 'Consumers' Evaluation of New Products: Learning from Self and Others', *Journal of Political Economy*, 104, pp683-703.

[30] Monteiro, Paulo. and Gonzalez, Jose. 1999, '"We Sold a Million Units" - The Role of Advertising Past-Sales', *Econpapers* no. 357.

[31] Reinganum, Jennifer. 1981,'Market Structure and the Diffusion of New technology', Bell Journal of Economics, 12, pp618-624.

[32] Rob, Rafael. 1991, 'Learning and Capacity Expansion under Demand Uncertainty', *Review of Economic studies*, 58, pp.655-675.

[33] Rohlfs, Jeffrey. 1974, 'A Theory of Interdependent Demand for a Communication Service', *Bell Journal of Economics and Management Science*, 5, pp16-37.

[34] Schmalensee, Richard. 1982, ‘Product Differentiation Advantages of Pioneering Brands’, *American Economic Review*, 72, pp.349-365.

[35] Shapiro, Carl. 1983, ‘Optimal Pricing of Experience Goods’, *Bell Journal of Economics*, 14, pp497-507.

[36] Stoneman, Paul. 2002, *‘The Economics of Technological Diffusion’*, Blakwell.

[37] Sutton, John. 1997, ‘Gibrat’s Legacy’, *Journal of Economic Literature*, 35, pp40-59.

[38] Vettas, Nikolaos. 1998, ‘Demand and Supply in New Markets: Diffusion with Bilateral Learning’ *Rand Journal of Economics*. 29, pp215-233.

[39] Vettas, Nikolaos. 2000, ‘Investment Dynamics in Markets with Endogenous Demand’ *Journal of Industrial Economics*. 48, pp189-203.