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Optimal Adjustment by Using Gradient Method

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Discussion Paper 07-17

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# Public Expenditure Composition and Economic Growth: Optimal Adjustment by Using Gradient Method †

by

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**Abstract:**
Previous researches studied how the components of fiscal spending affect the economic growth but did not explicitly enquire into how to adjust the components in order to achieve the highest rate of economic growth starting from the present shares of components. We investigate how to determine the optimal adjustment by introducing a gradient method which explicitly takes account for the adjustment cost and incorporates the constraint that shares of components are summed up to one. The resulting optimal adjustment shares are proportional to the deviations from the average over elements of a gradient vector and independent from the choice of regression equations. The optimal adjustment share is completely estimated by using the linear regression with any choice of omitted variable if the adjustment cost is given. The result is free from multicollinearity problem but is considering all adjustment costs unlike most of previous researches. The paper also provides an illustrative example taken from the annual panel data for the Japanese prefectural governments.

*JEL classification* : E62;E23;H50
*Keywords*: Economic growth; Public expenditure composition; Adjustment

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## 1. Introduction

Which component of government expenditure should be cut? Health?, security?, or education? Should tax rate be cut? These problems are important for advanced countries (Levine and Rehelt (1992), Mendoza, Milesi-Ferretti and Asea (1997), Kneller, Bleaney and Gemmell (1999) )and also for developing countries (Devarajan, Swaroop and Zou (1996), Gupta, Clement and Mulas-Granades (2005)). Most of empirical frameworks employ linear regressions in which the economic growth rate is regressed on the fiscal categories.

Davoodi and Zou (1998) and Xei, Davoodi and Zou (1999) have proposed an economic growth rate maximization problem in terms of fiscal category shares. They find out the optimal category share to maximize its growth rate. There are two approaches to solve this optimization problem. The first one is to solve this problem directly, given all necessary parameters. However, they cannot get sufficient kinds of data and then cannot estimate all necessary parameters to solve this problem. Therefore, they use the second approach, i.e., the gradient method (but implicitly use) and solve the problem starting from the existing category shares step by step. Then, in order to get the gradient vector at the existing category shares, they linearly approximate the objective function of the growth rate around the existing category shares. Thus, as a result, they have to estimate the coefficients for the fiscal category shares (which correspond to the gradient vector) in the linear regression, which coincides with the previous empirical framework and provides a growth theory with its framework.

However, there is widespread non-robustness about results from the linear regression analyses. The non-robustness may reflect two problems for a mulitcolinearity of the estimated coefficients and for the adjustment cost of gradient method. For the first problem, they must omit one particular category share in the regression due to the linear restriction on the fiscal category share: i.e., summing all shares equals one. Kneller, Bleaney and Gemmell (1999) have already pointed out this issue. Moreover, if the category chosen to be omitted is altered, the estimated coefficients of the included categories will change. Based on the facts, they insist that the investigator must choose a 'neutral' omitted category. However, Zhang and Zou (1998), Davoodi and Zou (1998), Xei, Davoodi and Zou (1999), and Jin, Qian and Weingast (2005) have dropped one or two fiscal share variables in an ad hoc manner without clearly stating any reason.[1] The second problem is for the adjustment cost of gradient method. One element of fiscal category shares must be omitted in the regression equation in order to avoid perfect muliticolinearity, as mentioned above. However, when we use the gradient method, we need to consider the adjustment cost for all adjusted category shares. Because, the correct interpretation of the coefficient on each fiscal category share is as the effect of a unit change in the relevant variable offset by a unit change in the omitted category share, which involves the adjustment cost of the omitted category. No paper pays attention on the adjustment cost for the omitted category. The estimated coefficients will change, depending on the adjustment cost of the omitted category.

The purpose of this paper is to propose a tractable theoretical framework linking the share of fiscal expenditures to economic growth, and to produce an empirical framework adjusting the present shares of component of fiscal spending toward the optimal ones. Most of previous researches did not explicitly construct an empirical framework for finding the optimal shares although they studied how the components of government expenditure affect the economic growth. We introduce a gradient method in

[1] Correctly speaking, they investigated the devolution of fiscal power from the national government to subnational governments to economic growth. They divided total government expenditure into the three levels of federal, state, and local government spending.

order to determine the optimal adjustment of shares. In this paper, the gradient method explicitly takes account for the cost of adjusting the share of component and incorporates the constraint that shares of components are summed up to one.

The resulting optimal adjustment shares are proportional to the deviations from the average over components of a gradient vector and independent from the choice of omitted variables from the regression equations. We can completely estimate the optimal adjustment share by using the linear regression with any choice of omitted variable if the adjustment size is given. Our result does not suffer from a multicollinearity but is considering all adjustment costs unlike most of the previous researches. We provide an illustrative example taken from the annual panel data of the Japanese prefectural governments.

Section 2 employs a theoretical framework proposed by Davoodi and Zou (1998) with a slight modification. Section 3 provides an empirical framework utilizing a gradient method, and proposes how to adjust the component shares of fiscal spending towards the optimal level. Section 4 gives an illustrative example taken from the annual panel data for the Japanese prefectural governments. Section 5 states concluding remarks.

## 2. A theoretical framework

Following Davoodi and Zou (1998), the growth model consists of a production function with two kinds of inputs: private capital and public spending, where the function exhibits constant returns to scale in the two kinds of inputs.[2] We depart a little

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[2] Davoodi and Zou (1998) basically follow Barro(1990) except for division of the public goods of government into the three categories of central, state and local governments. Barro (1990, p.107) discussed in detail the questions arising from the

from Davoodi and Zou's model with three public goods consisting of central, state and local government. In our model, public goods consisting of the four fiscal categories of government expenditure: h (health), s (security), e (education) and r (the remainder (including industrialization and management)). Let k be private capital stock, and g be total government spending. All variables are measured on a per capita basis and population is constant:

$$h + s + e + r = g\,. \tag{1}$$

The production function is a Cobb-Douglas:

$$y = Ak^{\alpha} h^{\beta} s^{\gamma} e^{\delta} r^{\lambda}\,, \tag{2}$$

where $y$ is per capita output, $\alpha$, $\beta$, $\gamma$, $\delta$ and $\lambda$ are all in [0,1] and $\alpha+\beta+\gamma+\delta+\lambda=1$.[3]

The allocation of total government spending to each category takes the following form:

$$h = \theta_h g\,,\ s = \theta_s g\,,\ e = \theta_e g\,,\ r = \theta_r g\,, \text{and } \theta_h + \theta_s + \theta_e + \theta_r = 1, \tag{3}$$

where $\theta_h$, $\theta_s$, $\theta_e$ and $\theta_r$ are respectively the shares of health, security, education and the remainder in the total spending on the interval [0,1]. The total government spending is financed by income tax at fixed rate $\tau$:

$$g = \tau y\,. \tag{4}$$

**Household behavior**

We consider a long-lived household who chooses the consumption path $\{c(t): t \geq 0\}$

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specification of public services as an input to production.

[3] The production function in (2) is plausible if we think of k to include human capital, or if we think of a production function $Y=AL^{1-\alpha}K^{\alpha}G^{1-\alpha}$ proposed by Barro and Sala-i-Martin (1995, p.153) where Y, L, K and G are aggregate output, labor force, capital and government expenditure.

to maximize his discounted utility,

$$U = \int_0^{\infty} \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \, , \tag{5}$$

subject to:

(i) the dynamic budget constraint of the household[4] [5]:

$$\dot{k} = (1-\tau) y k k^{-1} - c = (1-\tau)\tau^{(1-\alpha)/\alpha} k (A\theta_h^{\beta}\theta_s^{\gamma}\theta_e^{\delta}\theta_r^{\lambda})^{1/\alpha} - c \, , \tag{6}$$

(ii) the value constraints:

$$0 \le c \le y \; , \; k_0 \, given \; , \tag{7}$$

where σ is the inverse of the inter-temporal elasticity of substitution, and ρ is the rate of time preference. After taking the government's announcement on the tax rate τ and the share of each fiscal category $\theta_h, \theta_s, \theta_e$ and $\theta_r$, the household chooses the consumption path. We write the Hamiltonian:

$$H = \left[ \frac{c^{1-\sigma} - 1}{1-\sigma} \right] e^{-\rho t} + \phi\Big( (1-\tau)\tau^{(1-\alpha)/\alpha} k (A\theta_h^{\beta}\theta_s^{\gamma}\theta_e^{\delta}\theta_r^{\lambda})^{1/\alpha} - c \Big). \tag{8}$$

The necessary conditions for the optimal path of consumption are given by

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[4] We formulate that the household is able to perform the production process since the same equations as those in a decentralized economy emerge. Using (4) yields y=g/τ, which together with (2) can rewrite $k^{-1}$ by only g, τ, θ and A. We obtain (6) by inserting these relations into y and $k^{-1}$.

[5] In Davoodi and Zou (1998, p.247), they consider $\dot{k} = (1-\tau) y - c$, where y is exogenously given for a household. The difference between (6) and theirs comes from the fact whether or not the household already knows a production function and the tax to be financed for the government expenditure before the household decides a consumption plan. The household knows them in our model. This difference leads to $A^{1/\alpha}$ in (11) and $\alpha A^{1/\alpha}$ in Davoodi and Zou (1998, p.247), which has no influence on the analytical results but makes the expression clear.

$$\frac{\partial H}{\partial c} = e^{-\rho t} c^{-\sigma} - \phi = 0$$
$$\frac{\partial H}{\partial k} \equiv \phi(1-\tau)\tau^{(1-\alpha)/\alpha} A^{1/\alpha} \theta_h^{\beta/\alpha} \theta_s^{\gamma/\alpha} \theta_e^{\delta/\alpha} \theta_r^{\lambda/\alpha} = -\dot{\phi} \;. \tag{9}$$

Using the two equations above, we have[6]

$$\phi(t)k(t) \to 0 \quad \text{(The transversality condition)}, \tag{10}$$

$$\eta \equiv \frac{\dot{y}}{y} = \frac{\dot{c}}{c} = \frac{1}{\sigma}\left[(1-\tau)\tau^{(1-\alpha)/\alpha} A^{1/\alpha} \theta_h^{\beta/\alpha} \theta_s^{\gamma/\alpha} \theta_e^{\delta/\alpha} \theta_r^{\lambda/\alpha} - \rho\right], \text{(The Euler equation)}. \tag{11}$$

Since the transition dynamics for $\phi$ does not exist, we can obtain the optimal path directly, and the consumption growth coincides with the rate of the growth of output and capital. The equations (6), (7), (10) and (11) determine the optimal path of consumption and capital. We simply consider the Euler equation (11) as one of necessary condition for analysis.[7]

**Government behavior**

We assume that the government maximizes the growth rate of output in (11) by choosing $\omega = (\tau, \theta_h, \theta_s, \theta_e, \theta_r)$, while the value of $\omega$ is fixed for the household, subject to the constraint on the shares for fiscal category:

$$\underset{\omega}{\text{Max}}\ \eta \equiv F(\omega)\ , \tag{12}$$

subject to

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[6] By taking derivative of $\frac{\partial H}{\partial c} = 0$ in terms of t, $\dot{\phi}/\phi = -\rho - \sigma\dot{c}/c$ . We insert the relation of $\frac{\partial H}{\partial k} = 0$ into this relation. We obtain (11).

[7] We assume the following inequality:

$$\rho > \{(1-\sigma)/\sigma\}\{(1-\tau)\tau^{(1-\alpha)/\alpha} A^{1/\alpha} \theta_h^{\beta/\alpha} \theta_s^{\gamma/\alpha} \theta_e^{\delta/\alpha} \theta_r^{\lambda/\alpha} - \rho\}\ .$$

The inequality assures the transversality condition.

$$\theta_h + \theta_s + \theta_e + \theta_r = 1,\ 0 \le \theta_h, \theta_s, \theta_e, \theta_r, \tau \le 1 \ .$$

The F is continuous, the constraints are compact sets, and then the maximum and minimum exist. The F is also a strictly quasi-concave (while the constraint for θ is convex) and then the maximum is unique. Then, the Lagrange equation and the Kuhn-Tucker conditions hold (assuming that there is no corner solution):

$$L = F(\omega) + \varphi(1 - \theta_h - \theta_s - \theta_e - \theta_r)\ ,$$
$$\frac{\partial L}{\partial \omega} = 0,\ \frac{\partial L}{\partial \varphi} = 0\ . \tag{13}$$

The solutions for the shares of health, security, education and the remainder, and for the tax rate are given by

$$\theta_h^* = \frac{\beta}{\beta+\gamma+\delta+\lambda},\ \theta_s^* = \frac{\gamma}{\beta+\gamma+\delta+\lambda},\ \theta_e^* = \frac{\delta}{\beta+\gamma+\delta+\lambda},$$
$$\theta_r^* = \frac{\lambda}{\beta+\gamma+\delta+\lambda},\ \tau^* = \beta+\gamma+d+\lambda. \tag{14}$$

If the actual expenditure for each share and the actual tax rate do not coincide with the solutions of (14), reallocation of resources will enhance growth rate. We need all parameters in (14) in order to numerically determine the solutions. In order to estimate those parameters, we need a data set of either (θ, τ , ρ, σ, y) or (k, h, s, e, r, y). Howeever, it is not easy to obtain the data of (ρ, σ, k) in any country.[8] It is reasonable that Davoodi and Zou (1998) and Xei, Davoodi and Zou (1999) linearly approximated the function $F(\omega)$ at the present level or the average of data for ω. They came to use the gradient method as a result to solve this problem step by step and sought the gradient vector, i.e., the estimated coefficients for ω in the linear regression. They proposed an empirical framework in which growth rate is linearly regressed on ω, which is the same framework as the previous one without the growth theory.

[8] In Appendix 1, we explain how to estimate (β, γ, δ, λ) by using these data.

## 3 An empirical framework

### 3.1 An optimal adjustment of shares

This section investigates how to adjust the shares of fiscal expenditure starting from the present level of $\omega = (\omega_\tau, \omega_\theta) = (\tau, \theta_h, \theta_s, \theta_e, \theta_r)$ to the optimal one in which the rate of economic growth is maximized. We introduce a gradient method for solving (12) step by step which provides the optimally adjusting shares, and relate this method to the empirical framework of regression analysis.[9] The gradient method extends the previous researches by Davoodi and Zou (1998), Xei, Zou and Davoodi (1999), and Kneller, Bleaney and Gemmell (1999) among others in the sense that it brings them a framework to optimally adjust the shares of fiscal components.

We formulate a gradient method for maximizing a linearized function of $\eta \equiv F(\omega)$ in (12) at the present level of $\omega$ as follows:

$$\underset{z}{Max} \;\; dF = \Gamma' Z \tag{15}$$

subject to

$$Z'Z = \xi^2 \quad for \;\; \xi > 0, \quad \text{and} \;\; J_4' Z_\theta = 0,$$

where the partial differential coefficients $\Gamma' = (\Gamma_\tau, \Gamma_\theta') = (F_\tau, F_{\theta h}, F_{\theta s}, F_{\theta e}, F_{\theta r})$ with $F_\nu = \dfrac{\partial F}{\partial \nu}$ for $\nu = \tau, \theta_h, \theta_s, \theta_e$ and $\theta_r$ are called a gradient vector, the differentials $Z' = (Z_\tau, Z_\theta') = (d\tau, d\theta_h, d\theta_s, d\theta_e, d\theta_r)$ evaluated at the present level of $\omega = (\omega_\tau, \omega_\theta)$ denotes the adjustment of shares, the constant scalar value $\xi$ represents a norm of Z, and $J_4 = (1,1,1,1)'$ is a $4 \times 1$ unit vector. The second constraint in (15) indicates that the components of the differentials of share variables add up to zero. This condition is necessary because both the present and optimal shares must

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[9] The gradient method is a standard tool for stepwise maximizing a nonlinear function. See, for instance, Bazaraa, Sherali, and Shetty (1993) for detailed explanation.

satisfy $\theta_h + \theta_s + \theta_e + \theta_r = 1$.

The norm ( $\xi$ ) of Z can be interpreted as a size of adjustment. If the cost for adjusting the shares of fiscal spending is proportional to the size of adjustment, the size of $\xi$ can be a measure of adjustment cost. The adjustment cost consists of equally weighted components in Z. For any given $\xi > 0$, the problem of (15) determines the optimal adjustment in the sense that the solution provides the highest growth rate differential of dF(= dη) in (12). For easy of expositions, we introduce a notation $(\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4) \equiv (\Gamma_\tau, \Gamma'_\theta)$ which plays an important role in the following discussion.

The problem in (15) is maximized at

$$Z^* = \begin{pmatrix} Z^*_\tau \\ Z^*_\theta \end{pmatrix} = \xi \begin{pmatrix} \Gamma_\tau \\ \Gamma^*_\theta \end{pmatrix} \left(\Gamma_\tau^2 + \Gamma^*_\theta{}'\Gamma^*_\theta\right)^{-1/2} \text{ with its maximum } dF^* = \xi\left(\Gamma_\tau^2 + \Gamma^*_\theta{}'\Gamma^*_\theta\right)^{1/2}, \quad (16)$$

where $\Gamma^*_\theta = (\gamma_1 - \bar{\gamma}, \ldots, \gamma_4 - \bar{\gamma})'$, *and* $\bar{\gamma} = \frac{1}{4}\sum_{j=1}^{4} \gamma_j$. The derivation of (16) is given in Appendix 2. The solution $Z^*_\theta$ is proportional to $\Gamma^*_\theta$ which represents the partial differential coefficients of shares measured from their averaged value. The vector Z* determines the highest economic growth rate differential at the present level of ω. The share of government expenditure is adjusted from ω to ω* ≡ ω + Z* and the economic growth rate is adjusted from $F(\omega)$ to $F(\omega^*) \cong F(\omega) + dF^*$.[10]

Figure 1 illustrates how the gradient method works started from the present level of ω at the point P. The objective function $dF = \Gamma' Z$ of (15) indicates the tangent surface to $\eta = F(\omega)$ at the point of P. The optimal vector Z* is indicated by the arrow starting from the point P.[11]

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[10] We can choose any value of $\xi$ as long as ω* satisfies the constraint for the problem of (12).

[11] Unlike the standard gradient method, the optimal differential vector Z* is not necessarily orthogonal to the tangent line q because of the constraint $J'_4 Z_\theta = 0$.

**[Insert Figure 1]**

In order to estimate the gradient vector from the observed data of $\omega = (\omega_\tau, \omega_\theta)$, following Davoodi and Zou (1998), and Kneller, Bleaney and Gemmell (1999) we linearly approximate the function F(ω) and set up the linear regression:

$$\eta_{it} = A_0 + \gamma_0 \tau_{it} + \gamma_1 \theta_{h,it} + \gamma_2 \theta_{s,it} + \gamma_3 \theta_{e,it} + \gamma_4 \theta_{r,it} + B' D_i + C' T_t + u_{it} \qquad (17)$$

where $i$ = 1,…, $I$ and $t$ = 1,..., $N$ refer to prefecture $i$ and time $t$; $A_0, \gamma_0, \gamma_1, \gamma_2, \gamma_3$, and $\gamma_4$ are the parameters for constant term, tax rate, health, security, education, and the remainder respectively. B and C are $(I\text{-}1) \times 1$ and $(N\text{-}1) \times 1$ coefficient vectors; prefecture dummy $D_i$ is a $(I\text{-}1) \times 1$ vector of prefecture fixed-effects, time dummy $T_t$ is a $(N\text{-}1) \times 1$ vector of time fixed-effects, and $u_{it}$ are the *iid* random errors with normal distribution $N(0, \sigma_u^2)$.The coefficient vector of $(\gamma_0, \gamma_1, ..., \gamma_4)$ in (17) represents the gradient vector in (15). We cannot estimate the parameters by using the ordinary least square (OLS) because the regression (17) has a perfect multicollinearity problem due to the restriction of $\theta_h + \theta_s + \theta_e + \theta_r = 1$. Kneller, Bleaney and Gemmell (1999) pointed out that previous researches had suffered from this difficulty.[12]

An alternative formulation of (15) is to directly incorporate the restriction on the share variables by deleting one of $\mathrm{d}\theta_j$ ($j = h, s, e,$ and $r$). We delete $\mathrm{d}\theta_r$ by inserting the constraint $d\theta_r = - d\theta_h - d\theta_s - d\theta_e$ into (15). The maximization problem becomes

$$\underset{W}{Max}\ dF = \Delta' W$$

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[12] The fiscal variables in our model are h (health), s (security), e (education), r (the remainder), and π(≡ τy, amount of tax). The budget constraint is satisfied since the equation τ (=π/y) = (h + s + e + r)/y for defining the tax rate is employed in (17) and in an empirical study in Section 4. The formulation in (17) has no multicollinearity due to the budget constraint for fiscal variables, but has another muliticolliniearity due to the share variables.

subject to $$W'\begin{pmatrix}1 & 0' \\ 0 & I_3+J_3J_3'\end{pmatrix}W=\xi^2, \quad \xi>0\,, \tag{18}$$

where $\Delta'=(\Delta_\tau,\Delta_\theta')=(\gamma_0,\gamma_1-\gamma_4,\gamma_2-\gamma_4,\gamma_3-\gamma_4)$, $W'=(W_\tau,W_\theta')=(d\tau,d\theta_h,d\theta_s,d\theta_e)$, and $I_3$ is a $3\times 3$ unit matrix. We note that $\Delta_\theta$ represents the partial differential coefficients measured from the value of $\gamma_4$, while $\Gamma_\theta$ in (16) represents the coefficients measured from their averaged value. Though the formulation of (18) has a different expression, it is equivalent to that of (15).

The problem is maximized at

$$W^{**}=\begin{pmatrix}W_\tau^{**}\\ W_\theta^{**}\end{pmatrix}=\xi\begin{pmatrix}\Delta_\tau\\ \Delta_\theta^{**}\end{pmatrix}\left(\Delta_\tau^2+\Delta_\theta^{**}{}'\Delta_\theta^{**}+\frac{9}{16}\overline{\Delta}_\theta^2\right)^{-1/2} \tag{19}$$

with $dF^{**}=\xi\left(\Delta_\tau^2+\Delta_\theta^{**}{}'\Delta_\theta^{**}+\frac{9}{16}\overline{\Delta}_\theta^2\right)^{1/2}$, where $\Delta_\theta^{**}=\Delta_\theta-\frac{3}{4}J_3\overline{\Delta}_\theta,\ and\ \overline{\Delta}_\theta=\frac{1}{3}J_3{}'\Delta_\theta$.

The derivation is given in Appendix 2. Plugging W** into the constraint $d\theta_r = -d\theta_h - d\theta_s - d\theta_e$, we have the optimal adjustment of Z as

$$Z^{**}=\begin{pmatrix}Z_\tau^{**}\\ Z_\theta^{**}\end{pmatrix}=\xi\begin{pmatrix}\Delta_\tau\\ \Delta_\theta^{**}\\ -J_3'\Delta_\theta^{**}\end{pmatrix}\left(\Delta_\tau^2+\Delta_\theta^{**}{}'\Delta_\theta^{**}+\frac{9}{16}\overline{\Delta}_\theta^2\right)^{-1/2}. \tag{20}$$

The solution in (20) is identical to that of (16), and independent from the choice of omitted variables as proved in Appendix 2. The solution $W_\theta^{**}$ is proportional to $\Delta_\theta^{**}$. An advantage of the expression in (20) over (16) is that we do not need each parameter of $\gamma_j\ (j=h,s,e,\text{and } r)$ since Z** is expressed only in terms of $(\Delta_0,\Delta_\theta')=(\gamma_0,\gamma_1-\gamma_4,\gamma_2-\gamma_4,\gamma_3-\gamma_4)$.

In order to estimate the values of $\Delta_0$ and $\Delta_\theta$, we can use the linear regression omitting $\theta_{r,it}$ from (17):

$$\eta_{it}=A_0+\gamma_0\tau_{it}+(\gamma_1-\gamma_4)\theta_{h,it}+(\gamma_2-\gamma_4)\theta_{s,it}+(\gamma_3-\gamma_4)\theta_{e,it}+B'D_i+C'T_t+u_{it}\,. \tag{21}$$

Unlike the regression of (17), the multicollinearity does not exist in (21) and the OLS method is applicable. We can compute the solution (19) by using the estimates of $\Delta$ and determine the optimal adjustment Z**. The share of government expenditure is adjusted from $\omega$ to $\omega^{**} \equiv \omega + Z^{**}$. We can say that when a particular component of Z** is positive, the share of its fiscal category is below the optimal level and we should increase it towards the optimal level.

We can apply the other three regressions analogous to equation (21), depending on what variables we omit from the regression equation. In fact, Kneller, Bleaney and Gemmell (1999) examined the regression analyses with changing omitted variables. However, once we choose a particular equation, say (21) for example, the additional use of other regression equations does not provide further information in the sense that the estimates of all other regressions can be computed by using only the estimates of the originally chosen regression equation.[13] This fact will be empirically illustrated in Section 4.

### 3.2 Comparison with the framework of previous researches

We review the previous researches of Davoodi and Zou (1998), Xei, Zou and Davoodi (1999), and Kneller, Bleaney and Gemmell (1999) from the view point of optimal adjustment to maximize the differential rate of economic growth and clarify the significance of their contributions. Most of previous researches understood the problem along the following line of arguments. The estimated positive coefficient of a particular variable means that the present level of its category is below the optimal and the present

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[13] The claim above mentioned is proved in Appendix 3. This claim may provide a caution against some previous researches. Kneller, Bleaney and Gemmell (1999), for example, seem carried out the regression analyses with changing omitted variables without being aware of this fact.

level of the variable should be increased toward the optimal with the same amount of decrease in the omitted variable as illustrated in Figure 2. However, they did not explicitly examine what combination of shares induces the maximum rate of economic growth.

**[Insert Figure 2]**

We formulate the maximization problem:

$$\underset{W}{Max}\ dF = \Delta' W \tag{22}$$

subject to $W'W = \xi'^2$, $\xi' > 0$,

where $\xi'$ is positive constant but not necessarily equal to $\xi$ in (18). This problem is identical to (18) except for the constraint. The equation (22) imposes a constraint directly on the norm of the vector W after deleting one of the share variables. On the other hand, (18) essentially imposes a restriction on the original vector Z. The solution to (22) is as follows:

$$W^{***} = \begin{pmatrix} W_\tau^{***} \\ W_\theta^{***} \end{pmatrix} = \xi' \begin{pmatrix} \Delta_\tau \\ \Delta_\theta \end{pmatrix} (\Delta_\tau^2 + \Delta_\theta{}'\Delta_\theta)^{-1/2}, \text{ with } dF^{***} = \xi'(\Delta_\tau^2 + \Delta_\theta{}'\Delta_\theta)^{1/2}, \tag{23}$$

$$Z^{***} = \begin{pmatrix} Z_\tau^{***} \\ Z_\theta^{***} \end{pmatrix} = \xi' \begin{pmatrix} \Delta_\tau \\ \Delta_\theta \\ -J_3{}'\Delta_\theta \end{pmatrix} (\Delta_\tau^2 + \Delta_\theta{}'\Delta_\theta)^{-1/2}.$$

The derivation is given in Appendix 2. The optimal adjustment of $W_\theta^{***} = (d\theta_h, d\theta_s, d\theta_e)'$ is proportional to the vector of $\Delta_\theta$. The result of (23) implies that the share of an element of $W_\theta^{***}$ should be increased when the sign of corresponding element of $\Delta_\theta$ is positive. Though the argument illustrated in Figure 2 is consistent with the optimal adjustment of $W_\theta^{***}$ as far as the sign of adjustment for each component is concerned, the previous researches did not discuss how to obtain the optimal share of adjustment even within the framework of (22). And the framework of (22) is itself not preferable to that of (15) as will be explained later in this section.

The result of (23) implies that the optimal adjustment vector depends on the choice of omitted variable, and it gives a different vector from (23) if we choose an alternative omitted variable in the regression. For instance, if the variable $\theta_e$ is dropped instead of $\theta_r$, $\widetilde{\Delta}_\theta = (\gamma_1 - \gamma_3, \gamma_2 - \gamma_3, \gamma_4 - \gamma_3)'$ is the gradient vector for the corresponding maximization problem and the solution $\widetilde{W}_\theta^{***} = (d\theta_h, d\theta_s, d\theta_r)'$ is proportional to $\widetilde{\Delta}_\theta$. The optimal adjustment of components ($d\theta_h$, $d\theta_s$) can be different in sign among the two regressions. Kneller, Bleaney and Gemmell (1999) tried to find the regression which would provide a plausible solution of Z by selecting appropriate omitted variables. On the other hand, the optimal differential $\Delta_\theta^{**} = \Delta_\theta - \frac{3}{4} J_3 \overline{\Delta}_\theta$ for the problem of (18) is not necessarily equal to $\Delta_\theta$. Even the sign of element of $\Delta_\theta^{**}$ could be opposite to that of the corresponding element of $\Delta_\theta$. The solution W*** is not necessarily justified from the analysis of (18).

An underlying key idea that distinguishes the two problems lies in how to measure the cost for adjusting the fiscal shares. The maximization problem in (15) or equivalently in (18) measures the norm of the original variables before omitting one of the variables while the problem in (22) does it after omitting one variable. The former formulation equally reflects the adjustment of each component of the shares in the cost, while the latter one takes account for only the components in the regression but ignores the cost for adjusting the omitted variable. The solution to the former problem does not depend on which variable we omit from the regression equation. But, the latter actually does because the ignored costs differ among the regressions. We believe that the former solution is more appropriate than the latter one.

## 4. An illustrative example by using the panel data for Japanese prefectures

We provide an empirical example from the annual panel data for Japanese prefectures for the purpose of illustrating the discussion in section 3.[14]

### *4.1 Data sources*

The data are take from *"Annual Report of Local Finance"* published by the Institute of Local Finance (Chihou-zaimu -kyokai, in Japanese), and *"Annual Report of Prefectural Account"* published by the Cabinet Office of the Japanese government. The data for the 47 prefectures are available during the fiscal years 1981-2002. Each variable of the government expenditures are compiled from "Annual Report of Local Finance" (Settlement of Expenditure by Purpose) and defined as:

*h*: (*Health*) = welfare + hygiene;

*s*: (*Security*) = police + fire fighting;

*e*: (*Education*) = education;

*r*: (*Remainder*) = Commerce and manufacturing + Civil engineering works + Agriculture, forestry and fisheries + the others;

*g* : (*Total government spending*) = total value of the items above.

[14] Davoodi and Zou (1998) work with time-averaged data since the benefits of fiscal decentralization are not expected to affect year-to-year fluctuations in growth. The growth regression is estimated on data averaged over five- and ten-year periods. Accordingly, the dependent variable is the average growth rate over these two periods. However, we use annual data as Zhang and Zou(1998), Xie, Zou and Davoodi (1999) and Gupta, Clements, Baldacci and Mulas-Granados (2005) did. For the sake of simplicity, without implementing the Hausman test with a null hypothesis of no correlation among the individual effects and the error term, we adopt the fixed effects model in (21).

The shares of h, s, e and r to the g are given by $\theta_h \equiv h/g$ , $\theta_s \equiv s/g$ , $\theta_e \equiv e/g$ , $\theta_r \equiv r/g$ . The output and tax are taken from “Annual Report of Prefecture Account” (Summary Tables)[15]:

y: (*Output*) = Gross Prefecture Domestic Product at constant prices;

$\tau$ : (*Tax rate*) ≡ g*/y, g* = total government expenditure at constant prices;

$\eta$: (*Growth rate of real output*) = the growth rate of y.

These variables are measured in per capita base at constant prices and the population data are compiled from the latter data base.

### 4.2 Estimated results and illustration

Table 1 indicates descriptive statistics for the data set. During the period of 1981-1990, the GDP of prefectures grew, on average, at the rate around 3.78% per capita per annum, and the average tax rate was 12.85%. Among the expenditures of fiscal variables, the share of health, education, security, education, remainder categories accounted for 9.38, 5.63, 27.21, and 57.78%, respectively. On the other hand, during the period of 1991-2002, the growth rate of GDP dropped down to 0.83% while the tax rate increased to 13.8%. The share of health increased to 10.15%, but the shares of security and education decreased. Examining Table 1, we suppose that there exists a structural break between the periods 1981-1990 and 1991-2002, and divide the sample periods into the two parts.

**[Insert Table 1]**

Table 2 summarizes the results of estimating equation (21). The values in the

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15 The data of 1981-1990 are based on 1963SNA standard and the data of 1991-2002 based on 1993SNA standard, and measured at 1997 prices. The Cabinet Office of the Japanese government does not provide the re-estimated data prior to 1990 based on the 1993SNA standards.

columns 1 to 4, respectively, show the estimates with the omitted variables, h, s, e, and r. The estimated coefficients are different depending what variable is dropped from the regression equation. More precisely, the estimated coefficients in each panel are symmetric about the diagonal elements as expected from proposition 1 in Appendix 3. For example, if we take m = 4 and n = 1 in Appendix 3, we have $\hat{\gamma}_{1,4} = -0.372$ (the coefficient of health when the remainder is omitted), and $\tilde{\gamma}_{4,1} = -\hat{\gamma}_{1,4} = 0.372$ (the coefficient of the remainder when health is omitted). Similarly, we have $\hat{\gamma}_{2,4} = 0.534$ and $\tilde{\gamma}_{2,1} = \hat{\gamma}_{2,4} - \hat{\gamma}_{1,4} = 0.534 - (-0.372) = 0.906$ for m = 4 and n = 1. According to the framework of Section 3.2, the adjustment shares of W** in (23) are proportional to the vector of $\Delta_\theta$, which are given by the estimates in each column depending on the omitted variable. Kneller, Bleaney and Gemmell (1999) tried to find the regression which would provide a plausible solution of differentials by selecting the variable to be omitted.

**[Insert Table 2]**

In order to evaluate the optimal adjustment share of Z**, we assume that the government wants to adjust the shares of component by 2% points as a total from the present level, that is $\xi = 0.02$. Moreover, we assume the present level is equal to the sample means for each period. Table 3 shows $\Delta'^{**} = (\Delta_\tau, \Delta_\theta'^{**}, -J_3'\Delta_\theta^{**})$, $\omega$ (present levels of the share), Z** (optimal adjustment vector), and $\omega^{**}$ (adjusted vector of the shares) for each sample period. The entries of Z** in Panels (a) and (b) satisfy the restriction $d\theta_h + d\theta_s + d\theta_e + d\theta_r = 0$ in (15), and are independent from the choice of omitted variables. For the first period (1981-1990), we obtain the largest increase (dF** = 1.70% points) of the growth rate from the level at the sample means of ω if we choose the adjustment vector of $Z'^{**}$ = (-0.79, -0.69, 1.40, -0.87, 0.17) % points. The vector of Z** neither coincide with nor are proportional to the columns (1) through (4) on Panel

(a) in Table 2.

**[Insert Table 3]**

## 5. Concluding remarks.

The previous researches proposed theoretical frameworks for analyzing the optimal share of fiscal spending and attempted to implement them into empirical studies by using the linear regressions, while they did not explicitly construct an empirical framework for finding the optimal share or the optimal adjustment starting from the given fixed share. They only argue that the present level of a particular category is below the optimal and the level of this category should be increased toward the optimal with the same amount of decrease in the omitted variable if the estimate of coefficient for the corresponding variable is positive. The result from this discussion crucially depends on what the omitted variable is.

This paper proposes a tractable theoretical framework linking the share of fiscal expenditures to economic growth and an empirical framework adjusting the present shares of component of fiscal expenditures toward the optimal ones. We introduce a gradient method in order to determine the optimal adjustment of shares. The gradient method explicitly takes account for the cost of adjusting the share of component and incorporates the constraint that shares of components are summed up to one. The optimal adjustment shares are proportional to the deviations from the average over components of a gradient vector and independent from the choice of omitted variables from the regression equations. The discussion of previous researches is not necessarily conformable to our result. We can completely estimate the optimal adjustment share by using the linear regression with any choice of omitted variable if the adjustment cost of $\xi$ is given. In this sense, our result does not suffer from a multicollinearity in the linear regression but is considering all adjustment costs unlike most of the previous researches. An empirical example from the annual panel data for Japanese prefectures illustrates how the procedure proposed in this paper numerically works.

***Appendix 1: Direct methods to find the optimal solution of (14)***

We need all parameters in (14) to find the optimal solution. The first way to obtain them is to estimate the regression for (11) with a slight modification. After taking the logarithm of (11), we move the first term of the right hand side to the left and estimate the regression:

$$\begin{aligned} G &\equiv \ln\left(\frac{\dot{y}}{y}+\frac{\rho}{\sigma}\right)-\ln\frac{1}{\sigma}(1-\tau) \\ &= \frac{1-\alpha}{\alpha}\ln\tau+\frac{1}{\alpha}\ln A+\frac{\beta}{\alpha}\ln\theta_h+\frac{\gamma}{\alpha}\ln\theta_s+\frac{\delta}{\alpha}\ln\theta_e+\frac{\lambda}{\alpha}\ln(1-\theta_h-\theta_s-\theta_e) \end{aligned} \tag{A1}$$

by using the data of $\theta_h, \theta_s, \theta_e, \theta_r$, τ , y and ρ, σ.

The second way is to estimate the following function:

$$y/k = A(h/k)^{\beta}(s/k)^{\gamma}(e/k)^{\delta}(r/k)^{\lambda}. \tag{A2}$$

where the first order homogeneous restriction α+β+ γ+ δ+ λ=1 is incorporated in the Cobb-Douglas production function (2).The second way needs the data of h, s, e, r, y and k. The main difference between the two formulations lies in the point whether we require the data of k in (A2) or (ρ, σ) in (A1). In general, the data of k is so much different in each prefecture, though the data of (ρ, σ) is not. The reliable data of k for each prefecture is necessary but not available for the most countries. The prefecture-basis capital stock k is not published in Japan. The second way may not be useful under this circumstance of data availability in Japan.

***Appendix 2 : Derivations of (16), (19), (23)***

***Derivations of (16):*** We can solve the problem of (15) by using the Lagrange multiplier method:

$$L = \Gamma' Z + \frac{1}{2}\varphi_1(\xi^2 - Z'Z) - \varphi_2(0, J_4')Z\,;$$

$$\text{(i) } \frac{\partial L}{\partial Z} = \Gamma - \varphi_1 Z - \varphi_2(0, J_4')' = 0, \text{ (ii) } \frac{\partial L}{\partial \varphi_1} = \xi^2 - Z'Z = 0, \text{(iii) } \frac{\partial L}{\partial \varphi_2} = J_4' Z_\theta = 0, \tag{A3}$$

where $\Gamma' = (\Gamma_\tau, \Gamma'_\theta)$ and $Z' = (Z_\tau, Z'_\theta)$ are evaluated at the present level of $\omega' = (\omega_\tau, \omega'_\theta)$. From (i), we have

$$Z = \begin{pmatrix} Z_\tau \\ Z_\theta \end{pmatrix} = \frac{1}{\varphi_1}\left(\Gamma - \varphi_2 \begin{pmatrix} 0 \\ J_4 \end{pmatrix}\right) = \frac{1}{\varphi_1}\begin{pmatrix} \Gamma_\tau \\ \Gamma_\theta - \varphi_2 J_4 \end{pmatrix} \quad . \tag{A4}$$

Using (iii) and (A4), we obtain $0 = J'_4 Z_\theta = \frac{1}{\varphi_1} J'_4(\Gamma_\theta - \varphi_2 J_4)$, and

$$\varphi_2 = (J'_4 J_4)^{-1} J'_4 \Gamma_\theta = \frac{1}{4}\sum_{j=1}^{4} \gamma_j = \bar{\gamma} \ . \tag{A5}$$

Substituting $\varphi_2$ in (A.4) by $\bar{\gamma}$, we have

$$Z_\theta = \frac{1}{\varphi_1}(\Gamma_\theta - \bar{\gamma} J_4) \equiv \frac{1}{\varphi_1}\Gamma^*_\theta . \tag{A6}$$

Then, due to (A4) and (A6), we have

$$Z^* = \begin{pmatrix} Z^*_\tau \\ Z^*_\theta \end{pmatrix} = \frac{1}{\varphi_1}\begin{pmatrix} \Gamma_\tau \\ \Gamma^*_\theta \end{pmatrix}, \tag{A7}$$

and $\varphi_1 = \frac{1}{\xi}(\Gamma^2_\tau + \Gamma'^*_\theta \Gamma^*_\theta)^{1/2}$ from (ii) and (A7). Finally, we obtain

$dF^* = \Gamma' Z^* = \xi(\Gamma^2_\tau + \Gamma'^*_\theta \Gamma^*_\theta)^{1/2}$ .

***Derivations of (19):*** In a similar manner to (15), we solve the problem of (18):

$$L = \Delta' W + \frac{1}{2}\varphi_1(\xi^2 - W'AW) ; \tag{A8}$$

(i) $\frac{\partial L}{\partial W} = \Delta - \varphi_1 AW = 0$, (ii) $\frac{\partial L}{\partial \varphi_1} = \xi^2 - W'AW = 0$,

where $A = \begin{pmatrix} 1 & 0' \\ 0 & I_3 + J_3 J'_3 \end{pmatrix}$ and $I_3$ is a $3 \times 3$ unit matrix. Then, due to (i), the solution is,

$$W = \frac{1}{\varphi_1}\begin{pmatrix} \Delta_\tau \\ (I_3 + J_3 J'_3)^{-1}\Delta_\theta \end{pmatrix} = \frac{1}{\varphi_1}\begin{pmatrix} \Delta_\tau \\ \Delta_\theta - \frac{3}{4}\bar{\Delta}_\theta J_3 \end{pmatrix} = \frac{1}{\varphi_1}\begin{pmatrix} \Delta_\tau \\ \Delta^{**}_\theta \end{pmatrix}, \tag{A9}$$

where a relation $(I_3 + J_3 J'_3)^{-1} = I_3 - \frac{1}{4} J_3 J'_3$ is used.

Due to (ii) and (A9), we have $\xi^2 = W'AW = \frac{1}{\varphi_1^2}(\Delta_0^2 + \Delta'^{**}_\theta (I_3 + J_3 J'_3)\Delta^{**}_\theta)$ and

$$\varphi_1 = \frac{1}{\xi}\left(\alpha_0^2 + \Delta_\theta^{**}{}'\Delta_\theta^{**} + \frac{9}{16}\overline{\Delta}_\theta^2\right)^{1/2} \qquad \text{(A10)}$$

The solution in (19) follows from (A.9) and (A.10).The last element of Z** in (20) is obtained from $d\theta_r^{**} + J_3'\Delta_\theta^{**} = 0$.

The equivalence of $\Gamma_\theta^* = \Delta_\theta^{**}$ follows from the fact: for j = 1, 2, 3

$$\Delta_{\theta,j}^{**} = (\gamma_j - \gamma_4) - \frac{3}{4}\overline{\Delta}_\theta = \gamma_j - \frac{1}{4}\sum_{i=1}^4 \gamma_i \quad \text{and} \quad \Delta_{\theta,4}^{**} = -\sum_{j=1}^3 \Delta_{\theta,j}^{**} = \gamma_4 - \frac{1}{4}\sum_{i=1}^4 \gamma_i .$$

***Derivations of (23):*** The problem of (22) can be understood as a special case of (18) in which the matrix A is replaced by a unit matrix:

$$L = \Delta' W + \frac{1}{2}\varphi_1(\xi'^2 - W'W); \qquad \text{(A11)}$$

$$\text{(i)}\ \frac{\partial L}{\partial W} = \Delta - \varphi_1 W = 0, \quad \text{(ii)}\ \frac{\partial L}{\partial \varphi_1} = \xi'^2 - W'W = 0 .$$

Then, in an analogous way to (18) we obtain the solution

$$W = \frac{1}{\varphi_1}\begin{pmatrix}\Delta_\tau \\ \Delta_\theta\end{pmatrix}, \qquad \text{(A12)}$$

Due to (ii) and (A12), we have $\varphi_1 = \frac{1}{\xi'}\left(\Delta_\tau^2 + \Delta_\theta{}'\Delta_\theta\right)^{1/2}$. The last element of Z*** in (23) is obtained from $d\theta_r^{***} + J_3'\Delta_\theta = 0$.

### *Appendix 3 : Effects of omitted variables on the estimates*

This appendix examines the effect of omitted variables on the estimates in a regression with perfect collinearity, and shows that any additional use of regressions with different omitted variables can not extract further information from the given data set since the estimates of all other regressions can be computed from the estimates of the originally chosen regression equation. We use a model and notation of Kneller, Bleaney and Gemmell (1999) in this appendix, which are not necessarily conformable to the main body, for the purpose of clarifying their discussion. Let a linear regression be

$$y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j Y_{jit} + \sum_{j=1}^{m} \gamma_j X_{jit} + u_{it}, \quad i = 1,...,I,\ t = 1,\dots,N \qquad \text{(A13)}$$

with a linear constraint among the variables: $\sum_{j=1}^{m} X_{jit} = 0$, where $Y_{jit}$ are non-fiscal variables and $X_{jit}$ denote fiscal variables . If we omit the variable $X_{mit}$ from (A13), the equation is expressed as

$$y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j Y_{jit} + \sum_{j=1}^{m-1} \gamma_{j,m} X_{jit} + u_{it}, \qquad \text{(A14)}$$

where $\gamma_{j,m} = \gamma_j - \gamma_m, \quad j = 1,\dots,m-1$. On the other hand, if we omit another variable $X_{n,it}$, the equation has the form:

$$y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j Y_{jit} + \sum_{j=1(j \neq n)}^{m} \gamma_{j,n} X_{jit} + u_{it}, \qquad \text{(A15)}$$

where $\gamma_{j,n} = \gamma_j - \gamma_n; \quad j = 1,\dots,m(j \neq n)$ .T he following relation among the coefficients of (A14) and (A15) holds:

$$\gamma_{j,n} = \gamma_{j,m} - \gamma_{n,m} \quad j = 1,\dots,m-1(j \neq n) \text{ ; and } \gamma_{m,n} = -\gamma_{n,m}. \qquad \text{(A16)}$$

We can utilize the OLS method to estimate the parameter of either (A14) or (A15), while it is not applicable for (A13). The next proposition indicates that the estimates of (A14) completely determine those of (A15).

***Proposition 1:*** Let the OLS estimates of (A14) and (A15) be $\hat{\gamma}_{j,m}$ and $\tilde{\gamma}_{j,n}$ respectively. Then, the estimates $\tilde{\gamma}_{j,n}$ are completely determined by the following equation:

$$\tilde{\gamma}_{j,n} = \hat{\gamma}_{j,m} - \hat{\gamma}_{n,m} \text{ and } \tilde{\gamma}_{m,n} = -\hat{\gamma}_{n,m}. \qquad \text{(A17)}$$

*Proof:* We can assume n = m-1 without loss of generality by arranging the order of variables. In matrix notations, equation (A14) and (A15) are respectively expressed as:

$$y = Y_\Delta \beta_\Delta + X_\Delta \gamma_{\Delta,m} + X_{m-1}\gamma_{m-1,m} + u = W\beta + u,$$

$$Y_\Delta = (Y_0, Y_1, \ldots, Y_k), X_\Delta = (X_1, \ldots, X_{m-2}), W = (Y_\Delta, X_\Delta, X_{m-1}), \tag{A18}$$

$$\beta' = (\beta'_\Delta, \gamma'_{\Delta,m}, \gamma_{m-1,m}),$$

and similarly

$$y = Y_\Delta \beta_\Delta + X_\Delta \gamma_{\Delta,m-1} + X_m \gamma_{m,m-1} + u = Z\delta + u,$$

$$Z = (Y_\Delta, X_\Delta, X_m), \ \delta' = (\beta'_\Delta, \gamma'_{\Delta,m}, \gamma_{m-1,m}). \tag{A19}$$

We have the relation among the matrices of independent variables W and Z:

$$W = ZQ, \quad Q = \begin{pmatrix} I_{k+1} & 0 & 0 \\ 0 & I_{m-2} & -J_{m-2} \\ 0 & 0 & -1 \end{pmatrix}, \tag{A20}$$

where the constraint $X_\Delta J_{m-2} + X_{m-1} + X_m = 0$ is used. From (A18), (A19) and (A20), we have $y = ZQ\beta + u$, implying $\delta = Q\beta$. Plugging $W = ZQ$ in the normal equation $W'W\hat{\beta} = W'y$ for (A18), we have $Q\hat{\beta} = (Z'Z)^{-1} Z'y = \tilde{\delta}$. This proves Proposition 1.

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**Table 1. Descriptive statistics**

Panel (a) (1981-1990)

| | | Mean (%) | Standard deviation | Minimum (prefecture/year) | Maximum (prefecture/year) |
|---|---|---|---|---|---|
| Growth rate of prefectural GDP | | 3.78 | 2.26 | -8.97(Wakayama/1983) | 9.92(Kagawa/1988) |
| Share of health | $\theta_h$ | 9.38 | 1.64 | 6.24(Niigata/1987) | 15.29(Fukuoka/1984) |
| Share of security | $\theta_s$ | 5.63 | 2.05 | 1.53(Shimane/1984) | 19.11(Tokyo/1986) |
| Share of education | $\theta_e$ | 27.21 | 4.52 | 15.08(Tokyo/1989) | 40.43(Kanagawa/1984) |
| Share of the remainder | $\theta_r$ | 57.78 | 6.32 | 38.06(Kanagawa/1982) | 72.92 (Shimane/1984) |
| Tax rate | $\tau$ | 12.85 | 4.50 | 5.33(Osaka/1984) | 26.23(Shimane/1984) |

Panel (b) (1991-2002)

| | | Mean (%) | Standard deviation | Minimum (prefecture/year) | Maximum (prefecture/year) |
|---|---|---|---|---|---|
| Growth rate of prefectural GDP | | 0.83 | 2.13 | -6.31(Iwate/2001) | 11.70(Shiga/1991) |
| Share of health | $\theta_h$ | 10.15 | 1.72 | 6.68(Nagano/1995) | 18.18(Hyogo/1994) |
| Share of security | $\theta_s$ | 5.57 | 2.00 | 3.35(Shimane/1999) | 13.75(Tokyo/2002) |
| Share of education | $\theta_e$ | 23.47 | 4.15 | 14.49(Tokyo/1992) | 36.45(Kanagawa/2002) |
| Share of the remainder | $\theta_r$ | 60.81 | 6.12 | 38.54(Kanagawa/2002) | 70.82(Shimane/2000) |
| Tax rate | $\tau$ | 13.87 | 6.27 | 5.31(Kanagawa/1991) | 38.75(Shimane/1999) |

**Table 2. Estimation results**

Panel (a) (1981-1990) (No. of observations=423)

| Omitted variables | | health (1) | security (2) | education (3) | the remainder (4) |
|---|---|---|---|---|---|
| Constant | | -12.616 | 77.979** | -20.735* | 24.549*** |
| | | (17.367) | (35.806) | (10.677) | (7.073) |
| Share of health | $\theta_h$ | ___ | -0.906** | 0.081 | -0.372* |
| | | | (0.412) | (0.231) | (0.200) |
| Share of security | $\theta_s$ | 0.906** | ___ | 0.987** | 0.534 |
| | | (0.412) | | (0.437) | (0.365) |
| Share of education | $\theta_e$ | -0.081 | -0.987** | ___ | -0.453*** |
| | | (0.231) | (0.437) | | (0.129) |
| Share of remainder | $\theta_r$ | 0.372* | -0.534 | 0.453*** | ___ |
| | | (0.200) | (0.365) | (0.129) | |
| Tax rate | $\tau$ | -0.343 | -0.343 | -0.343 | -0.343 |
| | | (0.231) | (0.232) | (0.232) | (0.231) |
| Adjusted R-square | | 0.408 | 0.408 | 0.408 | 0.408 |

Panel (b) (1991-2002) (No. of observations=564)

| Omitted variables | | health (1) | security (2) | education (3) | the remainder (4) |
|---|---|---|---|---|---|
| Constant | | -2.857 | 20.616 | -4.102 | 0.586 |
| | | (8.078) | (27.680) | (7.212) | (2.314) |
| Share of health | $\theta_h$ | ___ | -0.235 | 0.012 | -0.034 |
| | | | (0.295) | (0.124) | (0.089) |
| Share of security | $\theta_s$ | 0.235 | ___ | 0.247 | 0.200 |
| | | (0.295) | | (0.331) | (0.278) |
| Share of education | $\theta_e$ | -0.012 | -0.247 | ___ | -0.047 |
| | | (0.124) | (0.331) | | (0.085) |
| Share of remainder | $\theta_r$ | 0.034 | -0.200 | 0.047 | ___ |
| | | (0.089) | (0.278) | (0.085) | |
| Tax rate | $\tau$ | 0.015 | 0.015 | 0.015 | 0.015 |
| | | (0.017) | (0.017) | (0.017) | (0.017) |
| Adjusted R-square | | 0.438 | 0.438 | 0.438 | 0.438 |

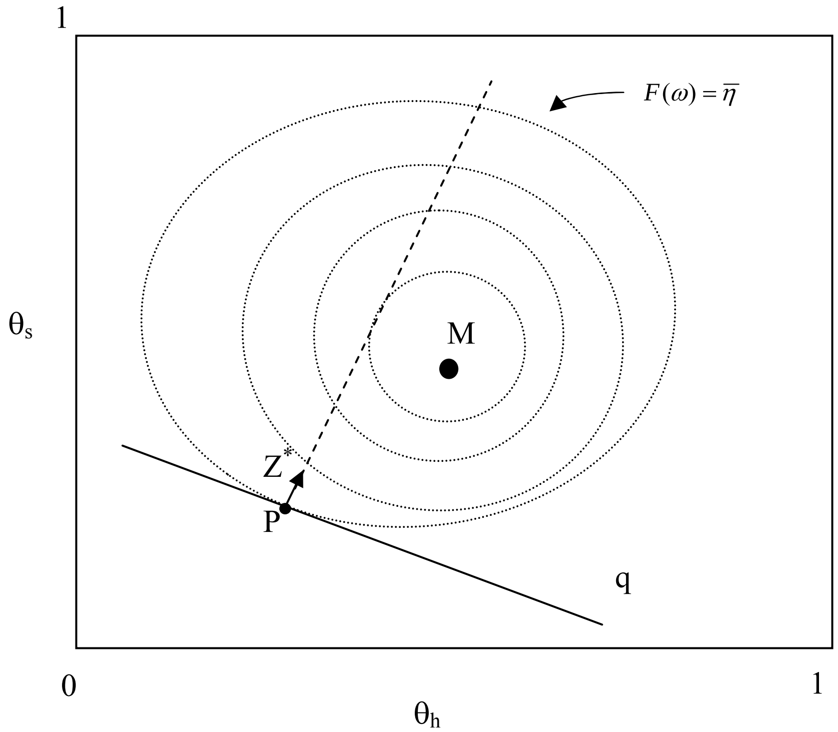
Note: All regressions include prefecture-specific dummies and time-specific dummies (fixed effect) although not reported here. Standard errors are in parentheses. Asterisks indicate variables significant levels; 10%(*), 5%(**) and 1%(***).

**Table 3. Optimal adjustment of $Z^{**}$ in (20)**

| Panel | (a)(1981-1990) | | | | (b)(1991-2002) | | | |
|---|---|---|---|---|---|---|---|---|
| | $\Delta^{**}$ | $Z^{**}$(%) | ω(%) | $\omega^{**}$(%) | $\Delta^{**}$ | $Z^{**}$(%) | ω(%) | $\omega^{**}$(%) |
| Tax rate | -0.343 | -0.79 | 12.85 | 12.06 | 0.015 | 0.15 | 13.87 | 14.02 |
| Health | -0.299 | -0.69 | 9.38 | 8.69 | -0.064 | -0.64 | 10.15 | 9.51 |
| Security | 0.607 | 1.40 | 5.63 | 7.03 | 0.171 | 1.71 | 5.57 | 7.28 |
| Education | -0.380 | -0.87 | 27.21 | 26.34 | -0.076 | -0.77 | 23.47 | 22.7 |
| Remainder | 0.073 | 0.17 | 57.78 | 57.95 | -0.030 | 0.30 | 60.81 | 61.11 |
| Growth rate | $dF^{**}$ = 1.70% | | | | $dF^{**}$ = 0.40% | | | |

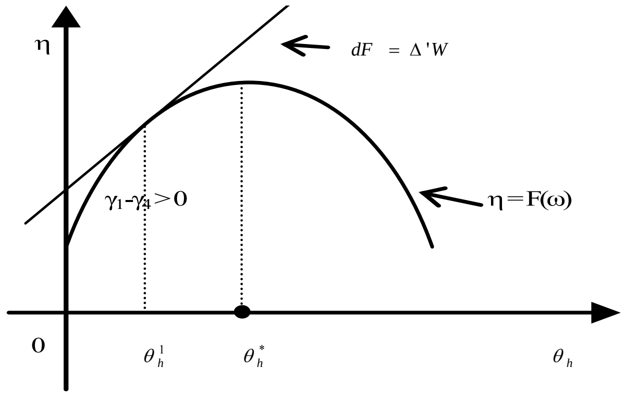
Note: The government is assumed to adjust the shares by 2% points as a total ($\xi = 0.02$). $\Delta^{**} = (\Delta_\tau, \Delta_\theta^{**}, -J_3'\Delta_\theta^{**})$, $\omega^{**} = \omega + Z^{**}$.

**Figure 1**. **Gradient Method**




Note: M: maximum point of $\eta = F(\omega)$, P: present level of ω, q: tangent line to the contour of $F(\omega) = \overline{\eta}$ at the present level of ω, which lies on the tangent surface of $dF \equiv \Gamma' Z$. The arrow starting from the point P stands for the optimal differential vector Z*.

**Figure 2**. **Linear approximation of the optimal shares**




Note: $\theta_h^*$ : element of $\omega^*$ at the maximum of $\eta^* = F(\omega^*)$.

$\theta_h^1$: present level, $\theta_h^1 < \theta_h^*$.