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Discussion Paper 07-26



Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

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# The Dynamic Welfare Cost of Stagnation: An Alternative Measure to the Lucas–Obstfeld Model*

by

Tatsuyoshi Miyakoshi [a], Masakatsu Okubo[b] and Junji Shimada [c]

[a] Osaka School of International Public Policy, Osaka University

[b]Graduate School of Systems and Information Engineering,University of Tsukuba

[c] School of Management, Aoyama Gakuin University

**Abstract**

This paper proposes an alternative measure to the Lucas–Obstfeld model to analyze the welfare costs of stagnation, and provides a practical illustration of both the Lucas–Obstfeld model and the alternative model. Compared with the Lucas–Obstfeld model, the alternative model can evaluate: (i) whether the policy was implemented in a timely fashion, (ii) whether the policy cost was expensive compared with the cost of stagnation, and (iii) whether the policy implemented was effective or whether an additional policy is required.

Keywords: Lucas model; dynamic welfare cost; time-varying parameters

JEL Classification Number: C32; C50; E32; E20

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**Correspondence:** Tatsuyoshi Miyakoshi, Osaka School of International Public Policy Osaka University,1-31, Machikaneyama-machi, Toyonaka,Osaka, 560-0043, Japan. tel:+81-6-6850-5638; fax:+81-6-6850-5656. E-mail: miyakoshi@osipp.osaka-u.ac.jp

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## 1. Introduction

Diagnoses of the seriousness of stagnation and the appropriate recovery process are important because they enable a government to evaluate its policy effects and implement additional policy if required. Several measures for diagnoses exist. A standard measure is the reduction in GDP (or its growth). Other measures are proposed by Chow and Kwan (1996) and Kwan and Chow (1996), who measured the effects of political movements in China (the Great Leap Forward Movement of 1958–1962 and the Cultural Revolution of 1966–1969) on output, consumption, and investment. They compared the 'hypothetical' time paths of these variables (i.e., the time paths in the absence of the political movements) with the actual time paths and computed the ratio between both time paths. This provided a measure of stagnation relating to the political movements. A third measure for diagnosis was developed by Lucas (1987) and Obstfeld (1994).

Lucas and Obstfeld measured compensations that would leave consumers indifferent to a decline in economic growth and an increase in economic instability. We refer to both types of compensation (for the decline in economic growth and the increase in instability) as the 'welfare cost of stagnation' and the model as the 'Lucas–Obstfeld model'. The welfare costs based on utility will equal the costs that people are willing to pay to prevent the outbreak of stagnation. Then, we can evaluate whether the cost of the policy to prevent stagnation, which a government would finance from tax revenues, is higher than the welfare cost of stagnation. The practical use of measuring this cost had not been explored previously. However, Lucas and Obstfeld did not define the cost of stagnation. Rather, their concern was with the latter compensation (the cost of the economic instability) and they did not provide any analysis of the cost of stagnation: they only derived the formulation for the cost of stagnation. This also applies to work by Dolmars (1998), Krusell and Smith (1999), Storestetten et al. (2001), Beaudry and Pages (2001), and Pallage and Robe (2003).

We consider that an alternative to the Lucas–Obstfeld model is needed to measure the cost of stagnation. Lucas and Obstfeld assumed that: (i) a stagnation economy is characterized by constant growth over time and instability, (ii) an agent knows the moments of the distribution for the economy (i.e., they assume rational expectations), and (iii) an agent measures the cost from the beginning of stagnation to the future as the 'whole cost' of stagnation compared with a hypothetical economy (i.e., an economy without stagnation). Alternatively, we argue that it is natural to assume that: (i) the stagnation economy has more complex processes of time-varying growth and instability, (ii) an agent knows the moments of the distribution for today's economy only and he/she expects (or believes) these moments will continue forever (i.e., myopic rational expectations, meaning that an agent bases tomorrow's moments on tomorrow's coming information, and the same for future moments), and (iii) an agent measures the cost of stagnation from period t to the future as a ' cost at period t' compared with a hypothetical economy. Both cost measures are based on different setups. The alternative model can reflect decreasing costs period by period in the recovery process of stagnation. Other alternatives combining the above three aspects (i)–(iii) seem implausible and are difficult to derive by applying the previous contributions of Lucas (1987), Obstfeld (1994), and Kwan and Chow (1996).

The purpose of this paper is to propose an alternative measure to the Lucas–Obstfeld model for analyzing the welfare cost of stagnation, and to provide a practical illustration of the alternative model compared with the Lucas–Obstfeld model by using Japanese data from 1975 to 2002.

By using this alternative model, we can evaluate: (i) whether the policy was implemented in a timely fashion, (ii) whether the policy cost was expensive compared with the cost of stagnation, and (iii) whether the policy was effective or whether additional policy is required. These specific exercises cannot be carried out under the existing frameworks, including the Lucas–Obstfeld model. However, how should we compare the Lucas–Obstfeld model and the alternative model? The 'whole cost' of stagnation by the Lucas–Obstfeld model is evaluated as being 25,390 yen per month (the cost ratio is 26% of the monthly consumption of 99,373 yen in January 1990, the starting period of stagnation). On the other hand, the alternative model evaluates the cost as being 35,798 yen per month (35%) in January 1992, and determines that the cost reaches a peak of 59,400 yen per month (51%) in December 2001, and falls to 55,610 yen per month (48%) in the most recent period, August 2002. The cost does not show any clear sign of a decrease.

This paper is organized as follows. In Section 2, we sketch the Lucas–Obstfeld model, adding the analyses for the properties of the welfare costs. In Section 3, we provide an alternative model. In Section 4, we show the practical use of both models. Section 5 concludes the paper.

## 2. The Lucas–Obstfeld Model

### 2.1. Framework

We sketch the Lucas–Obstfeld model.[1] Following Lucas (1987), the representative agent lives infinitely and maximizes an expected utility function V by choosing real consumption $C_t$ at time t. The agent has preferences specified by:

$$V = E\left[\sum_{t=0}^{\infty} \beta^{t} \frac{1}{1-\gamma} C_t^{1-\gamma}\right], \tag{1}$$

where $\beta \in (0,1)$ is a constant discount factor and $\gamma > 0$ is the constant coefficient of relative risk aversion. Here, we consider a pure exchange economy with no production, no storable goods, and no borrowing. Then, the optimal consumption $C_t$ for an agent is subject to exogenous income $I_t$ in each period and hence is equal to income: $C_t = I_t$ for all t.

Lucas and Obstfeld assumed a class of exogenous income. Hence, the optimal consumption streams $C_t$ with trend and cycle components, are given by:

$$\begin{aligned} &C_t = \lambda(1+\mu)^t e^{-\frac{1}{2}\sigma^2} z_t, \\ &i.\ e.,\ \ln C_t = \ln\lambda - \frac{1}{2}\sigma^2 + \mathrm{t}\cdot\ln(1+\mu) + \ln z_t : \ln z_t \sim N(0,\sigma^2)\text{'} \end{aligned} \tag{2}$$

where $\mu$ is the growth rate of consumption and $\ln z_t \sim N(0, \sigma^2)$. In addition, Lucas and

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[1] The Lucas–Obstfeld model was not constructed for practical use. Thus, it must be revised slightly, and so the cost that we derive is a little different from the original cost derived by Lucas and Obstfeld.

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Obstfeld assume that an agent has rational expectations, which implies that an agent knows those moments of the consumption distribution, and then maximizes an unconditional expectation of utility (1): the subscript of time t is not attached on V in (1). Owing to the property of the log-normal distribution, $E(z_t \bullet \exp(-\sigma^2/2)) = 1$, the mean consumption is:

$$E(C_t) = \lambda(1+\mu)^t, \tag{3}$$

where the mean consumption at t = 0 is λ.

Thus, Lucas and Obstfeld assumed that the stagnation process of exogenous income (consumption) can be expressed by constant moments over time of the distribution of consumption, λ, μ, and $\sigma^2$, and that an agent has rational expectations.

Under the above setup, we can calculate the indirect utility given the consumption process described by (2) and denote it by $V(\lambda, \mu, \sigma^2|\gamma,\beta)$. This is derived as follows:

$$V(\lambda,\mu,\sigma^2 \mid \gamma,\beta) = \frac{1}{(1-\gamma)(1-\phi)} \exp\left\{(1-\gamma)(\ln\lambda - \frac{\gamma}{2}\sigma^2)\right\} \quad \textit{if } \phi < 1,$$
$$\phi = \exp\{\ln\beta + (1-\gamma)\ln(1+\mu)\}. \tag{4}$$

Details of the derivation are given in the Appendix.

We consider two economies. One is called the *stagnation economy*, in which consumption growth $\mu_s$ and its variance $\sigma_s^2$ are calculated based on the data in the stagnation period. We denote the resulting indirect utility as $V(\lambda_S, \mu_S, \sigma_S^2 \mid \bar{\gamma}, \bar{\beta})$. The other economy is called the *hypothetical economy* (i.e., the economy without stagnation), which is based on expected consumption under the assumption that the growth rate and the variance in the prestagnation period are maintained during the stagnation period. The resulting indirect utility is $V(\lambda_H, \mu_H, \sigma_H^2 \mid \bar{\gamma}, \bar{\beta})$. The intuition behind this comparison is shown in Figure 1. Owing to (3), the $\lambda_H$ is mean consumption at the beginning of the stagnation period for the hypothetical economy. Thus, we compare both economies from the beginning of the stagnation period (denoted by t = 0 in Figure 1). Although $\gamma$ and $\beta$ may differ between the prestagnation and stagnation periods, we assume that they remain constant over time at $(\bar{\gamma}, \bar{\beta})$.

**[INSERT Figure 1]**

Using the indirect utilities under these economies, we define the 'whole cost' of stagnation as follows.

***Definition 1.*** The cost of stagnation is given by $\lambda^*$, which satisfies the following equation:

$$V(\lambda_S + \lambda^*, \mu_S, \sigma_S^2 \mid \bar{\gamma}, \bar{\beta}) = V(\lambda_H, \mu_H, \sigma_H^2 \mid \bar{\gamma}, \bar{\beta}), \qquad (5)$$

where the subscripts S and H denote the stagnation economy and the hypothetical economy, respectively.

The key concept relating to the whole cost of stagnation is the following. The consumption parameters are different between the stagnation and the hypothetical economies. Consumer preferences, given by $(\bar{\gamma}, \bar{\beta})$, transform the difference in consumption parameters into a difference in utility levels. The whole cost of stagnation is measured by the compensation required to leave consumers indifferent between the two economies. This compensation is uniform across all periods. The whole cost implies the cost from the beginning of the stagnation to the future.

The calculation of λ* is given by:

$$\lambda^* = \exp\{\Psi\} - \lambda_S ,$$
$$\Psi = \frac{1}{1-\bar{\gamma}}\left\{\ln\left(\frac{1-\phi_S}{1-\phi_H}\right) + (1-\bar{\gamma})(\ln\lambda_H - \bar{\gamma}\sigma_H^2/2)\right\} + \bar{\gamma}\sigma_S^2/2 \ , \quad (6)$$

where:

$$\phi_S = \exp\{\ln\bar{\beta} + (1-\bar{\gamma})\ln(1+\mu_S)\} \text{ and } \phi_H = \exp\{\ln\bar{\beta} + (1-\bar{\gamma})\ln(1+\mu_H)\}.$$

The derivation is given in the Appendix.[2]

## 2.2. Properties of Welfare Costs

We derive the partial derivatives of the cost λ*in terms of the time preference

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[2] Obstfeld (1994, pp. 1474–75) completely derived the cost of stagnation in the framework of Lucas, and the formulation of the costs is the mostly same as in (6). However, his concern was only with the cost of the economic instability. Obstfeld evaluated whether the persistency of consumption shocks affects Lucas's finding that the cost of economic instability may be slight. Therefore, in his application, only the cost of the economic instability was examined, and no analysis of the cost of the stagnation was provided (see Section 4 of Obstfeld, 1994).

$\bar{\beta}$ and the relative risk aversion $\bar{\gamma}$ by using equation (6) under $\bar{\gamma} > 1$:

$$\frac{\partial \lambda^*}{\partial \bar{\beta}} = \exp\{\Psi\} \frac{\phi_H - \phi_S}{(1-\bar{\gamma})\bar{\beta}(1-\phi_H)(1-\phi_S)}, \tag{7}$$

and

$$\frac{\partial \lambda^*}{\partial \gamma} = \exp\{\Psi\} \left\{ \frac{1}{(1-\bar{\gamma})^2} \{\ln \mu_S - \ln \mu_H\} + \frac{1}{2}(\sigma_S^2 - \sigma_H^2) \right\}, \tag{8}$$

where $\phi_H < 1$ *and* $\phi_S < 1$, as shown in equation (4).[3] When the growth rate of consumption falls in a stagnation period (i.e., $\mu_H > \mu_S$, which means that $\phi_H < \phi_S$), the increase in the time preference (i.e., the agent evaluates the future more) increases the welfare cost, which is uniform across all periods, to compensate for the reduced growth to the distant future: $\partial \lambda^* / \partial \bar{\beta} > 0$. When the reduced growth rate of consumption in a stagnation period is less than that in a prestagnation period plus the difference between both variances (which are estimated in this paper as mostly equal), a decrease in relative risk aversion increases the cost: $\partial \lambda^* / \partial \bar{\gamma} < 0$. In addition, the signs of the derivatives in terms of $\mu_H$, $\mu_S$ $\sigma_H$, $\sigma_S$, $\lambda_H$, and $\lambda_S$ are obvious: $\partial\lambda^*/\partial\mu_H>0$, $\partial\lambda^*/\partial\mu_S<0$, $\partial\lambda^*/\partial\sigma_H<0$, $\partial\lambda^*/\partial\sigma_S>0$, $\partial\lambda^*/\partial\lambda_H>0$, and $\partial\lambda^*/\partial\lambda_S<0$. The derivation is given in the Appendix.

## 3. An Alternative Model: The Dynamic Welfare Cost

Alternatively, it is natural to assume that the stagnation economy for consumption can

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[3] So far, Lucas (1987), Obstfeld (1994), and others have only roughly investigated the relationship between the cost and the preference parameters by calibration. Nevertheless, our

be expressed by the following time-varying intercept and slope:[4]

$$\ln C_t = a_t + b_t \cdot t + \ln z_t : \quad \ln z_t \sim N(0, \sigma^2)$$
$$a_t \equiv \ln\lambda_t - \frac{1}{2}\sigma^2; \quad b_t \equiv \ln(1+\mu_t) \tag{9}$$

$$\begin{bmatrix} a_t \\ b_t \end{bmatrix} = \begin{bmatrix} \alpha_0 \\ \beta_0 \end{bmatrix} + \begin{bmatrix} \alpha_1 & 0 \\ 0 & \beta_1 \end{bmatrix} \begin{bmatrix} a_{t-1} \\ b_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_{a,t} \\ \eta_{b,t} \end{bmatrix} : \quad \begin{bmatrix} \eta_{a,t} \\ \eta_{b,t} \end{bmatrix} \sim N\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} q_a^2 & 0 \\ 0 & q_b^2 \end{bmatrix} \right) \tag{10}$$

t=0,1,2,….

Then, we assume that an agent has myopic rational expectations, which implies that an agent knows only the moments of the distribution of today's consumption parameters, $\sigma^2$, at ($\lambda$t) and bt ($\mu$t). Moreover, an agent expects (or believes) these moments of today will continue forever for future consumption. Thus, based on today's information, an agent obtains the conditional moments for today and for future consumption (i.e., the stagnation economy). The indirect utility function of (4) conditioned on today's information includes the subscript of time t.

$$V_t(\lambda_t, \mu_t, \sigma^2 \mid \gamma, \beta) = \frac{1}{(1-\gamma)(1-\phi_t)} \exp\left\{(1-\gamma)(\ln\lambda_t - \frac{\gamma}{2}\sigma^2)\right\} \quad if\ \phi_t < 1,$$
$$\phi_t = \exp\{\ln\beta + (1-\gamma)\ln(1+\mu_t)\}. \tag{11}$$

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theoretical analysis requires that stringent results be obtained.

[4] We assume the variance of shocks is constant over time, considering Lucas's finding that the cost of business cycles may be slight. In addition, we assume that the preference parameters are constant over time, following the Lucas–Obstfeld model.

At period t, an agent gets these parameters and computes indirect utility from period t to the future, based on the coming information at period t. Here, we compare this indirect utility at period t with the indirect utility for the hypothetical economy mentioned in the Lucas–Obstfeld model. The cost of stagnation at period t is given by λt*, which satisfies the following equation:

$$V_t(\lambda_{S,t}+\lambda_t^*,\mu_{S,t},\sigma_{\mathrm{S}}^2\mid\bar{\gamma},\bar{\beta})=V_t(\lambda_{H,t},\mu_H,\sigma_{\mathrm{H}}^2\mid\bar{\gamma},\bar{\beta}),\qquad(12)$$

where the subscripts S and H denote the stagnation and the hypothetical economies, respectively. The parameters for the stagnation economy are changed period by period, whereas the parameters for the hypothetical economy are the same and constant over each period, as in the Lucas–Obstfeld model. The parameter $\lambda_{H,t}$ is the exception because it is an intercept of consumption and it changes at period t.

Then, an agent recomputes the 'cost at period t' of stagnation in each period, based on the updated information.

$$\lambda_t^*=\exp\{\Psi_t\}-\lambda_{S,\,t},$$
$$\Psi_t=\frac{1}{1-\bar{\gamma}}\left\{\ln\left(\frac{1-\phi_{S,t}}{1-\phi_H}\right)+(1-\bar{\gamma})(\ln\lambda_{H,t}-\bar{\gamma}\sigma_H^2/2)\right\}+\bar{\gamma}\sigma_S^2/2,\qquad(13)$$

where:

$$\phi_{S,\,t}=\exp\{\ln\bar{\beta}+(1-\bar{\gamma})\ln(1+\mu_{S,\,t})\}\text{ and }\phi_H=\exp\{\ln\bar{\beta}+(1-\bar{\gamma})\ln(1+\mu_H)\}.$$

Keeping the variance constant in the stagnation period but letting the consumption intercept and growth fluctuate, the agent recomputes the cost at period t, comparing with the hypothetical economy. This cost of stagnation is different period by period.

Figure 2 is a schematic diagram that describes this idea and summarizes our alternative model. An agent knows only the present period (T1)'s intercept $\lambda_{S,T1}$ and slope $\mu_{S,T1}$ in (11) from information at period T1, and he or she expects (or believes) these parameters $\lambda_{S,T1}$ and $\mu_{S,T1}$ will continue forever. On this basis, the agent constructs the stagnation economy from period t to the future. Thus, the stagnation economies expected from period T1 to the future and from period T2 to the future will be different, and as a result, the costs between T1 and T2 are different. The 'cost at period t' calculated in this way may be referred to as the dynamic cost of stagnation, in contrast to the 'whole cost' computed from the beginning of the stagnation to the future by the Lucas–Obstfeld model. In general, the cost at period t will be higher at the start and lower at the end of the stagnation period.

**[INSERT Figure 2]**

By using a 'cost at period t', we can evaluate: (i) whether the policy was implemented immediately in the period when the highest welfare cost arose, (ii) whether the cost of the implemented policies was expensive compared with the welfare costs of stagnation, and (iii) whether the cost of stagnation decreased gradually—that is, whether the policies were effective—or whether additional policies are required, which will be ascertained by determining the current welfare costs. These specific exercises cannot be undertaken under the existing frameworks, including under the Lucas–Obstfeld model.

Although an alternative model can compute the cost period by period, how can it derive the whole cost of stagnation that occurred in an era? We may consider the 'cost at the starting period t' as the 'whole cost' of stagnation. However, as an agent has myopic rational expectations about the stagnation economy expressed by time-varying parameters, he or she cannot predict the future economy. Thus, the 'cost at the starting period' cannot correctly capture the 'whole cost' from the starting period to the future. On the other hand, it is a very bold and implausible assumption that an agent has rational but not myopic rational expectations of the stagnation economy expressed by time-varying parameters. Moreover, it is technically difficult to derive the ' whole cost' and the 'cost at period t' by applying the Lucas–Obstfeld model to an economy expressed by constant parameters. Thus, the two cost measures are derived based on the two different conceptions of the stagnation economy. However, we propose a compromise. If we value a long-run viewpoint, it is natural to assume a stagnation economy with constant growth over time and instability. In contrast, if we value a short-run viewpoint, the assumption of a stagnation economy with time-varying parameters is more natural. In this sense, when we value the long-run viewpoint and evaluate the 'whole cost' of stagnation that has occurred in an era, the Lucas–Obstfeld measure may be appropriate.

## 4. A Practical Illustration of the Lucas–Obstfeld Model and the Alternative Model

### 4.1. Data, Unit Root, and Structural Change

The data used in this paper are monthly data from January 1975 to August 2002 (i.e., 1975:M1 to 2002:M8), which provides 332 observations. To estimate the parameters ($\lambda$, $\mu$, and $\sigma^2$) for consumption in the model, we use total consumption expenditure for workers' households from the *Monthly Report on Family Income and Expenditures Survey* (FIES). As the FIES reports nonseasonally adjusted data, we apply the census X-11 method to obtain the seasonally adjusted series. The per capita series is constructed by dividing consumption expenditure by the number of family members in the household. These data are converted to real values by using the consumer price index (for general prices in 2000) from the *Monthly Report of the Consumer Price Index*. All data are taken from the NIKKEI NEEDS CD-ROM.

We partition the whole sample (1975:M1 to 2002:M8) into two subsamples. The first subsample is from 1975:M1 to 1989:M12 and the second is from 1990:M1 to 2002:M8. The first subsample is the prestagnation period. Our objective is to estimate the cost of stagnation during the second period (the stagnation period) by comparing it with the hypothetical economy. This partition of periods seems appropriate, and is consistent with previous research, including Hayashi and Prescott (2002).[5] Moreover, we implement the tests for a unit root of consumption and for the structural change by using equation (14) of Perron (1989, p.1380):

$$\Delta \log C_t = a_1 + a_2 DU_t + a_3 D(TB)_t + b_1 t + b_2 DT_t + \rho \log C_{t-1} + \sum_{i=1}^{k} \xi_i \Delta \log C_{t-i} + \varepsilon_t \,, \quad (14)$$

[5] We suggest that stagnation began in 1990:M1 because the NIKKEI 225 peaked at a stock price of 38,926 yen in 1989:M12. Since that date, it has fallen gradually, as has the price of land. These events are said to define the collapse of the so-called Japanese bubble economy. In line with previous research, we maintain the importance of these events in defining the starting point of stagnation. Figure 2 plots consumption in logs, which shows the behavior of the data in the model. It suggests a trend break in the log of consumption (which reduces consumption growth) around 1990:M1. These figures suggest an obvious structural change.

where $DU_t=1$ if $t>T_B$ and 0 otherwise, $D(TB)_t=1$ if $t= T_B+1$ and 0 otherwise, $DT_t=t$ if $t>T_B$ and 0 otherwise. This model allows a sudden change in the level followed by a shift in the slope of the trend function at time $T_B$ ($1< T_B<T$). $T_B$=1989:M12. Both the null model of the unit root ($\rho=0, b_1=b_2=0$) and the alternative of trend stationarity ($\rho<0$) are nested in (14). At the same time, this nested model allow us to test the structural change at $t=T_B+1$. In fact, the structural change did occur because $a_2$ and $b_2$ seem to be significant, but $a_3$ is not significant and hence there is no jumping of data series. Table 1 rejects the null hypothesis of a unit root for consumption and the null hypothesis of no structural change at $T_B+1$=1990:M1.The visual inspection is given in Figure 3.

**[INSERT Figure 3 and Table 1]**

### 4.2. Preference Parameters

In order to compute the welfare cost of stagnation, the preference parameters are set exogenously, following convention. For the coefficient of relative risk aversion, we use values of $\bar{\gamma} \in \{0.5, 1.5, 2.5, 3.5, 5.0\}$. Previous studies suggest that, as long as the constant relative risk aversion preferences are used, the value of $\bar{\gamma}$ falls into this range (see, e.g., Kitamura and Fujiki (1997), Hamori (1998), and Nakano and Saito (1998)). We choose $\bar{\gamma} = 2.5$ as a base value and adopt a strategy that evaluates how the welfare cost is affected by the other choice of $\bar{\gamma}$. For the discount factor, we take values of $\overline{\beta} \in \{0.994, 0.995, 0.996, 0.997, 0.998\}$ and choose $\overline{\beta} = 0.996$ as a base value. Similarly, we check the robustness of the welfare cost to the other values of $\overline{\beta}$.

### 4.3 Estimated Cost for a Lucas-Obstfeld Model

**Normality of Stochastic Term and Estimated Parameters**

The parameters for consumption in the prestagnation period (1975:M1–1989:M12) and the stagnation period (1990:M1–2002:M8) are estimated by applying an ML methodology to the whole sample from 1975:M1 to 2002:M8:

$$
\begin{aligned}
\ln C_t &= \ln(\lambda_B(1-DU_t)+\lambda_S DU_t) - \frac{1}{2}(\sigma_B^2(1-DU_t)+\sigma_S^2 DU_t) \\
&\quad + \mathrm{t}\cdot\ln(1+\mu_B(1-DU_t)+\mu_S DU_t)+\ln z_t : \\
&\qquad\qquad \ln z_t \sim N(0,\sigma_B^2(1-DU_t)+\sigma_S^2 DU_t)
\end{aligned}, \quad (15)
$$

where $DU_t=1$ if $t>T_B$ (that is, $T_B$=1989:M12) and 0 otherwise. As shown in Table 2A, each test statistic for mean=0, skewness=0, excess kurtosis=0, and nonserial correlation of the squared error term with lags of up to 6 are not statistically significant at the 1% level, except for excess kurtosis in the stagnation period, supporting the normality of ln $z_t$. In Panel (A) of Table 2B, the parameters in both the prestagnation and the stagnation periods are estimated and denoted as the estimated parameters for the hypothetical economy and the stagnation economy. The exception is an estimate for λ in a hypothetical economy, which is replaced with one for a stagnation economy for the sake of simplicity (the difference is negligible because $a_3$=0 in Table 1, which means no jumping of the data series). Thus, an agent perceives that the estimated parameters in both periods are the consumption parameters for both economies. All parameter estimates for consumption are statistically significant at the 1% level. The estimated monthly consumption growth rate falls from 0.1059% in the prestagnation period to

–0.002018%. The variance (which represents instability) of the error term in the log of consumption increases from 2.339E(–4) to 3.927E(–4). The difference in consumption growth and variance between both economies is statistically significant at the 1% level, as is shown in Panel (B) of Table 2B.

**[INSERT Tables 2A and 2B]**

**Estimated Cost**

What is the estimated cost of stagnation according to this model? As Table 3 shows, by using these parameters in equation (5), we obtain a utility level of –3.813E(–06) for the hypothetical economy and one of –5.362E(–06) for the stagnation economy. This implies that stagnation reduces utility. Our cost measure $\lambda^*$ enables us to convert the reduction in the utility level into a level of compensation in Japanese yen, which is the same for all periods. The cost of stagnation is 25,390 yen. The cost ratio (cost/$\lambda_H$) is the ratio of the cost to the consumption of the hypothetical economy at the starting period. It is 26%, which shows the relative amounts and hence an actual impact.

To check the robustness of the results, we calibrate the preference parameters in (1). We use $\gamma$ = 0.5,1.5, 2.5, 3.5, and 5.0 and $\beta$ = 0.994, 0.995, 0.996, 0.997, and 0.998 around the base values in Table 4. Note here that $\gamma$>0 implies risk aversion. The ratios of costs range from 17% to 49% as $\beta$ increases at $\bar{\gamma}$=2.5, and from 20% to 33% as $\gamma$ decreases at $\bar{\beta}$=0.996, as the theoretical analysis suggests in (7) and (8). The ratios of costs reach a maximum at 87% and a minimum at 15%. These numbers are not small and the cost ratio seems to be relatively sensitive to the time preference.

Thus, the proposed ‘whole cost’ $\lambda^*$, which is particularly relevant to consumers, suggests that the cost of stagnation is large for Japanese consumers. As this represents what people are willing to pay to prevent the outbreak of stagnation, it indicates that people were prepared to pay a great deal to avoid the stagnation.

**[INSERT Tables 3 and 4]**

### 4.4. Estimated Cost for an Alternative Model

**Estimated Parameters**

The estimation results for $\alpha_0$, $\alpha_1$, $\beta_0$, and $\beta_1$ and other parameters in model (9) are obtained by the Kalman filter algorithm for the time varying parameter model (see the Appendix). The estimation results are shown in Table 5. Moreover, the estimated $a_t$ and $b_t$ over time, the actual values of log $C_t$, and the estimated $a_t + t{\cdot}b_t$ values over time are shown in Figure 4.[6] By using the estimated parameters in Table 5, the estimated coefficients over time are $a_t = 0.609 + 0.947\ a_{t-1}$ and $b_t = 0 + 0.496\ b_{t-1}$ , where some estimated parameters are not significant at the 5% or 10% levels. However, the dynamics of coefficients converge to $\lim_{t\to\infty} a_t = 0.609/(1-0.947) = 11.518$ and $\lim_{t\to\infty} b_t = 0$. In addition, the dynamic process of those coefficients are shown in Figure 4, and thus the time-varying coefficient estimates without the growth component ($b_t = 0$) can capture the actual values of consumption, as seen in the third column of Figure 4.[7] On date T1 in Figure 2, an agent can know the moments of distribution of the consumption

[6] In general, the normality test of a stochastic term is difficult. This will be a task for the future.
[7] Miyakoshi, Okubo, and Shimada (2006) have analyzed the welfare cost of the 1997 Asian crisis for the Asian countries by using this model. The time-varying parameter estimates cannot capture the recovery process for each country without the growth component because the Asian economy has grown strongly after the crisis.

process on period T1, that is, $a_{T1,}$ $b_{T1}$, and $\sigma^2$ in (9). The agent expects that these parameters will continue forever, and computes the welfare cost on this basis. However, on period T1+1, the agent recomputes a different welfare cost, based on $a_{T1+1,}$ $b_{T1+1,}$ and $\sigma$ on period T1+1. As seen in Figure 5, the estimated growth parameter $\mu_{ST}$ and the initial level of consumption $\lambda_{ST}$ are less than those values for the hypothetical economy for nearly all periods, which results in a higher cost of stagnation. On the other hand, the finding that the instability $\sigma^2$ (=0.013 *0.013 =0.00017 in Table 5) of the stagnation economy is smaller than that of the hypothetical economy $\sigma^2$ (=2.339E(–4) in Table 2B) results in a lower cost.

**[INSERT Table 5, Figure 4, and Figure 5]**

**Estimated Costs**

We can compute the welfare cost at each period by applying equation (13). As seen in Figure 6, the dynamic cost ratio (i.e., cost at each period/ $\lambda_H$, where $\lambda_H$ =99,373 yen is for 1990:M1 in (9)) shows the four peaks, which are 42% (44,260 yen per month) in February 1995, 42% (45,268 yen) in May 1997, 47% (51,992 yen) in February 1999, and 51% (59,399 yen) in December 2001. In addition, the cost remains high at 48% in the last period of August 2002. The cost of stagnation is large (more than 30% or 29,812 yen per month) for all periods and shows no sign of decreasing.[8]

**[INSERT Figure 6]**

[8] However, Miyakoshi, Okubo and Shimada (2006) have found that the welfare cost of the 1997

**Characteristics of the Alternative Model**

We evaluate whether the policy was implemented immediately around February 1995, when the first peak of welfare cost arose. Table 6 shows the rapid policy response soon after, which involved establishing a special bank to rescue failed banks. At the same time, the Ministry of Finance announced a decrease in the budget by 2.9%. Although these two policies seem to conflict, they were implemented immediately, and the recovery process seemed to proceed slowly until the second peak in May 1997. However, the policy did not prevent the subsequent accidental failures of two big financial companies (Hokkaido Takushyoku Bank and Yamaichi Security Company) in November 1997. The ineffectiveness of this policy induced the third peak in February 1999. The timing of policy implementation in February and March 1999 was rapid for the third peak, but ineffective, which induced the fourth peak of 51% in December 2001. Rather than the timing of policy implementation being the problem, the policies themselves were not effective.

We evaluate whether the cost of the policies implemented was cheap in comparison with the size of the welfare costs. That is, if the public money injection (the policy costs) of 7 trillion yen on March 19, 1999 could rescue the economy immediately, would people agree to pay this policy cost? The welfare costs (51,992 yen in February 1999) implies that people pay 51,992 yen per month *forever (many months)*. On the other hand, if 7 trillion yen in policy costs can sustain the hypothetical economy, how many months should people pay the cost of 51,992 yen per month to collect 7 trillion

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Asian crisis decreased gradually despite repeated upturns and downturns of costs.

yen? That is, (policy costs/number of workers in February 1999)/the welfare cost equals the number of months. Then, (7 trillion yen/53,250,000 workers)/51,992 yen/month = 2.5 months.[9] Less than three months is a very short period, meaning people will agree to pay the cheaper policy cost than the welfare cost. If the effectiveness of the policy is confirmed, the government can publicly announce the cheap cost of the policy. However, actually it is not confirmed.

We evaluate whether the cost decreased gradually, that is, whether the policies were effective, or whether additional policies are now required by checking the current welfare costs. The cost of stagnation at period t increased rapidly to 30% in June 1990, and did not show any sign of decreasing, suggesting that the policies were ineffective. Finally, the welfare cost remained high at 48% in the last period of August 2002. This suggests that an additional policy is required.

**[INSERT Table 7]**

## 5. Concluding Remarks

This paper has proposed an alternative measure to the Lucas–Obstfeld model to analyze the welfare cost of stagnation. In addition, it has provided a practical illustration of the alternative model in comparison with the Lucas–Obstfeld model, by using Japanese data from 1975 to 2002. The cost of the stagnation is worth measuring, but the practical use of measuring this cost had not been explored previously.

Using the Lucas–Obstfeld model, which is based on a long-run viewpoint and

[9] The data for 53,250,000 workers (employees) were compiled from the Labor Force Survey of

assumes a stagnation process with constant growth over time and instability, we found that the ratio of the ‘whole cost’ is high at 26% of the consumption at the starting period of stagnation (99,373 yen). We proposed an alternative model, which is based on a short-run viewpoint and assumes a stagnation process with time-varying consumption. Based on this model, the cost of the policies implemented immediately after the stagnation cost hit the third peak was 7 trillion yen in March 1999, which is not expensive compared with the corresponding welfare cost in February 1999. However, in the absence of effective policies, the cost did not decrease, but remained at 55,610 yen (48%) in August 2002, suggesting that an additional policy is required. Thus, two cost measures, the ‘whole cost’ and the ‘cost at period t’ of stagnation, proposed by both models, seem to be complementary in evaluating the cost of stagnation.

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the Statistic Bureau of Japan.

## Appendix

### Derivation of equation (4)

Taking the logarithm of consumption in (2) and defining the first three terms as $\alpha_t$, we obtain the following:

$$\ln C_t = \alpha_t + \ln z_t, \quad \alpha_t = \ln \lambda + t \cdot \ln(1+\mu) - \frac{1}{2}\sigma^2 . \tag{A-1}$$

Clearly, $E(\ln C_t) = \alpha_t$ and $Var(\ln C_t) = \sigma^2$. In addition, because of the property of log-normality, $E(C_t) = \exp\{\alpha_t + \sigma^2/2\}$. In the utility function (1), we take logarithms and obtain the following:

$$\ln E[\beta^t c_t^{1-\gamma}] = \ln E[\beta^t \eta_t^{1-\gamma} z_t^{1-\gamma}], \; \eta_t \equiv \lambda(1+\mu)^t e^{-\frac{1}{2}\sigma^2} . \tag{A-2}$$

Using the above property of log-normality, we can rewrite the above equation as:

$$\ln E[\beta^t \eta_t^{1-\gamma} z_t^{1-\gamma}] = E[\ln(\beta^t \eta_t^{1-\gamma} z_t^{1-\gamma})] + Var[\ln(\beta^t \eta_t^{1-\gamma} z_t^{1-\gamma})]/2 . \tag{A-3}$$

Extensive rearrangement yields the following:

$$E[\ln(\beta^t \eta_t^{1-\gamma} z_t^{1-\gamma})] = t\ln\beta + (1-\gamma)(\ln\lambda + t\ln(1+\mu) - \sigma^2\gamma/2),$$
$$\mathrm{Var}[\ln(\beta^t \eta_t^{1-\gamma} z_t^{1-\gamma})] = (1-\gamma)^2\sigma^2. \tag{A-4}$$

Hence,

$$\ln E[\beta^t \eta_t^{1-\gamma} z_t^{1-\eta}] = [\exp(\ln\beta + (1-\gamma)\ln\mu)]^t \exp\{(1-\gamma)(\ln\lambda - \sigma^2\gamma/2)\} . \tag{A-5}$$

Thus, inserting these relations into (1) yields the following indirect utility function:

$$V = \frac{1}{(1-\gamma)}\sum_{t=0}^{\infty} E[\beta^t \eta_t^{1-\gamma} z_t^{1-\gamma}] = \frac{1}{(1-\gamma)}\exp\{(1-\gamma)(\log\lambda - \sigma^2\gamma/2)\}\sum_{t=0}^{\infty}\phi^t,$$
$$\phi = \exp\{\ln\beta + (1-\gamma)\ln(1+\mu)\}. \tag{A-6}$$

■

**Derivation of equation (6)**

Substituting (4) into (5) yields:

$$\frac{1}{1-\phi_S}\cdot\exp\left\{(1-\bar{\gamma})(\ln(\lambda_S+\lambda^*)-\frac{\bar{\gamma}}{2}\sigma_{\mathrm{S}}^2)\right\}=\frac{1}{1-\phi_H}\cdot\exp\left\{(1-\bar{\gamma})(\ln\lambda_S-\frac{\bar{\gamma}}{2}\sigma_{\mathrm{H}}^2)\right\}. \quad \text{(A-7)}$$

Taking logarithms of (A-7) yields:

$$\ln(\lambda_S+\lambda^*)=\frac{1}{(1-\bar{\gamma})}\left\{\ln(\frac{1-\phi_S}{1-\phi_H})+(1-\bar{\gamma})(\ln\lambda_S-\frac{\bar{\gamma}}{2}\sigma_{\mathrm{H}}^2)\right\}+\frac{\bar{\gamma}}{2}\sigma_{\mathrm{S}}^2. \quad \text{(A-8)}$$

We replace the right-hand side of (A-8) with $\Psi$ to obtain equation (6).

**Derivation of equations (7) and (8)**

The $\Psi$ in equation (6) is rewritten as follows:

$$\Psi=\frac{1}{1-\bar{\gamma}}\{\ln(1-\phi_S)-\ln(1-\phi_H)\}+(1-\bar{\gamma})\ln\lambda_H+\frac{\bar{\gamma}}{2}(\sigma_S^2-\sigma_H^2)\ . \quad \text{(A-9)}$$

We take derivatives of equation (6). First, $\partial\phi_H/\partial\bar{\beta}=\phi_H/\bar{\beta}$, $\partial\phi_S/\partial\bar{\beta}=\phi_S/\bar{\beta}$. Second, using these relations, we obtain:

$$\frac{\partial\Psi}{\partial\bar{\beta}}=\frac{(\phi_H-\phi_S)}{(1-\bar{\gamma})\bar{\beta}(1-\phi_S)(1-\phi_H)}. \quad \text{(A-10)}$$

Finally, inserting this relation into $\partial\lambda^*/\partial\bar{\beta}=\exp(\Psi)\cdot\partial\Psi/\partial\bar{\beta}$, we obtain equation (7).

The $\phi_H$ and $\phi_S$ in (6) are approximated by a Taylor expansion around $\bar{\beta}=1$ and $\mu$=0. They then become:

$$\phi_H=\bar{\beta}+(1-\bar{\gamma})\mu_H\quad,\quad\phi_S=\bar{\beta}+(1-\bar{\gamma})\mu_S. \quad \text{(A-11)}$$

Then, we obtain the relation: $(1-\phi_H)\mu_S-(1-\phi_S)\mu_H=(1-\bar{\beta})(\mu_S-\mu_H)$. We obtain the following partial derivatives: $\partial\phi_H / \partial\bar{\gamma} = -\mu_H$ , $\partial\phi_S / \partial\bar{\gamma} = -\mu_S$ . Finally, we take the derivative of λ*. That is, $\frac{\partial\lambda^*}{\partial\bar{\gamma}} = \exp\{\Psi\} \cdot \frac{\partial\Psi}{\partial\bar{\gamma}}$ . Here:

$$\begin{aligned}\frac{\partial\Psi}{\partial\bar{\gamma}} &= \frac{1}{(1-\bar{\gamma})^2} \cdot \left\{ \frac{-\partial\phi_S / \partial\bar{\gamma}}{1-\phi_S}(1-\bar{\gamma}) + \frac{\partial\phi_H / \partial\bar{\gamma}}{1-\phi_H}(1-\bar{\gamma}) + \ln(1-\phi_S) - \ln(1-\phi_H) \right\} + (\sigma_S^2 - \sigma_H^2)/2 \\ &= \frac{1}{(1-\bar{\gamma})^2} \cdot \left\{ (1-\bar{\beta})(\mu_S - \mu_H)(1-\bar{\gamma}) + \ln(1-\phi_S) - \ln(1-\phi_H) \right\} + (\sigma_S^2 - \sigma_H^2)/2\end{aligned} \quad .(\text{A-12})$$

When we evaluate this derivative at $\bar{\beta} = 1$ (its common parameter between both paradigms), its derivative leads to equation (8).

However, the signs of derivatives in terms of the $\mu_H$, $\mu_S$ $\sigma_H$, $\sigma_S$, $\lambda_H$, and $\lambda_S$ are directly derived without approximation around $\bar{\beta} = 1$ and μ=0. ■

**The Kalman filter algorithm for the time-varying parameter model**

We briefly describe the Kalman filter algorithm for the time-varying parameter in our model's equations (9) and (10). We derive the smoothing estimates of the time-varying parameter.

The time-varying parameter model for (9) and (10) can be regarded as the state space model.

**The observation equation:**

$$y_t = H_t x_t + \varepsilon_t : \quad \varepsilon_t \sim N(0, \sigma^2), \qquad (\text{A-13})$$

where $y_t \equiv \ln C_t$, $\varepsilon_t \equiv \ln z_t$, $a_t \equiv \ln \lambda_t - \sigma^2/2$, $b_t \equiv \ln(1+\mu_t)$, $H_t = (1, t)$, and $x_t = (a_t, b_t)'$.

**The state equation:**

$$\mathrm{x}_{\mathrm{t}} = \alpha + \beta\, \mathrm{x}_{\mathrm{t\text{-}1}} + \eta_{\mathrm{t}}, \quad \eta_{\mathrm{t}} \sim \mathrm{NID}\left(\mathbf{0}, \Sigma_{\eta}\right), \tag{A-14}$$

where $\alpha = (\alpha_0, \beta_0)'$, $\beta = \begin{pmatrix} \alpha_1 & 0 \\ 0 & \beta_1 \end{pmatrix}$, $\eta_t = (\eta_{a,t}, \eta_{b,t})'$, $\mathbf{0} = (0, 0)'$, and $\Sigma_{\eta} = \begin{pmatrix} q_a^2 & 0 \\ 0 & q_b^2 \end{pmatrix}$.

We set the initial conditions $a_1$ and $b_1$ to be the estimates of regression model:

$$\ln C_t = a + b \cdot t + \ln z_t : \quad \ln z_t \sim N(0, \sigma^2), \tag{A-15}$$

obtained from the first four samples.

The unknown parameters are $\theta = (\alpha_0, \alpha_1, q_a, \beta_0, \beta_1, q_b, \sigma, a_0, \text{and } b_0)$. Applying a standard Kalman filter algorithm, we can obtain the estimates of parameters and state variables. See Hamilton (1994) and Harvey (1989) for a detailed explanation of the Kalman filter algorithm.

The smoothing densities of $a_t$ and $b_t$ given the full sample $Y_T = (y_T, ..., y_1)$ are:

$$a_t \middle| Y_T \sim N(a_{t|T}, P^a_{t|T}), \quad b_t \middle| Y_T \sim N(b_{t|T}, P^b_{t|T}), \tag{A-16}$$

where $\mathrm{a}_{\mathrm{t|T}} \equiv \mathrm{E}\left[\mathrm{a}_{\mathrm{t}} \middle| \mathrm{Y}_{\mathrm{T}}\right]$, $\mathrm{b}_{\mathrm{t|T}} \equiv \mathrm{E}\left[\mathrm{b}_{\mathrm{t}} \middle| \mathrm{Y}_{\mathrm{T}}\right]$, $\mathrm{P}^{a}_{\mathrm{t|T}} \equiv \mathrm{Var}\left[\mathrm{a}_{\mathrm{t}} \middle| \mathrm{Y}_{\mathrm{T}}\right]$, and $\mathrm{P}^{b}_{\mathrm{t|T}} \equiv \mathrm{Var}\left[\mathrm{b}_{\mathrm{t}} \middle| \mathrm{Y}_{\mathrm{T}}\right]$.

So, the smoothing estimates of $\lambda_t$ and $\mu_t$ are obtained by the expectations of log-normal density:

$$\mathrm{E}\left(\lambda_t \middle| \mathrm{Y}_T\right) = \exp\left(a_t + \frac{1}{2}\hat{\sigma}^2 + \frac{1}{2}P^a_{t|T}\right), \tag{A-17}$$

$$\mathrm{E}\left(\mu_t \middle| \mathrm{Y}_T\right) = \exp\left(b_t + \frac{1}{2}P^b_{t|T}\right) - 1. \tag{A-18}$$

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**Table 1. Perron's t test for Unit Root Hypothesis and Structural Change in the Trend Break at 1989:M12**

| $a_1$ | $a_2$ | $a_3$ | $b_1$ | $b_2$ | $\rho$ | $\xi_1$ | $\xi_2$ |
|---|---|---|---|---|---|---|---|
| 4.124 | 0.090 | 0.001 | 3.920E(–4) | –4.102E(–4) | –0.366 | –0.348 | –0.155 |
| (5.85) | (5.33) | (0.08) | (5.64) | (–5.43) | (–5.84) | (–5.30) | (–2.81) |

Note: E(–x) denotes $10^{-x}$. The numbers in parentheses denote t statistics. The order of the six lags is determined by SBIC and AIC, and the second lag is chosen unanimously by both criteria. The 1% significance point of the t statistic for $\rho$=0 is –4.90 from Table VI.B of Perron (1989, p. 1377), in view of the fact that 180/332≈0.5.

**Table 2A. Normality of the Stochastic Term**

| Period | NOBS | MEAN | STDEV | SKEW | EXKURT | $Q^2(6)$ |
|---|---|---|---|---|---|---|
| PS-Period | 180 | –0.000 | 0.015 | 0.230 | 0.800 | 2.898 |
| S-Period | 152 | 0.000 | 0.020 | 0.471 | 1.653* | 6.006 |

Note: PS-Period and S-Period denote the prestagnation period and the stagnation period, respectively. NOBS, MEAN, STDEV, SKEW, and EXKURT denote the number of observations, the mean, standard deviation, skewness, and excess kurtosis, respectively. The symbol ** denotes statistically significance at the 1% level for each test statistics for mean=0, skewness=0, and excess kurtosis=0. The Ljung-Box Q-Statistics, $Q^2(6)$, are distributed as $\chi^2$ (6) under the null hypothesis of nonserial correlation with up to six lags. The 1% critical value is 16.8.

**Table 2B. Estimated Parameters for Consumption**

| (A) Consumption Parameters | | | (B) Test Hypotheses | |
|---|---|---|---|---|
| | H-ECO | S-ECO | Hypothesis | $\chi$2(1) |
| $\lambda$ | 99373 yen | 99373 yen | | |
| | (619.43) | (619.43) | | |
| $\mu$ | 1.059E(–3) | –2.018E(–5) | H1: $\mu_H=\mu_S$, | 8049.48 |
| | (101.91) | (–3.33) | | |
| $\sigma^2$ | 2.339E(–4) | 3.927E(–4) | H2: $\sigma^2_H=\sigma^2_S$, | 15.54 |
| | (10.95) | (11.50) | | |

Note: H-ECO and S-ECO denote the hypothetical economy and the stagnation economy, respectively. The $\lambda$ is consumption in yen in 1990:M1 for both economies. All coefficients were significant at the 1% level. The critical value of the $\chi^2(1)$ distribution is 6.64 at the 1% level.

**Table 3. Welfare Costs ($\gamma$*=2.5, $\beta$*=0.996)**

| Parameters | Indirect Utility | | Cost | |
|---|---|---|---|---|
| | H-ECO | S-ECO | Level (yen) | Ratio=$\lambda^*/\lambda_H$(%) |
| $\gamma=\gamma^*,\beta=\beta^*$ | –3.813E(–06) | –5.362E(–06) | 25,390 | 26 |

**Table 4. Welfare Costs (ratio) based on Various Preference Parameters (%)**

| | | β | | | | |
|---|---|---|---|---|---|---|
| | | 0.994 | 0.995 | 0.996 | 0.997 | 0.998 |
| γ | 0.5 | 21 | 25 | 33 | 48 | 87 |
| | 1.5 | 19 | 23 | 29 | 39 | 61 |
| | 2.5 | 17 | 21 | 26 | 34 | 49 |
| | 3.5 | 16 | 19 | 23 | 30 | 42 |
| | 5.0 | 15 | 17 | 20 | 25 | 34 |

Note: The welfare cost (ratio) is $\lambda^*/\lambda_H$ (%).

**Table 5. Estimated Parameters in (10)**

| | $\sigma$ | $\alpha_0$ | $\alpha 1$ | $q$ a | $\beta 0$ | $\beta_1$ | $q$ b |
|---|---|---|---|---|---|---|---|
| Estimates | 0.013 | 0.609 | 0.947 | 0.003 | 0.000 | 0.496 | 0.000 |
| t values | (9.04) | (0.91) | (16.19) | (3.16) | (-0.40) | (1.15) | (2.71) |

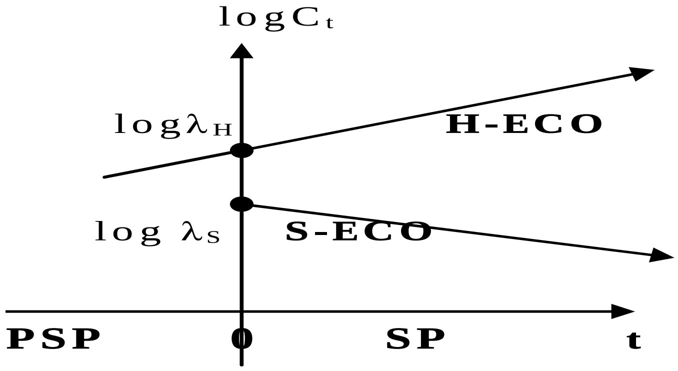
**Table 6. Main Economic Policies around the Peaks of Welfare Costs**

| Date | Events and the Contents of Policy |
|---|---|
| **12/1/1994** | **The credit unions began to fail.** |
| 12/9/1994 | The government decided to establish a special bank to rescue failed banks. |
| 12/20/1994 | The Ministry of Finance announced a decrease in the budget by 2.9% after a 40 year period without any reductions in the budget. |
| **5/1/1997** | **Security companies began to fail.** |
| 5/30/1997 | The Acts related to stock options and mortgage securitization were issued. |
| 6/27/1997 | The Fidelity and the Hyunde security companies (foreign companies) obtained security dealers' licenses from the Ministry of Finance. |
| **2/1/1999** | **The big banks began to fail.** |
| 2/26/1999 | The think tank of the prime minister announced the rescue packages. |
| 3/12/1999 | The government injected public money of 7 trillion yen to rescue the big banks. |
| **12/1/2001** | **Many private companies began to fail.** |
| 12/18/2001 | Of the companies having special status (Tokushu-hojin, in Japanese), only 17 were abolished and 45 privatized. |
| 2/28/2002 | The Bank of Japan increased the amount of its open market operations (purchasing), from 0.8 trillion yen per month to 1 trillion yen per month. |

Note: We checked policies implemented within two months of the peaks in costs. The date of the cost peaks and the main events following within two months are denoted in bold.

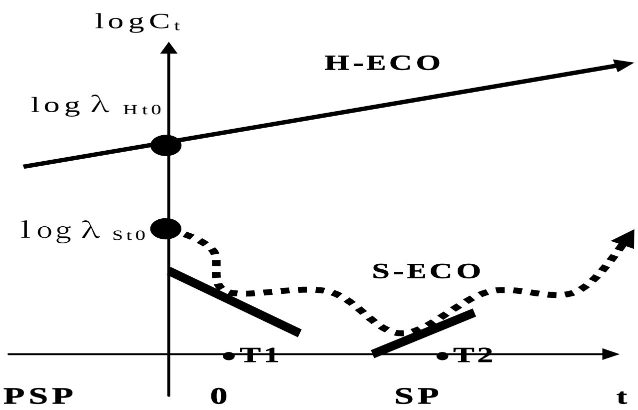
Sources: Nikkei Kinyu Nenpo 2005 (Summer), and Nippon Kezai Senbun Shya (in Japanese).

**Figure 1. Hypothetical and Stagnation Economies using the Lucas–Obstfeld Model**




Note: PSP (H-ECO) and SP (S-ECO) denote the prestagnation period (the hypothetical economy) and the stagnation period (the stagnation economy), respectively. Parts of the intercept, $-(1/2)\sigma^2_H$ and $-1/2)\sigma^2_S$, are neglected because they are negligible.

**Figure 2. Hypothetical and Stagnation Economies using the Alternative Model**




Note: See the note for Figure 1.

**Figure 3. Per Capita Total Consumption in Logarithm (Aggregate)**

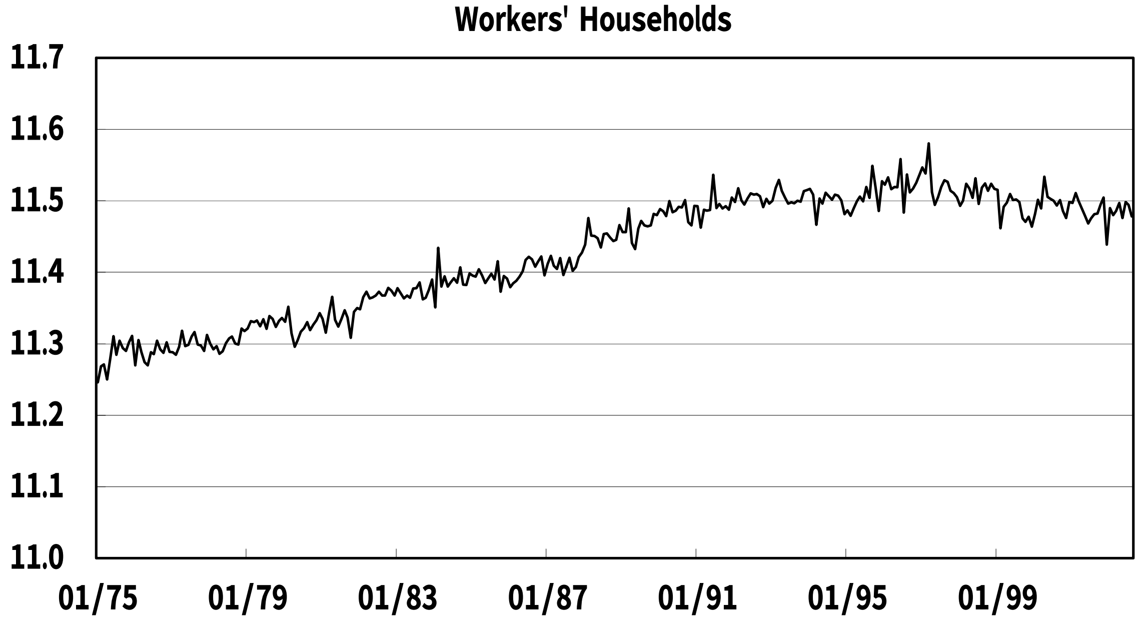
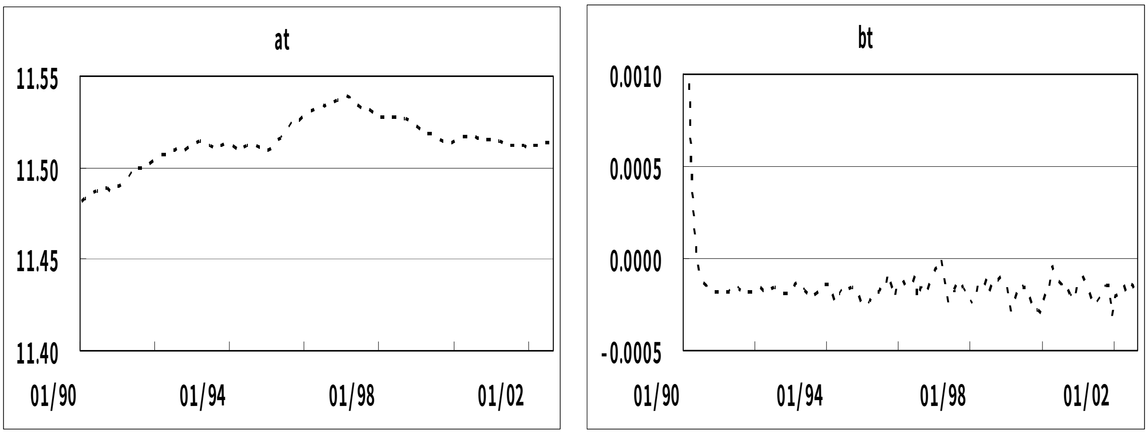



**Figure 4. Estimated $a_t$ and $b_t$ over Time: Estimation Values vs Actual Values**



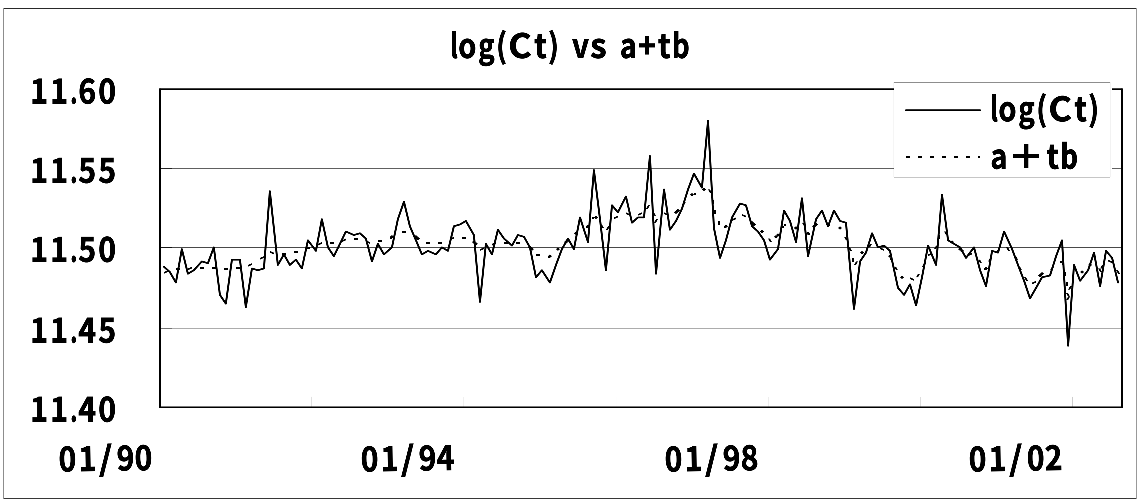
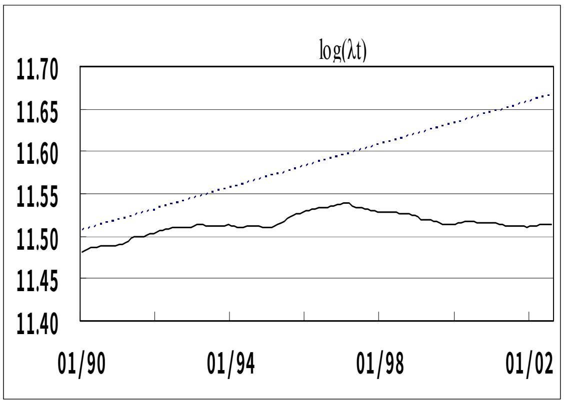
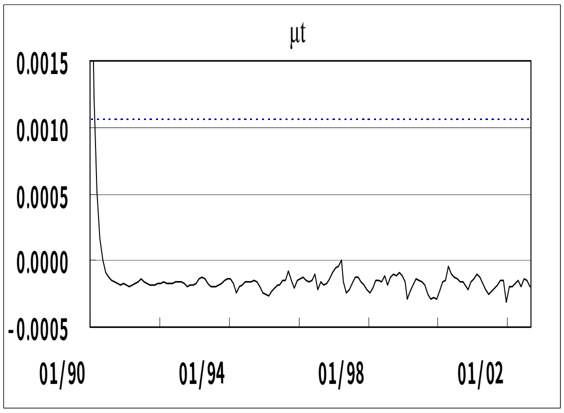



**Figure 5. Estimated $\mu_t$ and $\ln(\lambda_t)$ over time: Hypothetical vs Stagnation**




Note: The dotted and solid lines are for the hypothetical and stagnation economies, respectively.

**Figure 6. The Ratio of Cost at Each Period**