Discussion Papers In Economics And Business

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January 2008

この研究は「大学院経済学研究科・経済学部記念事業」
基金より援助を受けた、記して感謝する。

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Supply Chain Coordination Model with Retailer's Risk Attitudes*

Huy CHHAING†

Abstract

One of the major concerns in supply chain management is the coordination among various members of a supply chain comprising suppliers, manufacturers, distributors, wholesalers and retailers. We consider a newsvendor model in a two level supply chain with one supplier and one retailer. In this model, the retailer must order the item from the supplier prior to the selling season. Due to the short selling season and long replenishment lead time, the retailer is unable to reorder the item by using actual sales data generated from the early part of the season.

The purpose of this paper is to discuss the effect of the attitudes toward risk of the members on the coordination in a supply chain. Using the risk averse utility functions, we show that, the risk averse retailer's optimal order quantity is less than or equal to that of a risk neutral one, when the goodwill penalty cost is ignored. We also explore the relationship between the retailer's order quantity and the risk aversion function in a special case.

JEL Classification. C61, C63.
Key Words. Supply chain management, Newsvendor model, Risk aversion.

*The author would like to acknowledge Professor Eiji Takeda for helpful suggestions and comments during the course of this work.

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1 Introduction

One of the main issues of supply chain management is to find suitable mechanisms to coordinate the supplier-retailer relationship in order to achieve overall maximal profit. Traditional relationships between all parties in supply chains are the “trade-off relationship”, so the pricing problem must be the most sensitive factor among the relationships. Traditionally, many researches are taken from the viewpoint that a pricing problem can only benefit one party (that is, the party who is dominant in the chain). But if the two parties coordinate with each other, they may probably find out some pricing strategies so that both parties can improve their own performance (Li and Liu 2006).

Two important aspects of supply chain contracting are channel coordination and Pareto-efficiency. A supply chain is said to be coordinated when contractual terms between the members of the supply chain are set so as to ensure that the total expected profit of the supply chain is maximized. A contract is Pareto-efficient if all the members of the supply chain are no worse off (and at least one of them is strictly better off) with the contract in place than with some other default contract (Bose and Anand 2007).

This paper considers a two level supply chain in which there are one supplier and one retailer with a single perishable item which has a selling season with stochastic demand. Thus, the retailer faces the newsvendor's problem. The retailer must order the item from the supplier prior to the selling season. Due to the short selling season and long replenishment lead time, the retailer is unable to reorder the item by using actual sales data at the early part of the season.

Most of the previous literature assume that the decision maker (retailer) is risk-neutral. Recently, Keren and Pliskin (2006) study a newsvendor problem where the retailer is a risk averse decision maker, inwhich the demand distribution is a uniform distributed. This paper differs from Keren and Pliskin (2006) as follows: first, we study the problem in more general cases; second, we consider a special case where both the demand function and the retailer's utility function are exponential. We derive some interesting properties of various contracts such as, the wholesale price, the buyback and the revenue sharing contractss. The purpose of this paper is to discuss the effect of the attitudes toward risk of the retailer on the coordination in a supply chain.

A buyback contracts (also termed a returns contract) is a commitment by a supplier, service provider, or upstream distributor to accept products from excess inventory of a downstream channel member (Yao et al. 2008b). An earlier investigation of buyback contractss in distribution channels was conducted by Pasternack (1985). The practice of buyback contracts has been reported widely in both research literature and business. A supplier's returns contract is a common feature in the distribution of many products, such as books, CDs, and fashion apparel. An obvious explanation for a returns contract is risk-sharing; that is, the retailer returns the unsold products to the supplier or the supplier offers a credit on all unsold products to the retailer (Yue and Raghunathan 2007).

The revenue-sharing contract is also a way of coordinating the channel mechanisms, which has recently interested both academicians and practitioners. This contract form has become prevalent in the video-cassette rental industry, overtaking the relatively conventional wholesale price contracts (Yao et al. 2008a).

Through out this paper, we denote the subscripts ‘n’ and ‘a’ as risk-neutral and risk-averse, respectively. And the superscripts (W), (B) and (R) represent the wholesale price, buyback and revenue sharing contractss, respectively.

2 The case where the retailer is risk neutral

Consider a supply chain with two firms, a supplier and a retailer. The retailer faces the newsvendor's problem: the retailer must choose an order quantity before the start of a single selling season that has stochastic demand. Let $D > 0$ be the demand occurring in a selling season. Let F be the distribution function of demand and f its density function. F is differentiable, strictly increasing and $F(0) = 0$. Let $\bar{F}(D) = 1 - F(D)$ and $\mu = E[D]$. The supplier's production cost per unit is c. The retail price is p. For each demand the retailer does not satisfy the retailer incurs a goodwill penalty cost g. The retailer earns $\nu < c$ per unit unsold at the end of the season.

The sequence of events occurs in this game is as follows: the supplier offers the retailer a contract; the retailer accepts or rejects the contract; assuming the retailer accepts the contract, the retailer submits an order quantity, Q, to the supplier; the supplier produces and delivers the order to the retailer before the selling season begins; season demand occurs; and finally transfer payments are made between the firms based upon the agreed contract.

2.1 Wholesale price contracts

Under wholesale price contracts, the supplier charges the retailer w per unit purchased. The retailer gets a salvage value ν per unit unsold at the end of the season. For the shortage item the retailer pays a goodwill penalty g. It is reasonable to assume $\nu < w$. When the retailer is risk neutral, he/she wishes to maximize his/her profit function $\Pi_{\mathrm{r,n}}^{(\mathrm{W})}(Q)$,

$$\begin{aligned}\Pi_{\mathrm{r,n}}^{(\mathrm{W})}(Q) &= \int_0^Q [pD + \nu(Q-D) - wQ] f(D)\,dD + \int_Q^\infty [pQ - g(D-Q) - wQ] f(D)\,dD \\ &= (p+g-w)Q - g\mu - (p+g-\nu)\int_0^Q F(D)\,dD, \qquad (2.1)\end{aligned}$$

where $\mu = E(D)$. Taking the first derivative of $\Pi_{\mathrm{r,n}}^{(\mathrm{W})}(Q)$ with respect to Q yields

$$\frac{\partial \Pi_{\mathrm{r,n}}^{(\mathrm{W})}(Q)}{\partial Q} = (p+g-w) - (p+g-\nu)F(Q),$$

and the second derivative of $\Pi_{\mathrm{r,n}}^{(\mathrm{W})}(Q)$ with respect to Q yields

$$\frac{\partial^2 \Pi_{\mathrm{r,n}}^{(\mathrm{W})}(Q)}{\partial Q^2} = -(p+g-\nu)f(Q) < 0.$$

Then $\Pi_{\mathrm{r,n}}^{(\mathrm{W})}(\cdot)$ is concave in Q, so the retailer's maximum profit is attained at the order quantity $Q_{\mathrm{r,n}}^{(\mathrm{W})*}$,

$$Q_{\mathrm{r,n}}^{(\mathrm{W})*} = F^{-1}\left(\frac{p+g-w}{p+g-\nu}\right) = F^{-1}\left(1 - \frac{w-\nu}{p+g-\nu}\right). \qquad (2.2)$$

It shows that, as the goodwill penalty cost (g) increases, the retailer's optimal order quantity $Q_{\mathrm{r,n}}^{(\mathrm{W})*}$ placed by the retailer increases.

On the other hand, the supplier's expected profit function is

$$\Pi_{\mathrm{m,n}}^{(\mathrm{W})} = (w-c)Q, \qquad (2.3)$$

and the supply chain's expected profit is

$$\Pi_{\mathrm{sc,n}}^{(\mathrm{W})}(Q) = \Pi_{\mathrm{r,n}}^{(\mathrm{W})}(Q) + \Pi_{\mathrm{m,n}}^{(\mathrm{W})}(Q) = (p+g-c)Q - g\mu - (p+g-\nu)\int_0^Q F(D)\,dD, \qquad (2.4)$$

then the first order condition of $\Pi^{(\mathrm{W})}_{\mathrm{sc,n}}(Q)$ is

$$\frac{\partial \Pi^{(\mathrm{W})}_{\mathrm{sc,n}}(Q)}{\partial Q} = (p+g-c) - (p+g-\nu)F(Q) = 0,$$

which implies that

$$Q^{(\mathrm{W})*}_{\mathrm{sc,n}} = F^{-1}\left(\frac{p+g-c}{p+g-\nu}\right). \tag{2.5}$$

We can see that the wholesale price contracts does not coordinate the supply chain, since $Q^{(\mathrm{W})*}_{\mathrm{r,n}} = F^{-1}\left(\frac{p+g-w}{p+g-\nu}\right) < Q^{(\mathrm{W})*}_{\mathrm{n}} = F^{-1}\left(\frac{p+g-c}{p+g-\nu}\right)$, when $w > c$ (see, Cachon 2003).

2.2 Buyback contracts

With the buyback contracts, the supplier charges the retailer w per unit and pays b per unit unsold item. Assume that all unsold item are bought back by the supplier, and $b \geq \nu$ so the retailer resells all the unsold item to the supplier. The retailer pays a goodwill penalty g for the shortage item. The retailer expected profit, $\Pi^{(\mathrm{B})}_{\mathrm{r,n}}(Q)$, under the buyback contracts is

$$\begin{aligned}
\Pi^{(\mathrm{B})}_{\mathrm{r,n}}(Q) &= \int_0^Q [pD + b(Q-D) - wQ] f(D)\,dD + \int_Q^\infty [pQ - g(D-Q) - wQ] f(D)\,dD \\
&= (p+g-w)Q - g\mu - (p+g-b)\int_0^Q F(D)\,dD. \qquad (2.6)
\end{aligned}$$

The retailer wishes to maximize his/her expected profit. The first order condition of $\Pi^{(\mathrm{B})}_{\mathrm{r,n}}(Q)$ is

$$\frac{\partial \Pi^{(\mathrm{B})}_{\mathrm{r,n}}(Q)}{\partial Q} = (p+g-w) - (p+g-b)F(Q),$$

and the second order condition of $\Pi^{(\mathrm{B})}_{\mathrm{r,n}}(Q)$ is

$$\frac{\partial^2 \Pi^{(\mathrm{B})}_{\mathrm{r,n}}(Q)}{\partial Q^2} = -(p+g-b)f(Q) < 0.$$

Then $\Pi^{(\mathrm{B})}_{\mathrm{r,n}}(\cdot)$ is concave in Q, so the retailer's maximum profit is attained at the order quantity $Q^{(\mathrm{B})*}_{\mathrm{r,n}}$,

$$Q^{(\mathrm{B})*}_{\mathrm{r,n}} = F^{-1}\left(\frac{p+g-w}{p+g-b}\right) = F^{-1}\left(1 - \frac{w-b}{p+g-b}\right). \tag{2.7}$$

It shows that, as the goodwill penalty cost (g) increases, the retailer's optimal order quantity $Q^{(\mathrm{B})*}_{\mathrm{r,n}}$ placed by the retailer increases.

On the other hand, the supplier's expected profit function is

$$\begin{aligned}
\Pi^{(\mathrm{B})}_{\mathrm{m,n}}(Q) &= (w-c)Q - b\int_0^Q (Q-D) f(D)\,dD \\
&= (w-c)Q - b\int_0^Q F(D)\,dD, \qquad (2.8)
\end{aligned}$$

and the supply chain's expected profit is

$$\Pi^{(\mathrm{B})}_{\mathrm{sc,n}}(Q) = \Pi^{(\mathrm{B})}_{\mathrm{r,n}}(Q) + \Pi^{(\mathrm{B})}_{\mathrm{m,n}}(Q) = (p+g-c)Q - g\mu - (p+g)\int_0^Q F(D)\,dD, \tag{2.9}$$

then the first order condition of $\Pi^{(\mathrm{B})}_{\mathrm{sc,n}}(Q)$ is

$$\frac{\partial \Pi^{(\mathrm{B})}_{\mathrm{sc,n}}(Q)}{\partial Q} = (p+g-c) - (p+g)F(Q) = 0,$$

which implies that

$$Q^{(\mathrm{B})*}_{\mathrm{sc,n}} = F^{-1}\left(\frac{p+g-c}{p+g}\right). \tag{2.10}$$

2.3 Revenue sharing contracts

Though, Cachon (2003) considers a revenue sharing contracts in which the supplier charges the retailer w_{rs} per unit purchased plus the retailer gives the supplier a percentage of his/her revenue. In our model, we consider a revenue sharing contracts as follows, the supplier charges the retailer r per unit purchased. The retailer pays the supplier a $(w-r)$ per unit actually sold in addition to r, where $r \le w$, and gets a salvage value (ν) per unit unsold at the end of the season. Assume that $r > \nu$. The retailer pays a goodwill penalty (g) for the shortage item. The retailer wishes to maximize the following profit function $\Pi^{(\mathrm{R})}_{\mathrm{r,n}}(Q)$

$$\begin{aligned}\Pi^{(\mathrm{R})}_{\mathrm{r,n}}(Q) &= \int_0^Q [pD + \nu(Q-D) - rQ - (w-r)D] f(D)\, dD \\ &\quad + \int_Q^\infty [pQ - g(D-Q) - rQ - (w-r)Q] f(D)\, dD \\ &= (p+g-w)Q - g\mu - (p+g-w+r-\nu)\int_0^Q F(D)\, dD. \end{aligned} \tag{2.11}$$

Taking the first derivative of $\Pi^{(\mathrm{R})}_{\mathrm{r,n}}(Q)$ with respect to Q yields

$$\frac{\partial \Pi^{(\mathrm{R})}_{\mathrm{r,n}}(Q)}{\partial Q} = (p+g-w) - (p+g-w+r-\nu)F(Q),$$

and the second derivative of $\Pi^{(\mathrm{R})}_{\mathrm{r,n}}(Q)$ with respect to Q yields

$$\frac{\partial^2 \Pi^{(\mathrm{R})}_{\mathrm{r,n}}(Q)}{\partial Q^2} = -(p+g-w+r-\nu)f(Q) < 0. \qquad \text{since } r > \nu \text{ by assumption.}$$

Then $\Pi^{(\mathrm{R})}_{\mathrm{r,n}}(\cdot)$ is concave in Q, so the retailer's maximum profit is attained at the order quantity $Q^{(\mathrm{R})*}_{\mathrm{r,n}}$,

$$Q^{(\mathrm{R})*}_{\mathrm{r,n}} = F^{-1}\left(\frac{p+g-w}{p+g-w+r-\nu}\right) = F^{-1}\left(1 - \frac{r-\nu}{p+g-w+r-\nu}\right). \tag{2.12}$$

It shows that, as the goodwill penalty cost, g, increases, the retailer's optimal order quantity $Q^{(\mathrm{R})*}_{\mathrm{r,n}}$ placed by the retailer increases.

On the other hand, the supplier's expected profit function is

$$\begin{aligned}\Pi^{(\mathrm{R})}_{\mathrm{m,n}}(Q) &= (r-c)Q + \int_0^Q (w-r)Df(D)\, dD + \int_Q^\infty (w-r)Qf(D)\, dD \\ &= (w-c)Q - (w-r)\int_0^Q F(D)\, dD, \end{aligned} \tag{2.13}$$

and the supply chain's expected profit is

$$\Pi^{(\mathrm{R})}_{\mathrm{sc,n}}(Q) = \Pi^{(\mathrm{R})}_{\mathrm{r,n}}(Q) + \Pi^{(\mathrm{R})}_{\mathrm{m,n}}(Q) = (p+g-c)Q - g\mu - (p+g-\nu)\int_0^Q F(D)\, dD, \tag{2.14}$$

then the first order condition of $\Pi_{\mathrm{sc,n}}^{(\mathrm{R})}(Q)$ is

$$\frac{\partial \Pi_{\mathrm{sc,n}}^{(\mathrm{R})}(Q)}{\partial Q} = (p+g-c) - (p+g-\nu)F(Q) = 0,$$

which implies that

$$Q_{\mathrm{sc,n}}^{(\mathrm{R})*} = F^{-1}\left(\frac{p+g-c}{p+g-\nu}\right). \tag{2.15}$$

If $r = w$ the revenue sharing contracts model becomes a wholesale price contracts model.

3 The case where the retailer is risk averse

Let $u(x)$ be the utility function which represents the retailer's preferences. Assume that $u(x)$ is strictly increasing and twice continuously differentiable at all x, then

$$u'(x) > 0 \qquad \text{for all } x.$$

As is well-known, the decision maker is risk averse if and only if her utility function is concave. In what follows, assume that the retailer is risk averse. Thus

$$u''(x) \leq 0 \qquad \text{for all } x.$$

According to Pratt (1964), a measure of risk aversion to indicate the extent that the decision maker wants to averse the risk is

$$r(x) = -\frac{u''(x)}{u'(x)}.$$

3.1 Wholesale price contracts

If the retailer is risk averse, then his/her expected utility is

$$Eu_{\mathrm{r,a}}^{(\mathrm{W})}(Q) = \int_0^Q u[pD+\nu(Q-D)-wQ]f(D)\,dD + \int_Q^\infty u[pQ-g(D-Q)-wQ]f(D)\,dD. \tag{3.1}$$

Lemma 1. *$Eu_{r,a}^{(W)}(Q)$ is strictly concave in Q.*

Proof. We have

$$\begin{aligned} Eu_{\mathrm{r,a}}^{(\mathrm{W})}(Q) &= \int_0^Q u\left[pD+\nu(Q-D)-wQ\right] f(D)\,dD + \int_Q^\infty u\left[pQ-g(D-Q)-wQ\right] f(D)\,dD \\ &= \int_0^Q u\left[(p-\nu)D-(w-\nu)Q\right] f(D)\,dD + \int_Q^\infty u\left[(p+g-w)Q-gD\right] f(D)\,dD. \end{aligned}$$

By using Leibniz Integral Rule, the first order condition is

$$\begin{aligned} \frac{\partial Eu_{\mathrm{r,a}}^{(\mathrm{W})}(Q)}{\partial Q} &= -(w-\nu)\int_0^Q u'[(p-\nu)D-(w-\nu)Q]f(D)\,dD \\ &\quad + (p+g-w)\int_Q^\infty u'[(p+g-w)Q-gD]f(D)\,dD, \end{aligned} \tag{3.2}$$

and the second order condition is,

$$\frac{\partial^2 Eu_{\mathrm{r,a}}^{(\mathrm{W})}(Q)}{\partial Q^2} = (w-\nu)^2 \int_0^Q u''[(p-\nu)D-(w-\nu)Q]f(D)\,dD - (p+g-\nu)u'[(p-w)Q]f(Q) + (p+g-w)^2 \int_Q^\infty u''[(p+g-w)Q-gD]f(D)\,dD. \tag{3.3}$$

Since $u'(\cdot) > 0$ and $u''(\cdot) < 0$, it follows that $\frac{\partial^2 Eu_{\mathrm{r,a}}^{(\mathrm{W})}(Q)}{\partial Q^2} < 0$, for $Q > 0$. □

From Lemma 1, there exits a unique order quantity $Q_{\mathrm{r,a}}^{(\mathrm{W})*}$ such that it maximizes $Eu_{\mathrm{r,a}}^{(\mathrm{W})}(Q)$.

Theorem 1. *When $g = 0$, the optimal order quantity for a risk averse retailer is less than or equal to that of a risk neutral one; that is, $Q_{r,a}^{(W)*} \leq Q_{r,n}^{(W)*}$.*

Proof. We have

$$\frac{\partial Eu_{\mathrm{r,a}}^{(\mathrm{W})}(Q)}{\partial Q} = -(w-\nu)\int_0^Q u'\left[(p-\nu)D-(w-\nu)Q\right]f(D)\,dD + (p-w)\int_Q^\infty u'[(p-w)Q]f(D)\,dD,$$

then,

$$\frac{\partial Eu_{\mathrm{r,a}}^{(\mathrm{W})}\left(Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right)}{\partial Q_{\mathrm{r,n}}^{(\mathrm{W})*}} = -(w-\nu)\int_0^{Q_{\mathrm{r,n}}^{(\mathrm{W})*}} u'\left[(p-\nu)D-(w-\nu)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right]f(D)\,dD + (p-w)\int_{Q_{\mathrm{r,n}}^{(\mathrm{W})*}}^\infty u'\left[(p-w)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right]f(D)\,dD.$$

Noting that,

$$\left[(p-\nu)D-(w-\nu)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right] \leq \left[(p-\nu)Q_{\mathrm{r,n}}^{(\mathrm{W})*}-(w-\nu)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right], \qquad \text{for } D \leq Q_{\mathrm{r,n}}^{(\mathrm{W})*},$$

we have,

$$\begin{aligned} u'\left[(p-\nu)D-(w-\nu)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right] &\geq u'\left[(p-\nu)Q_{\mathrm{r,n}}^{(\mathrm{W})*}-(w-\nu)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right] \\ &= u'\left[(p-w)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right], \end{aligned}$$

because $u'(\cdot)$ is a decreasing function. Thus,

$$\begin{aligned} &\frac{\partial Eu_{\mathrm{r,a}}^{(\mathrm{W})}\left(Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right)}{\partial Q_{\mathrm{r,n}}^{(\mathrm{W})*}} \\ &\leq -(w-\nu)\int_0^{Q_{\mathrm{r,n}}^{(\mathrm{W})*}} u'\left[(p-w)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right]f(D)\,dD + (p-w)\int_{Q_{\mathrm{r,n}}^{(\mathrm{W})*}}^\infty u'\left[(p-w)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right]f(D)\,dD \\ &= -(w-\nu)u'\left[(p-w)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right]F\left(Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right) + (p-w)u'\left[(p-w)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right]\left(1-F\left(Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right)\right) \\ &= -(p-\nu)u'\left[(p-w)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right]F\left(Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right) + (p-w)u'\left[(p-w)Q_{\mathrm{r,n}}^{(\mathrm{W})*}\right], \end{aligned} \tag{3.4}$$

by substituting Eq. (2.2) into Eq. (3.4) yields

$$\begin{aligned}
&\frac{\partial Eu_{\mathrm{r,a}}^{(\mathrm{W})}\left(Q_{\mathrm{r,n}}^{(\mathrm{W})^*}\right)}{\partial Q_{\mathrm{r,n}}^{(\mathrm{W})^*}} \\
&\leq -(p-\nu)u'\left[(p-w)Q_{\mathrm{r,n}}^{(\mathrm{W})^*}\right]\left(\frac{p-w}{p-\nu}\right)+(p-w)u'\left[(p-w)Q_{\mathrm{r,n}}^{(\mathrm{W})^*}\right]=0.
\end{aligned} \tag{3.5}$$

Since $u'(\cdot)$ is a decreasing function and $0 = \frac{\partial Eu_{\mathrm{r,a}}^{(\mathrm{W})}\left(Q_{\mathrm{r,a}}^{(\mathrm{W})^*}\right)}{\partial Q_{\mathrm{r,a}}^{(\mathrm{W})^*}} \geq \frac{\partial Eu_{\mathrm{r,a}}^{(\mathrm{W})}\left(Q_{\mathrm{r,n}}^{(\mathrm{W})^*}\right)}{\partial Q_{\mathrm{r,n}}^{(\mathrm{W})^*}}$, we get $Q_{\mathrm{r,a}}^{(\mathrm{W})^*} \leq Q_{\mathrm{r,n}}^{(\mathrm{W})^*}$. □

3.2 Buyback contracts

Suppose all unsold items are bought back by the supplier. If the retailer is risk averse, then his/her expected utility is

$$Eu_{\mathrm{r,a}}^{(\mathrm{B})}(Q) = \int_0^Q u[pD + b(Q-D) - wQ]f(D)\,dD + \int_Q^\infty u[pQ - g(D-Q) - wQ]f(D)\,dD. \tag{3.6}$$

Lemma 2. . *$Eu_{r,a}^{(B)}(Q)$ is strictly concave in Q.*

Proof. Similar to the proof of Lemma 1. □

From Lemma 2, there exits a unique order quantity $Q_{\mathrm{r,a}}^{(\mathrm{B})^*}$ such that it maximizes $Eu_{\mathrm{r,a}}^{(\mathrm{B})}(Q)$.

Theorem 2.

1. *The retailer's order quantity is less than or equal to that of supply chain; that is, $Q_{r,n}^{(B)^*} \leq Q_{sc,n}^{(B)^*}$ for $\nu \leq b \leq \frac{(p+g)(w-c)}{p+g-c}$, and $Q_{r,n}^{(B)^*} = Q_{sc,n}^{(B)^*}$ if and only if $b = \frac{(p+g)(w-c)}{p+g-c}$.*
2. *When $g = 0$, the optimal order quantity for a risk averse retailer is less than or equal to that of a risk neutral one; that is, $Q_{r,a}^{(B)^*} \leq Q_{r,n}^{(B)^*}$.*

Proof.

(1): Since $F(\cdot)$ is increasing in Q, from Eqs. (2.7) and (2.10), it follows that $Q_{\mathrm{r,n}}^{(\mathrm{B})^*} = Q_{\mathrm{sc}}^{(\mathrm{B})^*}$ when $\frac{p+g-w}{p+g-\nu} = \frac{p+g-c}{p+g}$, which implies that $b = \frac{(p+g)(w-c)}{p+g-c}$. Noting from Eq. (2.7) that F is increasing in b, if $b < \frac{(p+g)(w-c)}{p+g-c}$ then $Q_{\mathrm{r,n}}^{(\mathrm{B})^*} < Q_{\mathrm{sc}}^{(\mathrm{B})^*}$.

(2): Similar to the proof of Theorem 1. □

3.3 Revenue sharing contracts

If the retailer is risk averse, then his/her expected utility is

$$Eu_{\mathrm{r,a}}^{(\mathrm{R})}(Q) = \int_0^Q u[(p-w+r-\nu)D - (r-\nu)Q]f(D)\,dD + \int_Q^\infty u[(p+g-w)Q - gD]f(D)\,dD \tag{3.7}$$

Lemma 3. *$Eu_{r,a}^{(R)}(Q)$ is strictly concave in Q.*

Proof. Similar to the proof of Lemma 1. □

From Lemma 3, there exits a unique order quantity $Q_{\mathrm{r,a}}^{(\mathrm{R})^*}$ such that it maximizes $Eu_{\mathrm{r,a}}^{(\mathrm{R})}(Q)$.

Theorem 3.

1. *The retailer's order quantity is less than or equal to that of supply chain; that is,* $Q_{r,n}^{(R)^*} \leq Q_{sc,n}^{(R)^*}$ *for* $r \geq \nu + \frac{(p+g-w)(c-\nu)}{p+g-c}$, *and* $Q_{r,n}^{(R)^*} = Q_{sc,n}^{(R)^*}$ *if and only if* $r = \nu + \frac{(p+g-w)(c-\nu)}{p+g-c}$.
2. *When* $g = 0$, *the optimal order quantity for a risk averse retailer is less than or equal to that of a risk neutral one; that is,* $Q_{r,a}^{(R)^*} \leq Q_{r,n}^{(R)^*}$.

Proof.

(1): Since $F(\cdot)$ is increasing in Q, from Eqs. (2.12) and (2.15), it follows that $Q_{\mathrm{r,n}}^{(\mathrm{R})^*} = Q_{\mathrm{sc}}^{(\mathrm{R})^*}$ when $\frac{p+g-w}{p+g-w+r-\nu} = \frac{p+g-c}{p+g-\nu}$. Thus, $r = \nu + \frac{(p+g-w)(c-\nu)}{p+g-c}$. Noting from Eq. (2.12) that F is decreasing in r, if $\nu + \frac{(p+g-w)(c-\nu)}{p+g-c} < r < w$ then $Q_{\mathrm{r,n}}^{(\mathrm{R})^*} < Q_{\mathrm{sc}}^{(\mathrm{R})^*}$.

(2): Similar to the proof of Theorem 1. □

4 A Special Case

In this section, we assume that the retailer is risk averse and his/her utility function is $u(x) = -a\,e^{-kx}$, where $a,\ k$ are positive numbers. Then, the risk aversion function is: $r(x) = k$.

4.1 Wholesale price contracts

The retailer's expected utility is

$$\begin{aligned} Eu_{\mathrm{r,a}}^{(\mathrm{W})}(Q) &= \int_0^Q u\left[(p-\nu)D - (w-\nu)Q\right] f(D)\,dD + \int_Q^\infty u\left[(p+g-w)Q - gD\right] f(D)\,dD \\ &= -a\,e^{k(w-\nu)Q} \int_0^Q e^{-k(p-\nu)D} f(D)\,dD - a\,e^{-k(p+g-w)Q} \int_Q^\infty e^{kgD} f(D)\,dD. \end{aligned} \tag{4.1}$$

The first order condition is

$$\begin{aligned} \frac{\partial Eu_{\mathrm{r,a}}^{(\mathrm{W})}(Q)}{\partial Q} &= -ak(w-\nu)\,e^{k(w-\nu)Q} \int_0^Q e^{-k(p-\nu)D} f(D)\,dD \\ &\quad + ak(p+g-w)\,e^{-k(p+g-w)Q} \int_Q^\infty e^{kgD} f(D)\,dD \end{aligned} \tag{4.2}$$

From (4.2), the optimal quantity $Q_{\mathrm{r,a}}^{(\mathrm{W})^*}$ satisfies the following equation:

$$(w-\nu)e^{k(w-\nu)Q} \int_0^{Q_{\mathrm{r,a}}^{(\mathrm{W})^*}} e^{-k(p-\nu)D} f(D)\,dD = (p+g-w)e^{p+g-w} \int_{Q_{\mathrm{r,a}}^{(\mathrm{W})^*}}^\infty e^{kgD} f(D)\,dD. \tag{4.3}$$

Eq. (4.3) gives:

$$e^{k(p+g-\nu)Q} \frac{\int_0^{Q_{\mathrm{r,a}}^{(\mathrm{W})^*}} e^{-k(p-\nu)D} f(D)\,dD}{\int_{Q_{\mathrm{r,a}}^{(\mathrm{W})^*}}^\infty e^{kgD} f(D)\,dD} = \frac{p+g-w}{w-\nu}. \tag{4.4}$$

We assume a situation where the random demand for the item is an exponential distributed function, $f(D) = \lambda e^{-\lambda D}$. By substituting this into the left hand side of Eq. (4.4), we get

$$Q_{\mathrm{r,a}}^{(\mathrm{W})^*} = \frac{1}{kp - k\nu + \lambda} \ln\left(\frac{(p+g-\nu)(kp-kw+\lambda)}{(\lambda-kg)(w-\nu)}\right), \text{ for } kg - \lambda < 0.$$

Theorem 4. *When $g = 0$, the more risk-averse the retailer is, the less the order quantity is. That is, the order quantity $Q^{(W)*}_{r,a}$ is decreasing in k.*

Proof. We have

$$Q^{(\mathrm{W})*}_{\mathrm{r,a}} = \frac{1}{kp - k\nu + \lambda}\left[\ln\left(\frac{p-\nu}{\lambda(w-\nu)}\right) + \ln(kp - kw + \lambda)\right],$$

then the first order conditon with respect to k is

$$\begin{aligned}\frac{\partial Q^{(\mathrm{W})*}_{\mathrm{r,a}}}{\partial k} &= \frac{1}{(kp - k\nu + \lambda)^2}\left[\frac{(p-w)(kp-k\nu+\lambda)}{kp-kw+\lambda}\right.\\ &\quad \left. -(p-\nu)\left(\ln\left(\frac{p-\nu}{\lambda(w-\nu)}\right) + \ln(kp-kw+\lambda)\right)\right].\end{aligned}$$

Let

$$\begin{aligned} g(k) &= \frac{(p-w)(kp-k\nu+\lambda)}{kp-kw+\lambda} - (p-\nu)\left(\ln\left(\frac{p-\nu}{\lambda(w-\nu)}\right) + \ln(kp-kw+\lambda)\right)\\ g'(k) &= \frac{(p-w)(p-\nu)(kp-kw+\lambda) - (p-w)^2(kp-k\nu+\lambda)}{(kp-kw+\lambda)^2} - \frac{(p-\nu)(p-w)}{kp-kw+\lambda}\\ &= \frac{-(p-w)^2(kp-k\nu+\lambda)}{(kp-kw+\lambda)^2} < 0,\end{aligned}$$

then $g(k)$ is decreasing in $k > 0$.

$$g(0) = (p-w) - (p-\nu)\ln\left(\frac{p-\nu}{w-\nu}\right).$$

Let

$$h(x) = (x-w) - (x-\nu)\ln\left(\frac{x-\nu}{w-\nu}\right), \qquad x \geq w,$$

then,

$$\begin{aligned} h'(x) &= 1 - \ln\left(\frac{x-\nu}{w-\nu}\right) - \left(\frac{x-\nu}{w-\nu}\right)\left(\frac{w-\nu}{x-\nu}\right)\\ &= \ln\left(\frac{w-\nu}{x-\nu}\right) \leq 0, \qquad \text{for } x \geq w.\end{aligned}$$

Thus, $h(x)$ is decreasing for $x \geq w$.

We have $h(w) = 0$, since $h(x)$ is decreasing for $x \geq w$, then $h(x) < 0$. For any $p > w$ we have $g(0) = h(p) < 0$ and $g(k)$ is decreasing in $k > 0$ then $g(k) < 0$. It implies that for $k > 0$ $Q^{*'}_{\mathrm{r,a}}(k) < 0$. □

An illustration is given in Figure 1. In this example we set the demand function $f(D) = \lambda e^{-\lambda D}$, then $Q^{(\mathrm{W})*}_{\mathrm{r,n}} = \frac{1}{\lambda}\ln\left(\frac{p+g-\nu}{w-\nu}\right)$ and $Q^{(\mathrm{W})*}_{\mathrm{sc,n}} = \frac{1}{\lambda}\ln\left(\frac{p+g-\nu}{c-\nu}\right)$. The utility function $u(x) = -e^{kx}$. The other parameters are $p = 1$, $w = 0.8$, $c = 0.6$, $\nu = 0.3$, $g = 0$. Let $\lambda = 0.01$, and k vary from 0.000 to 0.040. Figure 1 shows that the more risk averse retailer should set his/her $Q_{\mathrm{r,a}}$ to a lower value than the less risk averse one, i.e., the order quantity $Q_{\mathrm{r,a}}$ is decreasing in k.

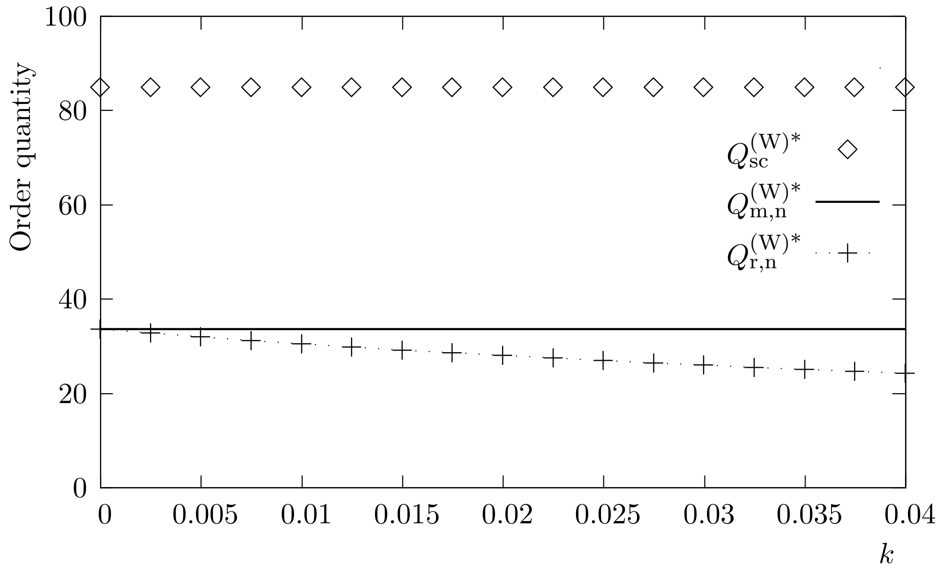

Figure 1: The order quantity (Q) versus risk aversion function (k)

4.2 Buyback contracts

The retailer's expected utility is

$$Eu^{(B)}_{r,a}(Q) = \int_0^Q u\left[(p-b)D-(w-b)Q\right] f(D)\,dD + \int_Q^\infty u\left[(p+g-w)Q-gD\right] f(D)\,dD$$
$$= -a\,e^{k(w-b)Q}\int_0^Q e^{-k(p-b)D} f(D)\,dD - a\,e^{-k(p+g-w)Q}\int_Q^\infty e^{kgD} f(D)\,dD. \qquad (4.5)$$

The first order condition is

$$\frac{\partial Eu^{(B)}_{r,a}(Q)}{\partial Q} = -ak(w-b)\,e^{k(w-b)Q}\int_0^Q e^{-k(p-b)D} f(D)\,dD$$
$$+\,ak(p+g-w)\,e^{-k(p+g-w)Q}\int_Q^\infty e^{kgD} f(D)\,dD \qquad (4.6)$$

From (4.6), the optimal quantity $Q^{(W)*}_{r,a}$ satisfies the following equation:

$$(w-b)e^{k(w-b)Q}\int_0^{Q^{(W)*}_{r,a}} e^{-k(p-b)D} f(D)\,dD = (p+g-w)e^{p+g-w}\int_{Q^{(W)*}_{r,a}}^\infty e^{kgD} f(D)\,dD. \qquad (4.7)$$

Eq. (4.7) gives

$$e^{k(p+g-b)Q}\frac{\int_0^{Q^{(W)*}_{r,a}} e^{-k(p-b)D} f(D)\,dD}{\int_{Q^{(W)*}_{r,a}}^\infty e^{kgD} f(D)\,dD} = \frac{p+g-w}{w-b}. \qquad (4.8)$$

We assume a situation where the random demand for the item is an exponential distributed function, $f(D)=\lambda e^{-\lambda D}$. By substituting this into the left hand side of Eq. (4.8), we get

$$Q^{(B)*}_{r,a} = \frac{1}{kp-kb+\lambda}\ln\left(\frac{(p+g-b)(kp-kw+\lambda)}{(\lambda-kg)(w-b)}\right), \text{ for } kg-\lambda<0.$$

Figure 2 shows the relationship between the buyback value (b) and risk aversion function (k). The demand function is $f(D)=\lambda e^{-\lambda D}$, then $Q^{(B)*}_{r,n} = \frac{1}{\lambda}\ln\left(\frac{p+g-b}{w-b}\right)$ and $Q^{(B)*}_{sc,n} = \frac{1}{\lambda}\ln\left(\frac{p+g}{c}\right)$. We

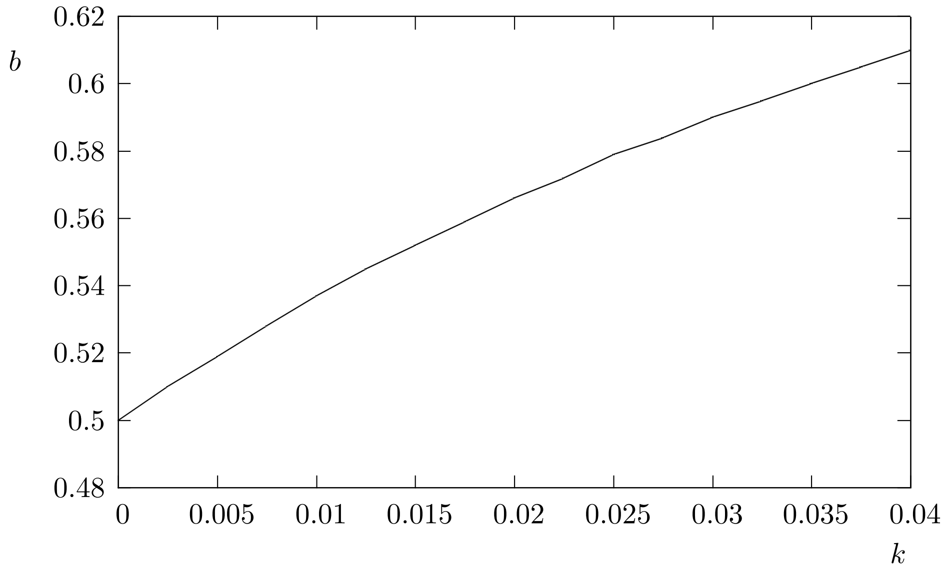

Figure 2: The buyback value (b) versus risk aversion function (k)

use the same parameters as in Figure 1. Figure 2 shows that the buyback value (b) is increasing function in risk aversion function (k).

Theorem 5. *When $g = 0$, the more risk-averse the retailer is, the less the order quantity is. That is, the order quantity $Q_{r,a}^{(B)*}$ is decreasing in k.*

Proof. Similar to the proof of Theorem 4. □

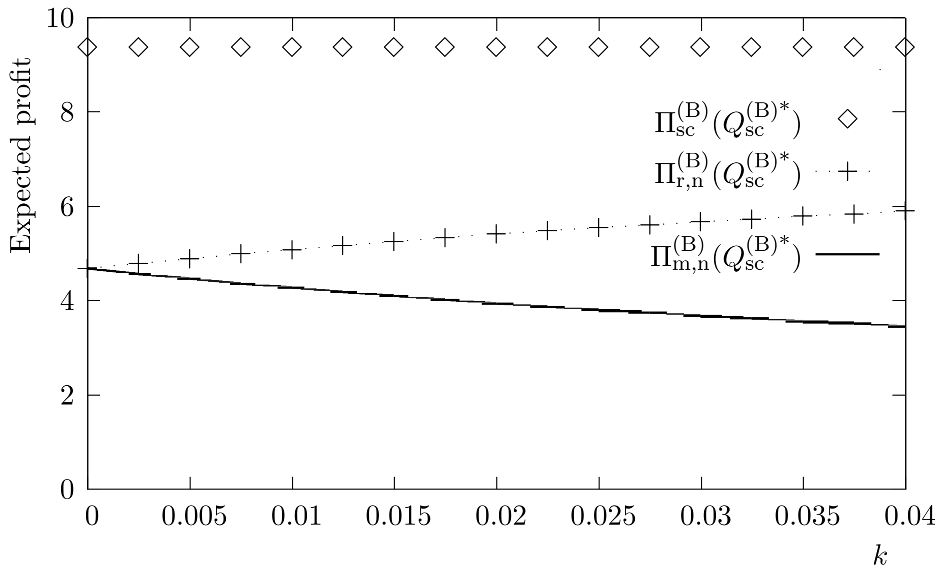

Figure 3: The expected profit ($\Pi^{(B)}$) versus risk aversion function (k)

We now use the same parameters presented in Figure 2. Figure 3 shows the relationship between each expected profit in the buyback contracts and the risk aversion function (k). The graph shows that as the risk aversion function (k) increases, the retailer's expected profit increases but the supplier's expected profit is going down.

4.3 Revenue sharing contractss

The retailer's expected utility is

$$
\begin{aligned}
Eu^{(\mathrm{R})}_{\mathrm{r,a}}(Q) &= \int_0^Q u\left[(p-w+r-\nu)D-(r-\nu)Q\right] f(D)\,dD + \int_Q^\infty u\left[(p+g-w)Q-gD\right] f(D)\,dD \\
&= -a\,e^{k(r-\nu)Q}\int_0^Q e^{-k(p-w+r-\nu)D} f(D)\,dD - a\,e^{-k(p+g-w)Q}\int_Q^\infty e^{kgD} f(D)\,dD. \quad (4.9)
\end{aligned}
$$

The first order condition is

$$
\begin{aligned}
\frac{\partial Eu^{(\mathrm{R})}_{\mathrm{r,a}}(Q)}{\partial Q} &= -ak(r-\nu)\,e^{k(r-\nu)Q}\int_0^Q e^{-k(p-w+r-\nu)D} f(D)\,dD \\
&\quad + ak(p+g-w)\,e^{-k(p+g-w)Q}\int_Q^\infty e^{kgD} f(D)\,dD. \quad (4.10)
\end{aligned}
$$

From (4.10), the optimal quantity $Q^{(\mathrm{R})*}_{\mathrm{r,a}}$ satisfies the following equation:

$$
e^{k(p+g-w+r-\nu)Q} \times \frac{\int_0^Q e^{-k(p-w+r-\nu)D} f(D)\,dD}{\int_Q^\infty e^{kgD} f(D)\,dD} = \frac{p+g-w}{r-\nu}. \quad (4.11)
$$

We assume a situation where the random demand for the item is an exponential distributed function, $f(D) = \lambda e^{-\lambda D}$. By substituting this into the left hand side of Eq. (4.11), we get

$$
Q^{(\mathrm{R})*}_{\mathrm{r,a}} = \frac{1}{k(p-w+r-\nu)+\lambda} \ln\left(\frac{(p+g-w+r-\nu)(kp-kw+\lambda)}{(r-\nu)(\lambda-kg)}\right), \text{ for } kg-\lambda<0.
$$

When demand function $f(D) = \lambda e^{-\lambda D}$, then $Q^{(\mathrm{R})*}_{\mathrm{r,n}}(Q) = \frac{1}{\lambda}\ln\left(\frac{p+g-w+r-\nu}{r-\nu}\right)$ and $Q^{(\mathrm{R})*}_{\mathrm{sc,n}}(Q) = \frac{1}{\lambda}\ln\left(\frac{p+g-\nu}{c-\nu}\right)$. We use the same parameters as in Figure 1. Figure 4 illustrates the relationship between the unit transfer price (r) and the risk aversion function (k). Figure 4 shows that as k increases the unit transfer price decreases.

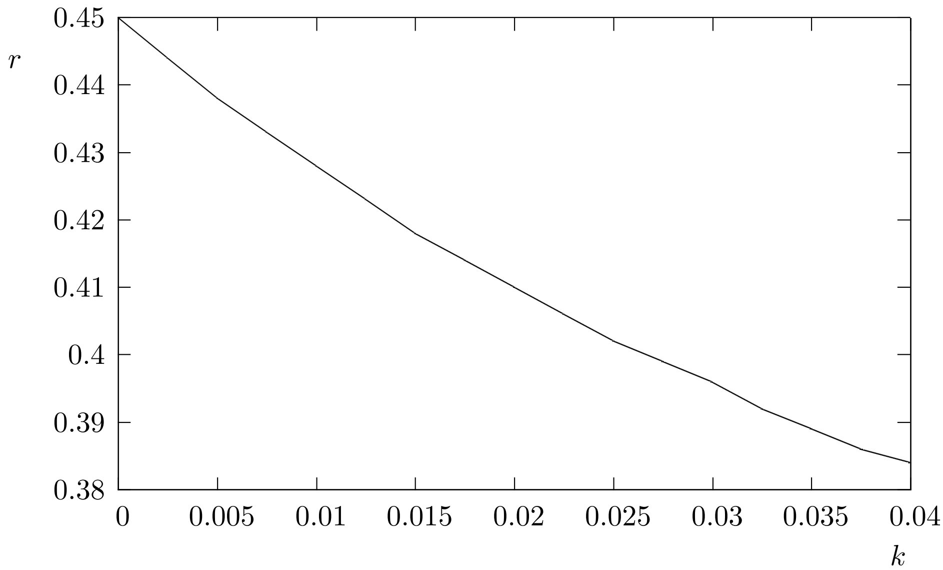

Figure 4: The wholesale unit transfer price (r) versus risk aversion function (k)

Theorem 6. *When $g = 0$, the more risk-averse the retailer is, the less the order quantity is. That is, the order quantity $Q_{r,a}^{(R)^*}$ is decreasing in k.*

Proof. Similar to the proof of Theorem 4. $\square$

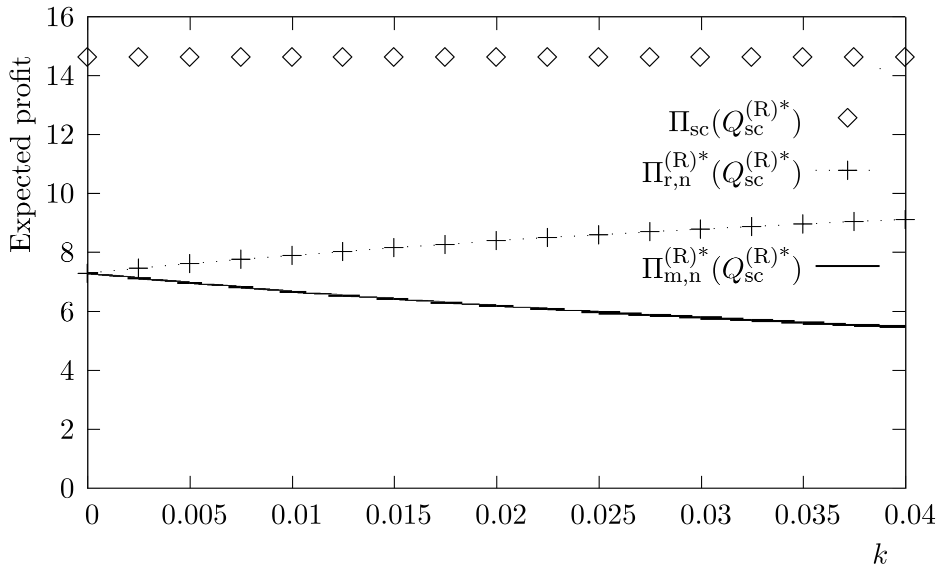

Figure 5: The expected profit ($\Pi^{(R)}$) versus risk aversion function (k)

We now use the same parameters presented in Figure 1. Figure 5 shows the relationship between each expected profit in the revenue sharing contractss and the risk aversion function (k). The graph shows that as the risk aversion function (k) increases, the retailer's expected profit increases, while the supplier's expected profit decreases.

5 Conclusions

We have considered a standard newsvendor problem consisting of a single supplier and single retailer. In this paper we derived some fundamental properties of the order quantity of risk averse retailer. In particular, we can conclude that when the goodwill penalty cost is ignored, the optimal order quantity of a risk averse retailer, $Q^*_{r,a}$, is less than or equal to that of a risk neutral one, $Q^*_{r,n}$, then the supply chain performance in the risk averse is worse off than that in the risk neutral case. Furthermore, the more risk averse the retailer is, the less the performance of the supply chain is.

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