



# **Discussion Papers In Economics And Business**

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in a Frictional Labor Market

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Discussion Paper 08-09

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この研究は「大学院経済学研究科・経済学部記念事業」  
基金より援助を受けた、記して感謝する。

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# Alteration in Skills and Career-Enhancing in a Frictional Labor Market\*

Yosuke Oda <sup>†</sup>

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## Abstract

This article constructs a job-search model in which worker's ability varies over time; a high-ability unemployed might lose her skills due to prolonged unemployment whereas a low-ability employed might acquire her skills due to (an implicit) on-the-job training. We numerically show that both pecuniary reward for short-term unemployed and reduction in unemployment benefits leads to lower unemployment rate, however, the former policy does stimulate career-enhancing of long-term unemployed whereas the latter does not. In addition, numerical analysis suggests that mixture of the two policy can lead to higher aggregate welfare than under a sole policy.

**Keywords:** job-search model; cyclical change in skills; career-enhancing separation

**JEL classification:** J64; J68

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\*I am grateful to Masaru Sasaki, Jun-ichiro Ishida, Noritaka Kudoh and seminar participants at Kansai university (Search Theory Workshop) and at Osaka university, and JEA Annual Meeting Fall 2006 held at Osaka City University for their helpful comments and discussions. I am particularly indebted to Ryo Horii for his helpful suggestions and comments.

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# 1 Introduction

This paper constructs a general equilibrium job-search model in which individual's their skills vary over time; high-skilled jobless workers lose skills due to prolonged unemployment whereas low-skilled employed workers acquire their skills due to (an implicit) on-the-job training. The aim of this article is, by making use of the model, to consider a policy that would overcome the serious issues below.

OECD (2002) documents that on average 30% of unemployed workers are long-term unemployed who has been unemployed for one year or more in 2000, and in ten countries of them, the proportions are over 40%. These proportions are historical high, and OECD (2002) concludes that "the long-term unemployed appear to be relatively more likely to go on to become very-long-term unemployed in some countries, and more likely to leave the labor force in others". As frequently discussed, the prolonged unemployment duration makes workers discouraged and/or less skilled, which makes again reemployment of the workers more difficult. Notably, such a negative spiral aggravate social welfare, in the sense of both that high unemployment rate worsens social welfare and that prolonged unemployment deteriorates welfare for long-term jobless workers. So it is worthwhile to consider a policy that overcomes the serious issues.

Heterogeneity in skills among individuals is treated in some earlier studies. For instance, Pissarides (1992) analyzes a worker's loss of skills in an overlapping generations framework. In his model, an old who was employed when young has higher productivity than an old who was unemployed when young. Thus individual's skill is changed for once in her life and the skill is not accumulated at all. As another example, Albrecht and Vroman (2002) studies a job-search model in which distribution of worker's skill is two-point (high or low). Since their paper focuses on firm's behavior, skill of each individual is assumed to be constant over time.

In contrast to these studies, this article assumes that individual's skill level varies over time. Following Albrecht and Vroman (2002), we assume that distribution of skills is of two-point (either high or low). Under the assumption, high-skilled unemployed loses her skills if she cannot find a job within certain duration while a low-skilled unemployed acquires her skills if she works at a job for certain duration. The rationale behind the assumption is that the former is due to prolonged unemployment while the latter is due to (an implicit) on-the-job training. Given the circumstance, we consider effects of two labor policies which are stated below on unemployment rate and on social welfare.

The policies considered here are quite simple. One is reduction in unemployment benefit and

the other is reemployment bonus. The former is straightforward. High unemployment benefits lead to high value of being unemployed, which results in high unemployment rate. Thus cutback in the benefits would decrease unemployment rate. The latter is originally planned as an economic experiment (for summary, see Meyer, 1995), which is a reward for workers who are hired within certain duration after fired. It directly increases worker's incentive to be employed, which results in lower unemployment rate. In summary, the former is a *stick* while the latter is a *carrot* as employment-boosting policy.

Theoretically, it is obvious that the two policies have similar effects on unemployment rate but not on social welfare, since the stick policy decreases welfare for jobless workers while the carrot one benefits for reward-qualified workers. This paper, however, focuses on another effect of the carrot policy on a worker's behavior. In this paper we do not treat on-the-job search, so if a worker wants to change her job, she must separate from the current job and once become unemployed to seek a new better job. In that context, reduction in unemployment benefits would make her discouraged to enhance her career since the value of unemployment is decreased.

Recalling that workers are either unemployed or employed and either high-skilled or low-skilled in our setting, a policy that reduces only the rate of unemployment would be insufficient from the point of view of social welfare. This is because the most socially desirable situation in the economy is to increase the number of high-skilled employed. So the policy really needed is such that not only the rate of unemployment is fallen off but low-skilled workers pursue a more productive job. In that context, reduction in unemployment benefits would make a worker discouraged to enhance her career since the value of unemployment is decreased, so a worker who is hired by low-skilled job does not intend to enhance her career. The pecuniary reward, however, can give a worker who is employed by low-skilled job an incentive to enhance her career. In summary, reduction in unemployment benefits does operate as an employment-boosting policy but does not as career-enhancing policy while implement of pecuniary bonus program operate as both policies. Notably, since the career-enhancement increases the number of individuals who are employed at a more productive job it has a positive effect on social welfare.

Given above, this paper shows examples of policy effects on unemployment rate and on social welfare. As predicted above, we numerically show that, if the tax that finances unemployment benefits and/or pecuniary bonus is not too high, (i) both lower unemployment benefits and higher pecuniary rewards lead to lower unemployment rate, (ii) both higher unemployment benefits and higher pecuniary bonus result in higher welfare, and (iii) if the two policies are implemented simul-

taneously, moderate level of unemployment compensation and reemployment bonus can achieve higher social welfare than when one of the two policies solely implemented.

The rest of the paper is composed as follows. Section 2 describes the model and defines the equilibrium, section 3 is devoted to numerical analysis, and section 4 concludes.

## 2 The Model

### 2.1 Basic Assumptions

#### *Workers*

This paper considers a continuous-time job-search model in which workers are infinitely-lived and risk-neutral. We focus only on a steady-state equilibrium. A measure of workers is fixed and normalized to one. Workers are either employed or unemployed and either high-skilled or low-skilled. Let  $u$  be a rate of unemployment. All jobless workers receive unemployment benefit  $z$ .

Following Albrecht and Vroman (2002), our model assumes that a distribution of skills among individuals is a two-point distribution; a fraction  $\gamma$  of the unemployed workers are high-skilled, a fraction  $1 - \gamma$  of them are low-skilled, a fraction  $\phi$  of employed workers are high-skilled, and a fraction  $1 - \phi$  of them are low-skilled. Note that, unlike Albrecht and Vroman (2002), we assume that workers' skills level vary over time so that  $\gamma$  and  $\phi$  are endogenously determined in equilibrium as described in detail below.

This paper assumes that a high-skilled unemployed worker becomes low-skilled unemployed at a Poisson rate  $\lambda$ , which implies that a high-skilled unemployed worker might lose skills if she does not work for certain duration (on average  $1/\lambda$ ).<sup>1</sup> The paper also presumes that a low-skilled employed acquires skills at a Poisson rate  $\mu$ . For simplicity, the paper does not treat on-the-job search. Thus, a low-skilled employed who acquired skills must become a high-skilled unemployed once to improve her career.<sup>2</sup> Suppose that high-skilled unemployed workers (whose unemployment

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<sup>1</sup>Regarding the assumption, it would be appropriate to assume a time-varying unemployment benefit rather than a constant unemployment benefit, however, such an assumption requires a more complicate setup. See footnote 2.

<sup>2</sup>Note that high-skilled unemployed workers are necessarily short-term unemployed worker but low-skilled unemployed workers are not necessarily long-term unemployed worker in the model. This is because low-skilled unemployed workers include a worker who has just lost her job (that is, they are short-term unemployed).

Given the fact, if we assume a time-varying unemployment benefits, the model needs three states of unemployment; high-skilled short-term unemployment, low-skilled short-term unemployment, and low-skilled long-term unemployment. To avoid the complexity, the paper regards all low-skilled unemployed as substantively long-term unemployed even if they are short-term unemployed. Hence, throughout the paper, the words *high-(low-)skilled* are

duration is necessarily short as in described in footnote 2) can receive reemployment bonus  $B$  if they are hired.

### *Firms*

Jobs are either filled or vacant. For simplicity, unlike Albrecht and Vroman (2002), assume that there are firms which hire only high-skilled worker and only low-skilled worker. The former is called type  $h$  firm and the latter type  $l$  firm. Thus there exists practically two labor markets in the model. For convenience they are called type  $h$  market and type  $l$  market, respectively.

When a job is filled, the job produces output  $y_i$  and payes wage  $w_i$  in the type  $i(= h, l)$  firm. Assume that  $y_h > y_l$ . The wage is determined by bilateral Nash bargaining as described below. Filled jobs break up at an exogenous Poisson rate  $\delta$ . If a job is vacant, the type  $i$  firm incurs cost  $c_i$  and the cost is assumed  $c_h > c_l$ . The markets are assumed to be free entry/exit so that firms enter or exit the market so as to maximize their profits, as described below.

### *Government*

The role of government in this model is to collect tax to finance unemployment benefit  $z$  and reemployment bonus  $B$ . The tax rate is endogenously determined to hold balanced finance at any moment. The details are in subsection 2.4.

### *Matching Technology*

Workers seeking a job and firms recruiting a worker meet randomly through a matching process. The matching technology in each market are specified as follows:

$$M(\gamma u, v_h) = (\gamma u v_h)^{\frac{1}{2}},$$

$$M((1 - \gamma)u, v_l) = [(1 - \gamma)u v_l]^{\frac{1}{2}},$$

where  $v_i(i = h, l)$  denotes a measure of vacancies. Given the matching technology, rates of matching for workers in each market are given by  $(\gamma u v_h)^{\frac{1}{2}}/\gamma u = \theta_h^{\frac{1}{2}}$  and  $[(1 - \gamma)u v_l]^{\frac{1}{2}}/(1 - \gamma)u = \theta_l^{\frac{1}{2}}$ , where  $\theta_h \equiv v_h/\gamma u$  and  $\theta_l \equiv v_l/(1 - \gamma)u$  are known as labor market tightness. Similarly, matching rates for firms in each market are given by  $(\gamma u v_h)^{\frac{1}{2}}/v_h = \theta_h^{-\frac{1}{2}}$  and  $[(1 - \gamma)u v_l]^{\frac{1}{2}}/v_l = \theta_l^{-\frac{1}{2}}$ , respectively.

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used rather than *short-(long-)term*.

## 2.2 Asset Value Equations

Before describe asset value equations, recall the assumption that a low-skilled employed acquires skills at a Poisson rate  $\mu$ . Then, does the worker immediately separate from the current job? In other words, does the worker necessarily become a high-skilled unemployed worker to improve her career? The answer is *no*. If the worker behaves rationally, she would consider whether such a career-enhancing separation is beneficial or not. If so, the worker separates from the current job to improve her career as soon as she has acquired skills. If not, she still works at the current job until an exogenous job destruction occurs even if she has acquired skills.

Given the fact, we should consider both economy with and without career-enhancing separation (hereafter, abbr. CES). Note that CES arises or not is endogenously determined because value of each state is endogenously determined. To do the analysis, the paper proceeds in two steps, guess and verify: the following subsections in this section describe CES economy supposing that CES condition holds (the guess), and the next section examines whether the condition meet or not by using numerical calculus (the verify). The economy with no-CES is summarized in Appendix A.1.

### *Asset Value Equations for Workers*

We use the following notations:  $U_h$  ( $U_l$ ) is the present-discounted value of high- (low-) skilled unemployment,  $W_h$  is the value of high-skilled employment and  $W_l^h$  ( $W_l^l$ ) is the value of being employed at type  $l$  firm where a worker has acquired skills (where a worker is still low-skilled), respectively. Assume that all individuals are levied capitation tax  $\tau$  which is used to finance unemployment benefit  $z$  and bonus  $B$ . Suppose also that all jobless workers receive unemployment benefit  $z$  regardless of their skills. Recall that a high-skilled unemployed loses skills at the rate  $\lambda$  whereas a low-skilled employed acquires at the rate  $\mu$  and that all jobs face to an exogenous job destruction at the rate  $\delta$ . Letting  $r$  be a discount factor which is common to all individuals, the value functions are given by the following equations:

$$rU_h = z - \tau + \theta_h^{\frac{1}{2}}(W_h + B - U_h) + \lambda(U_l - U_h), \quad (1)$$

$$rU_l = z - \tau + \theta_l^{\frac{1}{2}}(W_l - U_l), \quad (2)$$

$$rW_h = w_h - \tau + \delta(U_h - W_h), \quad (3)$$

$$rW_l^h = w_l - \tau + \delta(U_h - W_l^h), \quad (4)$$

$$rW_l^l = w_l - \tau + \delta(U_l - W_l^l) + \mu \max\{U_h - W_l^l, W_l^h - W_l^l\}. \quad (5)$$

Note that the fourth term in (5) represents a low-skilled employed worker's decision whether the



worker separates from the current job if she acquired skills ( $U_h - W_l^l$ ) or the worker still works at the current job even if she acquired skills ( $W_l^h - W_l^l$ ). The former corresponds to CES whereas the latter no-CES. Using these notations, CES condition is given by  $U_h \geq W_l^h$ .<sup>3</sup>

### *Asset Value Equations for Firms*

Firms discount the future at the rate  $r$  as well as workers. Let  $V_i (i = h, l)$  denotes a present-discounted value of vacancy for type  $i$  firms and let  $J_i (i = h, l)$  stands for a present-discounted value of filled job for type  $i$  firms. The values of vacancy and filled job are recursively represented as follows:

$$rV_h = -c_h + \theta_h^{-\frac{1}{2}}(J_h - V_h), \quad (6)$$

$$rV_l = -c_l + \theta_l^{-\frac{1}{2}}(J_l - V_l), \quad (7)$$

$$rJ_h = y_h - w_h + \delta(V_h - J_h), \quad (8)$$

$$rJ_l = y_l - w_l + (\delta + \mu)(V_l - J_l). \quad (9)$$

Eq. (9) shows that type  $l$  firms face to an exogenous job destruction at the rate  $\delta + \mu$  because the worker-firm match breaks up not only when a shock occurs at the rate  $\delta$  but when the employee acquires skills at the rate  $\mu$ . Supposing that the labor market is free entry, firms post a vacancy until the expected value of job offer equals to zero, which implies that  $V_h = V_l = 0$  holds in equilibrium (the free entry/exit condition).

## **2.3 Equilibrium**

In this subsection we characterize the equilibrium in CES economy. We begin with description of flow conditions which determines distribution of workers.

### *Flow Conditions*

In the steady state, population in each state does not vary over time so that an inflow and outflow in each state must be equal. Recall that  $u$  represents the rate of unemployment,  $\gamma$  denotes the ratio of high-skilled in unemployed workers and  $\phi$  indicates the ratio of high-skilled in employed workers in CES economy.

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<sup>3</sup>More precisely, CES condition should be stated as  $U_h \geq W_l^l$  since the expression  $W_l^h$  is not needed in CES economy, however, to emphasize the fact that CES condition implies that the state value of high-skilled unemployment is higher than (or equal to) the state value of low-skilled employment *after skill acquisition*, we use  $W_l^h$ .

First, consider the flow condition on high-skilled unemployment state. The inflow consists of workers from high-skilled employment due to an exogenous job destruction,  $\phi(1-u)\delta$ , plus workers from low-skilled employment due to the workers' career-enhancing separation,  $(1-\phi)(1-u)\mu$ , while the outflow consists of workers to low-skilled unemployment due to prolonged unemployment,  $\gamma u\lambda$ , plus high-skilled employed who find a job,  $\gamma u\theta_h^{\frac{1}{2}}$ . The flow condition on high-skilled unemployment is thus  $\phi(1-u)\delta + (1-\phi)(1-u)\mu = \gamma u\theta_h^{\frac{1}{2}} + \gamma u\lambda$ . Similarly, the condition on low-skilled unemployment is given by  $\gamma u\lambda + (1-u)(1-\phi)\delta = (1-\gamma)u\theta_l^{\frac{1}{2}}$ , which states that the inflow (workers from high-skilled unemployment due to prolonged unemployment plus workers from low-skilled employment due to an exogenous job destruction) equals to the outflow (workers who are employed). Finally, the condition on high-skilled employment state is  $\gamma u\theta_h^{\frac{1}{2}} = \phi(1-u)\delta$ , which indicates that the inflow which is high-skilled workers who are employed equals to the outflow which is composed by high-skilled workers who lose a job due to a job destruction shock.

By using the three conditions, we obtain the rate of unemployment  $u$ , the ratio of high-skilled in unemployed workers  $\gamma$ , and the fraction of high-skilled in employed workers  $\phi$  in steady state, which are arranged as follows:

$$u = \frac{\delta(\lambda\delta + \lambda\mu + \mu\theta_l^{\frac{1}{2}})}{\theta_l^{\frac{1}{2}}(\lambda\delta + \mu\theta_h^{\frac{1}{2}}) + \delta(\lambda\delta + \lambda\mu + \mu\theta_l^{\frac{1}{2}})}, \quad (10)$$

$$\gamma = \frac{\mu\theta_l^{\frac{1}{2}}}{\lambda\delta + \lambda\mu + \mu\theta_l^{\frac{1}{2}}}, \quad (11)$$

$$\phi = \frac{\mu\theta_h^{\frac{1}{2}}}{\lambda\delta + \mu\theta_h^{\frac{1}{2}}}. \quad (12)$$

### *Wage Determination*

When a match is formed, the wage  $w_i$  ( $i = h, l$ ) is determined so as to maximize a matching surplus:  $w_h = \arg \max(W_h + B - U_h)^\beta (J_h - V_h)^{1-\beta}$  and  $w_l = \arg \max(W_l - U_l)^\beta (J_l - V_l)^{1-\beta}$ , where  $\beta$  denotes a bargaining power for workers. The sharing rules are given by  $(1-\beta)(W_h + B - U_h) = \beta(J_h - V_h)$  and  $(1-\beta)(W_l - U_l) = \beta(J_l - V_l)$ . Using these conditions, state values (1)-(9), and free entry/exit conditions  $V_h = V_l = 0$ , we obtain the following expressions:

$$w_h = \beta y_h + (1-\beta)z - (1-\beta)(r+\delta)B + \frac{\beta(rc_h\theta_h + \lambda c_l\theta_l)}{r+\lambda}, \quad (13)$$

$$w_l = \beta y_l + (1-\beta)z + \frac{\beta[(r+\lambda+\mu)c_l\theta_l - \mu c_h\theta_h]}{r+\lambda}. \quad (14)$$

Eq.(13) shows that  $w_h$  is increasing in both  $\theta_h$  and  $\theta_l$ . Since higher  $\theta_h$  implies larger vacancies relative to high-skilled jobless workers, it is difficult for firms to recruit a worker, which makes workers more advantageous, which leads to higher wage. In addition, since higher  $\theta_l$  implies larger employment opportunity in type  $l$  labor market, high-skilled jobless workers do not care whether the bargaining is approved or not, which also makes workers ascendant. In contrast, as (14) shows,  $w_l$  is increasing in  $\theta_l$  but decreasing in  $\theta_h$ . Since higher  $\theta_h$  indicates that it is easy to find a job in type  $h$  market, which makes the state value of high-skilled unemployment higher. Low-skilled jobless workers want to move the state, however, since they have to be employed once in type  $l$  firm, they would accept lower wage. Hence higher  $\theta_l$  leads to lower  $w_l$ . Note that, from (13),  $w_h$  is decreasing in bonus level  $B$ . This implies that since higher  $B$  makes workers more hunger to get the bonus, they are willing to accept much lower wage.

#### *Job Creation*

As noted above, firms open vacancy until the expected profit equals to zero in the steady state. The fact is represented by the free entry/exit conditions,  $V_h = V_l = 0$ . Using them and eliminating  $J_h$  and  $J_l$  from (6)-(9), we have the following expressions:

$$\theta_h = [(y_h - w_h)/c_h(r + \delta)]^2, \quad (15)$$

$$\theta_l = [(y_l - w_l)/c_l(r + \delta + \mu)]^2. \quad (16)$$

By making use of (13)-(16), we can obtain the equilibrium values of  $w_h, w_l, \theta_h$ , and  $\theta_l$ .

#### *Government Budget*

As described in Section 2.1, the expenditure for unemployment benefits  $z$  and reemployment bonus  $B$  is financed by capitation tax and the tax is determined so as to balance the government budget at any moment. The tax is determined by the following budget constraint:

$$\tau = uz + \gamma u \theta_h^{\frac{1}{2}} B, \quad (17)$$

where the first term of the right hand side indicates the expenditure for unemployment benefit  $z$  and the second term represents the expenditure for reemployment bonus  $B$ .

#### *Characterization of Equilibrium*

Up to this point, we have obtained all expressions that characterize the equilibrium in our model. The equilibrium consists of 8-tuple,  $\{u, \gamma, \phi, \theta_h, \theta_l, w_h, w_l, \tau\}$ . They are successively derived as

follows. First, the wage  $w_i$  and tightness of each market  $\theta_i$  ( $i = h, l$ ) is determined by wage bargaining and firm's optimal entry strategy (13)-(16). Second, the rate of unemployment  $u$ , the fractions of high-skilled in jobless workers  $\gamma$  and in employed workers  $\phi$  are derived by flow conditions (10)-(12). Finally, the capitation tax  $\tau$  is determined so as to satisfy the government budget (17). We can confirm that the equilibrium is uniquely determined, which is shown in Appendix A.2.

### 3 Numerical Analysis

In the previous section we describe an economy with career-enhancing separation by assuming that such a behavior arises: i.e., the CES condition  $U_h \geq W_l^h$  is assumed to hold. However, the guess may not be true since the state values in the condition are endogenously determined in a general equilibrium. In other words, the equilibrium stated in the previous section is just a candidate, but not a certifiable equilibrium. Hence, before the analysis, we must rule out candidates of equilibrium if the CES condition does not hold. We examine whether the CES condition holds or not with varying policy variable, unemployment benefits  $z$ , pecuniary rewards  $B$  or both since we focus on the policies.

Before the examination, we can confirm that two employment policies, one is increase in pecuniary bonus  $B$  and the other is decrease in unemployment benefits  $z$ , have a different effect on the decision about career-enhancing separation. Regarding the CES condition,  $U_h \geq W_l^h$ , the former policy directly increases the left-hand side whereas the latter one directly decreases the left-hand side (of course, both  $B$  and  $z$  indirectly affect the state values). This suggests that the both policies can operate as employment-boosting program, however, the policy that cuts unemployment benefits is less apt to be career-enhancing policy. Since career-enhancing separation increases the number of employed in high-skilled job (which is the most valuable state in our economy), consequently, pecuniary reward policy seems to be more desirable from a point of view of social welfare.

#### *Social Welfare Function and Parameters*

As preparation for our analysis, define the measure of social welfare and set parameter values. Following Cahuc and Lehmann (2000), we use an expected utility for each type of individual as a measure of welfare (for example, the measure of welfare for a high-skilled unemployed is  $rU_h$ ). In line with this manner, we define a measure of aggregate welfare by the weighted sum of all

individual's welfare:

$$\Omega = r\{(1 - u)[\phi W_h + (1 - \phi)W_l^l] + u[\gamma U_h + (1 - \gamma)U_l]\}. \quad (18)$$

It is difficult to obtain some of parameter values we need, in particular, we do not have decisive evidence on the rate of loss and acquirement of individual's ability,  $\lambda$  and  $\mu$ . In consideration of plausibility, we set  $\lambda = 1.0$  and  $\mu = 0.4$ . This implies that the average duration of being high-skilled when unemployed is 12 months ( $12 \times (1/1.0) = 12.0$ ) and that the average duration of being low-skilled when she works at type  $l$  firm is 30 months ( $12 \times (1/4.0) = 30.0$ ).<sup>4</sup> In other words, a high-skilled worker loses her skills if she cannot find a job within one year on average after fired, and a low-skilled worker acquires her skills if she works at a certain job for two and a half years on average. Regarding the rest of parameter values, we set  $y_h = 5.0$ ,  $y_l = 3.0$ ,  $\beta = 0.5$ ,  $r = 0.05$ ,  $c_h = 1.0$ ,  $c_l = 0.5$ ,  $\delta = 0.2$ ,  $z \in [0, 3.5]$ , and  $B \in [0, 10]$ .<sup>5</sup>

In the rest of the paper, we focus on the economy with CES and show results under (i) pecuniary reward policy where  $B$  is policy variable given  $z$ , (ii) reduction in unemployment benefit where  $z$  is policy variable given  $B$ , and (iii) a mixture of the two policies where both  $B$  and  $z$  are policy variables in order.

### *Pecuniary Reward*

Here we examine effects of reemployment bonus program on the unemployment rate and on social welfare. To focus on this policy and since the level of unemployment benefit is taken as given here, we assume that  $z = 3.5$ . This implies the policy that reduces unemployment benefit is not implemented at all.

Numerical results are placed in Figure 1. (1-i) represents social welfare defined in (18) with varying bonus levels. It has hump-shaped, which implies that welfare improves as bonus increases and after that worsens. This is because higher reward benefits for bonus-qualified workers, however, it heavily burdens as the tax that finances increases. (1-ii) and (1-iii) represents unemployment rate for high- and low-skilled, respectively. These are monotonically decreasing in bonus level as predicted. Comparing (1-ii) to (1-iii), one can see that the number of low-skilled jobless workers are more decreased than high-skilled. This causes through two channels. First, since higher bonus

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<sup>4</sup>These calculations are followed from Albrecht and Vroman (2002).

<sup>5</sup>Note that since the elasticity of the matching technology with respect to vacancy is 0.5,  $\beta = 0.5$  implies that we focus on an efficient economy in the sense that Hosios condition holds (see Hosios, 1990).

makes bonus-qualified workers more apt to get a job, inflow to low-skilled unemployment (i.e., prolonged unemployment) reduces. Second, since low-skilled jobless workers are also induced to get a job as discussed before, the outflow from low-skilled unemployment increases and part of them move to high-skilled unemployment state due to career-enhancing separation, which results in increase in high-skilled unemployed. Consequently, these two effects extremely decrease the number of low-skilled unemployed and moderately decrease the number of high-skilled unemployed. This result would suggest that, if tax burden is not too heavy, pecuniary bonus program seems to quite a policy as employment-boosting and career-enhancing.

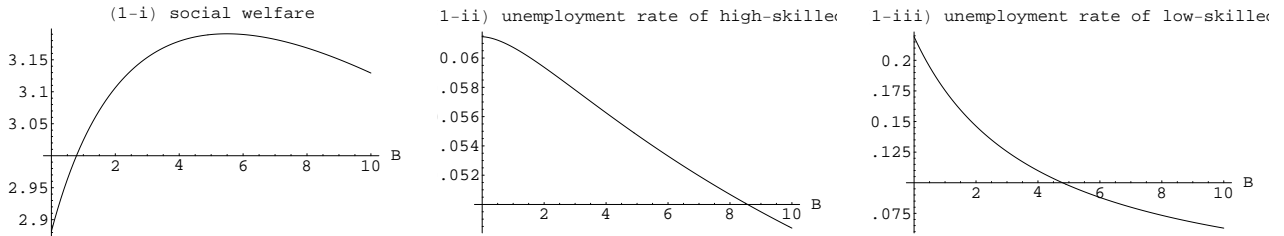


Figure 1: reemployment bonus ( $z = 3.5$ )

### *Reduction in Unemployment Benefit*

We examine effects of reduction in unemployment benefit on social welfare and on unemployment rate for high- and low-skilled here. To focus on this policy, we assume that there is no bonus program ( $B = 0$ ). Results are summarized in Figure 2, which are quite similar to the case of reemployment bonus. Social welfare (2-i) has hump-shaped, which suggests that moderate level of unemployment benefit maximizes social welfare. If the level is too high, heavy tax burden worsens individual's welfare while if the level is too low, it directly reduces individual's welfare. (2-ii) and (2-iii) show unemployment rate for high- and low-skilled unemployed, respectively. As discussed above, lower benefit leads to lower unemployment rate because jobless workers are more apt to get a job to escape from the current state. <sup>6</sup> Note that generous benefit decreases the number of high-skilled unemployed, which results in drastic increase in low-skilled unemployed as (2-iii) indicates.

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<sup>6</sup>In a model with search efforts of workers, it can be easily confirmed that lower unemployment benefits leads to higher job-search efforts, which results in lower unemployment rate.

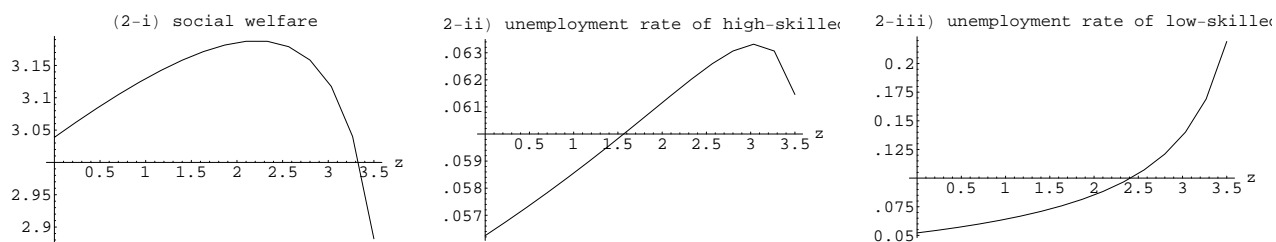


Figure 2: reduction in unemployment benefits ( $B = 0$ )

### *Mixture of the Two Policies*

Finally we analyze effects of mixture of the two policies on social welfare and on unemployment rate. To capture the shape of plane by numerical calculation, levels of pecuniary reward and unemployment benefit are divided into 10 grids, which are shown in Figure 3. (3-i) shows welfare level under arbitrary pairs of bonus  $B$  and unemployment benefit  $z$ .

When  $z$  is low, welfare is monotonically decreasing in bonus level. Such a counterintuitive situation occurs because bonus-qualified workers (high-skilled) can gain the bonus if they are hired whereas no-bonus-qualified workers (low-skilled) are heavily imputed the tax burden though they do not gain benefits from pecuniary reward. In aggregate, reduction in welfare of low-skilled workers dominates increase in welfare of high-skilled workers, which results in worsening of social welfare. When  $z$  is high, however, welfare is hump-shaped with respect to bonus level. Consider an extreme case,  $z = 3.5$ . Under such a generous unemployment compensation, there are many unemployed workers. Given the situation, an increase in pecuniary reward has two opposite effects on tax rate. First, it straightforwardly increases the tax rate, which worsens welfare. Second, since there are a lot of jobless workers with generous unemployment compensation, reduction in the number of jobless workers by bonus program drastically decreases the tax burden due to unemployment compensation, which improves welfare (even if the level of pecuniary reward is decent). When the bonus level is not too high, the second effect dominates the first one, which improves aggregate welfare, and vice versa.

Note that, by comparing (3-i) with (1-i) and (2-i), the maximized level of welfare under mixture of the two policies is higher than welfare level under a sole policy. The reason is straightforward. Since the tax burden due to unemployment benefit is reduced if the number of unemployed decreases, the two positive effect on social welfare. Reduction in jobless worker's welfare due to curtailed unemployment benefit is dominated by the positive effect, so aggregate welfare improves.

(3-ii) and (3-iii) show the unemployment rate of high- and low-skilled, respectively. Both rates are nearly increasing in unemployment benefit and decreasing in pecuniary reward. From (3-ii), high unemployment benefit and low bonus lead to reduction in the number of high-skilled unemployed, because it drastically increases the number of low-skilled jobless workers as (3-iii) indicates. Note that, by comparing (3-iii) with (1-iii) and (2-iii), the number of low-skilled unemployed under mixture of the policies is greatly lower than under a single policy, which suggests the usefulness of mixture of the two policies.

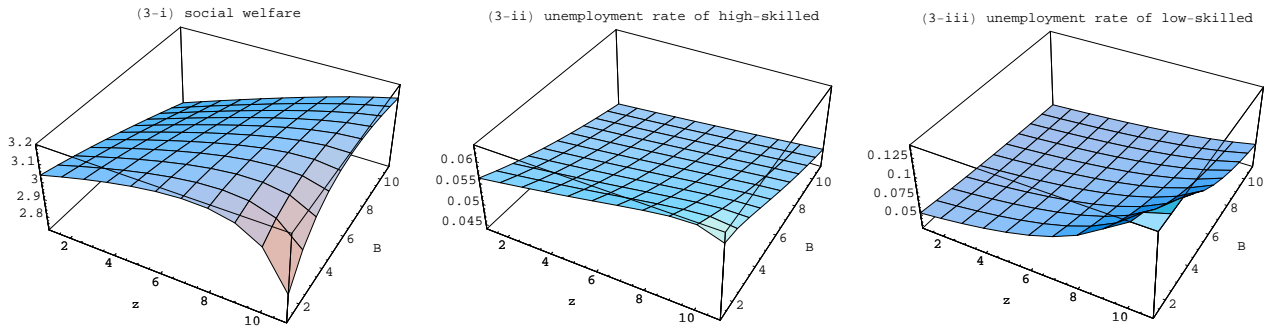


Figure 3: mixture of the two policies

## 4 Conclusion

This paper constructs a job-search model in which individual's skills vary over time and examines effects of employment-boosting policy on unemployment rate and on social welfare. We show that, (i) lower unemployment benefit and/or higher pecuniary bonus result in lower unemployment rate, (ii) higher unemployment benefit and/or higher reward lead to higher social welfare, and (iii) implement of the two policies can achieve higher social welfare than implement of single policy, if the tax finances these transfers is not too high. These are because pecuniary bonus program induces workers to seek a more productive job which is socially beneficial. In summary, reduction in unemployment benefit decreases unemployment rate at the cost of individual's welfare, however, pecuniary bonus operates as both employment-boosting policy and career-enhancing policy, which results in much higher social welfare.



# Appendix

## A.1. No-CES Economy

Here we describe the economy with no career-enhancing separation (no-CES). To distinguish the endogenous variables in this economy from CES economy, we denote the variables with tilde in no-CES economy. Matching technologies are given by  $M(\tilde{\gamma}, \tilde{u}) = (\tilde{\gamma}\tilde{u}\tilde{v}_h)^{\frac{1}{2}}$  and  $M((1 - \tilde{\gamma}), \tilde{u}) = [(1 - \tilde{\gamma})\tilde{u}\tilde{v}_l]^{\frac{1}{2}}$ . So the rate of matching for type  $i$  ( $i = h, l$ ) workers (firms) is  $\tilde{\theta}_i^{\frac{1}{2}}$  ( $\tilde{\theta}_i^{-\frac{1}{2}}$ ).

Asset value equations for workers and for firms are represented as follows:

$$r\tilde{U}_h = z - \tilde{\tau} + \tilde{\theta}_h^{\frac{1}{2}}(\tilde{W}_h + B - \tilde{U}_h) + \lambda(\tilde{U}_l - \tilde{U}_h), \quad (1')$$

$$r\tilde{U}_l = z - \tilde{\tau} + \tilde{\theta}_l^{\frac{1}{2}}(\tilde{W}_l^l - \tilde{U}_l), \quad (2')$$

$$r\tilde{W}_h = \tilde{w}_h - \tilde{\tau} + \delta(\tilde{U}_h - \tilde{W}_h), \quad (3')$$

$$r\tilde{W}_l^h = \tilde{w}_l - \tilde{\tau} + \delta(\tilde{U}_h - \tilde{W}_l^h), \quad (4')$$

$$r\tilde{W}_l^l = \tilde{w}_l - \tilde{\tau} + \delta(\tilde{U}_l - \tilde{W}_l^l) + \mu(\tilde{W}_l^h - \tilde{W}_l^l), \quad (5')$$

$$r\tilde{V}_h = -c + \tilde{\theta}_h^{-\frac{1}{2}}(\tilde{J}_h - \tilde{V}_h), \quad (6')$$

$$r\tilde{V}_l = -c + \tilde{\theta}_l^{-\frac{1}{2}}(\tilde{J}_l - \tilde{V}_l), \quad (7')$$

$$r\tilde{J}_h = y_h - \tilde{w}_h + \delta(\tilde{V}_h - \tilde{J}_h), \quad (8')$$

$$r\tilde{J}_l = y_l - \tilde{w}_l + \delta(\tilde{V}_l - \tilde{J}_l). \quad (9')$$

Flow conditions are given by:

$$\tilde{u} = \frac{\delta(\lambda\delta + \lambda\mu + \mu\tilde{\theta}_l^{\frac{1}{2}})}{\tilde{\theta}_l^{\frac{1}{2}}(\lambda\delta + \lambda\mu + \mu\tilde{\theta}_h^{\frac{1}{2}}) + \delta(\lambda\delta + \lambda\mu + \mu\tilde{\theta}_l^{\frac{1}{2}})}, \quad (10')$$

$$\tilde{\gamma} = \frac{\mu\tilde{\theta}_l^{\frac{1}{2}}}{\lambda\delta + \lambda\mu + \mu\tilde{\theta}_l^{\frac{1}{2}}}, \quad (11')$$

$$\tilde{\phi} = \frac{\mu\tilde{\theta}_h^{\frac{1}{2}}}{\lambda\delta + \lambda\mu + \mu\tilde{\theta}_h^{\frac{1}{2}}}. \quad (12')$$

Wage equations and job-creation conditions that determine equilibrium wage and market tightness are derived as follows:

$$\tilde{w}_h = \beta y_h + (1 - \beta)z - (1 - \beta)(r + \delta)B + \frac{\beta(rc_h\tilde{\theta}_h + \lambda c_l\tilde{\theta}_l)}{r + \lambda}, \quad (13')$$

$$\tilde{w}_l = \beta y_l + (1 - \beta)z + \frac{\beta\{(r + \lambda)(\delta + \mu) + \delta\mu\}c_l\tilde{\theta}_l - \delta\mu c_h\tilde{\theta}_h}{(r + \lambda)(\delta + \mu)}, \quad (14')$$

$$\tilde{\theta}_h = [(y_h - \tilde{w}_h)/c_h(r + \delta)]^2, \quad (15')$$

$$\tilde{\theta}_l = [(y_l - \tilde{w}_l)/c_l(r + \delta)]^2. \quad (16')$$

The capitation tax is determined so as to balance the following government budget:

$$\tilde{\tau} = \tilde{u}z + \tilde{\gamma}\tilde{u}\theta_h B. \quad (17')$$

Welfare function in no-CES economy is given by:

$$\tilde{\Omega} = r\{(1-u)[\phi W_h + \frac{(1-\phi)\delta}{\delta+\mu}W_l^l + \frac{(1-\phi)\mu}{\delta+\mu}W_l^h] + u[\gamma U_h + (1-\gamma)U_l]\}. \quad (18')$$

### A.2. Uniqueness of Equilibrium

As stated in section 2.3, the endogenous variables which construct the equilibrium are successively determined. Recall that wage  $w_i$  and market tightness  $\theta_i$  ( $i = h, l$ ) are firstly determined by (13)-(16). Eliminating  $w_h$  and  $w_l$ , we have the following expressions:

$$(r + \delta)(r + \lambda)c_h\theta_h^{\frac{1}{2}} + \beta(rc_h\theta_h + \lambda c_l\theta_l) - (1 - \beta)(r + \lambda)[y_h - z + (r + \delta)B] = 0, \quad (A.1)$$

$$(r + \delta + \mu)(r + \lambda)c_l\theta_l^{\frac{1}{2}} + \beta[(r + \lambda + \mu)c_l\theta_l - \mu c_h\theta_h] - (1 - \beta)(r + \lambda)(y_l - z) = 0, \quad (A.2)$$

which are simultaneous equations with respect to  $\theta_h$  and  $\theta_l$ . By implicit function theorem, we can easily obtain the shape of these expressions:

$$\frac{\partial\theta_h}{\partial\theta_l} = -\frac{\beta\lambda c_l}{\frac{1}{2}(r + \delta)(r + \lambda)c_h\theta_h^{-\frac{1}{2}} + \beta r c_h} < 0, \quad (A.1')$$

$$\frac{\partial\theta_h}{\partial\theta_l} = \frac{\frac{1}{2}(r + \lambda)(r + \delta + \mu)c_l\theta_l^{-\frac{1}{2}} + \beta c_l(r + \lambda + \mu)}{\beta\mu c_h} > 0. \quad (A.2')$$

Thus in  $\theta_l$ - $\theta_h$  plane, (A.1) is monotonically decreasing while (A.2) is monotonically increasing, which guarantees uniqueness of the solution. Given the values of  $\theta_h$  and  $\theta_l$ , wages are determined by (15)  $w_h = y_h - (r + \delta)c_h\theta_h^{\frac{1}{2}}$  and by (16)  $w_l = y_l - (r + \delta + \mu)c_l\theta_l^{\frac{1}{2}}$ , which is obvious that these values are also uniquely determined. Since the rest of endogenous variables are evidently unique, uniqueness of the equilibrium is proved.

## References

- [1] Albrecht, James. and Vroman, Susan. (2002). "A Matching Model with Endogenous Skill Requirements", *International Economic Review*, vol.43, No.1, pp.283-305.
- [2] Albrecht, James. and Vroman, Susan. (2005). "Equilibrium search with time-varying unemployment benefits", *Economic Journal*, vol.115 (July), pp.631-48.
- [3] Anderson, Patricia M. (1992). "Time-varying Effects of Recall Expectation, a Reemployment Bonus, and Job Counseling on Unemployment Durations", *Journal of Labor Economics*, vol.10, No.1, pp.99-115.
- [4] Cahuc, Pierre. and Lehmann, Etienne. (2000). "Should unemployment benefits decrease with the unemployment spell?" *Journal of Public Economics*, vol.77, pp.135-53.
- [5] Corson, Walter, Paul Decker, Shari Dunstan, and Stuart Kerachsky. (1992). "Pennsylvania Reemployment Bonus Demonstration: Final Report", Unemployment Insurance Occasional Paper 92-1. Washington D.C.: U.S.Department of Labor.
- [6] Davidson, Paul, and Stephen A. Woodbury. (1993). "The Displacement Effects of Reemployment Bonus Programs", *Journal of Labor Economics*, vol.11, No.4, pp.575-605.
- [7] Davidson, Paul, and Stephen A. Woodbury. (1997). "Optimal unemployment insurance", *Journal of Public Economics*, vol.64, No.3, pp.2.5-87.
- [8] Decker, Paul T., and Christopher J. O'Leary. (1995). "Evaluating Pooled Evidence from the Reemployment Bonus Experiments", *Journal of Human Resources*, vol.30, No.3, pp.534-50.
- [9] Decker, Paul T. (1994). "The Impact of Reemployment Bonuses on Insured Unemployment in the New Jersey and Illinois Reemployment Bonus Experiments", *Journal of Human Resources*, vol.29, No.3, pp.718-41.
- [10] Hosios, Arthur J. (1990). "On the Efficiency of Matching and Related Models of Search and Unemployment", *Review of Economic Studies*, vol.57, No.2, pp.279-98.
- [11] Meyer, Bruce D. (1990). "Unemployment Insurance and Unemployment Spells", *Econometrica*, vol.58, No.4, pp.757-82.

- [12] Meyer, Bruce D. (1995). “Lessons from the U.S. Unemployment Insurance Experiments”, *Journal of Economic Literature*, vol.33, No.1, pp.91-131.
- [13] OECD. (2000). “OECD Employment Outlook”.
- [14] O’Leary, Christopher J., Spiegelman, Robert G., and Kline, Kenneth J. (1995). “Do Bonus Offers Shorten Unemployment Insurance Spells? Results from the Washington Experiment”, *Journal of Policy Analysis and Management*, vol.14, No.2, pp.245-69.
- [15] O’Leary, Christopher J., Decker, Paul T., and Wander, Stephen A. (2005). “Cost-Effectiveness of Targeted Reemployment Bonuses”, *Journal of Human Resources* 40, No.1, pp.270-79.
- [16] Petrongolo, Barbara, and Pissarides Christopher A. (2001). “Looking into the Black Box: A Survey of the Matching Function”, *Journal of Economic Literature*, vol.39, No.2, pp. 390-431.
- [17] Pissarides, Christopher A. (1992). “Loss of Skill During Unemployment and the Persistence of Employment Shocks”, *Quarterly Journal of Economics*, vol.107, No.4, pp.1371-91.
- [18] Pissarides, Christopher A. (2000). “ Equilibrium Unemployment Theory”, 2nd edition, Cambridge, MA: MIT Press.
- [19] Woodbury, Stephen A., and Robert G. Spiegelman. (1987). “Bonuses to Workers and Employers to Reduce Unemployment: Randomized Trials in Illinois”, *American Economic Review*, vol.77, No.4, pp.513-30.