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Abstract

We introduce a common-pool contest into a continuous-time, differential game setting to model the dynamic behavior of agents facing a trade-off between socially productive activities and appropriation. We are able to identify multiple Markov perfect equilibrium strategies that are nonlinear in a state space, thus leading the economy to a state where 'partial cooperation' occurs. We show that such cooperation can be seen as a response to conflict. We also discuss the consequences of changes in the effectiveness of appropriation, the number of contenders, and the rate of time preferences on contest equilibria.

Keywords: Conflict, Cooperation, Differential Game, Markov Perfect Equilibrium, Nonlinear Markov strategy

JEL classifications: D 74, L 11

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1 Introduction

There is a relatively small but growing literature in political economics initiated by Hirshleifer (1991, 1995), Skaperdas (1992) and Grossman and Kim (1995). Their models share four common features. First, they postulate that conflict arises from the choice of rational and self-interested agents. Second, a well-defined and enforced property right over, at least, some goods do not exist. Third, the agents are assumed to be myopic in a way that they maximize only the current payoff. Fourth, their model is static. This paper conducts the analysis of conflict by extending their static models to a dynamic one.

Hirshleifer (1995) takes an initial step towards a dynamic approach by recognizing successive iterations of the one-shot game, and focuses on the convergent point of such iterations (he calls such a fixed point 'a steady state'). Nevertheless, Maxwell and Reuveny (2005, p.31) correctly point out that "However, this approach is not fully dynamic: it does not specify equations of motion for any variables, time is not a variable in the model, and the condition for dynamic stability is not derived based on standard dynamic analysis".

In response to such long-term desires, there have been several papers which attempt to construct a dynamic variation of the one-shot conflicting game analyzed by the above-mentioned authors. Garfinkel (1990) examines a dynamic model in which agents make choices between productive and fighting activities. She uses a repeated game setting where threats and punishments are available. Existence of cooperative (or disarmament) equilibria can be established using Folk Theorem arguments. Skaperdas and Syropoulos (1996) discuss a two-period model of conflict in which time-dependence is introduced by the assumption that second period resources of each agent are increasing in first-period's payoff. As a result, 'the shadow of the future' may impede the possibilities for cooperation. In other words, competing agents engage more in appropriation in order to capture a bigger share of today's pie. The equilibrium solution concept we employ in this paper allows us to identify possible cooperative outcomes as a result of decentralized decision-making by agents, without having to rely on the Folk Theorem of repeated games or enforceable commitments. Nevertheless, since the one-shot game is repeated every period due to the nature of the repeated game, it would be unsatisfactory to describe true dynamic situations which are not 'stationary'. More recently, Maxwell and Reuveny (2005) construct a conflict model with two competing groups in which each group's population and a stock of common (natural)-resources both change over time. Since three non-linear differential equations characterizing the dynamic paths of these stock variables do not allow an analytical solution, they resort to numerical simulations. These exercises reveal that *mild* conflict activity depresses the use of natural resources for production, thus possibly creating a Pareto improvement compared to cooperative situations where there is no appropriate activity, and, moreover, tends to reduce the volatility of those stocks through the transition. Although their model generates interesting insights, they still assume that agents are myopic. The authors in the literature have called for a full dynamic and multi-period model of the Skaperdas-Hirshleifer-Grossman and Kim-based literature which incorporates the behavior of non-myopic agents who taking into account the consequences of their future actions, which is also left as an open question in Maxwell and Reuveny (2005).[1] The goal of this paper is to accomplish this task.

We develop a forward-looking agent-based infinite horizon general-equilibrium model to study the dynamic evolution of self-enforcing property rights. There are various ways of extending one-shot, static models of Skaperdas, Hirshleifer, and Grossman and Kim to a dynamic setting. Following their models, we first assume that the initial resource endowment is fixed over time. This assumption would be defended either by interpreting the initial resource endowment as a time or labor supply, or by assuming the fixed population in order to keep the model tractable. The relevant state variable in our dynamic model is a durable stock which accumulates through time according to the production process using collective efforts of all parties involved. This durable stock is exhaustible or rival in the sense that one agent's

[1] More recently, there is another class of dynamic conflicting models that include, e.g., Gradstein (2003) and Gonsalez (2007). There are several important differences between the models in these papers and the one in ours. First, in their models a flow of the output produced each period is subject to predation, while in our model a stock variable is subject to predation. Secondary and more importantly, those papers investigate the relationship between conflict and economic growth in the standard growth model based explicitly on the investment and saving decisions of a large number of economic agents. Hence, their models are mostly concerned with the macroeconomic consequences, such as growth effects of insecure property rights. Since our model is a straightforward dynamic extension of Grossman, Hershleifer and Skaperdas which allows for static interaction among a small number of economic agents, it enables one to directly compare our results with those in static conflicting models and thus to highlight the strategic role of appropriation among those few agents in the intertemporal context.

use of the stock *does* diminish its availability to other agents, and is open to appropriation by rivals due to the lack of well-defined or enforceable property rights. Hence each of the agents is tempted by the immediate benefit attainable from capturing the stock. Natural resources and land in primitive historical societies are examples of such durable stocks or disputed wealth.

We model the incentives of agents to exert effort in an attempt to defend their claims on the stock and challenge the claims of others. All agents who succumb to the temptation reduce their help in production of the common-pool stock to increase their efforts to convert claims on the common stock into effective property rights. More specifically, agents derive utility (or a payoff) from owning the stock of durable good and, at every instant in time, choose how to allocate an endowment between appropriation of the common-pool stock (creating property rights) and participating in the production process to accumulate the common-pool stock in the economy. The production and appropriation decisions made independently and noncooperatively by each of the contenders jointly determine the evolution of the commonly accessible stock.

We present a tractable version of a differential game formulation of this model of conflict between several agents who attempt to appropriate a common-pool durable stock over an infinite horizon. The solution concept employed is Markov perfect (MP) equilibrium, restricting strategies to be functions of the current payoff-relevant state variable. Not all the strategies that describe a solution of the intertemporal optimizing problem of an agent are MP equilibria. The key to determining which describe equilibrium outcomes is subgame perfection over the global domain of a state variable. In spite of this natural but stringent requirement, there are multiple non-linear MP equilibrium strategies in our model. This multiplicity of strategies has the following characteristics and implications. First, most of those solutions commonly reveal that initially poor countries will exhibit an increase in appropriation as the aggregate stock of durable good gets larger until a steady state is reached. Second, on the other hand, in economies with an affluent endowment of natural resources the 'marginal gain' of appropriation is high and agents substitute appropriation for production for a while until the state variable reaches a threshold level. From that threshold onwards, agents choose to engage in production activity to some extent until a steady state is reached where the output of production is only just sufficient to replace the stock of durable goods. This result relates to the observation that rent-seeking activities in rich countries may result in deindustrialization as suggested by the literature on the resource curse (e.g., Sachs and Warner, 1999; Auty, 2001).[2] Third, our results also reveal that even if two economies are 'similar' in terms of the initial levels of common-pool endowment, production technology or preferences of agents, the economies converge to different steady states as well as follow different transitional paths. Put it differently, the model predicts that 'similar' countries converge to a low-income steady state with more unstable property rights, and some converge to a high income equilibrium with more stable property rights.[3] Which equilibria is realized in the long run depends on whether the coordination regarding the expectations formed players successfully is achieved or not. Neither of the above-mentioned one-shot models has addressed this indeterminacy feature.

Fourth, in the long run property rights may be 'partially' enforced in the sense that appropriation and productive activities coexist, so that neither a totally peaceful (disarmed) equilibrium nor a full-fighting equilibrium emerges as a long run outcome. The degree of 'partially' cooperation would vary depending on which steady state is reached among a continuum of steady states.

The organization of the paper is as follows. The next section describes the basic model. Section 3 conducts comparative static analysis with respect to several principle structural parameters. Section 4 derives an efficient solution (i.e., cooperative solution) as a reference path. Section 5 concludes the paper. Some mathematical proofs will be given in the appendices.

2 The Model

Consider an infinite horizon economy populated by $n \geq 2$ agents who strategically interact. Each of the agents derives utility from the consumption (or services) of a common-pool as-

[2] There is evidence that resource abundance in the definition used by Sachs and Warner (1999) is associated with civil war (e.g., Collier and Hoeffler, 2004; Hodler, 2006).

[3] Auty (2001) argues that experiences in different countries are complex and diverse. Some countries like Malaysia, Australia, Norway, Botswana and Canada appear to have used their resources judiciously, whereas countries like Nigeria, Mexico and Venezuela seem to have squandered their oil windfalls. According to Acemoglu *et. al.* (2001) the limiting force of conflict is institutional quality as a key driver for economic growth and prosperity.

set (such as land territories and natural resources) or (tangible and intangible) the stock of durables. We want our model to capture the role of productive and aggressive activities with the understanding that aggressive investment causes an inward shift of the aggregate production possibility frontier. Accordingly, we use a setup where appropriation and production are two substitutable investment choices. Specifically, let an individual decide at each point in time how much resources to devote for appropriation $a_i \geq 0$ and production $l_i \geq 0$. The individual resource (e.g. time) constraint is:

$$a_i + l_i = e_i, \tag{1}$$

where e_i is the endowment of a fill-in activity that is not subject to appropriation.[4] We will set $e_i = 1$ for ease of exposition. The time arguments have been suppressed in this and all subsequent equations.

The common-pool stock is subject to appropriation. The stock is generated by accumulation of output. Output is produced with a linear production technology:

$$Y(l_1, \ldots, l_n) = \sum_{j=1}^{n} l_j, \tag{2}$$

which captures the idea that higher productive efforts by agents cause an outward shift of the production possibility frontier for the economy as a whole. The output of production can be stored to augment the common-pool stock. However, storage entails costs such that the stock Z evolves according to

$$\dot{Z} = Y(l_1, \ldots, l_n) - \delta Z, \tag{3}$$

where $\delta \in (0, 1)$ is the rate at which output will depreciate if stored for future consumption, $\dot{Z}$ denotes the change of Z over time and $Z(0) \geq 0$ is the initial stock.

A main ingredient of the model is the conflict technology which, for any given values of $a_1, \ldots, a_n$, determines each agent's probability of winning sole possession in obtaining the

[4] The standard assumption that each agent has some essential property rights is implicit in this formulation. Individual's labor supply is such an example. Maxwell and Reuveny (2005) further assume that labor supply is growing over time as a result of the growth of population. However, in order to keep the model simple, we assume that the population of agents remains constant over time.

stock Z in a given period. To model this probability for agent i, a natural assumption is that the probability is increasing in aggressive investment of agent i, the fraction of time player i devotes to aggression, and decreasing in the sum of aggressive investment of all agents. A plausible form of the conflict technology is the Tullock contest success function (Tullock, 1980; Hirshleifer, 1991 and 1995; Gonzalez, 2007). In its standard formulation this function reads:

$$p_i(a_1, \ldots, a_n) = \begin{cases} a_i^r \Big/ \left(a_i^r + \sum_{j \neq i}^n a_j^r\right) & \text{for } a_i > 0 \\ 1/n & \text{for } a_i = 0 \ \forall i \end{cases} \tag{4}$$

where the parameter r captures the effectiveness of aggression. From the contest success function (4) we obtain the relative success of contender i in the contest. Alternatively, the contest success function (4) may be interpreted as a sharing rule, or ownership of assets that depends on the respective efforts of aggression. It is natural to assume in the analysis that each agent has an equal access to the prize when agents do not engage in aggressive behavior; hence the assumption that $p_i(0, \ldots, 0) = 1/n$ will be in force throughout the analysis.

The instantaneous expected payoff to each agent is given by $p_i(a_1, \ldots, a_n) Z$.[5] Each of the agents chooses the streams of a_i and l_i to maximize the discounted value of total expected payoffs subject to the feasibility conditions introduced in (1)-(4):

$$\begin{aligned} &\max_{a_i} \int_0^\infty p_i(a_1, \ldots, a_n) Z e^{-\rho t} dt \quad \text{subject to} \\ &\dot{Z} = \sum_{j=1}^n (1 - a_j) - \delta Z, \ Z(0) = Z_0 \geq 0, \\ &0 \leq a_i(t) \leq 1 \quad \text{for} \quad \forall t \in [0, \infty), \end{aligned} \tag{5}$$

where $\rho > 0$ is the rate of time preference.

[5] Alternatively, one may view the prize as flow services, such as output or utility from the stock variable Z rather than Z itself. To clarify this we need to introduce a concave function, say $u(Z)$ instead of Z. However, this complication does not affect our results at all.

2.1 Solution Concept

We solve the differential game using the notion of a (stationary) MP Nash equilibrium, because we think that this equilibrium concept captures the essential strategic interactions over time. MP strategies are decision rules such that each agent's decision is the best response to those of the other players, conditional on the current payoff-relevant state variable Z (see, e.g., Chapter 4 in Docker et al., 2000). Markovian strategies rule out path dependence in the sense that they depend only on the current values of the state variables rather than strategy choices in history. As a result, it does not matter how one gets to a particular point, only that one gets there.

MP equilibrium strategies must satisfy the Hamiltonian-Jacobi-Bellman equation given by:

$$\rho V_i(Z) = \max_{a_i \in [0,1]} \left[p_i(a_1, \ldots, a_n) Z + V_i'(Z) \left\{ \sum_{j=1}^{n} (1 - a_j) - \delta Z \right\} \right], \tag{6}$$

where V_i denotes the maximum value agent i attributes to the game that starts at Z. Notice that

$$\frac{\partial^2 p_i}{\partial a_i^2} Z = r(n-1) \frac{n(r-1) - 2r}{n^3 a_i^2} Z < 0 \quad \text{for} \quad \begin{cases} n = 2 \wedge r > 0, \\ n > 2 \wedge 0 < r < n/(n-2), \end{cases} \tag{7}$$

implying that the r.h.s. of (6) is concave in $a_i \in [0,1]$. We assume that $r < 1/(n-1)$ in what follows,[6] guaranteeing not only that the second-order condition (7) holds but also that the linear strategy of each agent, which plays an important role in the later analysis, is a nonnegative value. The first-order necessary condition for agent's choice of appropriation is given by

$$\frac{\partial p_i}{\partial a_i} Z - V_i'(Z) \begin{cases} = 0 \implies a_i \in [0,1], \\ > 0 \implies a_i = 1, \\ < 0 \implies a_i = 0, \end{cases} \tag{8}$$

the l.h.s. of which is evaluated for all $a_i \in [0,1]$. According to (8), each agent, when choosing a_i,

[6] Tullock (1980) assumes the same condition in his two-agent, rent-seeking game. Hirshleifer (1991, 1995) and Gonzalez (2007) also assume that $r < 1$ in their two-agent games. Condition $r < n/(n-1)$ reduces to $r < 1$ when $n = 2$.

trades the marginal increase in expected payoff from an increase in appropriation against the marginal loss in the discounted value of the future stream of payoffs which results from a reduction of productive effort. If the payoff gain from an increase in a_i is larger than the payoff loss implied by the decrease in l_i for all levels of a_i, then agent i will rationally devote all resources to appropriation. In contrast, the agent chooses $a_i = 0$ in cases where the discounted marginal gain from productive investment exceeds the instantaneous marginal gain from aggressive behavior for all levels of a_i.

2.2 Equilibrium

We can then make use of (8) to characterize subgame perfect equilibria of the differential game. Since we have started our analysis assuming identical agents, a natural focus is on symmetric equilibria. The symmetry assumption allows us to drop the subscript i in the subsequent discussion, and we will suppress this index unless strictly necessary for expositional clarity.

Let us first analyze interior solutions of a_i. Differentiation of the interior first-order condition in (8) gives

$$V''(Z) = \sum_{j=1}^{n} \frac{\partial^2 p_i}{\partial a_i \partial a_j} a_j'(Z) Z + \frac{\partial p_i}{\partial a_i} = -\frac{r(n-1)}{n^2 a^2} a'(Z) Z + \frac{r(n-1)}{n^2 a}. \tag{9}$$

At an interior solution of $a(Z)$ we may apply the envelope theorem to characterize $a'(Z)$. Using the symmetry assumption, we obtain

$$a'(Z) = \frac{\dfrac{1}{n} + \dfrac{r(n-1)}{n^2 a(Z)} \left[(1 - a(Z)) n - (\rho + 2\delta) Z\right]}{\dfrac{r(n-1)}{n^2 a(Z)^2} Z (n - \delta Z)}. \tag{10}$$

We will employ phase-plane methods to characterize the qualitative solution of the nonlinear differential equation (10) and the associated MP strategies. For this purpose we have to identify the steady state locus where $\dot{Z} = 0$, called C_1 in the following. Let us denote by C_2 the loci where $a'(Z)$ goes to plus infinity, and by C_3 the loci where $a'(Z)$ equals zero in the

(Z, a) space:

$$
\begin{aligned}
&C_1 := \{(Z, a) : \dot{Z} = (1 - a(Z))\, n - \delta Z = 0\}, \\
&C_2 := \{(Z, a) : a'(Z) \to \pm\infty\}, \\
&C_3 := \{(Z, a) : a'(Z) = 0\}.
\end{aligned}
\tag{11}
$$

The steady-state line C_1 is a downward-sloping, straight line in the (Z, a) space. It intersects the vertical axis at point $(0, 1)$ and the horizontal axis at point $(n/\delta, 0)$. Turn to C_2. Setting the denominator in (10) equal to zero, we obtain a vertical line at point $(n/\delta, 0)$. The locus C_3 is obtained by setting the numerator in (10) equal to zero. Solving for a gives the following locus:

$$
a = -\frac{r(n-1)}{1 - r(n-1)} + \frac{r(n-1)}{1 - r(n-1)} \frac{\rho + 2\delta}{n} Z. \tag{12}
$$

Using $1 - r(n-1) > 0$, (12) shows that the straight line C_3 has a positive slope and a negative intercept on the vertical axis, as shown in Figs. 1 and 2. Moreover, the point of intersection between the straight lines C_2 and C_3, labelled E, is situated in the nonnegative region of the (Z, a) plane:

$$
(Z_E, a_E) = \left(\frac{n}{\delta}, \frac{r(\rho + \delta)(n-1)}{[1 - r(n-1)]\,\delta} \right), \tag{13}
$$

which is called 'a singular point'. Note, however, that since point E may be located below or above the resource constraint (1), the value of a_E may or may not be less than 1. Depending on this value we can draw two diagrams such as in Figs.1 and 2. Moreover, it follows from (3) that any strategy $a(Z)$ above line C_1 implies that Z declines in time, while any strategy $a(Z)$ below line C_1 entails an increase of Z over time.

Collecting the arguments, we can illustrate an uncountable number of the hyperbolic curves corresponding to the solutions satisfying the HJB equation (6) in Figs. 1 and 2. These figures display representatives of those integral curves that are divided into five types of the families of strategies. Arrows on the *families* of integral curves a_j, $j = 1, \ldots, 4$, and a_L illustrate the evolution of Z over time. In particular, by direct integration of (10) and manipulating we can

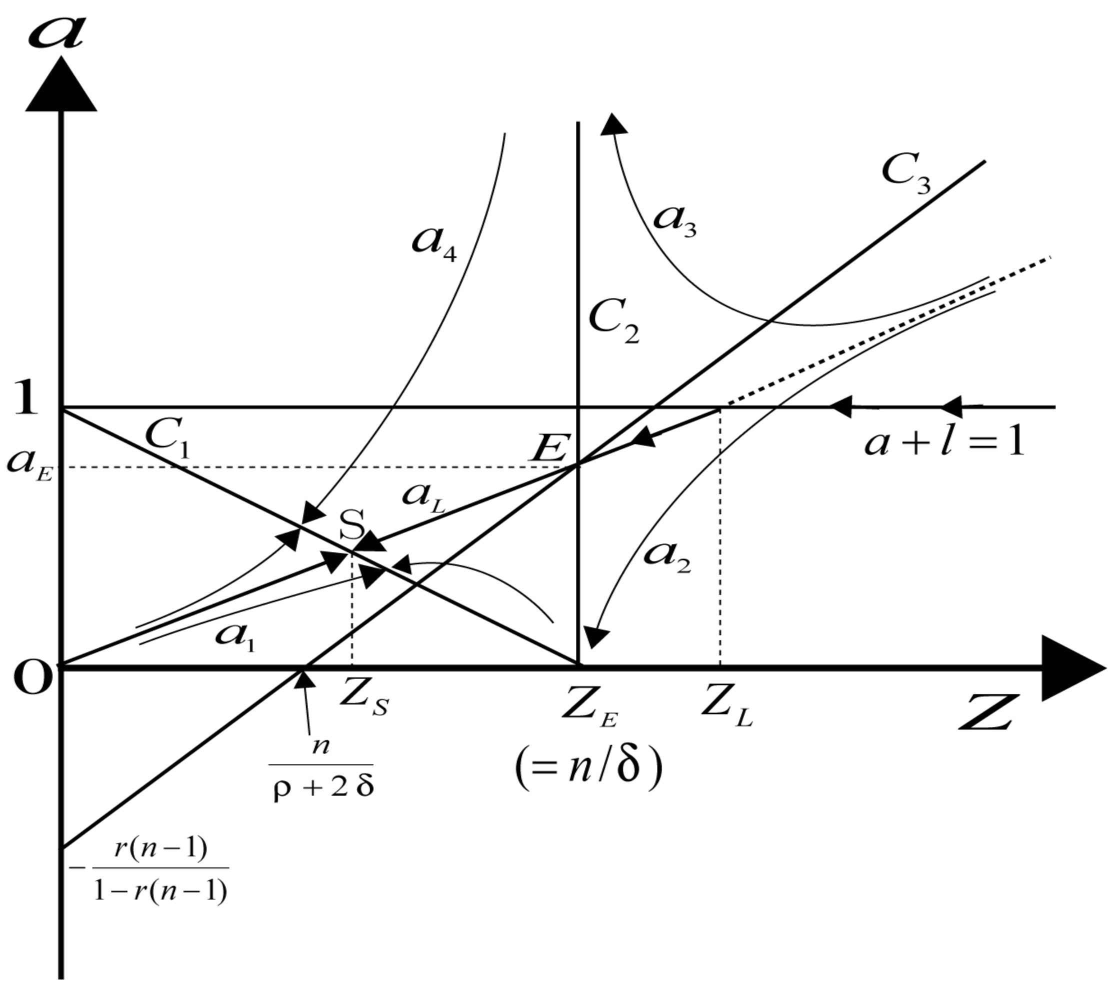

Figure 1: **Phase diagram when $a_E < 1$.**

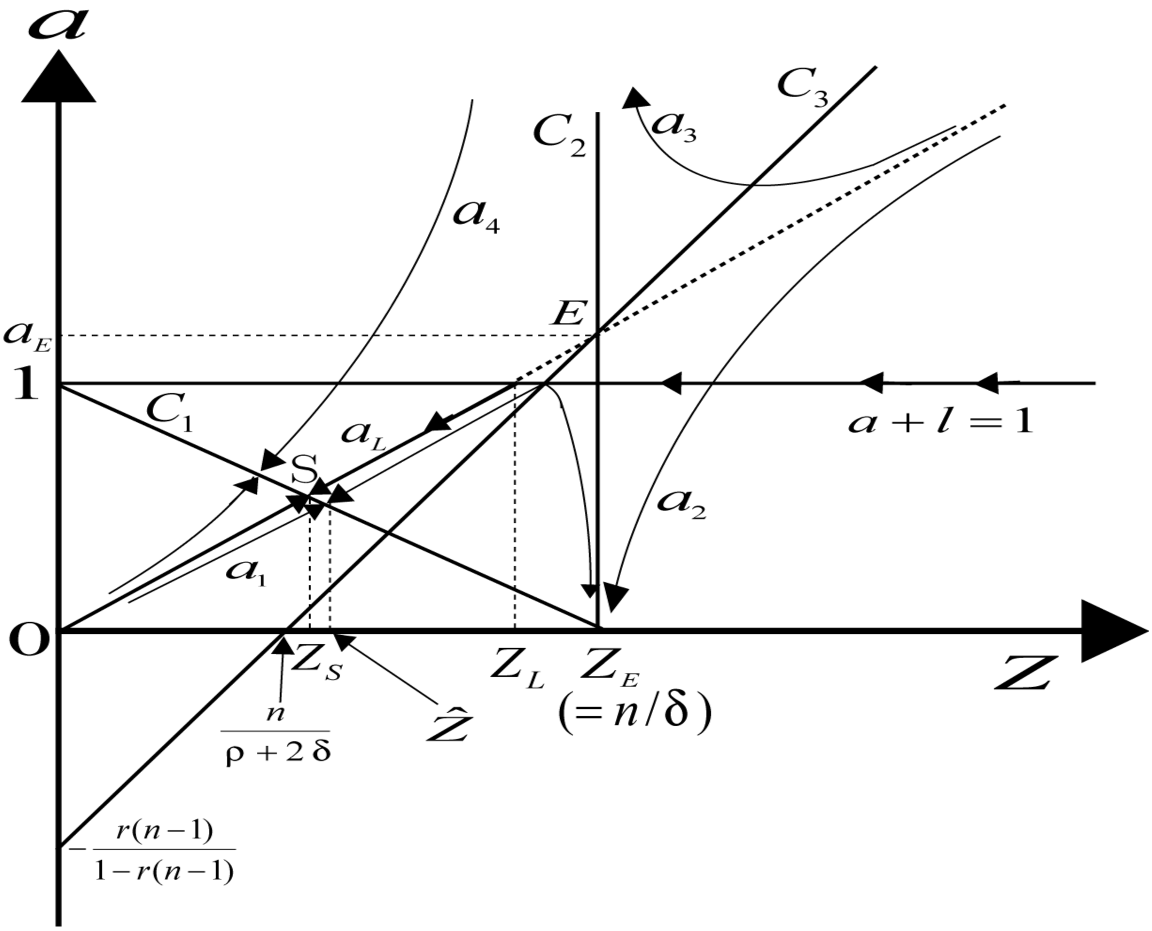

Figure 2: **Phase diagram when $a_E > 1$.**

obtain a general solution to (10):

$$a(Z)=\frac{(n-1)\,rZ\,(n-\delta Z)^{\frac{\rho+\delta}{\delta}}\,(\rho+\delta)}{n\,[1-r\,(n-1)]\,(n-\delta Z)^{\frac{\rho+\delta}{\delta}}+(n-1)\,r\,(\rho+\delta)\,c_1}, \tag{14}$$

where c_1 represents an arbitrary constant of integration and may take a positive, zero or negative value. When $c_1=0$, (14) simplifies to

$$a_L(Z)=\frac{r\,(n-1)\,(\rho+\delta)}{[1-r\,(n-1)]\,n}Z. \tag{15}$$

It is easy to confirm by the well-established guessing method for a value function that strategy a_L stands for the linear strategy (see Appendix B). Moreover, the left branch of the linear strategy a_L to the left of the steady state line C_1 starts from the origin, and then reaches point S on the steady state line C_1, while its right branch starts from any initial value $Z_0>Z_S$ [if we do not here take into account the resource constraint (1)], then reaching point S also. Moreover, it can be verified by substitution that the linear strategy a_L also goes through the singular point E.

The a_4- (a_3-) *family* of strategies represents the solution curve of (14) coupled with $c_1>0$ when $n<\delta Z$ ($n>\delta Z$), while the a_1- (a_2-) *family* of strategies represents the solution curve of (14) coupled with $c_1<0$ when $n<\delta Z$ ($n>\delta Z$). Moreover, the left-branch of the a_4-*family* of strategies also starts from the origin, while its right-branch starts from any initial value $Z_0<Z_E$, both of which reach the same point on line C_1. The left branch of the a_1-*family* of strategies starts from the origin and reaches a point on the steady state line C_1, while its right branch starts from point $(n/\delta,0)$ and reaches the same point on line C_1; therefore, those strategies never hit the horizontal axis except for the origin and point $(n/\delta,0)$. On the other hand, when the a_2- and a_3-families of strategies start from any initial value $Z_0>Z_E$, the a_2-*family* of strategies approaches point $(n/\delta,0)$, while the a_3-*family* of strategies goes to plus infinity, as illustrated in Figs.1 and 2.

Nevertheless, not all integral curves in Figs.1 and 2 are qualified as MP equilibrium strategies. There are three additional requirements which have to be met. The first prerequisite is that strategies should not violate the resource constraint (1). This implies that $a(Z)$ should be bounded to the nonnegative region below a horizontal line with intercept 1 in Figs.1 and 2.

The second requirement is that strategies should cover the entire (or global) domain $[0, \infty)$ in a continuous way.[7] At first glance this requirement seems to eliminate all strategies a_j, $j = 1, \ldots, 4$, and a_L. Nevertheless, strategies can potentially be continuously extended either by the cornered strategy $a = 1$ along the resource constraint (1), and/or by the non-aggressive strategy $a = 0$ on the horizontal axis (see Rowat, 2007). Both potential extensions are triggered by the corner solutions where the equality in (8) does not apply. Despite this, the strategies of the a_3-family that does not reach the resource constraint (1) are immediately eliminated because they can neither cover the global domain $[0, \infty)$ by themselves nor be extended by any strategy in a continuous way.

Furthermore, the non-aggressive strategy $a(Z) = 0$ on the horizontal axis is eliminated, since this strategy does not satisfy (8) for $Z \in (0, \infty)$, as shown in Lemma 1 in Appendix C. As a result, extending the a_1- and a_2-families of strategies by the patching strategy $a = 0$ to the global domain is not possible.

Turn to the linear strategy $\widehat{a}_L$, where the hat indicates those strategies extended by the patching cornered strategy $a = 1$. Since strategy $\widehat{a}_L$ can continuously pass through point E in Fig. 1, the coordinates of which are given by (13), strategy $\widehat{a}_L$ is continuos over the entire domain $[0, \infty)$. This property is also obtained in the case illustrated in Fig. 2 where strategy $\widehat{a}_L$ does not go through point E. Here, the patching strategy $a = 1$ instead of the interior strategy a_L will cross locus C_2 and thus strategy $\widehat{a}_L$ is continuos over the entire domain of Z in Fig.2 as well. Taken together, the extended linear strategy $\widehat{a}_L$ survives as a candidate for a subgame perfect strategy which will be discussed below.

The third and final requirement is subgame perfection. We have to show that there do not exist profitable deviations from strategy $\widehat{a}_L$. Strategy $\widehat{a}_L$ is stable in the sense that from an arbitrary initial value of Z strategy $\widehat{a}_L$ can reach the steady state point S in the long run. As a result, the convergence towards the finite steady state point S, together with (B4), ensures that

[7] Tsutsui and Mino (1990), Itaya and Shimomura (2001) and Rubio and Casino (2002) restrict the state space in order to get continuos, stable and subgame perfect equilibrium strategies. In particular, Tsutsui and Mino treat the domain of a state variable as endogenous to get different stable Markov perfect strategies associated with different steady states. Unfortunately, this approach prevents comparison of payoffs between strategies.

the value function associated with strategy $\widehat{a}_L$ is bounded. Hence we can apply the sufficiency conditions in Theorem 3 of Rowat's (2007) to demonstrate its subgame perfectness.

Applying the above sufficiency conditions and using the cornered strategy $a = 1$, we can construct more subgame perfect equilibrium strategies. As shown in Lemma 2 of Appendix C, the range of the state variable Z where the cornered strategy $a = 1$ remains as an optimal one (i.e., $(\partial p_i/\partial a_i) Z > V_i'(Z)$) depends on the chosen value of c_3 in (C6). In particular, the cornered strategy $a = 1$ remains subgame perfect over the domain $(0, \infty)$ if the constant c_3 is larger than the infimum of the set of c_3 satisfying inequality (C7) in Appendix C (see Fig. 3). When c_3 is smaller than that infimum, more of the a_4-families of strategies patched by strategy $a = 1$ will be subgame perfect as c_3 increases to that infimum, as shown in Fig.3.

In Fig.1 (i.e., when $a_E < 1$) the strategy a_4 patched by the cornered strategy $a = 1$ is qualified as a subgame perfect equilibrium strategy over the domain $(0, \infty)$ if c_3 is greater than that infimum. If this is a case, the linear MP strategy ends up with the highest level of steady state durable goods stock. On the other hand, in Fig.2 the patched strategy $\hat{a}_4$ is still qualified if c_3 is greater than that critical value. In addition, the strategies left off the strategy $\hat{a}_1$ which is tangent to the constraint $a = 1$ and then connected with the strategy $a = 1$ at that tangent point in Fig.2 are also subgame perfect. As a result, there are non-linear MP strategies which entail higher levels of durable goods stock in the steady state such as $\hat{Z}$ as compared to that supported by the linear strategy $\hat{a}_L$.

We summarize with the following theorem:

Proposition 1 *Consider the dynamic game defined by* (5) *and assume that the constant of integration* c_3 *is chosen so as to satisfy inequality (C7) in Appendix C. Symmetric Nash equilibria in continuos and asymptotically stable Markov perfect strategies must satisfy the following:*

(i)If $a_E \leqq 1$, *thus* $r(\rho + \delta)(n-1) \leqq [1 - r(n-1)]\delta$, *then there exist uncountably many non-linear Markov perfect strategies and the unique linear Markov perfect equilibrium strategy, coupled with strategy* $a(Z) = 1$, *that are globally defined over the entire domain of a state space, leading to steady state equilibria ranging over* $(0, Z_S]$; *and*

(ii)If $a_E > 1$*, thus* $r(\rho+\delta)(n-1) > [1-r(n-1)]\delta$*, then there exist uncountably many non-linear Markov perfect strategies, coupled with strategy* $a(Z) = 1$*, that are globally defined over the entire domain of a state space, leading to steady state equilibria ranging over* $\left(0, \hat{Z}\right]$ *with* $Z_S < \hat{Z} < Z_E$*, where*

$$Z_E = \frac{n}{\delta} \text{ and } Z_S = \frac{r(n-1)(\delta+\rho)}{(n-1)r\rho+\rho}. \tag{16}$$

Proposition 1 implies that there exist multiple MP strategies in the space of nonlinear strategies including the linear strategy, even in the case where the domain is globally defined. It can be best understood that multiplicity of MP strategies and of the associated steady state equilibria arise from the incomplete transversality condition, as pointed out by Tsutsui and Mino (1990). Since there are unaccountably many asymptotically stable nonlinear Markov strategies that converge to different steady states, it is impossible to uniquely pin down the constant of integration c_1 in (14) in addition to the constant c_3 in (C6) of Appendix C, so that we are unable to identify the unique path supported by a particular nonlinear MP strategy which converges the unique steady state. The emergence of a continuum of steady states supported by multiple non-linear MP equilibrium strategies significantly distinguishes our results not only from the results of Hirshleifer (1991, 1995) and Skaperdas (1992) using a static model but also from those of Maxwell and Reuveny's (2005) using a dynamic model with myopic agents in which the unique one-shot Nash equilibrium prevails or is repeated every period.

Another important aspect of Proposition 1 is that given any initial stock of Z, the economy approaches the range of steady state equilibria where the common-pool stock takes a positive value and individual aggressiveness takes an intermediate value between zero and one. In this sense, (implicit) 'partial cooperation' can be seen as a best response to the risk of appropriation. In affluent economies where the level of the stock variable is sufficiently large, investment in aggression reaches the maximum possible level (i.e., $a = 1$) in finite time. It then is decreasing until the steady state S is reached. Put differently, in affluent societies where there is a large amount of the common pool stock, a full fighting strategy will be rationally and inevitably chosen during the transition to the steady state. On the other hand, if the initial stock level is relatively low at the start of the game investment in aggressive behavior monotonically increases toward the steady state S over time. That is, as the common-pool stock Z gets larger over time, the contenders will become greedier, because the marginal gain of appropriation will be higher. In the long run the economy will reach a situation where 'partial cooperation' prevails in the sense that every agent chooses to contribute to the production of the common-pool stock Z to some extent. Although such 'partial cooperation' is also found in the static models of Hirshleifer and Skaperdas, differing degrees of 'partial cooperation' depending on the coordination of expectations formed by agents in the present model never emerge in their models.

3 Comparative Static Analysis

In this section we discuss the effects of a change in the model parameters on the transition path of the linear strategy a_L as well as on the associated long-run equilibrium point S, since other non-linear MP strategies display almost the same comparative statics properties. Consider first the effects of a change in the productivity (or effectiveness) of conflict technology. The recent developments in computer networks and their applications mentioned in the introduction are examples of technological change that potentially puts at risk the intellectual property rights of the software and music industry. In the model, a change in the productivity of the conflict technology is captured by a change in r. The shift of point S can be calculated by differentiating (15) and Z_S in (16) with respect to the parameter r, respectively:

$$\begin{aligned} \frac{da_S}{dr} &= \delta\,(\delta+\rho)\,(n-1)\,\Delta^{-2} > 0, \\ \frac{dZ_S}{dr} &= -n\,(n-1)\,(\delta+\rho)\,\Delta^{-2} < 0, \end{aligned}$$

where $a_S \equiv a_L(Z_S)$ and $\Delta \equiv (n-1)\,r\rho + \delta > 0$. Although an increase in r does not affect the line C_1, this increase strengthens the intensity of appropriation associated with every level of the common-pool stock Z during the transition path, thus making the linear strategy (15) steeper. Since the productivity of appropriation becomes more effective with higher r, all competing agents engage in more aggressive behavior in the hope of capturing more resources.

This finding is quite intuitive, and is also consistent with the static conflict models of Hirshleifer (1991, 1995).

An increase in the number of agents augments the aggregate endowment in proportion to n, which causes an outward shift of the aggregate resource constraint C_1 (i.e., scale effect), since each entrant provides one additional unit of the endowment. The larger aggregate endowment will increase the payoff each agent can expect to obtain from a given investment in aggression, thereby intensifying each agent's aggressive behavior and thus making the linear strategy a_L steeper. Since the former effect causes a counter-clockwise turn of line C_1 around point $(0, 1)$, these two effects together intensify individual appropriation, but the long run effect on the common-pool stock Z is ambiguous:[8]

$$\begin{aligned} \frac{da_S}{dn} &= r(\delta+\rho)\,\delta\Delta^{-2} > 0, \\ \frac{dZ_S}{dn} &= \left[\{1 - r(n-1) - nr\}(-r\rho+\delta) - \rho n^2 r^2\right]\Delta^{-2} \gtreqless 0. \end{aligned}$$

A higher depreciation rate causes a reduction in the level of the common-pool stock Z available to contenders, thereby discouraging appropriation. This negative prize effect causes a clockwise turn of line C_1 around point $(0, 1)$ (i.e., the aggregate resource constraint C_1 moves inward toward the origin). At the same time, a higher δ implies that the cost of reproducing the common-pool stock increases relative to the cost of aggressive behavior, which in turn strengthens an incentive for aggressive behavior, thus making the linear strategy a_L steeper. Although these two effects on appropriation operate in opposite directions, the following result indicates that the former effect will outweigh the latter effect in the long run:

$$\begin{aligned} \frac{da_S}{d\delta} &= -\rho r(n-1)\left[1 - r(n-1)\right]\Delta^{-2} < 0, \\ \frac{dZ_S}{d\delta} &= -n\left[1 - r(n-1)\right]\Delta^{-2} < 0. \end{aligned}$$

A decrease of the subjective rate of time preference makes the linear strategy a_L steeper,

[8] This effect has been also found in Result 4B of Hirshleifer (1995).

but it has no effect on line C_1. Hence we obtain the following long run effects:

$$\begin{aligned}\frac{da_S}{d\rho} &= r(n-1)\left[1-r(n-1)\right]\delta\Delta^{-2} > 0,\\ \frac{dZ_S}{d\rho} &= -n(n-1)\left[1-r(n-1)\right]r\Delta^{-2} < 0.\end{aligned}$$

The economic explanation is that as contenders become more patient (i.e., smaller ρ), they put more weight on future stocks of durable goods rather than the current one, and thus tend to spend more resources on production activity rather than on aggressive investment. This result has apparently not been addressed by Hirshleifer (1991, 1995) and Skaperdas (1992), who use the static conflict models. It stands in contrast to Skaperdas and Syropoulos's (1996) result in which the higher is the valuation of the future (i.e., smaller ρ), the stronger is the intensity of fighting. The reason for this difference is that in their two-period's model agent's first-period expenditure on appropriation increases agent's second-period payoff. Rather, our result is similar to Garfinkel's (1990) Folk Theorem type result in repeated games where a higher discount factor (i.e., a smaller ρ) makes it easier to sustain cooperative outcomes. An interpretation of our result is that long-sighted agents become less aggressive because they are more concerned about the future.

We may then summarize the discussion in the following proposition:

Proposition 2

(i) An increase in the effectiveness of aggression leads to a higher level of aggression and to a lower level of the common-pool stock;

(ii) an increase in the number of agents leads to a higher level of aggression, but the effect on the common-pool stock is ambiguous;

(iii) an increase in the depreciation rate leads to lower levels of aggression and of the common-pool stock; and

(iv)a decrease in the subjective rate of time preference (i.e., agents become more patient) leads to a lower level of aggression and to a higher level of the common-pool stock.

4 The Cooperative Solution

We will characterize the explicit cooperative solution as a benchmark steady state in the following. Assume an outside enforcer or centralized agency has the power to enforce every contender to execute its command. The cooperative strategy is one for which a centralized agency chooses the infinite-horizon planning profile of strategy $a \in R_+^n$ at the outset of the game so as to maximize $\int_0^\infty Ze^{-\rho t}dt$ subject to $\dot{Z} = n - \sum_{j=1}^n a_j - \delta Z$ where $a_j \in [0, 1]$ for all j. Clearly, this optimization yields a totally peaceful solution, that is, $a_j(t) = 0$ for $t \in [0, \infty)$ and all j. The result is understood by noting that expenditure on appropriation is socially wasteful in the sense that it causes a deadweight loss because of the non-productive use of resources. This deadweight loss should be zero in the hypothetical case where a central agency can directly control the allocation between productive and appropriation. As a result, the superior authority should establish point $(n/\delta, 0)$ in the long run. Agents would benefit from an enforced peaceful resolution because socially wasteful aggressive activity is completely eliminated.

Combined with the comparative static results in the previous section, we obtain the following results:

Proposition 3 *Assume that a centralized agency chooses an allocation between aggressive and productive investment so as to maximize aggregate payoff. The resulting allocation dictates that agents devote all resources to the socially productive activity to obtain the Pareto efficient point* $(n/\delta, 0)$ *in the long run. In an anarchic situation where every agent follows the Markov perfect equilibrium behavior described in Sections 3 and 4, a decrease in either the productivity of aggressiveness, the depreciation rate, or the subjective rate of time preference moves the resulting long run equilibrium closer to the first-best one.*

The nuclear nonproliferation treaty which deters the development of nuclear weapons (i.e., aggressive technology) would be socially desirable in a way that makes the long run outcome resulting from a non-cooperative equilibrium behavior closer to the peaceful and efficient one. Another example is patent law, which aims at enforcing property rights on investment return and thus limits socially wasteful activities. Patent law potentially prevents a rapid fall in the expected return from new innovation, which would be a consequence of imitation by rivals. The increase in return on investment caused by secure property rights is approximately captured by the effect of a lower depreciation rate in our model.

The problem with using the cooperative solution as a benchmark is that the socially attractive steady state is not self-enforcing because it does not usually constitute a subgame perfect (Nash) equilibrium. Nevertheless, agents have to be confronted with a coordination problem in order to select the most efficient strategy in non-cooperative environments where there are multiple equilibrium strategies. Accordingly, governments or central agencies can play an important role in solving the coordination problem stated above. Multiplicity entails that a window of opportunity is available for an anarchic society in the sense that the equilibrium is not predetermined only by the stock of durable goods Z_0. A coincidental start abstracts from all means of communication and trust between agents. But agents have an interest in avoiding the loss caused by aggression and may transcend their lack of confidence to achieve the Pareto-dominant equilibrium. Benevolent governmental institutions, even when they cannot enforce full property rights, might be important devices for solving the coordination problem, such that the society in the best circumstance is able to coordinate on the preferred among all feasible equilibria.

5 Conclusions

The first message of this paper is that completely aggressive behavior is not necessarily a rational strategy for an agent in anarchic situations. Rather, every agent will voluntarily and uniquely choose 'partial cooperation', in which each agent devotes his individual resource both to productive and appropriation at the same time, even though agents act fully rational and are guided by their self-interest. The primary driving force is the durability of the common-pool stock in conjunction with the forward looking behavior of agents. These intrinsically dynamic ingredients induce each contender to behave 'partially cooperatively', even without punishments and threats, unlike Garfinkel (1990). In other words, either if the stock depreciates completely each period or if contenders have myopic foresight, they are less motivated to follow a cooperative behavior in producing a commonly-accessible good.

The second message is that the use of nonlinear MP strategies would provide multiplicity of equilibrium strategies and uncountably many long run equilibria including the better outcome supported by the linear MP strategy, which is consistent with Dockner and van Long (1993) and Rowat (2007) in a differential game of international pollution control between two countries.[9] In other words, contenders may be able to reach a self-enforcing agreement or implicit tacit collusion which makes the long-run outcome closer to the first-best one as compared to the linear MP strategies. However, the problem of selecting among infinitely many nonlinear strategies clearly requires some coordination, or more explicitly, some preplay communication. In non-cooperative environments which do not allow such preplay communication or negotiation it may be one of the primary functions of centralized institutions to play a critical role to resolve such a coordination problem, as stated in Section 4.

In addition to the above-mentioned role of the government, the results of the present paper, which would be the third message, suggest that the government (or central agency) should also play the following roles in order to achieve a socially desirable outcome, and thus move the long run outcome closer to the more efficient ones. To do this, governments should attempt to deter the development of the conflict technology, reduce the depreciation rate of common-pool assets or induce people to have longer sight. Such structural or institutional reforms, including laws or institutional schemes, could reduce the likelihood of aggression, and thus lead to peaceful and more efficient outcomes in the long run.

The model presented in this paper should be developed further in several directions. In particular, introducing asymmetry among agents would enable us to compare the results of the present model with those static models which do incorporate asymmetric agents. The 'paradox of power' (Hirshleifer, 1991) may be generated in such an asymmetric dynamic conflicting model.

[9] Dockner and Long (1993) find that the first-best pollution stock can be approximated through an appropriate choice of non-linear MP strategies, *when the rate of time preference should be sufficiently small.* Our result slightly differs from theirs in that the linear MP strategy itself becomes closer to the first-best outcome as the rate of time preference is smaller.

Appendix A: Derivation on the HJB equation

In this appendix we show how to derive (10) in the text. Assuming an interior solution, we solve (8) for each agent to get the optimal strategy $a_i = a_i(Z)$. By substituting this optimal strategy into (6), the HJB equation (6) associated with agent i is transformed into

$$\rho V_i(Z) = p_i(a_1(Z), ..., a_n(Z))Z + V_i'(Z)\left[\sum_{j=1}^{n}(1 - a_j(Z)) - \delta Z\right]. \tag{A.1}$$

By differentiating (A.1) with respect to Z and applying the envelope theorem to the resulting expression, we obtain

$$\begin{aligned}\rho V_i'(Z) &= \sum_{j=1}^{n}\frac{\partial p_i}{\partial a_j}a_j'(Z)Z + p_i(.) + V_i''(Z)\left[\sum_{j=1}^{n}(1 - a_j(Z)) - \delta Z\right] \\ &\quad + V_i'(Z)\left[-\sum_{j=1}^{n}a_j'(Z) - \delta\right].\end{aligned} \tag{A.2}$$

Substituting (8) and (9) into $V_i'(Z)$ and $V_i''(Z)$ in (A.2), respectively, and exploiting symmetry yields

$$\begin{gathered}0 = (n-1)\left[\frac{\partial p_i}{\partial a_k}Z - \frac{\partial p_i}{\partial a_i}Z\right]a'(Z) + p(.) + \\ \left[\frac{\partial^2 p_i}{\partial a_i^2}a_i'(Z) + (n-1)\frac{\partial^2 p_i}{\partial a_k \partial a_i}a_k'(Z)\right]Z\left[n(1 - a(Z)) - \delta Z\right] \\ + \frac{\partial p_i}{\partial a_i}\left[n(1 - a(Z)) - \delta Z\right] - (\delta + \rho)\frac{\partial p_i}{\partial a_i}Z,\ k \neq i.\end{gathered} \tag{A.3}$$

Since the assumption of symmetry further allows us to make use of the following simple expressions:

$$\begin{aligned}p_i &= \frac{1}{n},\ \frac{\partial p_i}{\partial a_i} = \frac{r(n-1)}{n^2 a},\ \frac{\partial p_i}{\partial a_k} = -\frac{r}{an^2}, \\ \frac{\partial^2 p_i}{\partial a_i^2} &= r(n-1)\frac{n(r-1) - 2r}{n^3 a^2},\ \frac{\partial^2 p_i}{\partial a_k \partial a_i} = \frac{r^2(-n+2)}{n^3 a^2},\end{aligned} \tag{A.4}$$

we substitute those expressions into (A.3) yielding

$$
\begin{aligned}
0 = \frac{n-1}{an^2}\left[-r - r\left(n-1\right)\right] Za'(Z) + \frac{1}{n} + \\
\frac{r\left(n-1\right)}{n^3a^2}\left[r\left(-n+2\right) + n\left(r-1\right) - 2r\right] Z\left[n\left(1-a(Z)\right) - \delta Z\right] a'(Z) \\
+ \frac{r\left(n-1\right)}{n^2a}\left[n\left(1-a(Z)\right) - \delta Z\right] - \left(\delta+\rho\right)\frac{r\left(n-1\right)}{n^2a}Z.
\end{aligned}
\tag{A.5}
$$

Further rearranging (A.5) gives rise to (10) in the text.

Appendix B: Linear Strategy

We will show below that (15) represents a linear strategy. Under symmetry, rewrite the HJB equation (6) as follows:

$$
\rho V(Z) = \max_{a_i \in [0,1]} \left[p\left(a_1, a_2, ..., a_n\right) Z + V'(Z)\left\{n\left(1-a\right) - \delta Z\right\}\right]. \tag{B.1}
$$

Suppose that the value function is linear, that is, $V(Z) = A + BZ$, where A and B are unknown constants. Substitute this hypothetical value function into the above HJB equation to get

$$
\rho\left[A + BZ\right] = \max\left[\frac{1}{n}Z + B\left\{n\left(1-a\right) - \delta Z\right\}\right]. \tag{B.2}
$$

Substituting further the (interior) first-order condition (8), that is, $a = r\left(n-1\right)Z/Bn^2$ into a in (B.2), we obtain

$$
\rho A + \rho BZ = \frac{1}{n}Z + B\left\{n\left(1 - \frac{r\left(n-1\right)}{Bn^2}Z\right) - \delta Z\right\}. \tag{B.3}
$$

Comparing the coefficient of Z and the constant in both sides of (B3) yields

$$
\rho A - Bn = 0 \ \text{ and } \ \rho B - \frac{1}{n} + \frac{r\left(n-1\right)}{n} + B\delta = 0.
$$

Solving the above simultaneous system of equations in terms of A and B to yield

$$A = \frac{n}{\rho} B \ \text{ and } B = \frac{1 - r(n-1)}{(\rho + \delta) n}.$$

Further substitution of these expressions into the first-order condition, $a = r(n-1)Z/Bn^2$, yields (15). The corresponding value function is given by

$$V(Z) = \frac{1 - r(n-1)}{(\rho + \delta) n} \left(\frac{n}{\rho} + Z \right). \tag{B.4}$$

Appendix C: Subgame Perfect Solutions

To prove the existence of MP Nash equilibria we apply a sufficiency theorem stated in Theorem 3 of Rowat's (2007). To do this, we first have to show that the strategy is feasible, which is defined in Definition 1 of Rowat's (2007). Although it may be appropriate to distinguish between the value function associated candidates strategies and that associated with the well-defined strategies like those of Rowat's, it is omitted for the sake of notational simplicity but with understanding that the solutions have to pass the further tests to qualify for an equilibrium strategy satisfying the properties stated in the text.

Lemma 1 *The strategy $a(Z) = 0$ for $Z \in (0, \infty)$ is not an equilibrium strategy.*

Proof. Suppose that $(\partial p_i / \partial a_i) Z - V_i'(Z) < 0$. Then it follows from (8) that $a_i = 0$. In this case, the HJB equation (6) becomes

$$\rho V_i(Z) = \frac{1}{n} Z + V_i'(Z)(n - \delta Z). \tag{C.1}$$

By integration and imposing symmetry, we have

$$V(Z) = \frac{(n - \delta Z)^2 [n + (\rho + 2\delta) Z]}{n(\rho + 3\delta)(\rho + 2\delta)} + c_2 (n - \delta Z)^{-\frac{\rho}{\delta}}, \tag{C.2}$$

where c_2 represents a constant of integration. When $c_2 \neq 0$, $\lim_{Z \to n/\delta} V(Z) = \pm\infty$. This implies that the strategy $a(Z) = 0$ for $Z \in [0, \infty)$ is not an equilibrium strategy whenever $c_2 \neq 0$, because the value function (C.2) associated with this strategy is unbounded at $Z = n/\delta$ and thus the strategy $a(Z) = 0$ for $Z \in [0, \infty)$ ceases to be continuous at this point.

Next, consider the case where $c_2 = 0$. In this case it turns out that the derivative of the resulting value function with respect to Z is bounded above:

$$V'(Z) = \frac{n - \delta Z}{n(\rho + 3\delta)} \left[(n - 3\delta Z) - \frac{2\delta n}{\rho + 2\delta} \right] < 0 \text{ for } \forall Z > 0. \tag{C.3}$$

On other hand, since

$$\lim_{a_i \to 0} \frac{\partial p_i}{\partial a_i} Z = \lim_{a_i \to 0} \frac{r(n-1)}{n^2 a} Z \to \infty \text{ for } \forall Z > 0, \tag{C.4}$$

inequality $(\partial p_i / \partial a_i) Z < V_i'(Z)$ never holds except for $Z = 0$. Hence, the cornered strategy $a(Z) = 0$ is not an equilibrium strategy for $Z \in (0, \infty)$. ■

Lemma 2 *There exists a constant of integration which makes the strategy $a(Z) = 1$ an equilibrium strategy over $Z \in (0, \infty)$.*

Proof. When all players play strategy $a_i = 1$, the HJB equation (6) becomes

$$\rho V(Z) = \frac{1}{n} Z + V'(Z)(-\delta Z). \tag{C.5}$$

By integration and imposing symmetry, we have

$$V(Z) = \frac{Z + Z^{-\frac{\rho}{\delta}} n(\rho + \delta) c_3}{n(\rho + \delta)}, \tag{C.6}$$

where c_3 represents a constant of integration. When setting $a = 1$ in $\partial p_i / \partial a_i$ yields $(\partial p_i / \partial a_i) Z \equiv r(n-1) Z / n^2$, the first-order condition $(\partial p_i / \partial a_i) Z \geq V_i'(Z)$ only allows (C.6) to hold for the values of Z satisfying

$$\frac{\delta}{\rho n} \left[\frac{1}{\rho + \delta} Z^{\frac{\rho + \delta}{\delta}} - \frac{r(n-1)}{n} Z^{\frac{\rho + 2\delta}{\delta}} \right] \leq c_3. \tag{C.7}$$

Since the first exponent inside the brackets on the left-hand side of (C.7) is smaller than the second exponent term, it dominates for smaller values of Z, whereas for larger values of Z

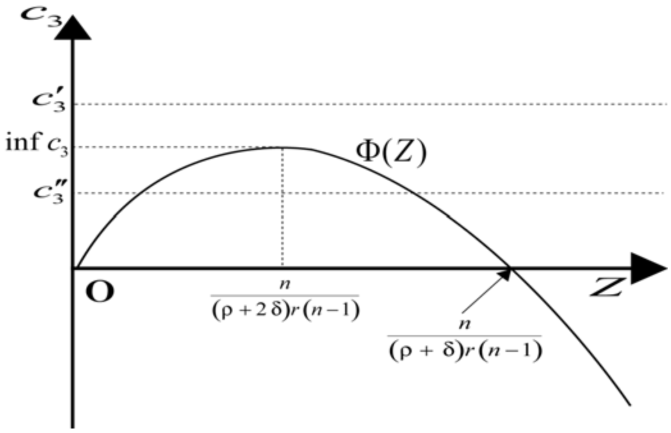

Figure 3: Transitions between the corner and interior solutions.

the second term overpowers it. Indeed, differentiation of the left-hand side of (C.7), which is denoted by $\Phi(Z)$, with respect to Z yields

$$\frac{d\Phi(Z)}{dZ} = \frac{1}{\rho n} Z^{\frac{\rho}{\delta}} \left[1 - \frac{r(n-1)(\rho+2\rho)}{n} Z \right],$$

which implies that for smaller values of Z the left-hand side of (C.7) takes a positive slope, while for larger values of Z that takes a negative slope. Noting that $\Phi(0) = 0$, these results together imply that the function $\Phi(Z)$ displays a U-shaped curve, as illustrated in Fig.3. This diagram shows that for larger values of c_3, as happens with c_3' in Fig.3, (C.7) is satisfied at any value of Z and thus $a(Z) = 1$ is a global solution to the HJB equation (C.5), whereas for smaller values of c_3 as happens with c_3'' in Fig.3, it is satisfied for smaller Z, is then violated, and finally is satisfied for larger values of Z again. ■

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