Discussion Papers In Economics And Business

Consumption Externalities and Equilibrium Dynamics with Heterogenous Agents

Kazuo Mino and Yasuhiro Nakamoto

Discussion Paper 08-30

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Consumption Externalities and Equilibrium Dynamics with Heterogenous Agents

Kazuo Mino and Yasuhiro Nakamoto

Discussion Paper 08-30

September 2008

この研究は「大学院経済学研究科・経済学部記念事業」
基金より援助を受けた、記して感謝する。

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Consumption Externalities and Equilibrium Dynamics with Heterogenous Agents*

Kazuo Mino† and Yasuhiro Nakamoto‡

Abstract

This paper explores the effect of consumption externalities on equilibrium dynamics of a standard neoclassical growth model in which there are two types of agents. To emphasize the presence of heterogenous agents, we distinguish intergroup consumption externalities from intragroup consumption externalities. We show that if the intragroup externalities dominates the intrergroup external effects, then the steady state equilibrium satisfies saddle-point stability and the equilibrium path of the economy is uniquely determined. In contrast, if the intergroup external effects of consumption are strong enough, the steady-state equilibrium is either unstable or locally indeterminate. Based on the analytical as well as numerical considerations, we give intuitive implications of stability conditions.

Keywords: Consumption externalities, Equilibrium determinacy, Heterogeneous agents, Progressive taxation

JEL Classification Code: E52, O42

---

*The first and second authors respectively acknowledge financial support by Grant-in-Aid for Scientific Research (No.17730139) and the Research Fellowships of the Japanese Society for the Promotion of Science for Young Scientists.

†Graduate School of Economics, Osaka University, 1-7 Machikaneyma, Toyonaka, 563-0043 Japan, e-mail: mino@econ.osaka-u.ac.jp

‡Graduate School of Economics, Osaka University, 1-7 Machikaneyma, Toyonaka, 563-0043 Japan, e-mail: ege006ny@mail2.econ.osaka-u.ac.jp

1 Introduction

Recently, there is a renewed interest in consumption external effects in dynamic macroeconomics. While the earlier contributions such as Abel (1990) and Galí (1991) focus on the role of consumption externalities in the asset-pricing models, the recent studies treat a wider class of issues. For example, the recent investigations consider external effects of consumption on optimal taxation (Ljungqvist and Uhlig 2000), on the relation between savings and long-term economic growth (Carroll et al. 1996 and 2000) as well as on the efficiency of equilibrium (Liu and Turnovsky 2005). A common feature of this literature is that most studies employ the representative agent frameworks. In this literature the consumption external effect is formulated in such a way that an individual consumer's felicity depends on the average level of consumption in the economy as well as on her own consumption. In the equilibrium of representative-agent economies the individual and the average levels of consumption coincide each other and, therefore, the presence of consumption externalities generally produces quantitative effects rather than qualitative effects: the equilibrium dynamics and the steady state characterization are usually the same as those of the models without consumption externalities.

Unlike the mainstream literature mentioned above, this paper examines the role of consumption externalities in the presence of heterogenous agents. Since the external interactions among the consumers tend to be much more complex in an economy with heterogenous agents than in the representative-agent counterpart, the presence of consumption external effects would yield fundamental impacts on the dynamic behavior of the economy if we consider heterogeneity of consumers. Using a simple neoclassical growth model with two types of agents, we confirm our prediction. We show that even in the symmetric steady state where every agent has the same levels of income and wealth, the dynamic behavior of the economy may not exhibit a regular saddle point stability. The equilibrium path of the economy could be either unstable or indeterminate. Thus consumption externalities, together with heterogeneity of agents, would yield a variety of dynamic behaviors, even if there is neither production external effect nor complex preference structure associated with labor-leisure choice.

The analytical framework of this paper is the standard neoclassical growth model with infinitely-lived agents. In this setting it has been well known that there exists a continuum of steady states if all the agents have an identical time discount rate, while the agent with the lowest time discount rate ultimately owns the entire capital stock if the time discount rate of each agent is not identical: see, for example, Becker (1980) and Soger (2002). To avoid those extreme outcomes, we introduce nonlinear income taxation into the base model. As pointed out by Sarte (1997), a nonlinear income tax scheme may yield a unique interior steady state in which every agent holds a positive amount of capital, even though the agents have heterogenous rates of time preferences. In this paper, we assume that the tax rate levied on an individual consumer depends on her income relative to the average level of income in the economy at large. This assumption, which follows Guo and Lansing (1998) and Lie and Sarte (2004), enables us to establish the symmetric steady state equilibrium in which wealth and income are equally distributed among the households. Owing to these two assumptions, the steady-state equilibrium of our economy with heterogenous agents is essentially the same as the stationary equilibrium of the representative agent economy. Hence, we may elucidate how the introduction of heterogeneity of agents affect the role of consumption externalities in the transition process of an economy.[1]

Our study presents two main findings. First, either if there are only intragroup consumption externalities or if the magnitude of intragroup externality denominates the intergroup effects, then a uniquely given steady state exhibits a regular saddle point property. In this case, the equilibrium path is determinate and it converges to the symmetric steady state equilibrium. Our second finding is that if the intergroup external effects have larger impacts on the individual consumption decision than the intragroup external effects, then the symmetric steady state equilibrium is either

[1] In a related study project, Mino (2007) and (2008) introduce consumption externalities into overlapping generations models where intragroup and intergroup externalities are respectively replaced with intragenerational and intergenerational external effects.

totally unstable or locally indeterminate. In the latter case, there exists a continuum of converging paths around the steady state, so that expectations-deriven economic fluctuations may emerge.

In the existing literature, García-Peñalosa and Turnovsky (2007) also study a neoclassical growth model with heterogenous agents and consumption externalities. The key assumption in their investigation is that every agent has a quasi-homothetic preference so that the aggregate behavior of the economy is independent of wealth distribution. Therefore, the role of consumption externalities in their model is essentially the same as that in the representative agent models. On the other hand, because of the introduction of nonlinear taxation, the macroeconomic stability of our model depends on wealth distribution and distribution dynamics are affected by external interactions among consumers. As a result, the presence of consumption externalities plays a more prominent role in our model than in García-Peñalosa and Turnovsky (2007).

It is also to be noted that Alonso-Carrera et al. (2008) and Chen and Hasu (2007) reveal that equilibrium indeterminacy may hold in the representative agent models with consumption externalities. Alonso-Carrera et al. (2008) show that if labor-leisure choice is allowed and if the utility function is not homothetic with respect to private and average consumption levels, then the one-sector growth model with consumption externalities may generate indeterminacy of equilibrium.[2] Chen and Hasu (2007) examines a two-sector growth model and shows that the presence of consumption externalities affects resource allocation between two production sectors, which may cause multiple equilibria.[3] Indeterminacy shown in these studies is, therefore, partially depends on the complex preference structure or on the production side of the model economy. In contrast, our study uses a one-sector neoclassical

[2] More precisely, the presence of indeterminacy requires that the marginal substitution between private and average consumption is not constant along the equilibrium path where the average consumption of the economy at large coincides with the level of private consumption.

[3] Weder (2000) also examines equilibrium indeterminacy in growth models with consumption externalities, but his model also involves production externalities.

growth model with fixed labor supply, so that the presence of heterogenous agents is the main source of multiple equilibria.

The next section sets up the analytical framework. Section 3 examines the dynamic behavior of our model economy and presents intuitive implication of the stability conditions. Section 4 presents numerical examples. Concluding remarks are given in Section 5.

2 The Model

2.1 Households

Suppose that there are two groups of infinitely-lived agents. Each group consists of a continuum of identical households. The felicity function and the initial holding of wealth of the representative household in each group are different from each other, but all the agents in the economy has the same rate of time preference. For simplicity, we assume that population in the economy is constant over time, so that the mass of each group will not change. We also assume that the economy is closed and the government does not issue interest bearing bonds. Thus the stock of capital is the only net asset held by the agents.

The representative agent in group i $(i = 1, 2)$ supplies one unit of labor in each moment and maximizes a discounted sum of utilities over an infinite time horizon. The objective functional of the representative agent in group i is given by

$$U_i = \int_0^{+\infty} e^{-\rho t} u^i(c_i, C_i, C_j) dt, \quad \rho > 0, \quad i, j = 1, 2, \quad i \neq j. \tag{1}$$

In the above, ρ denotes a given rate of time discount, c_i private consumption of group i agent, and C_i and C_j respectively represent the average levels of consumption in groups i and j. The instantaneous utility function, $u^i(\cdot)$, is assumed to be monotonically increasing and strictly concave in private consumption, c_i. It is also assumed that in the symmetric equilibrium where $c_i = C_1 = C_2$, the utility function holds the Inada conditions: $\lim_{C\to 0} u_1^i(C, C, C) = \infty$ and $\lim_{C\to\infty} u_1^i(C, C, C) = 0$, where $u_m^i(\cdot)$ $(m = 1, 2, 3)$ denotes the partial derivative of the utility function with respective to the m-th variable in $u^i(\cdot)$.

The key assumption about the instantaneous felicity function in (1) is that we distinguish intragroup externalities from intergroup externalities. Namely, an agent's concern with the consumption levels of members in her own group may be different from the concern with consumption of agents in the other group. The presence of intergroup external effects produces the outcomes specific to models with heterogenous agents.

According to the taxonomy given by Dupor and Liu (2003), the external effect of consumption on an individual utility may be either negative (jealousy) or positive (admiration). In addition, each consumer would be a conformist who likes being similar to others (keeping up with the Joneses) or an anti-conformist who wants to be different from others (running away from the Joneses). We allow, for example, an agent in a particular group feels jealousy as to consumption of others in her group but admires consumption of agents belongs to the other group. Such a situation may emerge, the agents in the rich group admire an increase in the benchmark level of consumption in the poor group, whereas they have jealousy as to the consumption level of other members in her group. In a similar vein, it is possible to assume that an agent wants to conduct the similar consumption as her own group's members, but she stays away from consumption behavior of the other group's agents. Hence, even though there are only two types of agents, the external effects among the consumers cover a richer class of situations than that treated in the representative-agent economy.[4]

As usual, the negative externality (jealousy) is expressed by $u_j^i(\cdot)\,(= \partial u^i / \partial C_j) < 0 \quad (i = 1, 2,\ j = 2, 3)$, while the positive externality (admiration) means that $u_j^i(\cdot)$ has a positive value. Similarly, if the marginal utility of private consumption increases with external effects, that is, $u_{1j}^i(\cdot)\,(= \partial^2 u^i / \partial C_j \partial c_i) > 0$, then the consumer's pref-

[4]Collier (2004) and Garriga (2006) present careful dissections on the formulation of consumption external effects. Frank (2005) interprets the households' concern about relative consumption based on a behavioral economics consideration.

erence exhibits conformism: the consumer likes being similar to others. In contrast, the consumer is anti-conformist if $u^i_{1j}(\cdot)\,(=\partial^2 u^i/\partial C_j \partial c_i) < 0$. In what follows, we assume that, regardless of the forms of external effects, the effects of a change in the private consumption dominate the impact on her utility caused by external effect. More specifically, the utility function is assumed to satisfy the following properties:

$$u^i_1(\cdot) + u^i_2(\cdot) > 0, \tag{2a}$$

$$u^i_1(\cdot) + u^i_3(\cdot) > 0, \tag{2b}$$

$$u^i_{11}(\cdot) + u^i_{12}(\cdot) < 0, \tag{2c}$$

$$u^i_{11}(\cdot) + u^i_{13}(\cdot) < 0, \tag{2d}$$

$$u^i_1(\cdot) + u^i_2(\cdot) + u^i_3(\cdot) > 0, \tag{2e}$$

$$u^i_{11}(\cdot) + u^i_{12}(\cdot) + u^i_{13}(\cdot) < 0, \tag{2f}$$

where $i = 1$ and 2. Conditions $(2a)$ and $(2b)$ mean that the marginal utility of own consumption dominates impacts produced by consumption externalities. Conditions $(2c)$ and $(2d)$ show that the diminishing marginal utility of own consumption dominates the outward looking conformism. Assumptions $(2e)$ and $(2f)$ ensure that, in a social symmetric equilibrium $C_1 = C_2$, the sign conditions given by $(2a)$ and $(2c)$ still holds even considering the intragroup external effects.

The flow budget constraint for each agent is

$$\dot{k}_i = \hat{r}_i k_i + \hat{w}_i - c_i + T_i, \quad i = 1, 2, \tag{3}$$

where, k_i is capital stock owned by an agent in group i, c_i consumption, $\hat{r}_i$ after-tax rate of return to asset, $\hat{w}_i$ the after-tax real wage rate and T_i expresses a transfer from the government. The initial holding of capital, $k_i(0)$, is exogenously given.

2.2 Production

The representative firm produces a single good by use of a constant-returns-to-scale technology expressed by

$$\bar{Y} = F\left(\bar{K}, N\right).$$

Here, $\bar{Y}$, $\bar{K}$ and N denote the output, capital and labor, respectively. We normalize the number of firms to unity so that $\bar{Y}$, K and N represent their aggregate values as well. Using the homogeneity assumption, we write the production function as follows:

$$Y = f(K),$$

where $Y \equiv \bar{Y}/N$ and $K \equiv \bar{K}/N$. The productivity function, $f(K)$, is assumed to be monotonically increasing and strictly concave in the capital-labor ratio, K, and fulfills the Inada conditions. The commodity market is competitive so that the before-tax rate of return to capital and real wage are respectively determined by

$$r = f'(K), \quad w = f(K) - Kf'(K). \tag{4}$$

For simplicity, we assume that capital does not depreciate.

If we denote the number of agents in group i by N_i $(i = 1, 2)$, then the full-employment conditions for labor and capital are:

$$N_1 + N_2 = N,$$

$$N_1 k_1 + N_2 k_2 = \bar{K}.$$

Letting $\theta_i = N_i/N$, we can rewrite the full-employment conditions as follows:

$$K = \theta_1 k_1 + \theta_2 k_2, \quad 0 < \theta_i < 1, \quad \theta_1 + \theta_2 = 1. \tag{5}$$

For notational simplicity, in the following we normalize the total population, N, to one. Thus θ_i represents the mass of agents of type i as well as the population share of that type.

2.3 Fiscal Rules

The government levies distortionary income tax and distributes back its tax revenue as a transfer to each agent. We assume that the same rate of tax applies to both capital and labor incomes. The rate of tax applies to income of an agent in group i is

$$\tau_i = \tau\left(\frac{y_i}{Y}\right), \quad i = 1, 2,$$

where τ_i is the rate of tax and y_i $(= rk_i + w_i)$ denotes the total income of an agent in group i. Namely, the tax rate applied to each agent depends only on its standing in the economy.[5] The tax function $\tau(y_i/Y)$: $\Re_+ \to \Re_+$ is continuous, monotonically increasing, a twice differentiable function and satisfies $0 < \tau(y_i/Y) < 1$.

Denoting the amount of tax payment by $T(y_i, Y) = \tau\left(\frac{y_i}{Y}\right) y_i$, the average rate of tax is $T(y_i, Y)/y_i = \tau(y_i/Y)$ and the marginal tax payment is

$$\frac{\partial T(y_i, Y)}{\partial y_i} = \tau\left(\frac{y_i}{Y}\right) + \tau'\left(\frac{y_i}{Y}\right)\frac{y_i}{Y} \equiv T_m\left(\frac{y_i}{Y}\right).$$

Note that the ratio of marginal and average tax payments expresses the degree of progressiveness of taxation. When this measure is higher (resp. lower) than one, taxation is progressive (resp. regressive). In our formulation, progressiveness of taxation is represented by

$$\frac{T_m(y_i/Y)}{\tau(y_i/Y)} = 1 + \frac{\tau'(y_i/Y)}{\tau(y_i/Y)}\frac{y_i}{Y} > 1, \tag{6}$$

implying that taxation is progressive. We also assume that the marginal tax payment monotonically increases with the relative income y_i/Y, so that

$$T_m'\left(\frac{y_i}{Y}\right) = 2\tau'\left(\frac{y_i}{Y}\right) + \tau''\left(\frac{y_i}{Y}\right)\frac{y_i}{Y} > 0. \tag{7}$$

The after-tax rate of return and real wage received by type i agents are respectively written as

$$\hat{r}_i = \left[1 - \tau\left(\frac{y_i}{Y}\right)\right] r, \qquad \hat{w}_i = \left[1 - \tau\left(\frac{y_i}{Y}\right)\right] w, \qquad i = 1, 2.$$

As a result, the flow budget constraint for the household (3) is rewritten as

$$\dot{k}_i = \left[1 - \tau\left(\frac{y_i}{Y}\right)\right] y_i - c_i + T_i, \qquad i = 1, 2.$$

We assume that the government follows the balanced-budget rule, so that its flow budget constraint (in per-capita term) is

$$\theta_1 T_1 + \theta_2 T_2 = \theta_1 \tau\left(\frac{y_1}{Y}\right) y_1 + \theta_2 \tau\left(\frac{y_2}{Y}\right) y_2.$$

[5] This formulation is used by Guo and Lansing (1998) and Li and Sarte (2004).

In addition, if we assume that the government pays back an identical amount of transfer to each agent, the per-capita lump-sum transfer is given by the following:

$$T_1 = T_2 = \theta_1 \tau \left(\frac{y_1}{Y}\right) y_1 + \theta_2 \tau \left(\frac{y_2}{Y}\right) y_2. \tag{10}$$

2.4 Consumption and Capital Formation

Under the fiscal rules given above, the type i agent's flow budget constraint is expressed as

$$\dot{k}_i = \left[1 - \tau\left(\frac{y_i}{Y}\right)\right](rk_i + w) - c_i + T_i, \quad i = 1, 2, \tag{11}$$

where T_i is determined by (10). Following Guo and Lansing (1998), we assume that the households perceive the rule of progressive taxation on private income, but she takes the transfer payment, T_i, as given. Therefore, the household of type i maximizes (1) subject to (11), the initial holding of capital, $k_i(0)$ as well as to the anticipated, given sequences of $\{C_i(t), C_j(t), r(t), w(t), Y(t), T_i(t)\}_{t=0}^{\infty}$.

Since we have assumed that agents are identical in each group, the equilibrium conditions involve $c_1 = C_1$ and $c_2 = C_2$ for all $t \geq 0$. Taking these consistency conditions into account, we define the following elasticities:

$$\begin{aligned} \Omega_1^i &\equiv -\frac{(u_{11}^i(C_i, C_i, C_j) + u_{12}^i(C_i, C_i, C_j))C_i}{u_1^i(C_i, C_i, C_j)} > 0, \\ \Omega_2^i &\equiv -\frac{u_{13}^i(C_i, C_i, C_j)C_j}{u_1^i(C_i, C_i, C_j)}, \qquad i,\ j = 1, 2. \end{aligned} \tag{12}$$

Here, Ω_1^i denotes the elasticity of marginal utility of consumption within the agent's own group, which equals the inverse of an elasticity of intertemporal substitution in private consumption plus social consumption in its own group. This elasticity has a positive value due to condition $(2c)$. Additionally, Ω_2^i is the elasticity of marginal utility with respect to the other group's consumption. The sign of this term depends on how group i agents respond to the consumption level of group j agents. If agents are conformist to keep up with the consumption of the other group's members (so that $u_{13}^i > 0$), then Ω_2^i has a negative sign. On the other hand, if they do not like being similar to the other group's agents ($u_{13}^i < 0$), then Ω_2^i is strictly positive. Note that, from $(2f)$ the following is satisfied:

$$\Omega_1^i + \Omega_2^i > 0, \qquad i = 1, 2. \tag{13}$$

Solving the households' optimization problems yields a set of Euler equations for optimal consumption in such a way that

$$\underbrace{\begin{bmatrix} \Omega_1^1/C_1 & \Omega_2^1/C_2 \\ \Omega_2^2/C_1 & \Omega_1^2/C_2 \end{bmatrix}}_{M} \begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \end{bmatrix} = \begin{bmatrix} \{1 - \tau(y_1/Y) - (y_1/Y)\tau'(y_1/Y)\}r - \rho \\ \{1 - \tau(y_2/Y) - (y_2/Y)\tau'(y_2/Y)\}r - \rho \end{bmatrix},$$

where M represents the matrix with respect to the coefficients of $\dot{C}_1$ and $\dot{C}_2$. Solving this set of equations with respect to $\dot{C}_1$ and $\dot{C}_2$, we obtain

$$\begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \end{bmatrix} = \frac{C_1 C_2}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2} \begin{bmatrix} \Omega_1^2/C_2 & -\Omega_2^1/C_2 \\ -\Omega_2^2/C_1 & \Omega_1^1/C_1 \end{bmatrix} \begin{bmatrix} \{1 - \tau(y_1/Y) - (y_1/Y)\tau'(y_1/Y)\}r - \rho \\ \{1 - \tau(y_2/Y) - (y_2/Y)\tau'(y_2/Y)\}r - \rho \end{bmatrix}. \tag{14}$$

Equations (10) and (11) yield

$$\dot{k}_i = \left[1 - \tau\left(\frac{y_i}{Y}\right)\right] y_i - C_i + \theta_1 \tau\left(\frac{y_1}{Y}\right) y_1 + \theta_2 \tau\left(\frac{y_2}{Y}\right) y_2, \qquad i = 1, 2. \tag{15}$$

Summing up the flow budget constraint (15), we obtain

$$\theta_1 \dot{k}_1 + \theta_2 \dot{k}_2 = \theta_1 y_1 + \theta_2 y_2 - \theta_1 C_1 - \theta_2 C_2.$$

Thus, from $y_i = rk_i + w$ and (5), we obtain the final-good market equilibrium condition for the entire economy:

$$\dot{K} = f(K) - C,$$

where $C = \theta_1 C_1 + \theta_2 C_2$.

3 Macroeconomic Stability

3.1 Dynamic System

Equations (4) and (5) give

$$y_i = rk_i + w = f(K) + (k_i - K)f'(K),$$

leading to

$$\frac{y_i}{Y} = 1 + (k_i - K) \frac{f'(K)}{f(K)}, \quad i = 1, 2, \tag{16}$$

where $K = \theta_1 k_1 + (1 - \theta_1) k_2$. Plugging (16) into (14) and (15), we obtain a complete dynamic system that depicts the dynamic behaviors of k_1, k_2, C_1 and C_2.

The solution of this dynamic system that fulfills the initial conditions on $k_1(0)$ and $k_2(0)$ as well as the transversality conditions for the households' optimization problem, $\lim_{t\to\infty} u_1^i(C_i(t), C_i(t), C_j(t)) e^{-\rho t} k_i(t) = 0$ $(i = 1,\ 2)$, presents the perfect-foresight competitive equilibrium of our model economy.[6]

3.2 Steady-State Equilibrium

In the steady-state equilibrium, k_i and C_i $(i = 1, 2)$ stay constant over time. From (14) and (15), the steady-state conditions are given by

$$C_i^* = y_i^* + \theta_j \left[\tau\left(\frac{y_j^*}{Y^*}\right) y_j^* - \tau\left(\frac{y_i^*}{Y^*}\right) y_i^* \right], \quad i, j = 1, 2, \quad i \neq j, \tag{17}$$

$$\rho = f'(K^*) \left[1 - \tau\left(\frac{y_i^*}{Y^*}\right) - \frac{y_i^*}{Y^*} \tau'\left(\frac{y_i^*}{Y^*}\right) \right], \quad i, j = 1, 2, \tag{18}$$

where C_i^* and k_i^* denote steady-state levels of k_i and C_i.

To simplify analytical argument, we make the following assumption:

Assumption 1. $\tau\left(\frac{y_i}{Y}\right) + \frac{y_i}{Y}\tau'\left(\frac{y_i}{Y}\right)$ $(i = 1, 2)$ *is a monotonic function of the relative income, y_i/Y.*

Since the derivative of the above function with respect to y_i/Y is $2\tau'(y_i/Y) + (y_i/Y)\tau''(y_i/Y)$, from (7) Assumption 1 means that the marginal tax payment, $\partial^2(\tau y_i)/\partial y_i^2$, has the same sign for all feasible levels of y_i/Y. Given Assumption 1, it is easy to confirm the following fact:

Proposition 1. *There is a unique, symmetric steady state in which* $k_1^* = k_2^*$ and $C_1^* = C_2^*$.

[6] Mino and Nakamoto (2008) examine the role of nonlinear income taxation in a heterogenous-agent model of growth without consumption externalities.

Proof. Conditions displayed in (18) yield

$$\tau\left(\frac{y_1^*}{Y^*}\right)+\frac{y_1^*}{Y^*}\tau'\left(\frac{y_1^*}{Y^*}\right)=\tau\left(\frac{y_2^*}{Y^*}\right)+\frac{y_2^*}{Y^*}\tau'\left(\frac{y_2^*}{Y^*}\right).$$

By Assumption 1, the above equation holds if and only if $y_1^*=y_2^*$. Thus from (17) it holds that $C_1^*=C_2^*$. ■

Note that $y_1^*=y_2^*=Y^*$ and $k_1^*=k_2^*=K$ in the symmetric steady state, so that the rate of income tax in the steady-state equilibrium is a given constant, $\tau(1)$. To make the steady state feasible, from (18) we should assume the following:

Assumption 2. *Tax function $\tau(y_i/Y)$ satisfies*

$$1-\tau(1)-\tau'(1)>0. \tag{19}$$

3.3 Stability

Let us examine the local stability condition of the steady state equilibrium defined above. Linear approximation of dynamic system, (14) and (15), around the steady state equilibrium yields the following:

$$\begin{bmatrix}\dot{C}_1\\ \dot{C}_2\\ \dot{k}_1\\ \dot{k}_2\end{bmatrix}=J\begin{bmatrix}C_1(t)-C_1^*\\ C_2(t)-C_2^*\\ k_1(t)-k_1^*\\ k_2(t)-k_2^*\end{bmatrix},$$

where the coefficient matrix J is

$$J=\begin{bmatrix}0 & 0 & \partial\dot{C}_1/\partial k_1 & \partial\dot{C}_1/\partial k_2\\ 0 & 0 & \partial\dot{C}_2/\partial k_1 & \partial\dot{C}_2/\partial k_2\\ -1 & 0 & f'(k^*)[1-\theta_2(\tau(1)+\tau'(1))] & \theta_2 f'(k^*)[\tau(1)+\tau'(1)]\\ 0 & -1 & \theta_1 f'(k^*)[\tau(1)+\tau'(1)] & f'(k^*)[1-\theta_1(\tau(1)+\tau'(1))]\end{bmatrix}.$$

Each element in J is evaluated at the steady state. The precise expression of J is displayed in Appendix A of the paper.

Let us write the characteristic equation of J in such a way that

$$\lambda^4-\mathrm{Tr}J\lambda^3+\mathrm{W}J\lambda^2-\mathrm{Z}J\lambda+\mathrm{Det}J=0, \tag{20}$$

where

$$\mathrm{Tr}J = f'(k^*)[2 - \tau(1) - \tau'(1)] > 0, \tag{21}$$

$$\mathrm{W}J = f'(k^*)\rho + \frac{\partial \dot{C}_1}{\partial k_1} + \frac{\partial \dot{C}_2}{\partial k_2}, \tag{22}$$

$$\begin{aligned}\mathrm{Z}J = f'(k^*)\Bigg\{&[1 - \theta_1(\tau(1) + \tau'(1))]\frac{\partial \dot{C}_1}{\partial k_1} + [1 - \theta_2(\tau(1) + \tau'(1))]\frac{\partial \dot{C}_2}{\partial k_2} \\ &- (\tau(1) + \tau'(1))\left[\theta_1 \frac{\partial \dot{C}_1}{\partial k_2} + \theta_2 \frac{\partial \dot{C}_2}{\partial k_1}\right]\Bigg\},\end{aligned} \tag{23}$$

$$\mathrm{Det}J = -\frac{f(k^*)f'(k^*)f''(k^*)\rho}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}[2\tau'(1) + \tau''(1)]. \tag{24}$$

Note that this model involves two jumpable variables, C_1 and C_2. Thus the necessary and sufficient condition for local determinacy is that the characteristic equation (20) has two roots with negative real parts. Considering the form of (24), we see that the sign of the determinant depends on the households' preferences as well as on the income taxation scheme. In the subsequent discussion, we assume that the marginal tax payment increases with the relative income around at least the steady state.

Assumption 3. *Tax function $\tau(y_i/Y)$ satisfies*

$$2\tau'(1) + \tau''(1) > 0. \tag{25}$$

Inspecting the characteristic equation given above, we obtain the main result of this paper:

Proposition 2. *Given Assumptions 1, 2 and 3, if $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 > 0$ and $\theta_1(\Omega_1^2 - \Omega_2^1) + \theta_2(\Omega_1^1 - \Omega_2^2) > 0$, then the steady-state equilibrium satisfies local determinacy.*

Proof. Let us denote roots of the characteristic equation by λ_s $(s = 1, 2, 3, 4)$. As (21) shows, from Assumption 2 the sign of the trace of J, which equals $\Sigma_{s=1}^4 \lambda_s$, is strictly positive. Hence, at least one of the characteristic roots has positive real part. Since the marginal tax payment increases with the relative income under Assumption 3, the sign of the determinant J $(= \Pi_{s=1}^4 \lambda_s)$ is strictly positive if $\Omega_1^1\Omega_1^2 > \Omega_2^1\Omega_2^2$: see (24). This means that the number of characteristic roots with positive real parts is either two or four. Finally, we rewrite ZJ as follows :

$$ZJ = \frac{(f')^3}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}\Big\{\Gamma(k^*)\Delta^2\left[\theta_1(\Omega_1^2 - \Omega_2^1) + \theta_2(\Omega_1^1 - \Omega_2^2)\right]$$
$$- \left[\theta_2(\Omega_1^2 + \Omega_2^2) + \theta_1(\Omega_1^1 + \Omega_2^1)\right](2\tau'(1) + \tau''(1))\Big\}. \tag{26}$$

As (26) shows, if $\theta_1(\Omega_1^2 - \Omega_2^1) + \theta_2(\Omega_1^1 - \Omega_2^2) > 0$, then ZJ, which equals $\lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_4\lambda_1 + \lambda_3\lambda_4\lambda_1 + \lambda_2\lambda_3\lambda_4$, has a negative sign. Hence, there are at most two characteristic roots with positive real part. This demonstrates that there is a two–dimensional stable manifold around the steady state, implying that the competitive equilibrium path converging to the steady state is uniquely determined. ■

In order to interpret Proposition 2, it is useful to consider the following three special cases. First, the above result means that if $\Omega_2^1 = \Omega_2^2 = 0$, then the economy has a unique converging path towards the socially symmetric steady-state equilibrium, regardless of the initial distribution of wealth and form of utility function of each type of agents. That is, as long as households in a group have neither jealousy nor admiration about the average consumption level in the other group, the economy has saddle-path stability and the competitive equilibrium path is uniquely determined.

Second, even if $\Omega_2^1 \neq 0$ and $\Omega_2^2 \neq 0$ so that there are intergroup external effects, the economy has a unique converging path towards the socially symmetric steady state, as long as $\Omega_1^2 > \Omega_2^1$ and $\Omega_1^1 > \Omega_2^2$. This result holds regardless of the signs of Ω_2^1 and Ω_2^2. Therefore, even if individuals' preferences exhibit conformism or anti-conformism as to the other group's consumption behaviors, the economy satisfies the saddlepoint stability when the degree of intergroup external effects is small enough.

Finally, the economy satisfies saddlepoint stability if Ω_2^1 and Ω_2^2 have different signs. For example, assume that agents in group 1 are richer than agents in group 2. Then, it is plausible to assume that that agents in group 2 like being similar to the average consumption in the richer group (group 1), whereas agents in group 1 have anti-conformism as to the average consumption in the poorer agents (group 2). If this is the case, it holds that $\Omega_2^1 > 0$ and $\Omega_2^2 < 0$, which ensures that the economy has saddlepoint property.

The proof of Proposition 2 immediately yields the following result:

Proposition 3. *Given Assumptions 1, 2 and 3, if* $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 < 0$, *then the steady-state equilibrium is either locally unstable or indeterminate.*

Proof. Equation (24) shows that the determinant of J is strictly negative when $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 < 0$. In this case the number of characteristic roots with negative sign is either one or three. The former case means that the stable manifold is one dimensional around the steady state and thus no converging path can be selected for arbitrarily given levels of initial capital stocks, $k_1(0)$ and $k_2(0)$. If there are three stable roots, there exists a continuum of converging paths starting from the given initial distribution of capital stocks.■

The above proposition fails to specify when indeterminacy emerges. Since it is hard to present the analytical conditions for local indeterminacy (the sufficient conditions under which that the characteristic equation has three stable roots), we inspect numerical examples in Section 4.

3.4 Intuition

As shown by Propositions 2 and 3, the key to determine dynamic behavior of our model economy is the sign of $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2$. To present an intuitive implication of the stability conditions, it is useful to inspect the Euler equations given below:

$$\begin{aligned} \frac{\dot{C}_1}{C_1} &= \frac{\Omega_1^2}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}\left\{\left[1 - \tau\left(\frac{y_1}{Y}\right) - \frac{y_1}{Y}\tau'\left(\frac{y_1}{Y}\right)\right] f'(K) - \rho\right\} \\ &- \frac{\Omega_2^1}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}\left\{\left[1 - \tau\left(\frac{y_2}{Y}\right) - \frac{y_2}{Y}\tau'\left(\frac{y_2}{Y}\right)\right]]f'(K) - \rho\right\}, \end{aligned} \tag{27}$$

$$\begin{aligned} \frac{\dot{C}_2}{C_2} &= \frac{\Omega_1^1}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}\left\{\left[1 - \tau\left(\frac{y_2}{Y}\right) - \frac{y_2}{Y}\tau'\left(\frac{y_2}{Y}\right)\right] f'(K) - \rho\right\} \\ &- \frac{\Omega_2^2}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}\left\{\left[1 - \tau\left(\frac{y_1}{Y}\right) - \frac{y_1}{Y}\tau'\left(\frac{y_1}{Y}\right)\right] f'(K) - \rho\right\}. \end{aligned} \tag{28}$$

If there is no intergroup external effect, i.e. $\Omega_2^i = 0 \ (i = 1, 2)$, then the Euler equations become

$$\frac{\dot{C}_1}{C_1} = \frac{1}{\Omega_1^1}\left\{\left[1 - \tau\left(\frac{y_1}{Y}\right) - \frac{y_1}{Y}\tau'\left(\frac{y_1}{Y}\right)\right] f'(K) - \rho\right\}, \quad \Omega_1^1 > 0, \tag{29}$$

$$\frac{\dot{C}_2}{C_2} = \frac{1}{\Omega_1^2}\left\{\left[1 - \tau\left(\frac{y_2}{Y}\right) - \frac{y_2}{Y}\tau'\left(\frac{y_2}{Y}\right)\right] f'(K) - \rho\right\}, \quad \Omega_1^2 > 0. \tag{30}$$

Comparing those two sets of Euler equations, we may obtain intuitive implication as to why equilibrium indeterminacy could be present if $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2$ has a negative sign.

Suppose that the economy initially stays at the steady state equilibrium. Suppose further that all the agents anticipate that the before-tax rate of return to capital will rise so that the after-tax rate of return they receive will increase as well. In the absence of intergroup externalities, (29) and (30) state that the after-tax rate of return exceeds the time discount rate and, hence, consumption growth rate becomes positive. Namely, the current consumption is substituted with the future consumption, which raises the current saving to accelerate capital accumulation. A rise in capital stock, however, depresses the rate of return to capital due to our assumption of diminishing marginal returns. Consequently, if there are only intragroup externalities, the initial anticipation of a rise in the rate of return to capital will not be self-fulfilled, implying that equilibrium indeterminacy may not emerge.

In contrast, if there are intergroup externalities as well and if the intergroup effects dominate intragroup effects ($\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 < 0$), then an expected rise in the (after-tax) rate of return to capital will lower the growth rate of consumption within own group: see the first terms in the right- hand sides of (27) and (28) . In this situation, the future consumption is substituted with the current consumption, which depresses investment. As a result, the stock of aggregate capital will decline, so that the rate of return to capital will rise. This indicates that the initial change in expectations may be self-fulfilled and sunspot-deriven changes in expectations affect the equilibrium path.

The above intuition, however, ignores the cross effects on optimal consumption represented by the second terms in the right-hand sides of (27) and (28) . For example, suppose that agents in each group are conformists who like being similar to members of the other group (i.e. $\Omega_2^i < 0,\ i = 1, 2$) . Then the anticipated rise in the rate of return to capital also accelerates consumption, which is generated by the intergroup external effects: see the signs of coefficients of the second terms in the right-hand sides of (27) and (28) . Therefore, given our assumption of $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 < 0$, those additional adjustments in the same direction could enhance instability so that the economy diverges from the steady state. In contrast, if agents are anti-conformist as for other group's consumption $(\Omega_2^i > 0, \ i = 1, 2)$, the effects of an expected rise in the rate of return to capital on consumption caused by the own effect would be mitigated by the cross effect. This may prevent the economy's diverging behavior. Although such an intuitive discussion cannot present the precise mechanism that generates multiple equilibria, the numerical examples given in the next section suggest that our intuition at least partially characterizes equilibrium dynamics of our model economy.

4 Numerical Analysis

In the previous section we have confirmed that if the sign of $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2$ is negative, then the steady-state equilibrium is either locally indeterminate or unstable. For the purpose of distinguishing the conditions for indeterminacy from these for instability, this section conducts numerical experiments by specifying the utility, production and tax functions.

We use the following utility function:

$$u^i(c_i(t), C_i(t), C_j(t)) = \frac{1}{1-\gamma_i}\left(c_i C_i^{\phi_i} C_j^{\eta_i}\right)^{1-\gamma_i}, \quad i, j = 1, 2, \quad i \neq j. \tag{31}$$

Here, γ_i denotes the inverse of elasticity of intertemporal substitution in felicity. The parameter ϕ_i represents the extent of the intragroup consumption externalities, whereas η_i shows the intensity of intergroup externalities. From (31) we find that

$$\begin{aligned}&\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 \\ &= \{-\gamma_1 + \phi_1(1-\gamma_1)\}\{-\gamma_2 + \phi_2(1-\gamma_2)\} - \eta_1\eta_2(1-\gamma_1)(1-\gamma_2).\end{aligned} \tag{32}$$

In view of conditions $(2c)$ and $(2f)$, the following inequalities must be satisfied:

$$\Omega_1^i = \gamma_i - \phi_i(1-\gamma_i) > 0, \quad i = 1, 2,$$

$$\Omega_i^i + \Omega_j^i = \gamma_1 - (\phi_1 + \eta_1)(1-\gamma_1) > 0, \quad i, j = 1, 2, \quad i \neq j.$$

In addition, in the symmetric steady state where $C_1 = C_2$, condition $(2d)$ requires the following:

$$-\frac{u_{11}^i C}{u_1^i} - \frac{u_{13}^i C}{u_1^i} = \gamma_i - \eta_i(1-\gamma_i) > 0, \qquad i = 1, 2.$$

As for the production function, it is given by Cobb-Douglas one:

$$f(K) = AK^{\alpha}, \quad 0 < \alpha < 1, \quad A > 0, \tag{33}$$

where $K = \theta_1 k_1 + \theta_2 k_t$.

The tax function is specified as

$$\tau\left(\frac{y_i}{Y}\right) = \frac{(y_i/Y)^{\xi}}{b + m\,(y_i/Y)^{\xi}}, \tag{34}$$

where

$$b + m > 0, \quad b\xi > 0, \quad \text{and} \quad (b+\xi)^2 > b(1+\xi) + m.$$

It is to be noticed that (34) fulfills all of our assumptions on the tax function including Assumption 1.[7] Under this specification of tax function, the key values evaluated at the steady state equilibrium are given by the following:

$$\tau(1) = \frac{1}{b+m} > 0,$$

$$\tau'(1) = \frac{b\xi}{(b+m)^2} > 0,$$

$$\tau''(1) = \frac{b\xi\{b(\xi-1) - m(1+\xi)\}}{(b+m)^3},$$

$$1 - \tau(1) - \tau'(1) = \frac{(b+m)^2 - b(1+\xi) - m}{(b+m)^2} > 0.$$

[7] Guo and Lansing (1998) and Li and Sarte (2004) specify the tax function in such a way that

$$\tau\left(\frac{y_i}{Y}\right) = \tau_0\left(\frac{y_i}{Y}\right)^{\phi}, \qquad 0 < \tau_0 < 1, \quad \phi < 1.$$

This specification also yields:

$$\frac{\partial\left(\tau\left(y_i/Y\right)y_i\right)/\partial y_i}{\tau\left(y_i/Y\right)} = 1 + \phi > 1,$$

$$2\tau'(1) + \tau''(1) = \phi\,(\phi+1) > 0.$$

However, this specification may violate the feasibility condition, $0 < \tau\,(.) < 1$.

The magnitudes of parameters concerning production, population distribution and tax functions are given by

$$\theta_1 = 0.2, \quad A = 1, \quad \alpha = 0.3, \quad b = 4, \quad m = 3, \quad \xi = 2.5, \quad \rho = 0.025.$$

Then the before-tax rate of return to capital, r, is 0.0382813 and the rate of the income tax is 0.1428571 so that $1 - \tau(1) - \tau'(1)$ and $2\tau' + \tau''(1)$ have positive values.[8] As for the parameter values concerning the preference structure, we consider the following three sets:

$$\begin{aligned}
\text{(i) } \gamma_1 &= 0.3, \quad \gamma_2 = 0.6, \quad \phi_2 = 0.2, \quad \eta_2 = -0.8,\\
\text{(ii) } \gamma_1 &= 0.3, \quad \gamma_2 = 2.5, \quad \phi_2 = -0.9, \quad \eta_2 = 0.45,\\
\text{(iii) } \gamma_1 &= 1.8, \quad \gamma_2 = 4.5, \quad \phi_2 = -0.9 \quad \eta_2 = 0.6.
\end{aligned}$$

Example (i) assumes that

$$\begin{aligned}
\text{sign } u_2^2 &= \text{sign } \phi_2 > 0, \qquad \text{sign } u_{12}^2 = \text{sign } \phi_2 (1 - \gamma_2) > 0,\\
\text{sign } u_3^2 &= \text{sign } \eta_2 < 0, \qquad \text{sign } u_{13}^2 = \text{sign } \eta_2 (1 - \gamma_2) < 0.
\end{aligned}$$

Hence, the agents in group 2 have admiration as well as conformism about the consumption behavior of their own group's members, while they are jealous but anti-conformist about the consumption level of group 1's agent. Similarly, examples (ii) and (iii) assume:

$$u_2^2 < 0, \quad u_{12}^2 > 0, \quad u_3^2 > 0, \quad u_{13}^2 < 0,$$

implying that the agents of group 2 have jealousy and conformism about their own group's average consumption; and they admire but have anti-conformism as to the other group's consumption.

Given those parameter magnitudes, we change ϕ_1 and η_1 with an intervals of 0.01. Figures 1, 2 and 3 respectively depict the case with preference parameters (i),

[8] Since we have ignore capital depreciation, the before tax rate of return to capital in the steady state has a rather high value.

(ii) and (iii) displayed above. In these figures, we divide (ϕ_1, η_1) space according to the stability conditions. The areas with shadow between stable and unstable regions represent the combination of ϕ_1 and η_1 that yields local indeterminacy. As the figures demonstrate, although the parameter space for indeterminacy is relatively small, we can find the possibility of equilibrium indeterminacy for wide ranges of values of ϕ_1 and η_1.

More specifically, Figure 1 shows that the equilibrium indeterminacy in example (i) emerges when

$$\eta_1 < 0, \quad \phi_1 > 0, \quad \phi_1 (1 - \gamma_1) > 0, \quad \eta_1 (1 - \gamma_1) < 0$$

Thus in case (i) the existence of equilibrium indeterminacy requires that the agents in groups 1 and 2 have the same preference structure. In case (ii), as shown by Figure 2, the equilibrium indeterminacy again emerges if the following conditions hold:

$$\eta_1 < 0, \quad \phi_1 > 0, \quad \phi_1 (1 - \gamma_1) > 0 \text{ and } \eta_1 (1 - \gamma_1) < 0$$

and, hence, the preferences of group 1's agents are the same as those in case (i). Note that, as long as conformism and anti-conformism are concerned, group 2's agents have the same preference as that held by group 1's agents. Finally, consider case (iii). Figure 3 demonstrates that indeterminacy may be observed when

$$\eta_1 > 0, \quad \phi_1 < 0, \quad \phi_1 (1 - \gamma_1) > 0 \text{ and } \eta_1 (1 - \gamma_1) < 0.$$

The common feature of those examples is that when indeterminacy emerges, the agents in each group are conformist to their own group' consumption, but they have anti-conformism as for the other group's consumption behavior. Therefore, in all the examples it holds that

$$\Omega_2^i \equiv -\frac{u_{13}^i(C_i, C_i, C_j)C_j}{u_1^i(C_i, C_i, C_j)} > 0, \quad i \neq j, \quad i, j = 1, 2. \tag{35}$$

Since (32) shows that examples (i), (ii) and (iii) satisfy $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 < 0$, (35) means that all of the adjustment coefficients of cross terms in the right hand sides of Euler equations (27) and (28) have positive values. Hence, the intuitive argument in Section 3.4 may be supported by our numerical experiments.

Finally, it should be pointed out that our numerical examples assume that there is a considerable difference in population share of each group (our examples set $\theta_1 = 0.2$ and $\theta_2 = 0.8$).[9] When using alternative parameter values that are not displayed here, we have found that if each group has a similar population share (for example, $\theta_1 = \theta_2 = 0.5$), then indeterminacy does not hold under plausible values of other parameters. That is, if θ_1 is close to θ_2, the steady-state equilibrium is always unstable if $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 < 0$. This fact may come from our modeling strategy that focuses on the symmetric steady state. To see this, note that the magnitude of θ_i does not directly affect the optimal consumption decision of each group, but it affects accumulation of each group's capital stock. Since the steady state is symmetric and since the transfer for every agent is assumed to be identical ($T_1 = T_2$), if θ_1 is close to θ_2, then the dynamic behaviors of k_1 and k_2 are not so much different from each other near the steady-state equilibrium. That is, the behavior of individual capital is similar to that of aggregate capital, K. Baed on the intuitive discussion in Section 3.4., we may conjecture that in the case of $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 < 0$, the dynamic system tends to be unstable if the aggregate capital behaves like that in the representative agent economy. As a result, the emergence of indeterminacy needs that the behavior of individual capital is sufficiently different from each other so that the dynamic motion of aggregate capital is different from one observed in the representative agent model. In our setting, therefore, the presence of equilibrium indeterminacy requires a sufficient degree of heterogeneity in population distribution.

5 Concluding Remarks

We have shown that if there are heterogenous agents and if consumption external effects perceived by consumers are not uniform, then the equilibrium path of the standard Ramsey economy would not display a regular saddle point property. The equilibrium dynamics could be unstable or indeterminate if the intergroup externalities have distinctive effects on the consumers' behaviors. In order to facilitate

[9] Obviously, our results hold in the opposite situation such that θ_1 is large and θ_2 is small.

comparison between the heterogenous-agent economy with the representative agent counterpart, this paper introduces a specific form of nonlinear income taxation that ensures the presence of a unique and symmetric steady-state equilibrium.

It is worth emphasizing that our stability results do not rely on the fact that we have restricted our attention to the symmetric steady state in which income and wealth distribution are equalized among the consumers. In the symmetric steady state of our model with the utility function given by (31), equation (32) shows that the sign of $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2$ depends only on the parameter values representing consumption external effects. If the steady state equilibrium is not symmetric in the sense that an unequal wealth distribution between groups remains, then the steady-state expression of $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2$ depends on the steady-state values of C_1^* and C_2^*. As an example, let us assume that each type of agent has different rate of time discount rate. In this case, the steady state conditions (18) are replaced with

$$\rho_i = f'(K^*)\left[1 - \tau\left(\frac{y_i^*}{Y^*}\right) - \frac{y_i^*}{Y^*}\tau'\left(\frac{y_i^*}{Y^*}\right)\right], \quad i = 1, 2.$$

Since $\tau\left(y_i/Y\right) + \tau'\left(y_i/Y\right)\left(y_i/Y\right)$ is assumed to be monotonic function of y_i/Y, the above equations demonstrate that if $\rho_i > \rho_j$, then $y_i^*/Y^* < y_j^*/Y$ so that $y_i^* < y_j^*$. As a result, (17) means that $C_i^* < C_j^*$. This means that the magnitude of $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2$ is affected by the steady state values of C_1^* and C_2^*. Since C_1^* and C_2^* are determined by production technology, in the asymmetric steady state the parameter values depicting consumption externalities alone cannot determines the dynamic behavior of the economy. However, it is easy to see that the condition $\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2 > 0$ is still necessary for establishing a regular saddle-point stability in an asymmetric steady state equilibrium.

In this paper we have employed a simple Ramsey model with fixed labor supply and a constant returns to scale technology. It would be useful to reconsider our discussion in models with increasing returns and/or endogenous labor supply.

Appendix

In this appendix, we analyze the conditions under which the steady-state equilibrium exhibits saddle-path stability. Let be assuming that $\Delta \equiv 1 - \tau(1) - \tau'(1) > 0$. The coefficients of the matrix J are given by:

$$\frac{\partial \dot{C}_1}{\partial k_1} = \frac{C^*}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}\frac{(f')^2}{f}\left\{(\Omega_1^2 - \Omega_2^1)\theta_1\Gamma(k^*)\Delta - (\theta_2\Omega_1^2 + \theta_1\Omega_2^1)(\tau''(1) + 2\tau'(1))\right\},$$

$$\frac{\partial \dot{C}_1}{\partial k_2} = \frac{C^*}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}\frac{(f')^2}{f}\left\{(\Omega_1^2 - \Omega_2^1)\theta_2\Gamma(k^*)\Delta + (\theta_2\Omega_1^2 + \theta_1\Omega_2^1)(\tau''(1) + 2\tau'(1))\right\},$$

$$\frac{\partial \dot{C}_2}{\partial k_1} = \frac{C^*}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}\frac{(f')^2}{f}\left\{(\Omega_1^1 - \Omega_2^2)\theta_1\Gamma(k^*)\Delta + (\theta_1\Omega_1^1 + \theta_2\Omega_2^2)(\tau''(1) + 2\tau'(1))\right\},$$

$$\frac{\partial \dot{C}_2}{\partial k_2} = \frac{C^*}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}\frac{(f')^2}{f}\left\{(\Omega_1^1 - \Omega_2^2)\theta_2\Gamma(k^*)\Delta - (\theta_1\Omega_1^1 + \theta_2\Omega_2^2)(\tau''(1) + 2\tau'(1))\right\}.$$

Plugging the above expressions into ZJ and arranging terms, we obtain the following:

$$\begin{aligned} \mathrm{Z}J = {} & \frac{(f')^3}{\Omega_1^1\Omega_1^2 - \Omega_2^1\Omega_2^2}\left\{\Gamma(k^*)\Delta^2[\theta_1(\Omega_1^2 - \Omega_2^1) + \theta_2(\Omega_1^1 - \Omega_2^2)]\right. \\ & \left. -[\theta_1(\Omega_1^1 + \Omega_2^1) + \theta_2(\Omega_1^2 + \Omega_2^2)](2\tau'(1) + \tau''(1))\right\}. \end{aligned}$$

Since $\Gamma(k^*) < 0$ and $\tau''(1) > 0$ if $\theta_1(\Omega_1^2 - \Omega_2^1) + \theta_2(\Omega_1^1 - \Omega_2^2) > 0$, then Z$J$ is strictly negative. As a result, there are at most two roots that have positive real parts.

References

Abel, A.B. (1990), "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic Review* 80, 38-42.

Alonso-Carrera, J., J. Caballé and X. Raurich, (2008), "Can consumption spillovers be a source of equilibrium indeterminacy?", *Journal of Economic Dynamics and Control* 32, 2883-2902.

Becker, R.A. (1980), "On the Long-Run Steady–State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households", *Quarterly Journal of Economics* 95, 375–382.

Carroll, C.D., J. Overland, and D.N. Wiel, (1997), "Consumption Utility in a Growth Model", *Journal of Economic Growth* 2, 339-367.

Carroll, C.D., J. Overland and D.N. Weil. (2000), "Savings, Growth and Habit Formation", *American Economic Review* 90, 341-355.

Chen, Been-Lon and M. Hsu, (2007), "Admiration is a Source of Indeterminacy", *Economics Letters* 95, 96-103.

Collier, C. (2004), "Misery Loves Company: Equilibrium Portfolios with Heterogenous Consumption Externalities", *International Economic Review* 43, 1169-1192.

Dupor, B. and Liu, W. F. (2003), "Jealousy and Equilibrium Overconsumption", *American Economic Review,* 93, 423-428.'

Frank, R. (2005), "Positional Externalities Cause Large and Preventable Welfare Losses", *American Economic Review* 95, 137-141.

Garriga, C. (2006), "Overconsumption, Reference Groups, and Equilibrium Efficiency", *Economics Letters* 91, 420-424.

García-Peñalosa, C. and Turnovsky, S. (2007), "Consumption Externalities: A Representative Consumer Model when Agents are Heterogeneous", forthcoming in *Economic Theory.*

Galí, J. (1994), "Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice and Asset Prices", *Journal of Money, Credit and Banking* 26, 1-8.

Guo, J.-T., and Lansing, K. J. (1998), "Indeterminacy and Stabilization Policy", *Journal of Economic Theory* 82, 481-490.

Li, W. and, Sarte, P. (2004), "Progressive Taxation and Long-Run Growth", *American Economic Review* 94, 1705–1716.

Liu, W.F. and S.J. Turnovsky (2005), "Consumption Externalities, Production Externalities, and Long Run Macroeconomic Efficiency", *Journal of Public Economics* 89, 1098-1129.

Ljungqvist, L. and Uhlig, H. (2000), "Tax Policy and Aggregate Demand Management Under Catching Up with the Joneses", *American Economic Review* 90, 356-366.

Mino, K. (2007), "Equilibrium Dynamics of an Overlapping Generations Economy with Consumption Externalities", unpublished manuscript.

Mino, K. (2008), "Growth and Bubbles with Consumption Externalities", *Japanese Economic Review* 59, 33-53.

Mino, K. and Nakamoto, Y. (2008), "Progressive Taxation, Wealth Distribution, and Macroeconomic Stability", Discussion Paper No. 08-22, Graduate School of Economics, Osaka University.

Sarte, P. (1997), "Progressive Taxation and Income Inequality in Dynamic Competitive Equilibrium", *Journal of Public Economics* 66, 125–171.

Sorger, G. (2002), "On the Long-Run Distribution of Capital in the Ramsey Model", *Journal of Economic Theory* 105, 226–243.

Weder, M. (2000), "Consumption Externalities, Production Externalities and Indeterminacy", *Metroeconomica* 51, 435-453.

Figure 1: $\gamma_1 = 0.3$, $\gamma_2 = 0.6$, $\phi_2 = 0.2$ and $\eta_2 = -0.8$

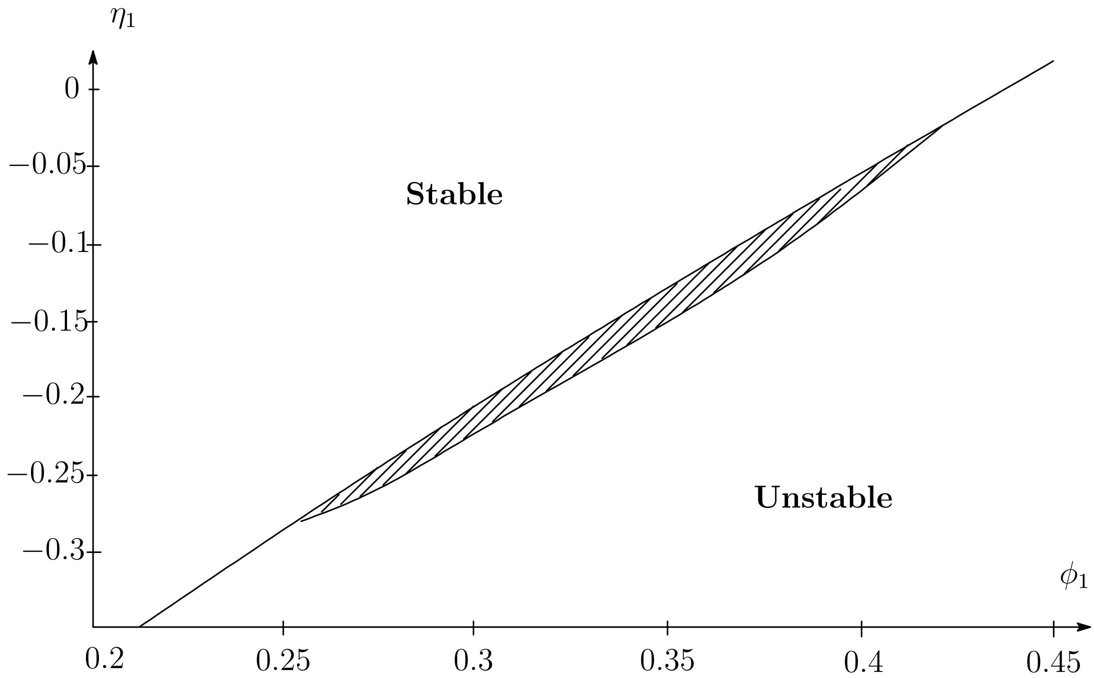

Figure 2: $\gamma_1 = 0.3$, $\gamma_2 = 2.5$, $\phi_2 = -0.9$ and $\eta_2 = 0.45$

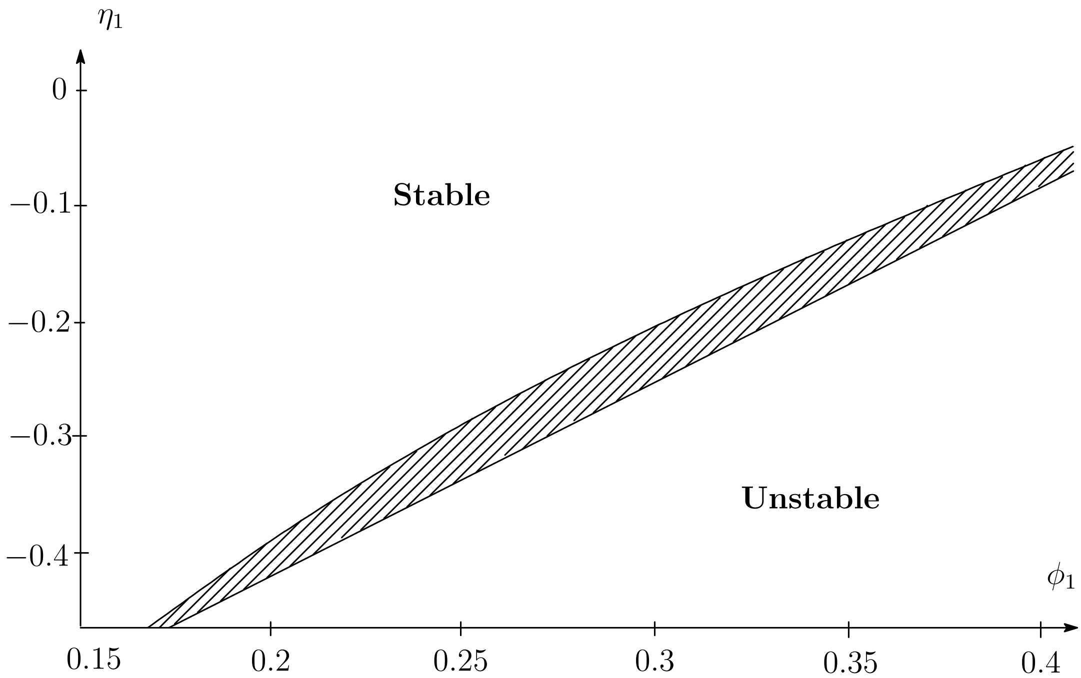

Figure 3: $\gamma_1 = 1.8$, $\gamma_2 = 4.5$, $\phi_2 = -0.9$ and $\eta_2 = 0.65$