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International Public Goods and Productivity Differentials

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Discussion Paper 08-38

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Abstract

The purpose of this paper, by introducing the adjustment expense of global income transfers across *N* countries, is to produce an explicit rule for the planner country regarding income transfers, and to investigate the effects of income transfers on each country's welfare. The findings are: (i) when country *i* has a productive advantage in producing public goods, country *i* becomes an income receiver; (ii) specifying the particular level of the adjustment expense for global income transfers, the planner can decide the values of income transfers for all countries; (iii) even though any country can become a planner of income transfers, all countries get the same utility level, while the low adjustment expense under a particular planner country leads to a Pareto-improving outcome; (iv) all conclusions are derived based on well-known information regarding the cost of producing public goods and income levels for all countries.

Keywords: international public goods; productivity differentials; planner; global income transfer; adjustment expense; welfare

JEL classification: H41, F13

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1. Introduction

Since Warr's (1983) seminal work, a large body of literature has evolved on the neutrality result that the real equilibrium is unaffected by income transfers when public goods are privately provided. This framework is generalized and reinforced by giving the proof of existence and uniqueness of the equilibrium by Bergstrom et al. (1986, 1992). However, income transfers are not completely neutral in some cases.[1] In one of these cases, Ihori (1996) investigates the welfare effects of changes in productivity differentials associated with the production of international public goods across countries.[2] It is shown that an income transfer from a country with low productivity (i.e., a high cost of producing public goods) to a country with high productivity (i.e., a low cost) produces a Pareto improvement. His analysis on income transfers used a two-country model.

After Ihori (1996), Boadway and Hayashi (1999), Ihori (1999), Caplan et al. (2000), Itaya et al. (2002) and Kim and Shim (2006) investigate the effect of income transfer under productivity or population differentials, among others. They mainly investigate the effects of *local income transfers* between two countries, i.e., the effects between two countries in an N-country model. However, when they investigate the effects of *global income transfers* across N countries, the previous analysis between the two countries cannot answer the following questions. First, how are the different productivities among the N countries compared, and which countries among the N countries are income senders and income receivers? Second, when some countries are income receivers, how much do they receive? Third, which country manages the income transfer? To my knowledge, no papers investigate the effects of *global income transfers* across N countries with an explicit planner rule.

The purpose of this paper is to develop an explicit rule for the planner country of global income transfers among N countries, and to investigate the effects of income transfers on each country's welfare, in order to answer the three questions above. We introduce the concept of adjustment expense for income transfers across countries. The finding for the first question is, when country i has a productive advantage in producing public goods, country i becomes an income receiver. The finding for the second question is, specifying the particular level of the adjustment expense, the planner can decide the size of income transfer for all countries. The finding for the third question is, even though any country can become the planner of income transfers, each country gets the same utility level, while the low adjustment expense under a particular planner country (the USA or the United Nations) leads to a Pareto improvement. All conclusions are derived based on well-known information on the cost of producing public goods and income levels for all countries.

The paper is organized as follows. Section 2 outlines the model with the noncooperative Cournot–Nash equilibrium of voluntary contributions to international public goods when global income transfers are implemented. Section 3 investigates the effects of global income transfers on national welfare under different productivities of the N countries, comparing the effects of local transfer between two countries. Section 4 concludes the paper.

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[1] A useful survey in this field is Ihori (1996).

[2] Murdoch and Sandler(1986,p.84) consider the weapons with the North Atlantic Treaty Organization's (NATO) allies as the international public good and investigate its demand..

2. The Model

Consider a model where there is one public good, one private good, and N countries (i = 1,2,…,N). Country i consumes an amount x_i of the private good and provides an amount g_i to the supply of the international public good. The total supply of international public good, G, is just the sum of g_i provided by each country. Country i's utility is given by $U_i = (x_i, G)$, where U_i is strictly increasing and quasi-concave, and G *and* x_i are normal goods for each country. Country i's budget constraint is given by $x_i + p_i g_i = y_i$, where $y_i > 0$ is the exogenously given national income of country i and $p_i > 0$ is the relative price (cost of production) of public goods in terms of private consumption in country i. As Ihori (1996) interpreted, low (high) p_i means a high (low) productivity in producing the public good. We also make the Cournot–Nash assumption that each country believes that the contributions of others are independent of its own. Then, we let $G = g_i + \sum_{j \neq i} g_j$ denote the total public good, where $\sum_{j \neq i} g_j$ is the sum of g_j provided by countries j other than i.

Definition 1. A Cournot–Nash equilibrium in this model is a vector of $\{ g_i^* : i = 1,...,N\}$, such that for each i, (x_i^*, g_i^*) solves the following:

$$\max_{x_i, g_i} \; U_i = U_i(x_i, g_i + \sum_{j \neq i} g_j^*)$$
$$s.t. \quad x_i + p_i g_i = y_i, \quad x_i, g_i \geq 0 \; : \; i = 1,2,.., N. \tag{1}$$

Each country consumes the same level of $g_i^* + \Sigma_{j \neq i} g_j^*$ at the equilibrium, which is a characteristic of the international public good. The solution to problem (1) yields the reaction function, which we can denote by $g_i(p_i, y_i, \sum_{j \neq i} g_j^*)$. The solution g_i^* (i = 1,2,…,N) satisfies $g_i^* = g_i(p_i, y_i, \sum_{j \neq i} g_j^*), i = 1, 2, ..., N$ at the equilibrium. Then, the solution g_i^* in (1) can be written as $g_i^*(p_1, p_2, ...p_N, y_1, y_2, ...y_N)$ by substituting the other solutions into it. Furthermore, by inserting g_i^* into the budget constraint, we get the solution of x_i^* . The existence and uniqueness of this solution is assumed here.[3] Because the reaction function $g_i(p_i, y_i, \sum_{j \neq i} g_j^*)$ derived from an unspecified utility function has a general form, the solution g_i^* is more general, and it is then difficult to investigate the

[3] Miyakoshi (2008) tries to prove the existence and the uniqueness of this equilibrium.

effect of income transfers on country i's utility. We use an alternative method, following Bergstrom et al. (1986, 1992).

Let us consider the maximization problem of a given country at a Cournot–Nash equilibrium in (1). We can reformulate (1) as follows, by adding $\sum_{j\neq i} g_j^*$ on both sides of the budget equation:

Definition 2. A Cournot–Nash equilibrium in this model is such that for each i, (x_i^*, G^*) i = l,2,...,N, solves:

$$\max_{x_i, G} U_i = U_i(x_i, G)$$
$$s.t. \quad x_i + p_i G = y_i + p_i \sum_{j\neq i} g_j^*,$$
$$x_i \geq 0, G \geq \sum_{j\neq i} g_j^* \geq 0, \; i = 1,2,..,N. \tag{2}$$

In (2), each country implicitly chooses not only its supply g_i^* of international public goods, but also the equilibrium level of G^* itself. Each country consumes the same level of G^*. The first order condition here is $p_i \partial U_i / \partial x_i = \partial U_i / \partial G$ (i = 1,2,..,N). Utilizing the implicit theorem function, these N equations can be solved to give x_i^* (i = 1,2,…,N).[4]

Based on Definition 2, we follow Ihori (1996), who uses the expenditure function to analyze the effect of the income transfers. We define the expenditure function at the Cournot–Nash equilibrium as follows:

$$\min_{x_i, G} E_i \equiv x_i + p_i G \quad s.t. \; U^i(x_i, G) = \overline{U}_i^*, \tag{3}$$

where $\overline{U}_i^* \equiv U_i(x_i^*, G^*)$ is the utility level at the equilibrium. Then, the expenditure function can be written as $E_i = E_i(\overline{U}_i^*, p_i)$. At the equilibrium, this expenditure is equal to the income in (2):

$$E_i(\overline{U}_i^*, p_i) = y_i + p_i \sum_{j\neq i} g_j^*, \tag{4}$$

[4] G^* is not decided. Warr (1983) sums the budget constraints of the N countries and then obtains G^* under $p_i = 1$ for all i. However, $p_i \neq 1$ in our model and then the equilibrium G^* is derived in (6) and (7).

where the income on the right-hand side contains actual income y_i and the externalities from the other country's provision of the public good. By using Shephard's Lemma on the expenditure function $E_i(\overline{U}_i^*, p_i)$ at the equilibrium, we can derive the following:

$$\frac{\partial E_i(\overline{U}_i^*, p_i)}{\partial p_i} \equiv G_i(\overline{U}_i^*, p_i), \quad G_i(\overline{U}_i^*, p_i) = G^*, \tag{5}$$

where $G_i(\overline{U}_i^*, p_i)$ is the compensated demand function of country i for the international public good, which must equal the same level of G^* for each country supported by this Lemma. From (4) and (5), the equilibrium can be summarized by the following equations:

$$\sum_{i=1}^{N} \Phi_{-i} E_i(\overline{U}_i^*, p_i) = \sum_{i=1}^{N} \Phi_{-i} y_i + \Phi(N-1)G^*, \tag{6}$$

$$G_i(\overline{U}_i^*, p_i) = G^*, \quad (i = 1, 2, \ldots, N), \tag{7}$$

where $\Phi = \prod_{i=1}^{N} P_i$, $\Phi_{-i} = \prod_{j \neq i}^{N} P_j$ and multiplying (4) by Φ_{-i} and summing yields (6). These N+1 equations decide the utilities $\overline{U}_i^*$ of the N countries and the amount of the international public good G^* at the equilibrium. This formulation is an extension of Ihori's (1996) two-country model to N countries.

3. Planner of Income Transfers: Local vs Global Transfers

We take the total derivatives of (6) and (7) consisting of N+1 equations:

$$\sum_{i=1}^{N} \Phi_{-i} E_{iU} d\overline{U}_i^* - \Phi(N-1) dG^* = \sum_{i=1}^{N} \Phi_{-i} dy_i, \tag{8}$$

$$G_{iu} d\overline{U}_i^* - dG^* = 0, \quad (i = 1, 2, .., N), \tag{9}$$

where $\partial E_i(\overline{U}_i^*, p_i) / \partial \overline{U}_i^* \equiv E_{iU}$ and $\partial G_i(\overline{U}_i^*, p_i) / \partial \overline{U}_i^* \equiv G_{iU}$. By using $G_{1u} d\overline{U}_1^* = dG^*$ in (9), we can delete dG^* from (8) and (9) and decrease the number of equations from N+1 to N. Moreover, by using the relationship of the expenditure function (3) in country 1,

$$E_{1U} = x_{1U} + p_1 G_{1U}, \tag{10}$$

we can rewrite (8) and (9) as follows:

$$\Phi_{-1}(x_{1U}+p_1G_{1U})d\overline{U}_1^* + \sum_{i=2}^{N}\Phi_{-i}E_{iU}d\overline{U}_i^* - \Phi(N-1)G_{1u}d\overline{U}_1^* = \sum_{i=1}^{N}\Phi_{-i}dy_i \ , \qquad (11)$$

$$G_{iu}d\overline{U}_i^* - G_{ju}d\overline{U}_j^* = 0 \ , \ (i,j = 1,\ldots,N), \qquad (12)$$

We arrange equation (11) and (12) in vector and matrix format,

$$\begin{bmatrix} \Phi_{-1}(x_{1U}+p_1G_{1U}(2-N)), \ \Phi_{-2}E_{2U}, \ldots\ldots\ldots, \Phi_{-N}E_{NU} \\ -G_{1U}, \quad G_{2U}, \ 0 \quad ,0\ldots\ldots,0 \\ 0 \qquad , \ -G_{2U}, G_{3U}, 0\ldots\ldots, 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 0 \qquad , \ 0 \quad , \ 0 \quad , 0\ldots -G_{(N-1)U}, G_{NU} \end{bmatrix} \bullet \begin{bmatrix} d\overline{U}_1^* \\ d\overline{U}_2^* \\ .. \\ .. \\ .. \\ d\overline{U}_N^* \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N}\Phi_{-i}dy_i \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix}, \qquad (13)$$

where the determinant Δ of the matrix on the left hand side of (13) is:

$$\Delta = \Phi_{-1}(x_{1U}+p_1G_{1U}(2-N))\Delta_{11} + G_{1U}\Delta_{21}$$
$$= \Phi_{-1}(x_{1U}+p_1G_{1U}(2-N))(G_{2U}\cdot\cdot G_{NU}) + G_{1U}\sum_{i=2}^{N-1}\Phi_{-i}E_{iU}\cdot(\Pi_{j\neq i}G_{2U}\cdot\cdot G_{jU}\cdot\cdot G_{NU}), \qquad (14)$$

where Δ_{11}, Δ_{12} are sub-determinants of Δ. Because of (10), $\Phi_{-i}E_{iU} = \Phi_{-i}x_{iU} + \Phi_{-i}p_iG_{iU}$. Inserting this relation into (14) yields:

$$\Delta = \Phi_{-1}(x_{1U}+p_1G_{1U}(2-N))(G_{2U}\cdot\cdot G_{NU}) + (G_{1U}\ G_{2U}\cdot\cdot G_{NU})N\Phi$$
$$+ G_{1U}\sum_{i=2}^{N-1}\Phi_{-i}x_{iU}(\Pi_{j\neq i}G_{2U}\cdot\cdot G_{jU}\cdot\cdot G_{NU}) \qquad (15)$$
$$= \Phi_{-1}(x_{1U}+2p_1G_{1U})(G_{2U}\cdot\cdot G_{NU}) + G_{1U}\sum_{i=2}^{N-1}\Phi_{-i}x_{iU}(\Pi_{j\neq i}G_{2U}\cdot\cdot G_{jU}\cdot\cdot G_{NU}).$$

The partial derivative of the compensated demand function, x_{iU} and G_{iU}, is positive in terms of utility because they are normal goods. Then, the determinant Δ is positive.

Local income transfers

The planner is interested in the local income transfer from country *j* to *i* under $p_j > p_i$, that is, in (13):

$$dy_i = -dy_j > 0, dy_s = 0 \text{ for all } s \neq i, j., \text{ then, } \sum_{i=1}^{N} \Phi_{-i} dy_i = (\Phi_{-i} - \Phi_{-j}) dy_i \quad (16)$$

The welfare effects of this transfer are derived using Cramer's rule on (13):

$$\frac{d\overline{U}_i^*}{dy_i} = \frac{\Phi(p_j - p_i)(-1)^{2i} \Pi_{S\neq i}^{N} G_{SU}}{p_j \cdot p_i \Delta} > 0, \ \frac{d\overline{U}_j^*}{dy_i} = \frac{\Phi(p_j - p_i)(-1)^{2j} \Pi_{S\neq j}^{N} G_{SU}}{p_j \cdot p_i \Delta} > 0 \quad (17)$$

We have derived the same effect of local income transfers as Ihori (1996) and others: a transfer from country *j* with low productivity (high price p_j) of providing the public good to country *i* with high productivity (low price p_i) improves welfare in both countries.

Here, we consider global income transfers among many countries by introducing the adjustment expense for income transfers, which to my knowledge has not been proposed yet.

Global income transfers

We first design the planner of global income transfers, which is only one country among *N* countries. The planner country *h* takes a first linear approximation around the present utility level, and approximates the utility increase $d\overline{U}_h^*$ from the present level by using the income transfers and the partial derivatives of utility in terms of each country's income (solved from (13)) as follows:

$$\overline{U}_h^*(y_1 + \Delta y_1, y_2 + \Delta y_2, ..., y_N + \Delta y_N) \cong \overline{U}_h^*(y_1, y_2, ..., y_N) + \sum_{i=1}^{N} (\partial \overline{U}_h^* / \partial y_i) \Delta y_i$$
$$i.e., \quad d\overline{U}_h^* \equiv \overline{U}_h^*(y_1 + \Delta y_1, y_2 + \Delta y_2, ..., y_N + \Delta y_N) - \overline{U}_h^*(y_1, y_2, ..., y_N) \cong \sum_{i=1}^{N} (\partial \overline{U}_h^* / \partial y_i) dy_i. \quad (18)$$

Next, the planner country *h* maximizes the utility $\overline{U}_h^*$ (or $d\overline{U}_h^*$), which is already maximized under a given income of each country, by transferring income to each country. Here, prices are always fixed. However, the income transfers among the *N* countries involve an expense for making the agreement between the income senders and income receivers, or an administration expense. Then, the maximization problem of planner country *h* is formulated as follows:

$$\underset{\{\tilde{y}_i\}}{Max} \ \tilde{U}_h = \sum_{i=1}^{N} a_{hi} \tilde{y}_i,$$
$$s.t. \quad \sum_{i=1}^{N} \tilde{y}_i = 0, \quad \sum_{j=1}^{N} \tilde{y}_i^2 \leq \zeta^2, \quad (19)$$

where

$$\tilde{U}_h \equiv d\overline{U}_h^*, \quad \tilde{y}_i \equiv dy_i, \quad a_{hi} \equiv \partial \overline{U}_h^* / \partial y_i \ (const.),$$
$$\zeta > 0 (const. fixed). \quad (20)$$

The first constraint in (19) means that the income transfers are implemented with fixed total income across the N countries. The second constraint means all income transfers are implemented with a given total expense ζ^2 >0. The unit expense for $\tilde{y}_i^2$ of the income transfer for any country i and for the positive transfer $\tilde{y}_i > 0$ (income receiver) and the negative transfer $\tilde{y}_i < 0$ (income sender) equals one for the sake of simplicity. Finally, a_{hi} is derived from (13) as follows:

$$a_{hi} \equiv \frac{\partial \overline{U}_h^*}{\partial y_i} = \frac{1}{\Delta}\left[\Phi_{-i}\Pi_{S\neq h}^{N} G_{SU}\right] > 0 \qquad (i = 1,2,.h,.,N). \tag{21}$$

Then, the Lagrange equation L and Kuhn-Tucker condition for (19) are as follows:

$$L = \tilde{U}_h + \lambda(0 - \sum_{i=1}^{N}\tilde{y}_i) + \phi(\zeta^2 - \sum_{i=1}^{N}\tilde{y}_i^2)$$

$$a_{hi} - \lambda - 2\phi\tilde{y}_i = 0,\ (i = 1,2,..,N),\ 0 = \sum_{i=1}^{N}\tilde{y}_i\ , \tag{22}$$

$$\zeta^2 \geq \sum_{i=1}^{N}\tilde{y}_i^2,\ \ \phi(\zeta^2 - \sum_{i=1}^{N}\tilde{y}_i^2) = 0,\ \ \ \phi \geq 0\ . \tag{23}$$

The multiplier ϕ is nonnegative and is depending on the exogenously given price parameters in (19). Note that ϕ is zero when all p_i are the same, while it is positive when all p_i are not the same.[5] Then, λ can be solved as follows, inserting the first conditions into the second condition in (22) when $\phi > 0$, that is, all prices are not the same. We consider the case of $\phi > 0$, that is, all prices are not the same. Then, the second condition in (23) can be rewritten as $\zeta^2 = \sum_{i=1}^{N}\tilde{y}_i^2$. By using the resolved λ, we get $\tilde{y}_i$.

[5] When $\phi > 0$, ϕ can be solved in (24) where all prices are not the same. If all prices are the same, a_{hi} are the same for all i because of (21) and then ϕ in (24) is zero. Conversely, when all prices are not the same (at least one is different) and then at least one a_{hi} for i is different in (21), $a_{hi} - \lambda \neq 0$ for i and then $\phi > 0$ and $\tilde{y}_i \neq 0$ in the first condition in (22). Similarly, we assume that all prices are the same and then a_{hi} for all i is the same as in (21). Aggregating the first condition and using the second condition in (22), we can get $\sum_{i=1}^{N} a_{hi} - N\lambda - 2\phi\sum_{i=1}^{N}\tilde{y}_i = \sum_{i=1}^{N} a_{hi} - N\lambda = 0$. Then, because a_{hi} is the same for all i, $a_{hi} = \lambda$ for all i , which means $\phi = 0$ (in which case $\tilde{y}_i$ are not determined for all i) or $\tilde{y}_i = 0$ for all i in (22).

$$\sum_{i=1}^{N}\frac{a_{hi}-\lambda}{2\phi}=0, \quad \lambda=\frac{1}{N}\sum_{i=1}^{N}a_{hi}, \quad \tilde{y}_i=\frac{1}{2\phi}(a_{hi}-\frac{1}{N}\sum_{i=1}^{N}a_{hi}). \tag{24}$$

On the other hand, by inserting $\tilde{y}_i$ in (24) into the second condition in (23), ϕ can be solved as follows:

$$\phi=\frac{\sqrt{\sum_{i=1}^{N}(a_{hi}-\frac{1}{N}\sum_{i=1}^{N}a_{hi})^2}}{2\zeta}. \tag{25}$$

Finally we get $\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_N$ as follows, by inserting (25) into (24).

$$\tilde{y}_j=\frac{\zeta\left(a_{hj}-\frac{1}{N}\sum_{i=1}^{N}a_{hi}\right)}{\sqrt{\sum_{i=1}^{N}(a_{hi}-\frac{1}{N}\sum_{i=1}^{N}a_{hi})^2}}, \qquad (j=1,2,.h,.,N). \tag{26}$$

Then, by inserting (21) into (26), we can get the following solution $\{\tilde{y}_j\}$:

$$\tilde{y}_j=\frac{\zeta\frac{\Pi_{S\neq h}^{N}G_{SU}}{\Delta}\left(\Phi_{-j}-\frac{1}{N}\sum_{i=1}^{N}\Phi_{-i}\right)}{\sqrt{\sum_{i=1}^{N}(a_{hi}-\frac{1}{N}\sum_{i=1}^{N}a_{hi})^2}}=\frac{\zeta\left(\Phi_{-j}-\frac{1}{N}\sum_{i=1}^{N}\Phi_{-i}\right)}{\sqrt{\sum_{i=1}^{N}(\Phi_{-i}-\frac{1}{N}\sum_{i=1}^{N}\Phi_{-i})^2}}, \quad (j=1,2,.h,.,N). \tag{27}$$

We can easily confirm that the solution satisfies the conditions in (22) and (23).

If the planner can get more information about x_{1U} $(for\,\Delta)$ and $\{G_{1U}, G_{2U}, ..., G_{NU}\}$, the optimal utility level of country h is solved by inserting (27) into (19):

$$d\overline{U}_h^*=\sum_{j=1}^{N}\frac{1}{\Delta}\left[\Phi_{-j}\Pi_{S\neq h}^{N}G_{SU}\right]\frac{\zeta\left(\Phi_{-j}-\frac{1}{N}\sum_{i=1}^{N}\Phi_{-i}\right)}{\sqrt{\sum_{j=1}^{N}\left(\Phi_{-j}-\frac{1}{N}\sum_{i=1}^{N}\Phi_{-i}\right)^2}}. \tag{28}$$

We approximate the utility level of the other country T under the planner country h by using (18). Then, inserting (27) into (18) we have:

$$d\overline{U}_T^* = \sum_{j=1}^{N} \frac{1}{\Delta} \left[\Phi_{-j} \Pi_{S \neq T}^{N} G_{SU} \right] \frac{\zeta \left(\Phi_{-j} - \frac{1}{N} \sum_{i=1}^{N} \Phi_{-i} \right)}{\sqrt{\sum_{j=1}^{N} \left(\Phi_{-j} - \frac{1}{N} \sum_{i=1}^{N} \Phi_{-i} \right)^2}} . \tag{29}$$

Here, when the planner is changed from country *h* to country *T*, the optimal utility level of country *T* is the one replacing $\Pi_{S \neq h}^{N} G_{SU}$ with $\Pi_{S \neq T}^{N} G_{SU}$ in (28), which is the same as that in (29). That is, even though any country can become the planner of income transfers, each country gets the same maximum utility level, $d\overline{U}_i^*$. It is because the optimal income transfers do not depend on the planner as shown in (27). Moreover, when the unit adjustment expense for income transfers is decreased from one by a planner country with strong political power, that is, the ζ in (19) increases, the utility levels for all countries increase, showing a Pareto improvement. In this sense, the United Nations or the US may be appropriate for a planner. If it is the United Nations, it decides the unit expense (then ζ) and the optimal income transfers in (27).

Following the planner country *h*, which manages the global income transfers across all countries, we obtain the following policy conclusions.

(i) Because of (27), when the cross product of costs except for the cost of country *j* (Φ_{-j}) is larger than the average cross product of costs except for one country ($\frac{1}{N}\sum_{i=1}^{N}\Phi_{-i}$), (this positive difference is called the *productive advantage* of country *j*), country *j* is an income receiver ($\tilde{y}_j$ >0).

(ii) Because of (27), specifying the particular level of the adjustment expense ζ, the planner can decide the size of income transfers for all countries.

(iii) Because of the rule of the planner country *h* in (19) and the solution in (27), even though any country can become the planner, these conclusions hold, moreover because of (29), each country gets the same utility level. It does not matter which country becomes the planner. However, when the unit adjustment expense for income transfers is decreased from one by a planner country with strong political power, the utility levels for all countries increase, creating a Pareto improvement.

(iv) All conclusions are derived based on well-known information on the cost p_i of producing public goods, and the income level y_i for all countries.

An illustration of global income transfers

We provide an illustration of the planner's rule using the example of $N = 2$. Because of (19), the maximization problem of a planner country 1 is as follows:

$$\underset{\{\tilde{y}_1, \tilde{y}_2\}}{Max}\ \tilde{U}_1 = a_{11}\tilde{y}_1 + a_{12}\tilde{y}_2 , \tag{30}$$

$$s.t. \quad \tilde{y}_1 + \tilde{y}_2 = 0 , \tag{31}$$

$$\tilde{y}_1^2 + \tilde{y}_2^2 \leq \zeta^2 . \tag{32}$$

Moreover, because of (21):

$$a_{11} = \frac{1}{\Delta}\left[\Phi_{-1}G_{2U}\right] > 0 \text{ , } a_{12} = \frac{1}{\Delta}\left[\Phi_{-2}G_{2U}\right] > 0 \text{ , and } \Phi_{-1} = p_2 \text{ ,} \Phi_{-2} = p_1 \text{ .} \quad (33)$$

Then, the objective function in (30) can be rewritten, by using (33), as follows:

$$\tilde{U}_1 = \frac{1}{\Delta}G_{2U}\left[p_2\tilde{y}_1 + p_1\tilde{y}_2\right] = A\left[p_2\tilde{y}_1 + p_1\tilde{y}_2\right]: A \equiv \frac{1}{\Delta}G_{2U} > 0 \text{ .} \quad (34)$$

The maximization program (34), (31) and (32) is illustrated using Figure 1.

Insert Figure 1

The objective function (34) and constraints (31) and (32) are depicted in Figure 1, where the term $\tilde{U}_1 / P_1 A$ shows the utility level, constraint (31) is the solid line and constraint (32) is the region within the circle. In the case of $P_1 = P_2$, the objective function (34) coincides with the constraint (31), and then the optimum transfers are on the constraint (31) and within (32),but it is not uniquely decided. The optimum utility level is zero. These findings are already shown in footnote 4.

In the case of $P_1 \neq P_2$, the objective function (34) with $P_1 > P_2$ obtains the maximum utility for planner country 1 at point *C*. The optimum income transfer exists at point *C** with $\tilde{y}_1^* < 0$ and $\tilde{y}_2^* > 0$ and the utility level is $A\left[p_2\tilde{y}_1^* + p_1\tilde{y}_2^*\right]$. If there is no constraint (32), the optimal income transfer $\tilde{y}_1^* < 0$ endlessly decreases to obtain the maximum utility level, by moving upwards on the line (31). This formulation of maximization without (32) is used in previous studies. On the other hand, the non-planner country 2 has obtained their utility level by receiving income from planner country 1: $B\left[p_2\tilde{y}_1^* + p_1\tilde{y}_2^*\right]$ where $B \equiv G_{1U} / \Delta$. If country 2 becomes the planner, its objective function is $B\left[p_2\tilde{y}_1 + p_1\tilde{y}_2\right]$ where $B \equiv G_{1U} / \Delta$. When $P_1 > P_2$, the optimal income transfer is given at point *C** by using Figure 1 and its utility level is $B\left[p_2\tilde{y}_1^* + p_1\tilde{y}_2^*\right]$. The utility level for the planner country 2 is the same as that of the non-planner country 2. The reason is as follows. The optimal transfers are decided only by the constraints (31) and (32). Moreover, the increase in the utility is approximated by a linear function of the income transfers of all countries and then, whichever country become the planner, this country necessarily obtains the intersection of two constraints (31) and (32). Finally, when the unit adjustment expense of income transfers increases, constraint (32) becomes larger and the utility for each country increases.

4. Concluding Remarks

We have pointed out three problems regarding local income transfers. First, how are the different productivities among *N* countries compared, and which countries among the N countries are income senders and income receivers? Second, when some countries are income receivers, how much do they receive? Third, which country manages the income transfers? To resolve these issues, this paper proposes a planner of global income transfers and finds the optimal global income transfers for all countries. Only one planner, country h among the N countries, maximizes its utility, which is already maximized given the income levels of all countries, by transferring income to each country for a fixed adjustment expense. The income transfers among countries involve a fixed adjustment expense ζ for forming agreements among income-sender and income-receiver countries.

Using a planner country h, which manages global income transfers across all countries, we obtain the following policy conclusions.

(a) The finding for the first problem is, when country i has a productive advantage in producing the international public good, country i becomes an income receiver.
(b) The finding for the second problem is, specifying the particular level of the adjustment expense ζ, the planner country can decide the size of income transfers for all countries.
(c) The finding for the third problem is, even though any country can become a planner of income transfers, these conclusions hold. Whichever country becomes a planner, each country gets the same utility level. However, when the unit adjustment expense for income transfers is decreased from one by a planner country (the United Nations or the US) with strong political power, the utility levels for all countries increase, creating a Pareto improvement.
(d) All conclusions are derived based on well-known information on the cost of producing public goods and income levels for all countries.

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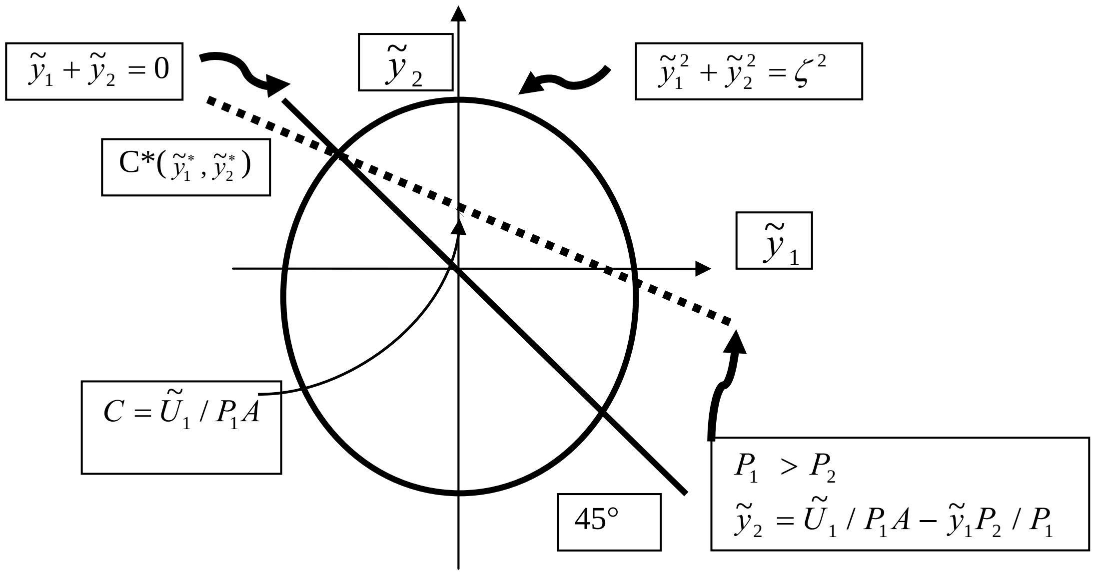

$\tilde{y}_1 + \tilde{y}_2 = 0$
$\tilde{y}_2$
$\tilde{y}_1^2 + \tilde{y}_2^2 = \zeta^2$
$C^*(\tilde{y}_1^*, \tilde{y}_2^*)$
$\tilde{y}_1$
$C = \tilde{U}_1 / P_1 A$
$P_1 > P_2$
$\tilde{y}_2 = \tilde{U}_1 / P_1 A - \tilde{y}_1 P_2 / P_1$
45°

Figure 1. N = 2