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Koichi Futagami[†] and Takeo Hori[‡]

Abstract

Do we have too few children? We intend to address this question. In developed countries, the fertility rate has declined since WWII. This may cause a slowdown in the growth of GDP in developed countries. However, important factors for the well-being of individuals are per capita variables, like per capita growth and per capita consumption. In turn, the rate of technological progress determines the growth rates of per capita variables. If the population size is increasing, the labour inputs for R&D activity increase, and thus speed up technological progress. As individuals do not take account of this positive effect when deciding the number of their own children, the number of children may become smaller than the socially optimal number of children. However, an increase in the number of children reduces the assets any one child owns: that is, there is a capital dilution effect. This works in the opposite direction. We examine this issue using an endogenous growth model where the head of a dynastic family decides the number of children.

Keywords: Technological Progress, Fertility, R&D

JEL Classification Numbers: J1, O30

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1 Introduction

Do we have too few children? We intend to address this question in this paper. Developed countries experienced a decline in the fertility rate after WWII. Consequently, the population size of developed countries will decrease dramatically. As is well known from standard growth theory, the natural rate of growth determines the growth rate of GDP: that is, the sum of the rate of technological progress and the rate of population growth. Therefore, it is often argued that the decline in population size due to the decrease in the fertility rate necessarily reduces the growth rate of GDP and potentially the level of welfare.[1] Some economists insist that an increase in the rate of technological progress—another factor that determines the growth rate of GDP—will counter the decline in the fertility rate, especially as policies are required to promote technological growth.

This dual argument at first appears convincing. However, it does not consider economic reasoning. First, the important variables for welfare are not GDP itself, but rather per capita variables like GDP per capita or consumption per capita. In fact, standard growth theory predicts that a decline in the population growth rate leads to an increase in per capita variables like these, and therefore the growth rate of per capita variables is determined by the rate of technological progress.[2]

Second, we need to consider the endogenous mechanisms affecting the determination of economic growth. The dualism maintains that if a government could increase the rate of technological progress, this would overcome the decline in the fertility rate, that is, the decrease in population growth. However, how should the government promote technological progress? Standard growth theory (that is, the Solow model), cannot answer this question because the technological progress is brought into the economy as manna from heaven. Therefore, we must resort to endogenous growth models to respond. In these models, the rate of technology progress is endogenously determined. The essential problem then becomes, when the population size is decreasing, can the government raise the rate of technological progress? Importantly, to boost technological progress there is a need for

[1] The Japanese government recently appointed a minister of state for special missions to address the declining fertility rate and undertake countermeasures.

[2] See any standard textbook; for example, Barro and Sala-i-Martin (2004).

many researchers. A decrease in the population size may then reduce the number of researchers working in research laboratories, and thus may lower the rate of technological progress.

To investigate this issue, we employ the model in Jones (1995). In this model, population growth invokes technological progress. Therefore, we can show that the decline in population growth decreases the rate of technological progress and thus decreases the growth rates of GDP and consumption per capita. Consequently, it is questionable whether the government can increase the rate of technological progress when the rate of population growth is in decline. If this is the case, should the government increase the rate of population growth to increase the rate of technological progress?

However, the Jones (1995) model does not directly indicate that the government should raise the rate of population growth. First, the purpose of economic policy is not to promote economic growth, but rather to increase the welfare level of individuals. Therefore, we first consider how individuals derive their utility. Individuals derive utility not only from the consumption of final goods, but also from having children. An increase in the number of children naturally raises their level of welfare. However, while children are a source of enjoyment for their parents, raising children invokes pecuniary costs along with opportunity costs because their parents usually stop working or reduce their working time to rear them. By taking account of these costs and the utility from having children, individuals make a decision on how many children they have.

Although this is a completely private decision, should the government intervene in this largely private decision? The answer is that if market failure exists, it is rational for a government to intervene in the decision-making process using taxes or subsidies. The question is whether there is any market failure in the decisions made by individuals on the number of children they will have. Consider now that researchers and engineers in private firms conduct research and development (R&D) activities. When parents decide upon the number of children, they do not take into account the positive effects of population size on these R&D activities in private firms. Consequently, because of this positive externality, the resource allocation of the market equilibrium may differ from the socially optimal allocation. That is, the number of children in the market equilibrium is smaller than in the socially optimal allocation. Thus, some scope may exist for government intervention by granting households a subsidy for having children. Nevertheless, it is not obvious whether the government should provide the subsidy to households that want to have more children, for there exists yet another cost for households to have a child: namely, if a household decides to have one more child, the amount of household capital that child owns becomes smaller. This reduces the capital income for each child. This is the *capital dilution effect*. Therefore, there is a need for further analysis based on the Jones (1995) model.

Jones (2003) has already examined some of these issues by modeling the household's decision on the number of children in Jones (1995). In fact, Jones (2003) also argues that the number of children in the market equilibrium is smaller than in the socially optimal allocation. However, Jones (2003) does not take into account those R&D activities that target profit. We incorporate profit-maximizing firms conducting R&D activities in our model. In particular, this modification can easily overturn Jones's (2003) result. Moreover, Jones's (2003) analysis is limited to steady state analysis. In contrast, the present analysis extends Jones (1995) by examining the transition paths to the steady state.[3] Because it takes a fairly long time until the economy approaches the steady state, it is also important to examine the character of the transition paths. [4] We construct a dynamic system for the model and conduct numerical simulations based on some plausible parameters. We show that the number of children in the market equilibrium can become larger than in the socially optimal allocation not only at the steady state, but also on the transition paths.[5] This implies that the government should not intervene in the decisions of families by giving them subsidies to increase the number of children.

The rest of the paper is structured as follows: Section 2 sets up the model. Section 3 constructs the dynamic system of the model. Section 4 derives the socially optimal allocation. By conducting a numerical simulation, we compare

[3]Jones (1995) also only examines the character of the steady state. Arnold (2006) examines the dynamics of Jones's (1995) growth model.

[4]See, for example, Steger (2003).

[5]Jones (2003) also suggests this result for the steady state. However, he does not conduct a formal analysis.

the number of children in the market equilibrium with that in the socially optimal allocation. Section 5 provides some concluding remarks.

2 The Model

In this section, we set up a model based on Jones (2003). A representative dynastic family populates the economy. Jones assumes that the government collects lump sum taxes from households and uses them to pay wages for researchers. In contrast to Jones (2003), we incorporate profit-maximizing private firms undertaking R&D activity. Consequently, there are three sectors: a final goods sector, an intermediate goods sector, and an R&D sector. First, we consider the final goods sector.

2.1 Final Goods Sector

The final good, Y_t, is produced by the following production function:

$$Y_t = L_{Y,t}^{1-\alpha} \int_0^{A_t} x_{j,t}^{\alpha} dj, \quad 0 < \alpha < 1, \tag{1}$$

where $L_{Y,t}$ and $x_{j,t}$ respectively, represent labour input and the input of the jth intermediate good at time t. A_t stands for the variety of intermediate goods at time t. If the R&D firms succeed in inventing a new variety, A_t increases. Perfect competition is supposed to prevail in the final goods market. Therefore, we obtain the following profit-maximization conditions:

$$(1-\alpha)\frac{Y_t}{L_{Y,t}} = w_t, \tag{2}$$

$$\alpha L_{Y,t}^{1-\alpha} x_{j,t}^{\alpha} = p_{j,t}, \tag{3}$$

where w_t and $p_{j,t}$ are the wage rate and the price of intermediate good j at time t, respectively. We normalize the price of final goods to one. From (3), we obtain the following demand function for intermediate good j:

$$x_{j,t} = \alpha^{\frac{1}{1-\alpha}} \frac{L_{Y,t}}{p_{j,t}^{1/(1-\alpha)}}. \tag{4}$$

2.2 Intermediate Goods Sector

A single firm produces each intermediate good. This firm is a monopoly and can set the price of the intermediate good that it supplies. The monopoly is protected by perfect patent protection. One unit of capital supplied by the family produces one unit of the intermediate good. Therefore, the producer of the jth intermediate good maximizes profit according to the following:

$$\pi_j = p_j x_j - r x_j,$$

subject to the demand function of the final good sector, (4), where r is the rental rate of capital. This results in the following pricing rule:

$$p \equiv p_j = \frac{1}{\alpha} r.$$

Hence, the price is the same for all intermediate goods j. Thus, the output levels of all intermediate goods j are the same and given by:

$$x \equiv x_j = \left(\frac{\alpha^2}{r}\right)^{\frac{1}{1-\alpha}} L_Y. \tag{5}$$

The profit of each intermediate good firm is given by:

$$\pi_x \equiv \pi_j = (1-\alpha)\left(\frac{\alpha^{1+\alpha}}{r^{\alpha}}\right)^{\frac{1}{1-\alpha}} L_Y. \tag{6}$$

2.3 R&D Sector

R&D activities are carried out using labour inputs according to the following technology:

$$\dot{A}_t = \widetilde{\delta} L_{A,t}. \tag{7}$$

where $L_{A,t}$ is the labour input for R&D activities at time t. $\widetilde{\delta}$ represents the productivity level of R&D activities.[6] $\dot{A}_t$ measures new intermediate goods. We assume that the accumulated knowledge positively affects productivity in the following manner:

$$\widetilde{\delta} = \delta A_t^{\phi}, \quad \delta > 0, \quad 0 < \phi < 1, \tag{8}$$

[6]If we incorporate the duplication effect into the innovation technology, the production function becomes $\dot{A}_t = \widetilde{\delta}(L_{A,t})^{\lambda}$, $0 < \lambda \leq 1$. We neglect this effect for analytical simplicity.

where ϕ represents a parameter that expresses the extent of knowledge spillover. $\phi < 1$ means there are decreasing returns in the production of new intermediate goods. Perfect competition prevails in R&D races. Each R&D firm maximizes its profit without considering this spillover effect. Therefore, the objective of the firm becomes:

$$\pi_{A,t} = P_{A,t}\dot{A}_t - w_t L_{A,t},$$

where $P_{A,t}$ is the price of a blueprint of a newly invented intermediate good. Free entry into the R&D race leads to the following zero-profit condition:

$$P_{A,t}\widetilde{\delta} = w_t.$$

By using (8), we obtain:

$$P_{A,t}\delta A_t^{\phi} = w_t. \tag{9}$$

The discounted sum of profit of the intermediate good firm buying the blueprint determines the price of the blueprint. That is, the following holds:

$$P_{A,t} = \int_{\tau=t}^{\infty} \pi_{x,\tau} e^{-\int_{u=t}^{\tau} r_u du} d\tau, \tag{10}$$

where r_u represents the return on assets at time u. By differentiating (10) with respect to time t, we obtain the following no-arbitrage condition:

$$r_t P_{A,t} = \pi_{x,t} + \dot{P}_{A,t}.$$

2.4 Dynastic Family

Individuals derive their utility not only from their own consumption but also from the utility of their children.[7] Thus, parents care about the number of their children, not just their own utility. We assume that the head of a representative dynastic family maximizes the following:

$$U_t = \int_{\tau=t}^{\infty} u(c_\tau, N_\tau) e^{-\rho(\tau-t)} d\tau, \tag{11}$$

where c_τ is the consumption of a member of the dynasty at time τ, $\rho(> 0)$ is the rate of time preference, and N_τ is the number of members of the dynasty at time

[7]See Barro and Becker (1989) for this approach.

τ. We further assume that the instantaneous utility function of the head takes the following form for analytical simplicity:[8]

$$u(c_\tau, N_\tau) = \log c_\tau + \theta \log N_\tau, \tag{12}$$

where $\theta(> 0)$ is the weight placed on the utility of the offspring.

We next formulate the budget constraint of the dynasty. Let n_t denote the number of children at time t for a member of the family. We assume that each member has one unit of time endowment and that rearing children requires time: that is, when rearing n_t children, an individual member of the family must devote $\beta(n_t)$ units of time and he/she must give up the corresponding wage income. The rearing cost function, $\beta(n_t)$ satisfies the following conditions; $\beta'(n_t) > 0$ and $\beta''(n_t) > 0$. Thus, the per capita stock of assets evolves according to the following equation:

$$\dot{a}_t = (r_t - n_t)a_t + w_t[1 - \beta(n_t)] - c_t. \tag{13}$$

Because there are N_t identical individuals and each individual has n_t children, the size of the family evolves according to the following equation:

$$\dot{N}_t = n_t N_t. \tag{14}$$

The head of the family maximizes (11) subject to (13) and (14). The first-order conditions are given by:

$$\frac{1}{c_t} = \lambda_t, \tag{15}$$

$$\mu_t N_t = \lambda_t[a_t + w_t\beta'(n_t)], \tag{16}$$

$$\dot{\lambda}_t = (\rho + n_t - r_t)\lambda_t, \tag{17}$$

$$\dot{\mu}_t = (\rho - n_t)\mu_t - \frac{\theta}{N_t}, \tag{18}$$

where λ_t and μ_t are costate variables associated with asset holding and the family size, respectively. Furthermore, the following transversality conditions must be satisfied: $\lim_{t\to\infty} \lambda_t a_t e^{-\rho t} = 0$ and $\lim_{t\to\infty} \mu_t N_t e^{-\rho t} = 0$. The left-hand side of (16) represents the shadow value of children in the family. The right-hand side of (16) represents the cost to have children. The first and second terms of (16) represent the capital dilution effect and the opportunity cost, respectively.

[8] See the appendix for the rationale underlying this functional form.

3 Market Equilibrium and Dynamics

Based on the model in the preceding section, we derive the market equilibrium and construct a dynamic system. The goods market equilibrium is given by:

$$Y_t = C_t + \dot{K}_t, \tag{19}$$

where C_t and K_t are aggregate consumption and capital stock, respectively. Denoting the per capita variables in lowercase, we can transform (19) into the following per capita terms:

$$\dot{k}_t = y_t - c_t - n_t k_t. \tag{20}$$

The equilibrium condition for capital is given by:

$$K_t = A_t x_t. \tag{21}$$

We note here that there exist A_t varieties of intermediate goods at time t and the quantity $x_{j,t}$ is the same for all intermediate goods. The market equilibrium condition of the labour market is given by:

$$L_{Y,t} + L_{A,t} = [1 - \beta(n_t)]\, N_t. \tag{22}$$

To derive the dynamic system of the economy, let us define the following variables: $\chi_t \equiv c_t/k_t$, $z_t \equiv A_t/k_t$, $\zeta_t \equiv \mu_t N_t$, $\nu_t \equiv A_t^{1-\phi}/N_t$, $\widetilde{P}_{A,t} \equiv P_{A,t}/N_t$, and $g_{A,t} \equiv \dot{A}_t/A_t$. In the appendix, we show that the following five equations constitute the dynamic system of the economy:

$$\dot{\chi}_t = \left\{\chi_t + \left(1 - \frac{1}{\alpha^2}\right) r_t - \rho\right\} \chi_t, \tag{23}$$

$$\dot{z}_t = \left(\chi_t + n_t + g_{A,t} - \frac{r_t}{\alpha^2}\right) z_t, \tag{24}$$

$$\dot{\zeta}_t = \rho\zeta_t - \theta, \tag{25}$$

$$\dot{\nu}_t = [(1-\phi) g_{A,t} - n_t]\, \nu_t, \tag{26}$$

$$\dot{\widetilde{P}}_{A,t} = \left(r_t - \frac{1-\alpha}{\alpha} \frac{r_t}{z_t \widetilde{P}_{A,t}} - n_t\right) \widetilde{P}_{A,t}. \tag{27}$$

The appendix also shows that r_t, n_t, and $g_{A,t}$ are given by the solutions of the following three equations:

$$\delta \frac{\widetilde{P}_{A,t}}{\nu_t} = (1-\alpha) \left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}}, \tag{28}$$

$$\frac{1}{z_t}\left(\frac{\alpha^2}{r}\right)^{\frac{1}{1-\alpha}} + \frac{1}{\delta}\nu_t g_{A,t} = 1 - \beta(n_t), \tag{29}$$

$$\beta'(n_t) = \frac{1}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}\frac{\zeta_t \chi_t - 1}{z_t} r_t^{\frac{1}{1-\alpha}}. \tag{30}$$

We now derive the balanced growth path (hereafter BGP). Section 5 examines the full dynamics. In this section, we characterize the BGP. The BGP, $\{\chi^*, z^*, \widetilde{P}_A^*, r^*, \nu^*, \zeta^*, g_A^*, n^*\}$ is determined by the following equations:

$$\chi^* = \rho - \left(1 - \frac{1}{\alpha^2}\right) r^*, \tag{31}$$

$$\chi^* = \frac{r^*}{\alpha^2} - n^* - g_A^*, \tag{32}$$

$$\zeta^* = \frac{\theta}{\rho}, \tag{33}$$

$$g_A^* = \frac{n^*}{1-\phi}, \tag{34}$$

$$n^* = r^*\left(1 - \frac{1-\alpha}{\alpha}\frac{1}{z^*\widetilde{P}_A^*}\right), \tag{35}$$

and the three equations, (28), (29), and (30). From (31), (32), and (34), the following holds:

$$r^* = \rho + \frac{2-\phi}{1-\phi} n^*. \tag{36}$$

Then, substituting (36) into (31), we obtain:

$$\chi^* = \frac{\rho}{\alpha^2} + \left(\frac{1}{\alpha^2} - 1\right)\frac{2-\phi}{1-\phi} n^*. \tag{37}$$

When $\theta = 1$, we obtain a clear result for the decision of the dynastic family. By using (29) and (30), we obtain the following equation:

$$\frac{\beta'(n^*)}{1 - \beta(n^*) - \frac{1}{\delta}\nu^* g_A^*} = \frac{\alpha^2}{1-\alpha}\frac{\zeta^*\chi^* - 1}{r^*}.$$

Due to (37) and (A8), we can rearrange this into the following:

$$\frac{\beta'(n^*)}{1 - \beta(n^*) - l_A^*} = \frac{1+\alpha}{\rho}. \tag{38}$$

If the allocation of labour input to the R&D sector is determined, (38) gives the steady state number of children in the market equilibrium.

4 Socially Optimal Allocation

By solving the social planner's problem, we can derive the socially optimal allocation, especially the optimal number of children. The social planner maximizes the following welfare level of the dynastic family:

$$U_t = \int_{\tau=t}^{\infty} [\log c_\tau + \theta \log N_\tau]\, e^{-\rho(\tau-t)} d\tau,$$

subject to the resource constraint and the production function of new varieties:

$$\dot{k}_t = k_t^\alpha \left[A_t \{1 - \beta(n_t) - l_{A,t}\}\right]^{1-\alpha} - c_t - n_t k_t, \tag{39}$$

$$\dot{A}_t = \delta A_t^\phi L_{A,t} = \delta A_t^\phi l_{A,t} N_t, \tag{40}$$

and (14). The head of the dynastic family does not take account of (40) when he/she optimizes. This is the source of the externality.

The first-order conditions of this maximization problem are given by:

$$\frac{1}{c_t} = \lambda_t, \tag{41}$$

$$\mu_{1,t} N_t = \lambda_t \left[(1-\alpha)\frac{y_t \beta'(n_t)}{1-\beta(n_t)-l_{A,t}} + k_t\right], \tag{42}$$

$$\mu_{2,t} \delta A_t^\phi N_t = \lambda_t (1-\alpha)\frac{y_t}{1-\beta(n_t)-l_{A,t}}, \tag{43}$$

$$\dot{\lambda}_t = \left(\rho + n_t - \alpha\frac{y_t}{k_t}\right)\lambda_t, \tag{44}$$

$$\dot{\mu}_{1,t} = (\rho - n_t)\mu_{1,t} - \frac{\theta}{N_t} - \mu_{2,t}\delta A_t^\phi l_{A,t}, \tag{45}$$

$$\dot{\mu}_{2,t} = \rho\mu_{2,t} - \lambda_t \frac{(1-\alpha)y_t}{A_t} - \mu_{2,t}\phi\delta A_t^{\phi-1} l_{A,t} N_t, \tag{46}$$

where λ_t, $\mu_{1,t}$, and $\mu_{2,t}$ are costate variables associated with capital, family size, and the varieties of intermediate goods, respectively. Furthermore, the following transversality conditions must be satisfied: $\lim_{t\to\infty} \lambda_t a_t e^{-\rho t} = 0$, $\lim_{t\to\infty} \mu_{1,t} N_t e^{-\rho t} = 0$, and $\lim_{t\to\infty} \mu_{2,t} A_t e^{-\rho t} = 0$.

From (41) and (44), we obtain:

$$\dot{c}_t = \left[\alpha(z_t l_{Y,t})^{1-\alpha} - \rho - n_t\right] c_t. \tag{47}$$

From (39) and (47), we obtain:

$$\dot{\chi}_t = \left\{\chi_t - (1-\alpha)\,(z_t l_{Y,t})^{1-\alpha} - \rho\right\}\chi_t. \tag{48}$$

Due to the definition of z_t and (39), we obtain the following dynamics for z_t:

$$\dot{z}_t = \left[\chi_t + n_t + g_{A,t} - (z_t l_{Y,t})^{1-\alpha}\right] z_t. \tag{49}$$

We here define $\zeta_t \equiv \mu_{1,t} N_t$ and $\psi_t \equiv \mu_{2,t} A_t$. Then, from (45), we obtain:

$$\dot{\zeta}_t = \rho\zeta_t - \theta - \psi_t g_{A,t}. \tag{50}$$

From (46), we obtain:

$$\dot{\psi}_t = \{(1-\phi)g_{A,t} + \rho\}\,\psi_t - (1-\alpha)\frac{(z_t l_{Y,t})^{1-\alpha}}{\chi_t}. \tag{51}$$

From (43), we obtain the following relationship:

$$\frac{(1-\alpha)(z_t l_{Y,t})^{1-\alpha}}{[1-\beta(n_t) - l_{A,t}]\chi_t} = \delta\frac{\psi_t}{\nu_t}. \tag{52}$$

Note that we have used the following definition of ν_t: $\nu_t \equiv A_t^{1-\phi}/N_t$. As for the dynamics of ν_t, we can obtain the same equation as (26). From (41) and (42), we obtain:

$$\zeta_t = \frac{1}{c_t}\left[(1-\alpha)\frac{y_t\beta'(n_t)}{1-\beta(n_t) - l_{A,t}} + k_t\right].$$

By using the definition of χ_t and z_t, we can rearrange this relationship as follows:

$$\zeta_t\chi_t - 1 = (1-\alpha)\frac{{z_t}^{1-\alpha}\beta'(n_t)}{[1-\beta(n_t) - l_{A,t}]^{\alpha}}. \tag{53}$$

Noting that (A8) and $l_{Y,t} = 1 - \beta(n_t) - l_{A,t}$, (48), (49), (50), (51), and (26), together with (52) and (53), constitute the dynamic system of the socially optimal allocation.

We next derive the BGP of the socially optimal allocation. From (48), (49), (50), (51), and (26), the BGP of the social optimal, $\{\chi^{OP}, z^{OP}, \psi^{OP}, \nu^{OP}, \zeta^{OP}, g_A^{OP}, n^{OP}\}$, is determined by the following: equations:

$$\chi^{OP} = (1-\alpha)\,(z^{OP} l_Y^{OP})^{1-\alpha} + \rho, \tag{54}$$

$$\chi^{OP} = (z^{OP} l_Y^{OP})^{1-\alpha} - n^{OP} - g_A^{OP}, \tag{55}$$

$$\rho\zeta^{OP} = \theta + \psi^{OP} g_A^{OP}, \tag{56}$$

$$\left\{(1-\phi) g_A^{OP} + \rho\right\} \chi^{OP} \psi^{OP} = (1-\alpha)(z^{OP} l_Y^{OP})^{1-\alpha}, \tag{57}$$

$$g_A^{OP} = \frac{n^{OP}}{1-\phi}, \tag{58}$$

and the relationship, $l_Y^{OP} = 1 - \beta(n^{OP}) - \frac{1}{\delta}\nu^{OP} g_A^{OP}$ and the two equations; (52) and (53).

When $\theta = 1$, we can obtain a clear result similar to (38) for the market equilibrium:

$$\frac{\beta'(n^{OP})}{1-\beta(n^{OP})} = \frac{1}{\rho}. \tag{59}$$

In order to compare the socially optimal number of children at the steady state with the number of children in the market equilibrium at the steady state, it is useful to rewrite (38) using the share of researchers allocated to R&D activities. This share is defined by $l_A \equiv s_A[1-\beta(n)]$. Then, (38) is transformed into the following form:

$$\frac{\beta'(n^*)}{1-\beta(n^*)} = \frac{(1-s_A^*)(1+\alpha)}{\rho}.$$

Because $\beta'(n)/[1-\beta(n)]$ is an increasing function of n, if the following inequality holds, $(1-s_A^*)(1+\alpha) > 1$, then the number of children in the market equilibrium at the steady state is larger than the socially optimal number of children at the steady state. That is, too many children exist in the market equilibrium at the steady state. However, because the share of researchers is an endogenous variable, we conduct a simulation approach to obtain a much clearer result. Moreover, we calibrate the transition paths based on some plausible parameters in the following section.

5 Simulation of Transition Paths

To analyse the model numerically, we specify the function $\beta(\cdot)$ as:

$$\beta(n) = \beta n^2, \tag{60}$$

where β is a positive constant.

As a benchmark, we choose the parameter values as follows. The time preference rate, ρ, is set to 0.05, which is a conventional value in the growth literature. The weight on the utility of offspring, θ, is equal to 1, because we analyse the case where $\theta = 1$ holds in Section 4. We assume that α is equal to 1/1.25, which implies that the mark-up of the intermediate goods sector is 1.25. The values of β and δ are chosen so that the population growth rate of the market equilibrium at BGP becomes sufficiently close to 0.01. Then, we obtain $\beta = 1300$ and $\delta = 0.3$. Finally, ϕ is set to 0.5 so that the annual growth rate of the economy, g_A^*, is around 0.02.

Under this parameter set, the population growth rate of the market equilibrium is equal to 0.0099, which is larger than that of the socially optimal allocation, $n^{op} = 0.0077$. In this benchmark case, the number of children in the market equilibrium at the steady state is larger than the socially optimal number of children at the BGP. That is, there are too many children in the market equilibrium at the steady state. Because the growth rates are given by (34) and (58), it is expected that in our numerical example, g_A^* is larger than g_A^{op}. As expected, g_A^* is 0.197 and larger than $g_A^{op} = 0.0147$. The market equilibrium attains a higher growth rate than the socially optimal growth rate. The share of labour allocated to R&D activities in market equilibrium $s_A^* = 0.185$ is smaller than the socially optimal share $s_A^{op} = 0.201$ and satisfies the condition $(1 - s_A^*)(1 + \alpha) > 1$.

The next question is whether the number of children in the market equilibrium is larger than the socially optimal number of children along all the transition paths to the BGP. For this purpose, we analyse the transition paths by using the relaxation algorithm.[9] In the following numerical examples, the initial values of the state variables are chosen as $z(\equiv A_t/k_t) = 0.015$ and $v(\equiv {A_t}^{1-\phi}/N_t) = 2.9$ so that in the benchmark market equilibrium, the population growth rate decreases over time and the share of labour allocated to R&D activities increases over time.

Figure 1 presents the results for the benchmark case. The upper panel shows the transitional paths of the number of children (the population growth rate). The

[9]Trimborn et al. (2008) detail the relaxation algorithm. They also provide MatLab programs for the relaxation algorithm, freely downloadable at http://www.rrz.uni-hamburg.de/IWK/trimborn/relaxation.htm.

solid line stands for the number of children in the market equilibrium while the dotted line stands for the socially optimal number of children. The panel shows that along all transition paths to the BGP, the number of children in the market equilibrium is larger than the socially optimal number of children in this benchmark economy.

The middle and lower panels of Figure 1 depict the transitional paths of g_A and s_A, respectively. From the lower panel, we know that the labour share allocated to R&D activities in the market equilibrium increases over time along the transition path whereas it decreases over time in the socially optimal allocation. The share of labour allocated to R$D activities is also smaller than the socially optimal share along the transition path. The growth rate of the market equilibrium is smaller than the socially optimal growth rate during the early stage of transition due to the smaller share of labour allocated to R&D activities in the market equilibrium. During the later stages of transition and at the BGP, however, the growth rate of the market equilibrium becomes higher than the socially optimal rate. This is because the differences between the shares of labour allocated to R&D activities of the market equilibrium and that of the social optimum decrease over time along the transition path, and because the higher population growth of the market equilibrium positively affects the growth rate.

In Figures 2–4, we present numerical examples other than the benchmark case. Figure 2 shows where the value of ρ is increased and decreased from the benchmark level. Figures 3 and 4 show where the values of ϕ and θ are increased and decreased from the benchmark levels. All figures show that not only at the BGP but also along the transition path to the BGP, the number of children in the market equilibrium is larger than the socially optimal number of children. In all of the figures, the growth rate and the share of labour allocated to R&D activities exhibit similar transitional paths. The only exception is when the value of ρ is increased from the benchmark. When ρ is 0.07, the share of labour allocated to R&D activities in the market equilibrium becomes higher than that of the social optimum in the later stage of transition (the lower left panel of Figure 2). However, even in this exceptional case, the number of children in the market equilibrium is larger than the socially optimal number of children along the transition path.

6 Concluding Remarks

In this paper, we incorporated profit-maximizing innovating firms and thus extended the model in Jones (2003). Importantly, the present model is much more realistic than Jones (2003), where the labour allocation is fixed. Moreover, we have examined transition paths by conducting simulation analysis. We have shown that the number of children in the market equilibrium is larger than in the socially optimal allocation. Based on plausible parameters, this is not only at the steady state but also on the transition paths.

The usual argument is that because the number of children that parents actually have is smaller than the number of children they want to have, the government must subsidize parents so they can have more children. However, our result implies that the government should be cautious with policy when intervening in the private decisions of parents with respect to the number of children.

Finally, we provide some directions for future research. First, in the present model, we assume that the number of children take real numbers. However, in reality the number of children must be nonnegative integers. Therefore, there may be a substantial difference between having two children and having three children. If so, we must investigate this issue by taking into account the integer problem. Second, a continuous decrease in the population leads to zero population size. Given at least some population size must exist for economic activities and human life, we must consider the possibility of an optimal population size. These are important issues to be considered in future research.

Appendix

A1

In this appendix, we show how the objective function (11) and the instantaneous utility function (12) are derived. Consumption at time t by a member of the dynastic family is given by c_t. Because a member has n_t children, the utility of this member, U_t is defined as:

$$U_t = u(c_t) \cdot dt + (1 - \rho dt)\Upsilon(n_t dt) \cdot (n_t dt) U_{t+dt}, \tag{A1}$$

where $\Upsilon(n_t dt)$ is the degree of altruism of parents toward their children. For simplicity, we assume the following functional forms: $u(c_t) = \frac{{c_t}^{1-\sigma}}{1-\sigma}$ and $\Upsilon(n_t dt) = (n_t dt)^{-\varepsilon}$.

We assume that the size of the dynastic family at time t is one, that is, $N_t = 1$. Then, we have $N_{t+dt} = n_t dt$, $N_{t+2dt} = n_t dt \times n_{t+dt} dt$, and so on. Therefore, by using (A1), we can obtain the following discounted sum of U_t:

$$U_t = u(c_t) \cdot dt + (1-\rho dt) u(c_{t+dt})(N_{t+dt})^{1-\varepsilon} \cdot dt + (1-\rho dt)^2 u(c_{t+2dt})(N_{t+2dt})^{1-\varepsilon} \cdot dt + \cdots . \tag{A2}$$

Because of the assumption of the functional form, we have:

$$u(c)N^{1-\varepsilon} = \frac{\left(cN^{\frac{1-\varepsilon}{1-\sigma}}\right)^{1-\sigma} - 1}{1-\sigma}.$$

As in Barro and Sala-i-Martin (2004, p410), we add the term -1 in the numerator so that we obatin the log-utility form as σ approaches 1. When σ approaches 1 by keeping the ratio $(1-\varepsilon)/(1-\sigma)$ constant, then its limit becomes:

$$\lim_{\sigma \to 1} \frac{\left(cN^{\frac{1-\varepsilon}{1-\sigma}}\right)^{1-\sigma} - 1}{1-\sigma} = \ln c + \theta \ln N, \tag{A3}$$

where $\theta \equiv (1-\varepsilon)/(1-\sigma)$. Consequently, (A2) can be rewritten as follows:

$$U_t = [\ln c_t + \theta \ln N_t] dt + (1-\rho dt)\{[\ln c_{t+dt} + \theta \ln N_{t+dt}] dt + (1-\rho dt)[\ln c_{t+2dt} + \theta \ln N_{t+2dt}] dt + \cdots\}.$$

Neglecting the higher-order terms of $(dt)^n$ $(n > 1)$, (A1) finally becomes:

$$U_t = [\ln c + \theta \ln N] \cdot dt + (1-\rho dt) U_{t+dt}.$$

Dividing both sides of this equation and taking the limit of $dt \to 0$, we obtain the following differential equation:

$$\dot{U}_t = \rho U_t - (\ln c_t + \theta \ln N_t).$$

By integrating this differential equation, we can obtain the desired result.

A2

In this appendix, we derive the dynamic system of the economy. Noting that the same amount of the intermediate goods is produced, we can transform (1) into the following production function in intensive form:

$$y_t = k_t^{\alpha}(A_t l_Y)^{1-\alpha}, \tag{A4}$$

where $l_Y \equiv L_Y/N$. Thus, (20) becomes:

$$\dot{k}_t = k_t^{\alpha}(A_t l_Y)^{1-\alpha} - c_t - n_t k_t. \tag{A5}$$

From (15) and (17), we obtain:

$$\dot{c}_t = (r_t - \rho - n_t)c_t. \tag{A6}$$

By using (5) and (21), and defining $z_t \equiv A_t/k_t$, we obtain:

$$z_t l_{Y,t} = \left(\alpha^2/r\right)^{\frac{1}{1-\alpha}}. \tag{A7}$$

By defining $\chi_t \equiv c_t/k_t$ and using (A7), we obtain (23). We next derive the dynamics of z_t, (24). By defining $\zeta_t \equiv \mu_t N_t$, we obtain (25). By similarly defining $\nu_t \equiv A_t^{1-\phi}/N_t$, we obtain (26). Again, by defining $\widetilde{P}_{A,t} \equiv P_{A,t}/N_t$ and using (6) and (A7), we obtain (27).

By using (2), (9), and (A7), we obtain (28).

From the definition of ν_t, (7), and (8), we obtain:

$$L_{A,t}/N_t \equiv l_{A,t} = \frac{1}{\delta}\nu_t g_{A,t}. \tag{A8}$$

Substituting (A7) and (A8) into (22), we obtain (29).

From (15) and (16), we obtain:

$$\zeta_t = \frac{k_t + w_t\beta'(n_t)}{c_t}. \tag{A9}$$

By using (2), and (A4), we obtain $(1-\alpha)\left(\frac{k_t}{A_t l_{Y,t}}\right)^{\alpha} = \frac{w_t}{A_t}$. Substituting this into (A9), using (A7), and rearranging, we obtain (30).

A3

In this appendix, we derive the equation that determines the socially optimal number of children at the steady state, (59).

From (54), (55), we obtain:

$$(1-\alpha)\,(z^{OP} l_Y^{OP})^{1-\alpha} + \rho = (z^{OP} l_Y^{OP})^{1-\alpha} - n^{OP} - g_A^{OP}.$$

By using (58), this relationship can be rearranged as follows:

$$\alpha (z^{OP} l_Y^{OP})^{1-\alpha} = \rho + \frac{2-\phi}{1-\phi} n^{OP}. \tag{A10}$$

Next, substituting (A10) into (55) and using (58), we obtain:

$$\chi^{OP} = \frac{1}{\alpha}\left[\rho + (1-\alpha)\frac{2-\phi}{1-\phi} n^{OP}\right]. \tag{A11}$$

From (57), (A10), and (A11), we obtain:

$$\psi^{OP} = \frac{(1-\alpha)\left(\rho + \frac{2-\phi}{1-\phi} n^{OP}\right)}{(\rho + n^{OP})\left[\rho + (1-\alpha)\frac{2-\phi}{1-\phi} n^{OP}\right]}. \tag{A12}$$

Substituting (A12) into (52) and noting that $l_Y = 1 - \beta(n) - l_A$ and (A11), we obtain:

$$\frac{(z^{OP})^{1-\alpha}}{(l_Y^{OP})^{\alpha}} = \frac{\delta}{\alpha} \frac{\rho + \frac{2-\phi}{1-\phi} n^{OP}}{(\rho + n^{OP})\nu^{OP}}. \tag{A13}$$

By multiplying both sides of (A13) by l_Y^{OP} and substituting(A10) into them, we obtain:

$$\nu^{OP} = \frac{\delta[1 - \beta(n^{OP})]}{\rho + \frac{2-\phi}{1-\phi} n^{OP}}. \tag{A14}$$

From (53) and (56), we obtain:

$$(1-\alpha)\frac{(z^{OP})^{1-\alpha}}{(l_Y^{OP})^{\alpha}}\beta'(n^{OP}) = \zeta^{OP}\chi^{OP} - 1 = \frac{\theta}{\rho}\chi^{OP} - 1 + \frac{g_A^{OP}}{\rho}\psi^{OP}\chi^{OP}.$$

Substituting (58), (A11), and (A12) into this relationship, we obtain:

$$\begin{aligned}(1-\alpha)\frac{(z^{OP})^{1-\alpha}}{(l_Y^{OP})^{\alpha}}\beta'(n^{OP}) =& \frac{\theta}{\rho\alpha}\left[\frac{\theta-\alpha}{\theta}\rho + (1-\alpha)\frac{2-\phi}{1-\phi} n^{OP}\right] \\ &+ \frac{1-\alpha}{\alpha\rho(1-\phi)}\frac{n^{OP}}{\rho + n^{OP}}\left(\rho + \frac{2-\phi}{1-\phi} n^{OP}\right).\end{aligned} \tag{A15}$$

When $\theta = 1$, we can rearrange (A15) as follows:

$$(1-\alpha)\frac{(z^{OP})^{1-\alpha}}{(l_Y^{OP})^{\alpha}}\beta'(n^{OP}) = \frac{1}{\rho}\left(\rho + \frac{2-\phi}{1-\phi}n^{OP}\right)\left(1 + \frac{1}{1-\phi}\frac{n^{OP}}{\rho + n^{OP}}\right).$$

Substituting (A13) into this and using (A14), we obtain (59).

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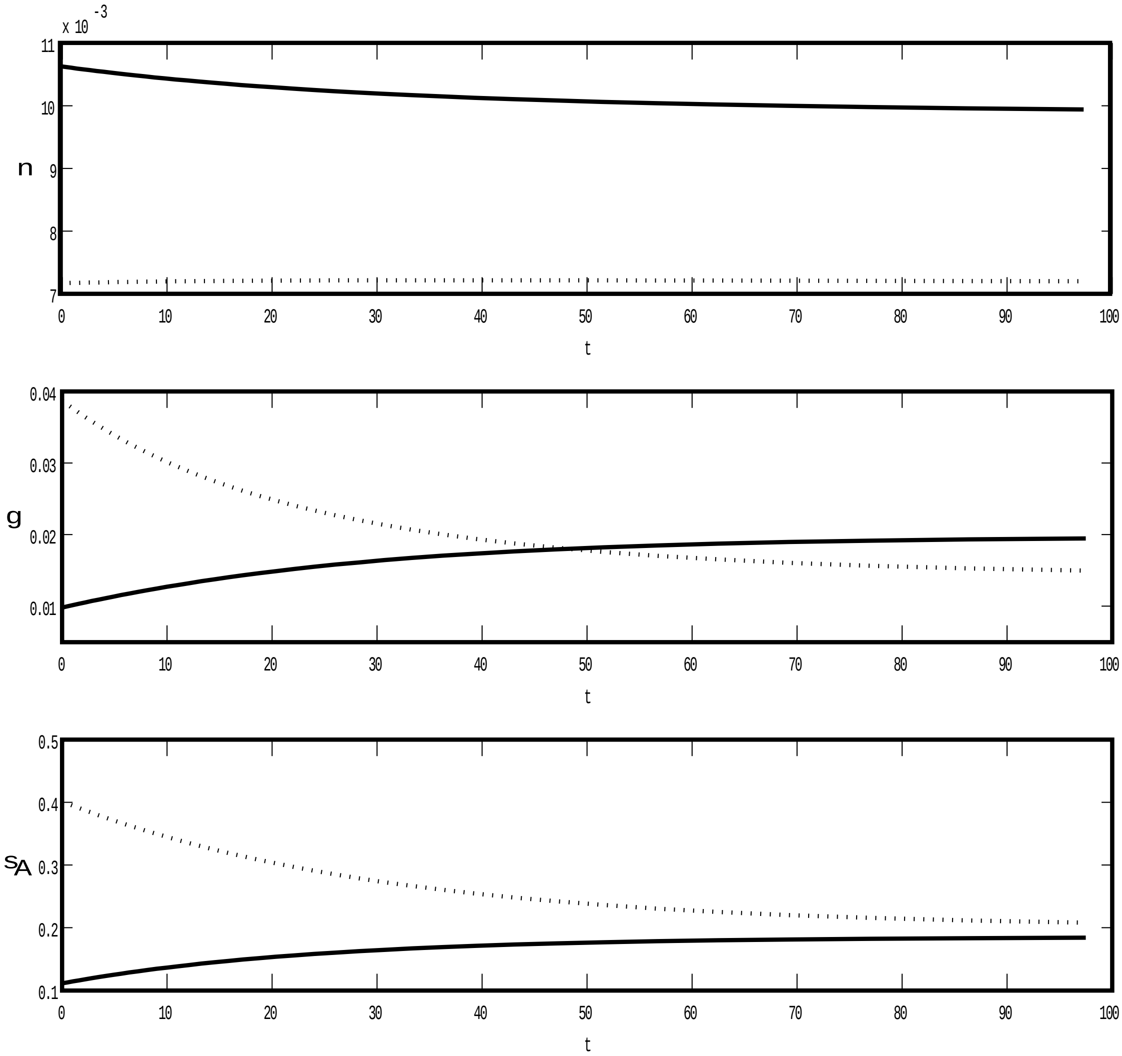
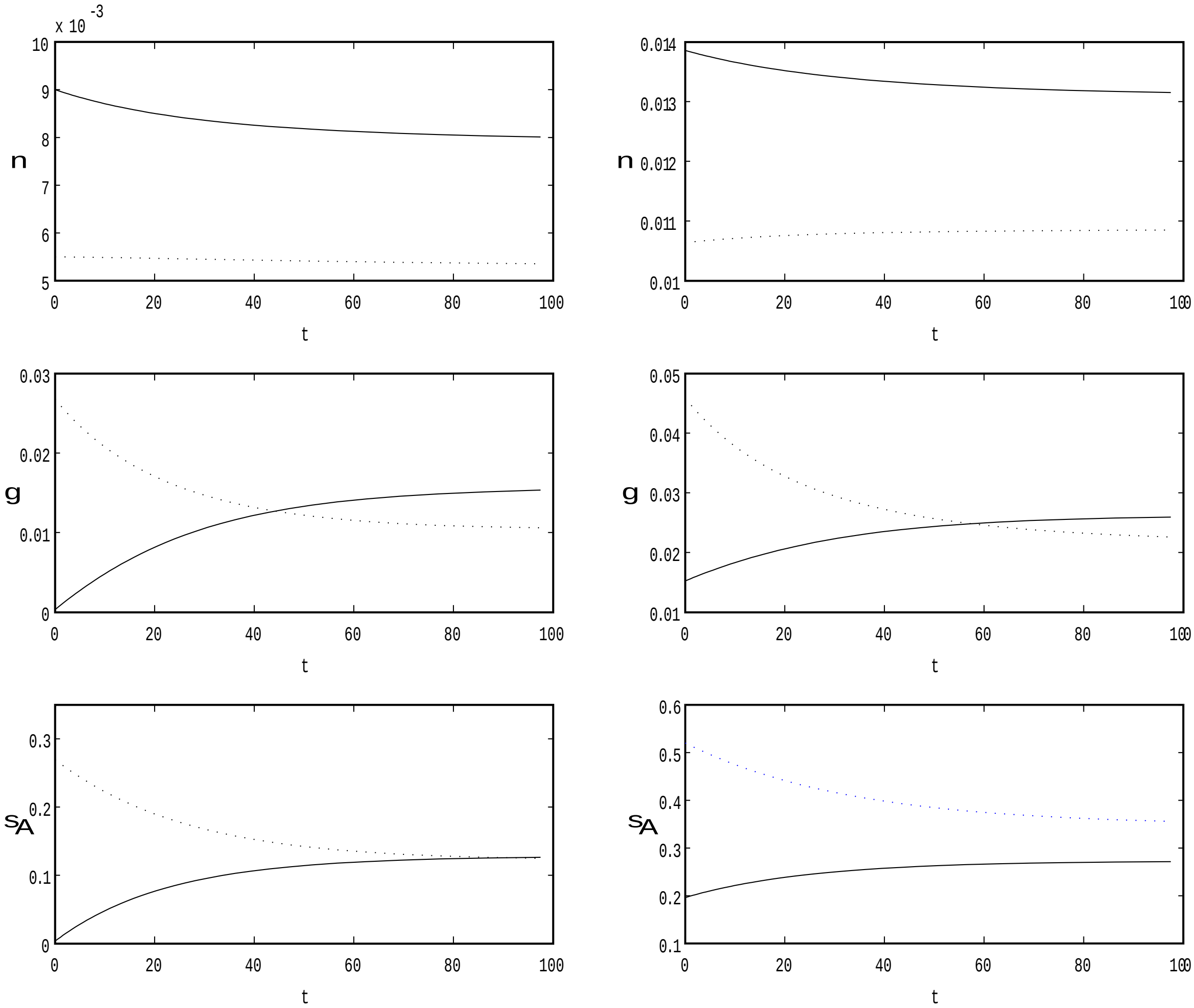
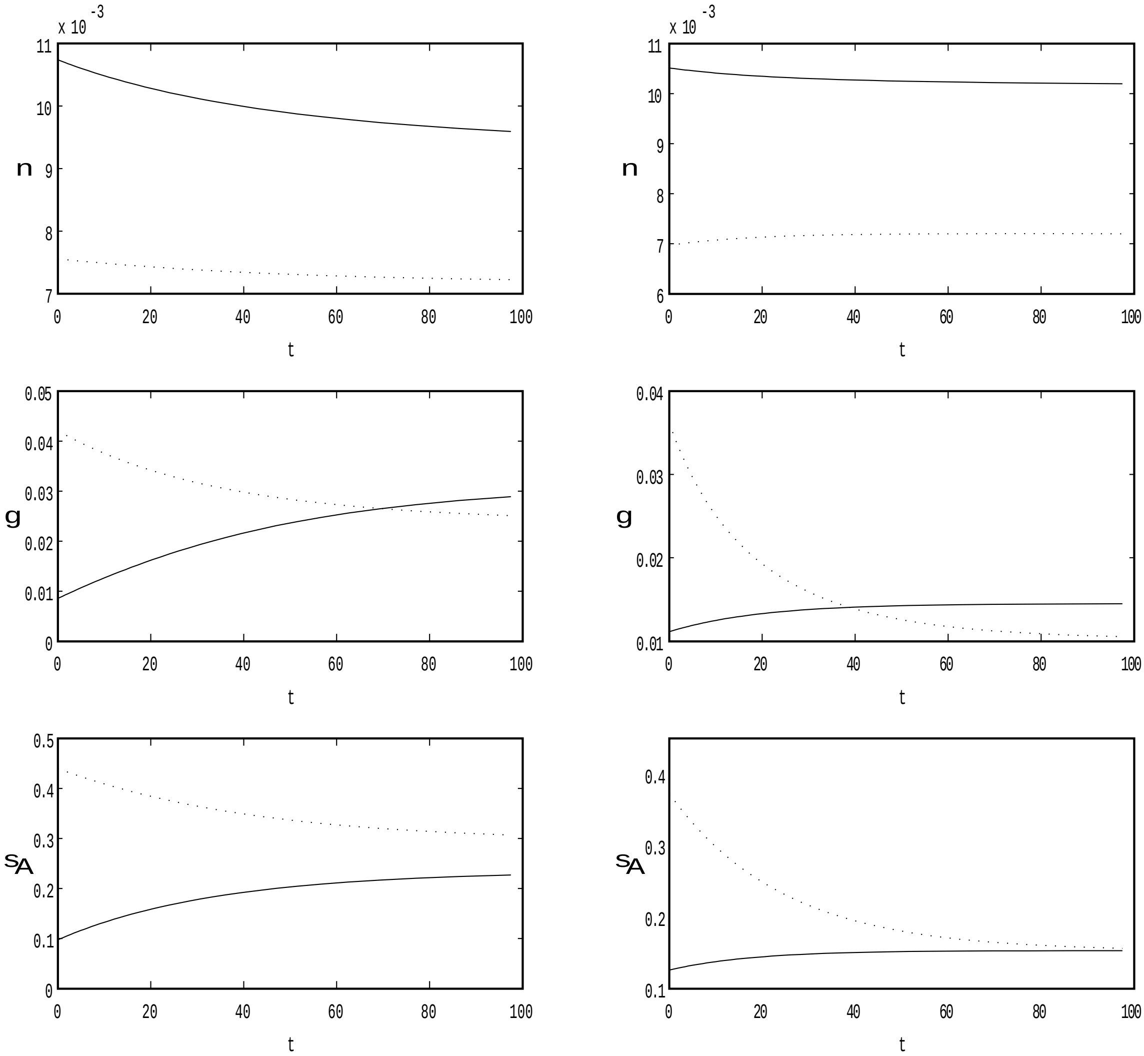


Figure 1. Benchmark Case



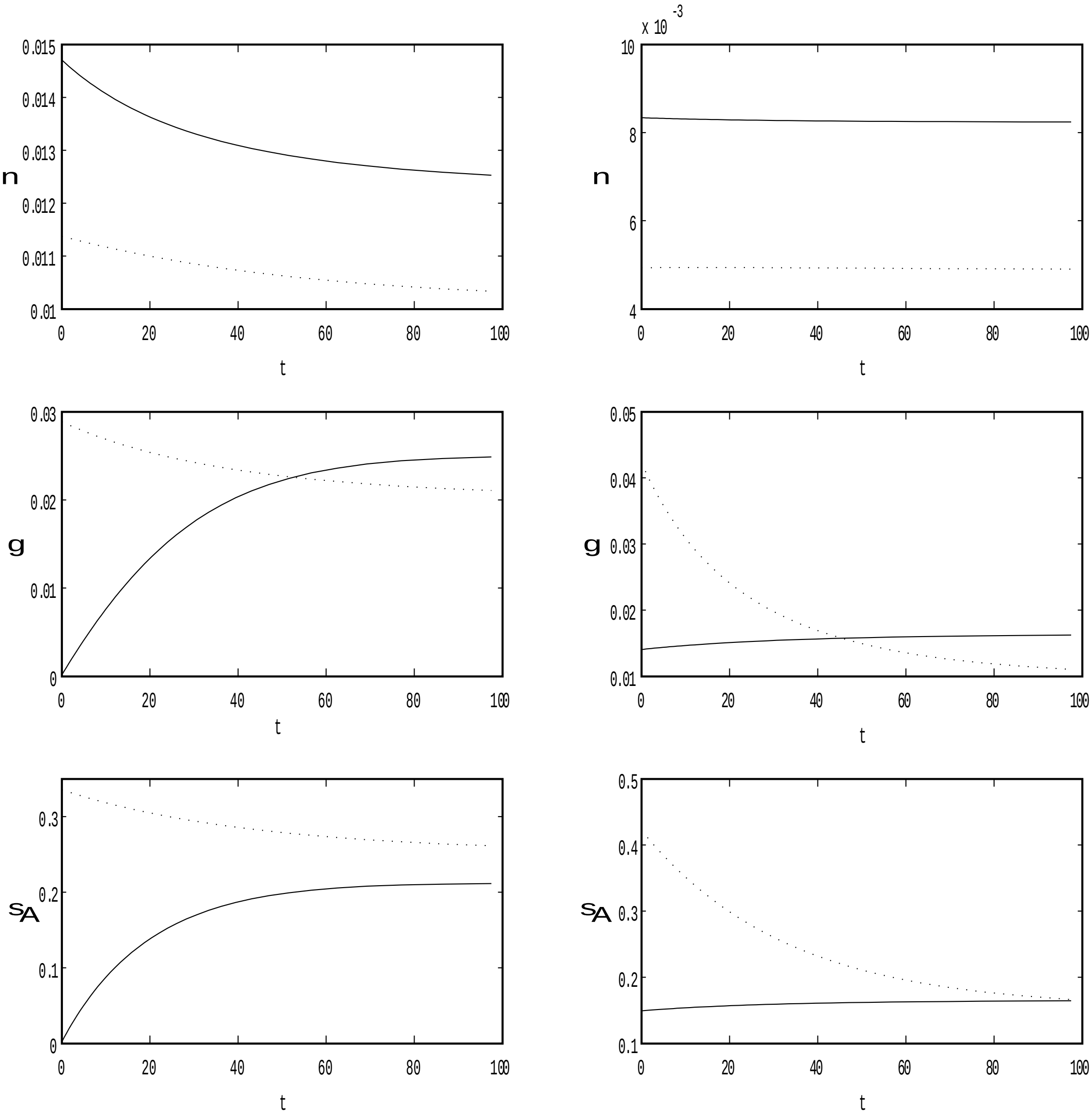
(a) ρ = 0.07 (b) ρ = 0.03

Figure 2. Effects of Changes in ρ



(a) $\phi = 0.7$ (b) $\phi = 0.3$

Figure 3. Effects of Changes in ϕ



(a) $\theta = 1.2$ (b) $\theta = 0.9$

Figure 4. Effects of Changes in θ