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Keisuke Kawata[†]

Abstract

This paper considers an on-the-job search model that includes wage bargaining and employer-employee mismatch. There are two states of workers in relationship to their fit for a particular job, good match versus bad match (mismatch). These states change in accordance with a stochastic process. There are two main results; the first is that the turnover level that workers find optimal is lower than the socially optimal level. The second is that the level of the firm's entry is not optimal even though the *Hosios condition* is hold. The first result is clearly distinct from previous studies.

JEL classification: J63; J81
Keywords: on-the-job search, wage bargaining, mismatch, turnover

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1. Introduction

Many authors have studied job-to-job transitions. Most of these studies are based on on-the-job search models. On-the-job search models have welfare implications for the efficiency of turnover decisions. In the case of a wage bargaining model where firms bargain a wage contract with an encountered worker (for example, [6], [11], and [13]), the turnover decisions by workers become socially optimal because high-productivity firms offer higher wages than low-productivity firms.

This paper's purpose is to show that the turnover decision is not socially optimal, even though a wage contract determined by the bargaining process, if it involves a *mismatch* between a worker and a firm in an on-the-job search model with wage bargaining. This study also shows that the levels of job-to-job transitions are underrepresented in a decentralized economy. The central assumption of this paper is that the worker's state is changed by a shock, which implies that the productivity of a worker-firm pair is either high or low. These results imply the necessity of a policy that induces workers to move to a more productive firm, which is clearly distinct with the result of previous literatures.

Several examples support this assumption. Young-aged workers have a comparative advantage for a physical work while older workers have a comparative advantage for skilled work. When a young worker matches with a firm specializing in physical tasks, it is a good match. However, if the worker grows old, this match is no longer good. Another example is a change of preferences for working hours. The worker's perceived disutility from working hours can be triggered by some shocks, such as a childbirth and marriage. If the disutility is low, she/he prefers to work long hours in a full-time job and earns a high wage. On the other hand, if the disutility is high, she/he chooses to work short hours in a part-time job earning low wages. In the latter case, childbirth switches her/his preference to work, motivating her/him to move to a part-time job from a full-time job. Some empirical studies ([3], [9], [7], and [12]) have suggested that, for working hours of one's choice, people switch between full-time jobs and part-time jobs.

I develop a tractable on-the-job search model with firms of type L or type S and workers of state l or state s. Let (i, j) denote a pair of a state i worker and a type j

worker. Pairs (l, S) and (s, L) suffer from mismatch and produce low output. Worker's states switch back and forth between type l and type s when a shock occurs, causing or eliminating a mismatch. A mismatched worker, while on the job, is allowed to search for a new firm and decides whether to quit and move to a new firm or stay in the current firm when finding the new firm. In this decision, the worker chooses either to eliminate the current mismatch by moving or to wait for a good match in the future by staying at the current firm. If the current pair is already a mismatch, the mismatch may be eliminated by a shock in the future. Alternatively, even through the current pair may not be mismatched at present, they become mismatched by a shock later. Moreover, a low-productivity pair may become a high-productivity pair after a shock, of vice versa.

In equilibrium, some workers stay in a low-productivity firm because they expect that the current pair will become a good-match in the near future. The worker's behavior regarding turnover is classified into three types; (i) they continue to stay type L firms regardless of their worker type, (ii) they continue to stay type S firms regardless of their worker type, and (iii) they continue to move no matter when the current pair becomes mismatched.

This paper finds two important implication from the present study. One regards a worker's decision on turnover. Fewer workers choose the third type of behavior in a decentralized economy than in a socially efficient economy. The reasoning behind this phenomenon is that workers do not take into consideration the future poaching firm's capital gains from the future turnover. When a worker moves to a well-matched firm, she/he will suffer from mismatch if her/his state changes after moving. She/he can then eliminate the new mismatch by moving to a new suitable firm again, but the value of this turnover is underestimated because a part of the capital gains from this turnover goes to the new poaching firm. Thus, the value of a well-matched pair is underestimated as well.

In contrast to this paper's findings, other research found the worker's decision on turnover to be optimal in on-the-job search models with wage bargaining ([6], [11], and [13]). Similarly to this paper's findings, however, they suggested that part of the capital gains from turnover went to the poaching firm. The difference between this paper's model and the previous models is that productivity is changed in accordance with a stochastic process in this model while productivity of a worker-firm pair was constant in the previous models.

The second implication is that the number -of firm entering the labor market is not socially optimal because both labor market tightness and the ratio of type L firms to type S firms are distorted. In this paper, besides the well -known congestion externalities, a new source of distortion occurs in that both firms and workers under estimate the firm's capital gain from poaching. This is the same intuition as the above proposition. This proposition implies that the number of a firm's entry is not socially optimal even if the *Hosios condition* is held.

As another source of distortion, some studies showed that a worker's turnover decision was inefficient. [11] showed that the level of turnover was inefficient in Nash bargaining in the double breach, and that they presented the efficiency wage bargaining model. Regardless of the single breach case or their model of wage bargaining, this paper shows that the level of turnover was inefficient. [5] showed that the level of turnover exceeded the optimal level because workers put excess effort into the- on-the-job search. This study's model assumes that effort for on-the-job search is exogenous. [2] showed that the level of turnover was below the optimal level when workers were risk-averse; this differs from the present study, in which workers are risk-neutral. In wage postings, where firms post a wage contract and commit to it before meeting workers (For example, [4]), the turnover decision by workers is not socially optimal. Because a wage is dispersed, high-productivity firms may post lower wage than low-productivity firms and workers may move to low-productivity firms from high-productivity firms.

This article is organized as follows. Section II presents the basic framework. Section III defines the partial equilibrium in which both the rate of matches for workers and the rate of matches for vacancies are exogenous, and then demonstrates the welfare implications. In section IV, the paper expands its consideration to the general equilibrium model in which, by the free entry condition, contract rates are endogenously determined. This section shows both the existence of this equilibrium and the two welfare implications for a firm's entry.

2. The Model

A. Environment

This study uses a continuous time model with search. For simplicity, the study discusses only a steady state. The number of workers is normalized to unity and they are infinitely lived. At any given point in time, workers are in one of two states: l state or s state. A worker switches back and forth between the two states via exogenous shocks: π_{ls} represents the rate of a shock that turns from state l to state s, while π_{sl} is the rate of a shock that turns in the reverse direction. These rates vary among workers. Firms choose either type L or type S at entry, given the two types of workers. The number of firms is determined by the free entry condition.

To focus attention on job-to-job transition patterns, no separation is assumed for employed workers[1], implying that in the steady-state, no one is unemployed.

A pair's output varies by the matched pattern. This study assumes that an L firm matched with an l worker can gain higher profit than one matched with an s worker, and in the same manner, an S firm matched with an s worker can gain higher profit than one matched with an l worker. Let an instantaneous surplus produced by an i state worker and a j type firm denote y_{ij}. This study assumes that the order of instantaneous surplus is defined by $y_{lL} > y_{lS}$ and $y_{sS} > y_{sL}$.

matching technology

To eliminate mismatches, workers are allowed to search while they are on the job. This study, for simplicity, assumes that the cost of the search is zero, which encourages workers to constantly look for better jobs[2]. Any vacancy and worker, regardless of being unemployed or employed, come together via a matching technology $\mu(1, v)$ where 1 is the number of workers and v is the number of vacancies. The function $\mu(1, v)$ is twice differentiable and increasing in its arguments, and it exhibits constant returns to scale.

[1]When separation occurs endogenously, the lower separation rate implies that the unemployment is lower. Even though the exogenous job separation is incorporated into the model, the main implications remain the same

[2][15] discusses the case of endogenous search intensity.

The flow rate of matches for a vacancy is then obtained $\mu(1, v)/v = q(\theta)$, where $q(\theta)$ is a differentiable decreasing function, and $\theta = v/1$ is the tightness of the labor market. It also immediately follows from the constant returns to scale assumption that the flow rate of matches for a worker is $\mu(1, v)/1 = \theta q(\theta) = p(\theta)$. In addition, this study also makes the standard Inada-type assumptions on $\mu(1, v)$, which ensures that $\theta q(\theta)$ is increasing function and that $\lim_{\theta\to\infty} q(\theta) = 0, \lim_{\theta\to 0} q(\theta) = \infty, \lim_{\theta\to\infty} \theta q(\theta) = 0$, and $\lim_{\theta\to 0} \theta q(\theta) = \infty$. Let γ denote a fraction of L type firms; accordingly, the effective arrival rate of a job offer from a L type firm is $\gamma\theta q(\theta)$. For convenience, I denote $\gamma\theta q(\theta)$ as p_L, $(1 - \gamma)\theta q(\theta)$ as p_S.

B. Wage Contract Determination

This study assumes that a wage contract is determined by the bargaining model offered by [6]. I discuss for other bargaining form in later. They constructed an explicit model including an on-the-job search that allowed the incumbent firm to pose a wage counter-offer to its worker. This study uses the following notations: U_i is the value for an unemployed worker of type i, W_{ij} is the value for a worker of type i working with a type j firm, V_j is the value of a vacancy owned by a firm of type j, and J_{ij} is the value of a type j job filled in by a worker of type i. Additionally, this study defines T_{ij} as the total value of match, which implies $T_{ij} = W_{ij} + J_{ij}$. Upon encountering a worker, a firm offers the worker a wage depending on the worker's type, which is then written into a wage contract.

The bargaining process for an employed worker is not as simple as the standard process. When an employed worker contacts an outside firm, a trilateral wage bargaining game among the poaching firm j, the employed worker i, and the incumbent firm j' occurs.[3] The game of bargaining has two steps described below.

The first step is to play a second price auction game between the two firms. There are two possible cases: the total surplus with the incumbent firm is higher than with the poaching firm, $T_{ij} > T_{ij'}$, or a reverse case, $T_{ij} < T_{ij'}$. Because no employer will pay more

[3] In [6], the wage is determined by the Rubinstein-type infinite-horizontal game of the strategic bargaining.

than match productivity, then the incumbent firm i can continue to employ the worker if $T_{ij} > T_{ij'}$, and the poaching firm i' can hire the worker if $T_{ij} < T_{ij'}$. Moreover, the firm with larger T does not have an incentive to pay its worker over the maximum wage that the counterpart firm with smaller T can pay, which ensures $W(w) = T_{ij'}$ if $T_{ij} > T_{ij'}$, or $W(w) = T_{ij}$ if $T_{ij} < T_{ij'}$.

At the second step, the employed worker is allowed to renegotiate with a firm, so the outcome of this renegotiation is determined through the standard *Nash bargaining* process, in which the threat value of the worker is the value of employment under the wage which is offered by another firm on first step or the total value of a match. Then, if $T_{ij} > T_{ij'}$, the bargaining over the wage solves:

$$W_{ij}(w_{ij}) = T_{ij'} + \beta(T_{ij} - T_{ij'}), \tag{1}$$

where β represents a worker's bargaining power.

(1) shows that the value of the employed worker contracting the wage w_{ij} consists of the outside option value, that is, the total value of matching between the i worker and the j' firm, plus a share β of the social capital gain $(T_{ij} - T_{ij'})$.

The result of the wage bargaining game is summarized as follows,

(i) If $T_{ij} \geq T_{ij'}$ and $W_{ij} \geq T_{ij'} + \beta(T_{ij} - T_{ij'})$, the worker keeps the current wage contract w with firm i, because the current wage is best for her/him. Then, after the bargaining, the worker's value $\bar{W}_{ij}$, the incumbent firm's value $\bar{J}_{ij}$, and the poaching firm's value $\bar{J}_{ij'}$ are,

$$\bar{W}_{ij} = W_{ij},\ \bar{J}_{ij} = J_{ij},\ \bar{J}_{ij'} = 0. \tag{2}$$

(ii) If $T_{ij} > T_{ij'}$ and $W_{ij} < T_{ij'} + \beta(T_{ij} - T_{ij'})$, the worker obtains a wage rise from her/his current employer because the new wage determined by the wage bargaining is more profitable for the worker than the old wage.

Then, after the bargaining new values are,

$$\bar{W}_{ij} = T_{ij'} + \beta(T_{ij} - T_{ij'}),\ \bar{J}_{ij} = (1-\beta)(T_{ij} - T_{ij'}),\ \bar{J}_{ij'} = 0. \tag{3}$$

(iii) If $T_{ij} < T_{ij'}$, the worker moves to the poaching firm. Then, after the bargaining values are,

$$\bar{W}_{ij} = T_{ij} + \beta(T_{ij'} - T_{ij}), \bar{J}_{ij} = 0, \ \bar{J}_{ij'} = (1-\beta)(T_{ij'} - T_{ij}). \quad (4)$$

The result (iii) means that the wage for the poaching firm is higher as the total value with the incumbent firm is higher.

3. Partial Equilibrium

This section's purpose is twofold; the first purpose is to define and characterize partial equilibrium of the decentralized economy in which the labor market tightness (θ) is taken as given. The second purpose is to show the novel result, which is different from [6], in that the level of turnover in market equilibrium is not efficient. Analyzing the general equilibrium framework with frictions when θ is determined endogenously is more complex; therefore, this study first reduces to the tractable partial equilibrium model, later extending to the general equilibrium model.

A. Basic Bellman Equation

This study now develops expressions for various value functions. Workers and firms discount the future at the common rate r. It is assumed, for convenience, that a worker's instantaneous utility is linear in the income flow(the wage for an employed worker).

It begins with the value of a worker of type i employed in a type j firm as follows:

$$rW_{ij} = w_{ij} + \pi_{ii'}(W_{i'j} - W_{ij}) + \sum_{j'=L,S} p_{j'}(\bar{W}_{ij} - W_{ij}), \quad i \neq i'. \quad (5)$$

The second term on the right-hand side represents the expected capital change with rate $\pi_{ii'}$ by switching to state i'. The final term is the expected capital gain with rate p_j by encountering a new firm through on-the-job search activities, and the values for each type of firms filling in a vacancy are given:

$$rJ_{ij} = y_{ij} - w_{ij} + \pi_{ii'}(J_{i'j} - J_{ij}) + \sum_{j'=\{L,S\}} p_{j'}(\bar{J}_{ij} - J_{ij}). \quad (6)$$

These value functions are constructed in a similar manner to those for an employee. The first terms on the right-hand side of (6) represents instantaneous profit, the second term is the expected capital change by switching to the worker's state, and the final term indicates the expected capital loss with the rate p_j throughout the wage competition against another firm.

To focus attention on job-to-job transition patterns, this study assumes that both the value of unemployment and the value of holding an open vacancy are sufficient low; this implies that no one chooses to be unemployed voluntarily.[4].

The next section incorporates the free-entry conditions into the model, which thereby ensures that the value of holding an open vacancy is zero.

B. Equilibrium in Decentralized Economy

There are three patterns of the worker's turnover behaviors, which is a policy function of her/his states and types of the current and new firms, depending on the combination of her/his parameters (π_{ls}, π_{sl}).

The first behavior pattern is called *"Perfect Separation Behavior"*(PSB) characterized by a worker always trying to match with a suitable firm. If an s type worker originally working in an S type firm and turns out to be l type, she/he quits the S type firm and starts to search for a job in L type firms on the job. Then, the condition for PSB is $T_{lL} > T_{lS}$ and $T_{sL} < T_{sS}$, which implies that the total value of a non-mismatched pair is higher than that of a mismatched pair.

The second pattern is referred to as *"Stay at L-firm Behavior"*(SLB) characterized by any worker's, regardless of worker type, preferring to stay in L firms. The s type worker who frequently switches to l type (high π_{sl}) chooses to stay in the L type firm. If a l type worker working in an L type firm turns out to be the s type, the worker remains in the L type firm. Then, the condition of SLB is $T_{lL} > T_{lS}$ and $T_{sL} > T_{sS}$, which implies that the total value of a worker-type L firm pair is higher than that of a worker-type S firm pair.

[4]In the next section, the value of holding an open vacancy (V_j) is zero with the free-entry condition. Meanwhile, the low value of unemployment is feasible if the benefit of unemployment is low.

The final one is symmetric to the second one, referred to as *"Stay at S-firm Behavior"*(SSB), which is characterized by any worker's, regardless of the worker type, preferring to stay in S firms. Then, the condition of SSB is $T_{lL} < T_{lS}$ and $T_{sL} < T_{sS}$, which implies that the value of a worker-type S firm pair is higher than that of a worker-type L firm pair.

According to the process of bargaining, the conditions determining these turnover patterns are summarized as follows:

- *If $T_{lL} > T_{lS}$ and $T_{sL} < T_{sS}$, the turnover pattern is PSB,*
- *If $T_{lL} > T_{lS}$ and $T_{sL} > T_{sS}$, the turnover pattern is SLB,*
- *If $T_{lL} < T_{lS}$ and $T_{sL} < T_{sS}$, the turnover pattern is SSB.*

Workers change jobs if and only if the total value of a match with a newly encountered firm exceeds that of the current firm.

Next this study calculates the total values of a match and illustrate the ranges of points (π_{ls}, π_{sl}) within which each turnover pattern occurs. The expression for the total value T_{lL} can be derived from the sum of (5) and (6),

$$rT_{lL} = y_{lL} + \pi_{ls}(T_{sL} - T_{lL}) + p_S \max\{\beta(T_{lS} - T_{lL}), 0\}. \tag{7}$$

The final term represents the expected capital gain by a type l worker's movement to a type S firm. The worker obtain the capital gain from her/his turnover if $\beta(T_{lS} - T_{lL})$ is positive. The new firm also obtains the capital gain, $(1 - \beta)(T_{lS} - T_{lL})$. Then, the total capital gain from her/his turnover is $T_{lS} - T_{lL}$.

Similarly, T_{sL}, T_{lS} and T_{sS} are given:

$$rT_{sL} = y_{sL} + \pi_{sl}(T_{lL} - T_{sL}) + p_S \max\{\beta(T_{sS} - T_{sL}), 0\}, \tag{8}$$

$$rT_{lS} = y_{lS} + \pi_{ls}(T_{sS} - T_{lS}) + p_L \max\{\beta(T_{lL} - T_{lS}), 0\}, \tag{9}$$

$$rT_{sS} = y_{sS} + \pi_{sl}(T_{lS} - T_{sS}) + p_L \max\{\beta(T_{sS} - T_{sL}), 0\}. \tag{10}$$

The turnover decision depends on the signs of the differences, $T_{lL} - T_{lS}$ and $T_{sS} - T_{sL}$. This study denote $y_{lL} - y_{lS}$ as $\Delta_l > 0$ and $y_{sS} - y_{sL}$ as $\Delta_s > 0$. The conditions for workers choosing the *PSB* are,

$$T_{lL} - T_{lS} = \frac{\Delta_l - \pi_{ls}(T_{sS} - T_{sL})}{r + \pi_{ls} + \beta p_L}, \tag{11}$$

$$T_{sS} - T_{sL} = \frac{\Delta_s - \pi_{sl}(T_{lL} - T_{sL})}{r + \pi_{sl} + \beta p_S}. \tag{12}$$

(11) and (12) imply the expected net gain by switching to a non-mismatched firm from a mismatched firm.

Using (7)-(10), a worker's pattern of turnover is determined.

Proposition 1 *The properties of the turnover patterns are illustrated as follows,*

(i) *In the $\pi_{ls} - \pi_{sl}$ space, there exists a threshold $(\pi^*_{ls}(\pi_{sl}), \pi^{**}_{ls}(\pi_{sl}))$ (formally defined $T_{lL} = T_{lS}$ and $T_{sL} = T_{sP}$) such that,*

$$\pi^*_{ls}(\pi_{sl}) = \frac{(y_{lL} - y_{lS})(\pi_{sl} + \beta p_S + r)}{y_{sS} - y_{sL}} \text{ and } \pi^{**}_{ls}(\pi_{sl}) = \frac{(y_{lL} - y_{lS})\pi_{sl} + (\beta p_L + r)(y_{sL} - y_{sS})}{y_{sS} - y_{sL}}.$$

(ii) *For any (π_{ls}, π_{sl}),*

- *If $\pi^*_{ls}(\pi_{sl}) > \pi_{ls} > \pi^{**}_{ls}(\pi_{sl})$, then the behavior of the worker is PSB*
- *If $\pi^{**}_{ls}(\pi_{sl}) > \pi_{ls}$, then the behavior of the worker is SLB*
- *If $\pi_{ls} > \pi^*_{ls}(\pi_{sl})$, then the behavior of the worker is SSB*

The proof of Proposition 1 appears in Appendix. This argument is illustrated in figure 1. $\pi^*_{ls}(\pi_{sl})$ and $\pi^*_{ls}(\pi_{sl})$ is increasing function, and $\pi^*_{ls} > \pi^{**}_{ls}$for any π_{sl}

Proposition 1 has two important implications. First, any worker who frequently changes from type l to type s (which means π_{ls} is quite high) but seldom changes from type s to type l (which means π_{sl} is quite low), chooses the *SSB*. Even through an l type worker moves to an L firm to eliminate the mismatch, because the worker more frequently changes to type s, the worker soon suffers from mismatch with the L firm by a change to type s. Therefore, this worker's optimal behavior is not to move to the L type firm. Meanwhile, workers who are high π_{sl} and low π_{ls} are more likely to choose the *SLB*.

Secondly, the comparative statics exercise illustrates that the higher bargaining power of workers (higher β) or the higher arrival rate for workers (higher p) shifts locus $\pi_{ls}^{**}(\pi_{sl})$ upward but $\pi_{ls}^{*}(\pi_{sl})$ downward, causing the dominance of *PSB*. This implies that the differences, $T_{lL} - T_{lS}$ and $T_{sS} - T_{sL}$, are increasing functions for β and p.

There are two effects of the bargaining power on the turnover decisions for workers whose optimal choice is *PSB*. The study is restricted to the turnover behavior of an l worker in an S firm. Currently, they are mismatched. According to the third term on the right-hand side of (9), an increase in the bargaining power raises the mismatched value T_{lS}. From (11), however, the total capital gain $T_{lL} - T_{lS}$ decreases. This effect is called the direct effect.

The second effect of the bargaining power comes through a channel of the direct effect for an s worker. According to (8), an increase in the bargaining power raises the mismatched value T_{sL}, which leads to an increase in $T_{sL} - T_{lL}$ from the second term on the right-hand side of (7), thereby the non-mismatched value T_{lL}. It implies a reduction of the extent of the capital loss. Then, the total capital gain $T_{lL} - T_{lS}$ increases. This effect is called the indirect effect.

Similarly, there are the direct and indirect effects for a s type worker; $T_{sS} - T_{sL}$ decreases from the direct effect but increases from the indirect effect. The sign of $T_{lL} - T_{lS}$ depends on the measure of $T_{sS} - T_{lS}$; that is, $T_{lL} - T_{lS}$ is positive if $\pi_{ls}(T_{sS} - T_{lS}) < \Delta_l$. The indirect effect has a significant role in efficiency (discussed in detail in subsection C). The same argument can be applied in a case of an increase in p.

The next section deals with the solution for a social planner problem, and then the study demonstrates that the turnover decision in the decentralized economy is different from the social optimal decision.

C. Social Planner Problem for the Turnover

The purpose of this sub-section is to illustrate the efficient conditions of the turnover and to show that the level of the turnover in a decentralized economy is lower than the level determined by the social planner.

The number of type l workers are denoted as E_l and type s workers similarly are denoted as E_s. For convenience, the numbers reach the level of steady state, then E_l equals $\pi_{sl}/(\pi_{sl}+\pi_{ls})$ and E_s equals $\pi_{ls}/(\pi_{sl}+\pi_{ls})$ with steady state conditions $\pi_{sl}(1-E_l) = \pi_{ls}E_s$. Let E_{ij} denote the number of type i workers in type j firms.

The total surplus can be written as:

$$TS = \int_{t=0}^{\infty} e^{-rt}\Big\{E_{lL}y_{lL} + E_{lS}y_{lS} + E_{sS}y_{sS} + E_{sL}y_{sL}\Big\}dt - c(v_L + v_S). \tag{13}$$

The total surplus is equal to the total flows of net output minus the vacancy cost. The efficient turnover is calculated to maximize the TS subject with the following equations:

$$\dot{E_{ij}} = \int_0^1 \int_0^1 \dot{e}_{ij} d\pi_{ls} d\pi_{sl},\ i = \{l, s\}, j = \{L, S\}. \tag{14}$$

where $\dot{e}_{ij}$ represents the law of motion of the number of type i workers working in a type j firm characterized by (π_{ls}, π_{sl}).

The law of motion of e_{lL} and e_{sS} are,

$$\dot{e_{lL}} = p_L\mu_L e_{lS} + \pi_{sl}e_{sL} - (\pi_{ls} + (1-\mu_L)p_S)e_{lL} \tag{15}$$

$$\dot{e_{sS}} = p_S\mu_S e_{sL} + \pi_{ls}e_{lS} - (\pi_{sl} + (1-\mu_S)p_L)e_{sS}, \tag{16}$$

where $\mu_L \in [0,1]$ is the probability that a type l worker who faces a choice between a L firm or a S firm decides to move to the L firm, and $\mu_s \in [0,1]$ is the probability that a type s worker facing the same choice decides to move to the S firm.

TS is maximized with respect to μ_L and μ_S, subject to (13), (14), $e_{ls} = e_l - e_{lL}$, and $e_{sL} = e_s - e_{sS}$. The optimal μ_L and μ_S are as follows,

$$\mu_L = \begin{cases} 1 \quad if\ \pi_{ls} < \dfrac{(\pi_{sl} + p_S + r)\Delta_l}{\Delta_s} \\ (0,1)\ \ if\ \pi_{ls} = \dfrac{(\pi_{sl} + p_S + r)\Delta_l}{\Delta_s} \\ 0 \quad if\ \pi_{ls} > \dfrac{(\pi_{sl} + p_S + r)\Delta_l}{\Delta_s}, \end{cases} \tag{17}$$

and

$$\mu_S = \begin{cases} 1 \quad if\ \pi_{ls} > \dfrac{\pi_{sl}\Delta_l + (p_L + r)\Delta_s}{\Delta_s} \\ (0,1)\ \ if\ \pi_{ls} = \dfrac{\pi_{sl}\Delta_l + (p_L + r)\Delta_s}{\Delta_s} \\ 0 \quad if\ \pi_{ls} < \dfrac{\pi_{sl}\Delta_l + (p_L + r)\Delta_s}{\Delta_s}. \end{cases} \tag{18}$$

(Appendix B contains more detail.)

Proposition 2 abstracts these conditions.

Proposition 2 *There are the two loci of points such that* $\hat{\pi}^{*}_{ls}(\pi_{sl}) = \Delta_l(\pi_{sl} + p_S + r)/\Delta_s$ *and* $\hat{\pi}^{**}_{ls}(\pi_{sl}) = (\Delta_l \pi_{sl} + \Delta_s(p_L + r))/\Delta_s$*. The efficient turnover patterns are characterized below;*

- *If* $\hat{\pi}^{*}_{ls}(\pi_{sl}) > \pi_{ls} > \hat{\pi}^{**}_{ls}(\pi_{sl})$*, then the turnover pattern of the worker is PSB*
- *If* $\hat{\pi}^{**}_{ls}(\pi_{sl}) > \pi_{ls}$*, then the turnover pattern of the worker is SLB*
- *If* $\pi_{ls} > \hat{\pi}^{*}_{ls}(\pi_{sl})$*, then the turnover pattern of the worker is SSB*

This argument is illustrated in figure 2. The social planner deals with the cost of mismatch because that if a worker's type changes, her/his productivity is lowered. If p_j is higher, the cost of the mismatch is lower because the worker's transition to a new job is very easy, and the expected period of mismatch is very short.

The study compares the social planner's solution and the decentralized economy's solution.

Proposition 3 *If firms have the bargaining power* ($\beta \neq 1$)*, the efficient domain of PSB is larger than the domain of PSB in the decentralized economy.*

The shaded areas in figure 2 represent differences between the domain of PSB in the social planner and the domain of PSB in the decentralized economy.

Proposition 3 is clearly different from the result of [6]. In their model, independently of the worker's bargaining power, the social planner's decision for the turnover is equal to the worker's decision. Proposition 3 implies that there exist workers who should have moved to non-mismatch poaching firms.

From (7) to (10), the worker's capital gains from turnover is part of the total capital gain. Meanwhile, the social planner considers not only the worker's capital gains but also the poaching firm's capital gains. Then, the value of turnover for the social planner is the total capital gain. This means that workers underestimate the value of the third term on the right-hand side of (7) to (10). Thus, from based upon Proposition 1, the differences, $T_{lL} - T_{lS}$ and $T_{sS} - T_{sL}$, are smaller than those determined by the social planner.

In a case where workers have monopolistic bargaining power $\beta = 1$, there is no difference between their decision and the social planner's decision. This results from the worker's gain being equal to the total capital gain from turnover (and the poaching firm's gain is zero), then the differences in the decentralized economy equal the one determined by the social planner.

In [6], similarly to this study's model, the worker's value of turnover is different from the social planner's value. However, their model showed only the direct effect on $T_{lL} - T_{lS}$ and $T_{sS} - T_{sL}$ through a change of β in the denominator of (11) and (12) because a productivity change resulting from a shock is not incorporated into their model. The direct effect does not change the sign of the differences, $T_{lL} - T_{lS}$ and $T_{sS} - T_{sL}$. Moreover, in their model, both workers and the social planner always look for a better match and move to a high productivity firm because the matching quality dose not switch back and forth between mismatch and non-mismatch in a given firm. Therefore, the order of the total values of each firm for a worker coincides with that for the social planner's.

This proposition implies that labor market policies that cause turnover may improve a social efficiency.

4. General Equilibrium

In this section, the study expands to consider the general equilibrium model in which labor market tightness is determined by the free entry condition and derives welfare implications for the firm's entry.

A. Free Entry Condition

The Inada condition for the matching function guarantees an existence of the firm's entry. This study demonstrates that there are both L type and S type firms.

A firm with an unfilled vacancy can gain a profit when the firm meets an employed worker who is hired by a different type of firm and it succeeds at poaching her/him. If the firm poaches an employed worker from the same type of firms, it does not gain at all because the firm has to compete with the incumbent firm, leading to zero profit.

Then, there are two cases where a type L firm can gain a profit. The first case is that the firm meets a type l worker who chooses PSB and works in a type S firm. The second case is that the firm meets any type of worker who chooses SLB and works in a type S firm.

Thus, the value of a type L firm with a vacancy, V_L is as follows:

$$\begin{aligned} rV_L =& - c + q(1-\beta)\Big[\int\int_{\pi_{ls}^{**}}^{\pi_{ls}^{*}} (T_{lL} - T_{lS})e_{lS}(\pi_{ls}, \pi_{sl}) d\pi_{ls} d\pi_{sl} \\ &+ \int\int^{\pi_{ls}^{*}} \big\{(T_{lL} - T_{lS})e_{lS}(\pi_{ls}, \pi_{sl}) + (T_{sL} - T_{sS})e_{sS}(\pi_{ls}, \pi_{sl})\big\} d\pi_{ls} d\pi_{sl}\Big], \end{aligned} \tag{19}$$

where c is the firm's instantaneous search cost. The first double integral term is the expected capital gain by meeting a type l employed worker who chooses PSB and works in a type S firm, and the second double integral term is the expected capital gain by meeting a worker who chooses SLB.

Similarly, the value of a type S firm with a vacancy, V_S is as follows,

$$\begin{aligned} rV_S =& - c + q(1-\beta)\Big[\int\int_{\pi_{ls}^{**}}^{\pi_{ls}^{*}} (T_{sS} - T_{sL})e_{sL}(\pi_{ls}, \pi_{sl}) d\pi_{ls} d\pi_{sl} \\ &+ \int\int_{\pi_{ls}^{*}} \big\{(T_{lS} - T_{lL})e_{lL}(\pi_{ls}, \pi_{sl}) + (T_{sS} - T_{sL})e_{sL}(\pi_{ls}, \pi_{sl})\big\} d\pi_{ls} d\pi_{sl}\Big]. \end{aligned} \tag{20}$$

This study considers the steady-state equilibrium where both L type and S type firms exist. There are two conditions that allow firms to enter freely. The first condition is that the expected value of a type L vacancy equals that of a type S vacancy, that is, $V_L = V_S$. From (7) to (10), this condition is rewritten,

$$\int\int^{\pi_{ls}^{**}} A d\pi_{ls} d\pi_{sl} + \int\int_{\pi_{ls}^{**}}^{\pi_{ls}^{*}} B d\pi_{ls} \pi_{sl} - \int\int_{\pi_{ls}^{*}} C d\pi_{ls} d\pi_{sl} = 0, \tag{21}$$

where

$$A = \frac{\{(\beta p_L + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s\}e_{lS} - \{(\beta p_L + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l\}e_{sS}}{(p_L + \pi_{ls} + r)(\beta p_L + \pi_{sl} + r) - \pi_{ls}\pi_{sl}},$$
$$B = \frac{\{(\beta p_S + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s\}e_{lS} - \{(\beta p_L + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l\}e_{sL}}{(\beta p_L + \pi_{ls} + r)(\beta p_S + \pi_{sl} + r) - \pi_{ls}\pi_{sl}},$$
$$C = \frac{\{(\beta p_S + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l\}e_{sL} - \{(p_S + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s\}e_{lL}}{(\beta p_S + \pi_{ls} + r)(\beta p_S + \pi_{sl} + r) - \pi_{ls}\pi_{sl}}.$$

The calculation of the number of each type worker e_{ij} in the steady state appears in Appendix C. In the steady state, no worker who chooses SLB works in S firms, and in a similar manner, no worker who chooses SSB works in L firms.

The proposition for existence of this equilibrium can be derived.

Proposition 4 *When there are workers who chooses PSB, there exist both type L firms and type S firms.*

The logic behind proposition is very simple. A firm can gain profit if and only if the firm can poach a worker from another type firm. Then, if the number of type L firms increases, the expected capital gain of a type S firm's vacancy increases because it is easier to find an employed worker working in a type L firm. Meanwhile, that of a type L firm's vacancy decreases because it it more likely to meet an employed worker from the same type firm. Moreover, if there are no type L firms, type S firms can not gain profits at all, which discourages them from entry. This mechanism guarantees that there exist the two types of firms.

The second free entry condition is that both expected values of type L vacancies and type S vacancies are zero. Then, from (19) and (20)

$$c = q(1-\beta)\Big[\int\int^{\pi_{ls}^*} A d\pi_{ls} d\pi_{sl} + \int\int_{\pi_{ls}^{**}}^{\pi_{ls}^*} \frac{\{(\beta p_S + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s\}e_{lS}}{(\beta p_L + \pi_{ls} + r)(\beta p_S + \pi_{sl} + r) - \pi_{ls}\pi_{sl}} d\pi_{ls} d\pi_{sl}\Big], \tag{22}$$

$$c = q(1-\beta)\Big[\int\int_{\pi_{ls}^*} C d\pi_{ls} d\pi_{sl} + \int\int_{\pi_{ls}^{**}}^{\pi_{ls}^*} \frac{\{(\beta p_L + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l\}e_{sL}}{(\beta p_L + \pi_{ls} + r)(\beta p_S + \pi_{sl} + r) - \pi_{ls}\pi_{sl}} d\pi_{ls} d\pi_{sl}\Big]. \tag{23}$$

The Inada condition ensures the existence of the interior of solution according to (22) and (23).

B. Social Planner Problem for the Firm Entry

This study compares the level of firm's entry in the decentralized economy to that of the social optimal level.

From $p_L = \gamma p(\theta)$ and $p_S = (1-\gamma)p(\theta)$, the social planner's problem for the ratio of the type L vacancies to type S vacancies ($= \gamma$) and the market tightness θ is defined as follows,

$$\max_{\gamma,\theta} \int_0^\infty e^{-rt}\Big\{\int\int \{y_{lL}e_{lL} + y_{lS}(e_l - e_{lL}) + y_{sS}e_{sS} + y_{sL}(e_s - e_{sS})\}d\pi_{ls}d\pi_{sl})\Big\}dt \quad (24)$$

$$s.t. \quad \dot{e_{lL}} = p_L\mu_L e_{lS} + \pi_{sl}e_{sL} - (\pi_{ls} + (1-\mu_L)p_S)e_{lL} \quad (25)$$

$$and \quad \dot{e_{sS}} = p_S\mu_S e_{sL} + \pi_{ls}e_{lS} - (\pi_{sl} + (1-\mu_S)p_L)e_{sS}, \quad (26)$$

The solution for the above problem appears in Appendices D and E.

The optimal level of γ is,

$$\int\int^{\hat{\pi}_{ls}^{**}} A'd\pi_{ls}d\pi_{sl} + \int\int_{\hat{\pi}_{ls}^{**}}^{\hat{\pi}_{ls}^{*}} B'd\pi_{ls}\pi_{sl} - \int\int_{\hat{\pi}_{ls}^{*}} C'd\pi_{ls}d\pi_{sl} = 0 \quad (27)$$

$$where, \quad A' = \frac{\{(p_L + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s\}e_{lS} - \{(p_L + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l\}e_{sS}}{(p_L + \pi_{ls} + r)(p_L + \pi_{sl} + r) - \pi_{ls}\pi_{sl}},$$

$$B' = \frac{\{(p_S + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s\}e_{lS} - \{(p_L + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l\}e_{sL}}{(p_L + \pi_{ls} + r)(p_S + \pi_{sl} + r) - \pi_{ls}\pi_{sl}},$$

$$C' = \frac{\{(p_S + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l\}e_{sL} - \{(p_S + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s\}e_{lL}}{(p_S + \pi_{ls} + r)(p_S + \pi_{sl} + r) - \pi_{ls}\pi_{sl}}.$$

Comparing with (21), $A = A'$, $B = B'$, and $C = C'$ when workers have a monopolistic bargaining power ($\beta = 1$). Then the study can obtain the following,

Proposition 5 *When workers have monopolistic bargaining power($\beta = 1$), the ratio of the type L vacancies to type S vacancies is optimal, but when the firms have any bargaining power($\beta < 1$), the ratio is not optimal.*

The logic is similar to that of proposition 2. In a decentralized economy, the values of poaching workers which are determined by, $T_{lL} - T_{lS}$ and $T_{sS} - T_{sL}$, are underestimated because workers do not consider a poaching firm's capital gain from the future turnover. In addition, the extent of the underestimation is different between $T_{lL}-T_{lS}$ and $T_{sS}-T_{sL}$. It implies that the difference between the optimal number of firms and the number of firms in a decentralized economy is different between the type L vacancies and type S vacancies. Then, the ratio of type L vacancies to type S vacancies in a decentralized economy does not equal the social optimal ratio.

In a search model with two sectors, [1] showed a similar proposition, the ratio of a firm's entry in the two sectors was not optimal. Unlike this study's model, he assumed that the fixed cost was different in each sectors. Then, the holdup problem lowered the firm's entry in the high fixed cost sector.

The optimal level of θ is,

$$c = \mu' \Big[\int \int^{\hat{\pi}^*_{ls}} A' d\pi_{ls} d\pi_{sl} + \int \int_{\hat{\pi}^{**}_{ls}}^{\hat{\pi}^*_{ls}} \frac{\{(p_S + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s\} e_{lS}}{(p_L + \pi_{ls} + r)(p_S + \pi_{sl} + r) - \pi_{ls}\pi_{sl}} d\pi_{ls} d\pi_{sl} \Big], \quad (28)$$

$$c = \mu' \Big[\int \int_{\hat{\pi}^*_{ls}} C' d\pi_{ls} d\pi_{sl} + \int \int_{\hat{\pi}^{**}_{ls}}^{\hat{\pi}^*_{ls}} \frac{\{(p_L + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l\} e_{sL}}{(p_L + \pi_{ls} + r)(p_S + \pi_{sl} + r) - \pi_{ls}\pi_{sl}} d\pi_{ls} d\pi_{sl} \Big]. \quad (29)$$

Comparing these two equations with (22) and (23) is more complicate. For the terms in the bracket, (28) and (29) equal (22) and (23) if $\beta = 1$. This follows a similar logic to that of Proposition 5.

Meanwhile, for the term outside the brackets, (28) and (29) equal (22) and (23), respectively when $\beta = -(q'(\theta)\theta)/q(\theta)$, which called *Hosios condition*[5].

Proposition 6 *In a decentralized economy, the level of market tightness cannot achieve the optimal level without unless the elasticity of the matching function is one.*

This proposition implies that the level of market tightness is not optimal even if the *Hosios condition* is held. This model has two sources of distortion for a firm's entry. The first is the congestion externality, which can be eliminated by the *Hosios condition.* The second is the underestimation of the total capital gain from poaching that is the sum of the worker's capital gain and firm's capital gain. According to proposition 2, the total capital gain from turnover is efficient only if workers have monopolistic bargaining power.

[5]See Hosios [10] and Pissarides [14].

Thus, if and only if workers have monopolistic bargain power and the elasticity of the matching function equals one, market tightness is the optimal level.

C. Alternative Bargaining Procedure

The main propositions, Proposition 3, 5, and 6, can be applicable to [8]'s, [11]'s, and [13]'s bargaining procedure. The wage is determined by standard general Nash bargaining in [8] and [13]. In [11][6], either workers or firms have monopolistic bargaining power. Their bargaining procedures have two properties similar to the procedure in this paper. (i) workers move to a firm of the higher total value. (ii) total value functions under [8]'s and [13]'s are the same form to this paper's , and total value functions under [11]'s are $\beta = \frac{1}{2}$ case of this paper's. Then, from the same logic in this paper, proposition 1 and later propositions hold under [8]'s, [11]'s, and [13]'s.

5. Conclusion

This study has developed an *on-the-job* search model with wage bargaining that involves a shock that changes the worker's states. The study demonstrated the welfare implication when the level of turnover is socially inefficient. This inefficiency results from a part of capital gain from turnover going to a poaching firm, discouraging a worker from switching firms. Besides the welfare implication on turnover decisions, this study demonstrated that the levels of the firm's entry were not optimal. Also, the firm's capital gain from poaching decreases because another poaching firm in the future may capture a part of future capital gain from the original turnover.

Finally, this study briefly discusses a policy implication regarding a hiring subsidy. A hiring subsidy is defined here as a temporary subsidy to firms in hiring a worker. This subsidy causes workers to turn over because the total value of a worker increases. According to proposition 2, the level of turnover is lower than the optimal level, making this effect positive for social welfare.

Beside the decision of turnover, the hiring subsidy affects the level of a firm's entry.

[6] Their model analyzes more general case, involving the double breach.

The hiring subsidy encourages the firm's entry, because the poaching firms take a part of the subsidy. It has an ambiguous effect on the social surplus because there are two possible cases of a firm's entry; one case is the over-entry and the other case is the under-entry, depending on the worker's bargaining power. In the case where the worker's bargaining power is greater than the *Hosios* condition, the hiring subsidy certainly improves social welfare according to Proposition 6. This results from the level of the firm's entry being lower than the optimal level from the two sources, the congestion externality and underestimation of the capital gain from turnover. Then, the subsidy can correct both the turnover decision and the level of the firm's entry.

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Appendix

A Proof of Proposition 1

Conditions for *PSB*

Conditions for PSB are $T_{lL} \geq T_{lS}$ and $T_{sL} \leq T_{sS}$. Total values of PSB are,

$$rT_{lL} = y_{lL} + \pi_{ls}(T_{sL} - T_{lL}), \tag{A-1}$$
$$rT_{sL} = y_{sL} + \pi_{sl}(T_{lL} - T_{sL}) + p_S\beta(T_{sS} - T_{sL}), \tag{A-2}$$
$$rT_{lS} = y_{lS} + \pi_{ls}(T_{sS} - T_{lS}) + p_L\beta(T_{lL} - T_{lS}), \tag{A-3}$$
$$rT_{sS} = y_{sS} + \pi_{sl}(T_{lS} - T_{sS}). \tag{A-4}$$

Conditions for PSB are rewritten by taking the following steps;

step 1 From (A-1) and (A-3), $T_{lL}(\pi_{ls}, \pi_{sl}) - T_{lS}(\pi_{ls}, \pi_{sl})$ is,

$$T_{lL}(\pi_{ls}, \pi_{sl}) - T_{lS}(\pi_{ls}, \pi_{sl}) = \frac{\Delta_l + \pi_{ls}(T_{sL}(\pi_{ls}, \pi_{sl}) - T_{sS}(\pi_{ls}, \pi_{sl}))}{r + p_L\beta + \pi_{ls}}, \tag{A-5}$$

where $\Delta_l = y_{lL} - y_{lS}$ and $\Delta_s = y_{sS} - y_{sL}$. From $T_{sL} \leq T_{sS}$, $T_{lL}(\pi_{ls}, \pi_{sl}) - T_{lS}(\pi_{ls}, \pi_{sl})$ is a decreasing function for π_{ls}. $T_{lL} - T_{lS} < 0$ if $\pi_{ls} \to \infty$, and $T_{lL} - T_{lS} > 0$ if $\pi_{ls} = 0$. Then there are unique $\pi^*_{ls}(\pi_{sl})$ as $T_{lL}(\pi^*_{ls}, \pi_{sl}) = T_{lS}(\pi^*_{ls}, \pi_{sl})$ and $T_{lL}(\pi_{ls}, \pi_{sl}) > T_{lS}(\pi_{ls}, \pi_{sl})$ if and only if $\pi^*_{ls}(\pi_{sl}) > \pi_{ls}$. Using (A-1) to (A-4), simple algebra gives,

$$\pi^*_{ls}(\pi_{sl}) = \frac{\Delta_l(\pi_{sl} + r + \beta p_S)}{\Delta_s}. \tag{A-6}$$

step 2 The condition $T_{sL} \leq T_{sS}$ is rewritten. From (A-2) and (A-4), $T_{sS}(\pi_{ls}, \pi_{sl}) - T_{sL}(\pi_{ls}, \pi_{sl})$ is;

$$T_{sS}(\pi_{ls}, \pi_{sl}) - T_{sL}(\pi_{ls}, \pi_{sl}) = \frac{\Delta_s + \pi_{sl}(T_{lS}(\pi_{ls}, \pi_{sl}) - T_{lL}(\pi_{ls}, \pi_{sl}))}{r + p_S\beta + \pi_{sl}} \tag{A-7}$$

Similarly to step 1, $T_{sS}(\pi_{ls}, \pi_{sl}) - T_{sL}(\pi_{ls}, \pi_{sl})$ is the decreasing function for π_{sl} from $T_{lS}(\pi_{ls}, \pi_{sl}) < T_{lL}(\pi_{ls}, \pi_{sl})$. $T_{sS}(\pi_{ls}, \pi_{sl}) - T_{sL}(\pi_{ls}, \pi_{sl}) < 0$ if $\pi_{sl} \to \infty$, and $T_{sS}(\pi_{ls}, \pi_{sl}) - T_{sL}(\pi_{ls}, \pi_{sl}) < 0$ if $\pi_{sl} = 0$. Then there are $\pi^*_{sl}(\pi_{ls}); T_{sS}(\pi_{ls}, \pi^*_{sl}) = T_{sL}(\pi_{ls}, \pi^*_{sl})$ Using (A-1) to (A-4), simple algebra gives:

$$\pi_{ls} = \frac{\Delta_l \pi^*_{sl}(\pi_{ls}) + \Delta_s(r + \beta p_L)}{\Delta_s}. \tag{A-8}$$

$T_{sS} > T_{sL}$ if the worker's parameters (π_{ls}, π_{sl}) satisfy $\pi^*_{sl}(\pi_{ls}) > \pi_{sl}$.

step 3 From step 1 and step 2, a worker's behavior is *PSB* if and only if her/his parameters (π_{ls}, π_{sl}) satisfy the conditions, $\pi_{ls} < \pi^*_{ls}(\pi_{sl})$ and $\pi_{sl} < \pi^*_{sl}(\pi_{ls})$. Thus, conditions for *PSB* are:

$$\frac{\Delta_l(\pi_{sl} + r + \beta p_S)}{\Delta_s} > \pi_{ls} > \frac{\Delta_l \pi_{sl} + \Delta_s(r + \beta p_L)}{\Delta_s}. \tag{A-9}$$

Conditions for ***SLB***

Conditions of *SLB* are $T_{lL} \geq T_{lS}$ and $T_{sL} \geq T_{sS}$. Total values of *SLB* are as follow:

$$rT_{lL} = y_{lL} + \pi_{ls}(T_{sL} - T_{lL}), \tag{A-10}$$
$$rT_{sL} = y_{sL} + \pi_{sl}(T_{lL} - T_{sL}), \tag{A-11}$$
$$rT_{lS} = y_{lS} + \pi_{ls}(T_{sS} - T_{lS}) + p_L\beta(T_{lL} - T_{lS}), \tag{A-12}$$
$$rT_{sS} = y_{sS} + \pi_{sl}(T_{lS} - T_{sS}) + p_L\beta(T_{sL} - T_{sS}). \tag{A-13}$$

$T_{lL} > T_{sL}$ from $y_{lL} > y_{sL}$, and $T_{lS} - T_{sS}$ is $\{p_L\beta(T_{lL} - T_{sL})\}/\{p_L\beta + \pi_{ls} + \pi_{sl} + r\}$ from (A-12) and (A-13). Then $T_{lS} - T_{sS} < T_{lL} - T_{sL}$, so this is rewritten as $T_{sL} - T_{sS} < T_{lL} - T_{lS}$. Thus $T_{lL} > T_{lS}$ must hold if $T_{sL} > T_{sS}$ hold. From simple algebra, $\pi_{ls} = \{\Delta_l \pi^*_{sl}(\pi_{ls}) + \Delta_s(r + \beta p_L)\}/\Delta_s$ and $T_{sL} \geq T_{sS}$ if $\pi_{ls} > \pi^*_{ls}(\pi_{sl})$. Thus, a worker's behavior is *SLB* if and only if the worker's parameters (π_{ls}, π_{sl}) satisfy $\pi_{ls} < \{\Delta_l \pi_{sl} + \Delta_s(r + \beta p_L)\}/\Delta_s$.

Conditions for ***SSB***

The conditions of *SSB* are $T_{lL} \leq T_{lS}$ and $T_{sL} \leq T_{sS}$. Then, total values of *SSB* are as follow,

$$rT_{lL} = y_{lL} + \pi_{ls}(T_{sL} - T_{lL}) + p_S\beta(T_{lS} - T_{lL}), \tag{A-14}$$
$$rT_{sL} = y_{sL} + \pi_{sl}(T_{lL} - T_{sL}) + p_S\beta(T_{sS} - T_{sL}), \tag{A-15}$$
$$rT_{lS} = y_{lS} + \pi_{ls}(T_{sS} - T_{lS}), \tag{A-16}$$
$$rT_{sS} = y_{sS} + \pi_{sl}(T_{lS} - T_{sS}). \tag{A-17}$$

By similar calculation to the condition for *SLB*, a worker's behavior is *SSB* if and only if, the worker's parameters (π_{ls}, π_{sl}) satisfy $\pi_{ls} \geq \{\Delta_l(\pi_{sl} + r + \beta p_S)\}/\Delta_s$.

B Proof of Proposition 2

The Hamiltonian function is as follows:

$$\begin{aligned} H =& e^{-rt}[e_l y_{lS} + \Delta_l e_{lL} + e_s y_{sL} + \Delta_s e_{sS}] \\ &-\lambda_1[p_L\mu_L(e_l - e_{lL}) + \pi_{sl}(e_s - e_{sS}) - (\pi_{ls} + (1 - \mu_L)p_S)e_{lL}] \\ &-\lambda_2[p_S\mu_S(e_s - e_{ss}) + \pi_{ls}(e_l - e_{lL}) - (\pi_{sl} + (1 - \mu_S)p_L)e_{sS}]. \end{aligned} \tag{B-1}$$

The optimal conditions are:

$$\mu_L = 1 \iff -\lambda_1(p_L(e_l - e_{lL}) + p_S e_{lL}) \geq 0, \tag{B-2}$$
$$\mu_S = 1 \iff -\lambda_2(p_S(e_s - e_{sS}) + p_L e_{sS}) \geq 0, \tag{B-3}$$
$$e^{-rt}(y_{lL} y_{lS}) + \lambda_1(p_L\mu_L + \pi_{ls} + (1-\mu_L)p_S) + \lambda_2\pi_{ls} - \dot{\lambda}_1 = 0, \tag{B-4}$$
$$e^{-rt}(y_{sS} - y_{sL}) + \lambda_1\pi_{sl} + \lambda_2(p_2\mu_S + \pi_{sl} + (1-\mu_S)p_L) - \dot{\lambda}_2 = 0. \tag{B-5}$$

According to (B-2) and (B-3), there are $\mu_L = 1$ if and only if $\lambda_1 \leq 0$ and $\mu_S = 1$ if and only if $\lambda_2 \leq 0$.

From (B-4) and (B-5), $\dot{\lambda}_1 = -r\lambda_1$, and $\dot{\lambda}_2 = -r\lambda_2$, the sign of both λ_1 and λ_2 are:

$$\lambda_1 < 0 \iff \pi_{ls} < \frac{(r + \pi_{sl} + \mu_S p_L + (1-\mu_S)p_S)\Delta_l}{\Delta_s}, \tag{B-6}$$
$$\lambda_2 < 0 \iff \pi_{ls} > \frac{\pi_{sl}\Delta_l - (r + \mu_L p_L + (1-\mu_L)p_S)\Delta_s}{\Delta_s}. \tag{B-7}$$

For any π_{sl}, the right hand of (B-6) is higher than the right hand of (B-7), then the optimal conditions for μ_L, μ_S are:

$$\mu_L = 1 \iff \pi_{ls} < \frac{(r + \pi_{sl} + p_S)\Delta_l}{\Delta_s}, \tag{B-8}$$
$$\mu_S = 1 \iff \pi_{ls} > \frac{\pi_{sl}\Delta_l - (r + p_L)\Delta_s}{\Delta_s}. \tag{B-9}$$

C Steady State Condition

The following calculation focuses on the flow of workers who have any combination of parameters (π_{ls}, π_{sl}). Let N_l be the fraction of state l workers and N_s be the fraction of state s workers. In the steady state, the fraction of changing type l equal the fraction of changing type s, then the steady state conditions are $\pi_{sl}N_s = \pi_{ls}N_l$ and $N_s = 1 - N_l$. Then, in the steady state, the fraction of type l workers is $N_l = \pi_{sl}/(\pi_{ls} + \pi_{sl})$ and the fraction of type s workers is $N_s = \pi_{ls}/(\pi_{ls} + \pi_{sl})$.

The fraction of state i workers working in type j firms is denoted as e_{ij}. According to the assumption that both the endogenously and the exogenously job separations do not occur, the proportion of unemployment also converge to 0. Then, $e_{lL} + e_{lS} = N_l$ and $e_{sL} + e_{sS} = N_s$. I present the steady-state condition for each worker's behavior as discussed below.

Perfect Separation Behavior

If the worker's behavior is *PSB*, the steady state condition for the e_{lL} is,

$$p_L e_{lS} + \pi_{sl} e_{sL} = \pi_{ls} e_{lL}. \tag{C-1}$$

The right hand of (C-1) is outflow to e_{lL}, which is the fraction of workers in type L firms who change to s state from l state from a shock. The left hand is inflow to e_{lL}, the first term in the left hand is the fraction of l state workers who move to type L firms from type S firms, and the second term in the left hand is the fraction of workers in type L firms who change to l state from s state from a shock.

Similarly, the condition for the e_{sS} is:

$$p_S e_{sL} + \pi_{ls} e_{lS} = \pi_{sl} e_{sS}. \tag{C-2}$$

By $e_{lS} = N_l - e_{lL}$ and $e_{sL} = N_s - e_{sS}$, the fraction e_{lL} and e_{sS} are:

$$e_{lL} = \frac{N_s \pi_{ls} \pi_{sl} + N_l(\pi_{ls}(p_L - \pi_{sl}) + p_L p_S)}{\pi_{ls} p_L + (\pi_{sl} + p_L) p_S}, \tag{C-3}$$

$$e_{sS} = \frac{N_l \pi_{ls} \pi_{sl} + N_s(\pi_{sl}(p_S - \pi_{ls}) + p_L p_S)}{\pi_{ls} p_L + (\pi_{sl} + p_L) p_S}. \tag{C-4}$$

Thus, the steady state condition of workers choosing *PSB* is characterized by N_l, N_s, (C-3), and (C-4).

Stay L Firm Behavior

In the steady state, the employed workers in type S firms converge to 0 if the workers's behaviors are *SLB*. This is because that workers in type S firms continue to move to L firms. Then $e_{lL} = N_l = \pi_{sl}/(\pi_{ls} + \pi_{sl})$, $e_{sL} = N_s = \pi_{ls}/(\pi_{ls} + \pi_{sl})$, and $e_{lS} = e_{sS} = 0$.

Stay S Firm Behavior

Similarly, the employed workers in type L firms converge to 0. Then $e_{lS} = N_l = \pi_{sl}/(\pi_{ls} + \pi_{sl})$, $e_{sS} = N_s = \pi_{ls}/(\pi_{ls} + \pi_{sl})$, and $e_{lL} = e_{sL} = 0$.

D The Social Planner Problem for the Type L Firms to Type S Firms Ratio

Let $\lambda_1(\pi_{ls}, \pi_{sl})$ and $\lambda_2(\pi_{ls}, \pi_{sl})$ as shadow values of each parameters π_{ls} and π_{sl}. The Hamiltonian function is,

$$\begin{aligned} H =& e^{-rt}[\int\int \{e_l y_{lS} + \Delta_l e_{lL} + e_s y_{sL} + \Delta_s e_{sS}\} d\pi_{ls}\pi_{sl}] \\ &- \int\int \lambda_1(\pi_{ls}, \pi_{sl})[p_L \mu_L e_{lS} + \pi_{sl} e_{sL} - (\pi_{ls} + (1-\mu_L) p_S) e_{lL}] d\pi_{ls} d\pi_{sl} \\ &- \int\int \lambda_2(\pi_{ls}, \pi_{sl})[p_S \mu_S e_{sL} + \pi_{ls} e_{lS} - (\pi_{sl} + (1-\mu_S) p_L) e_{sS}] d\pi_{ls} d\pi_{sl}. \end{aligned} \tag{D-1}$$

Given the optimal condition for μ_L, μ_S, the fist order condition for γ is,

$$\int\int^{\pi_{ls}^{**}} -\lambda_1 e_{ls} - \lambda_2 e_{sS} d\pi_{ls} d\pi_{sl} + \int\int_{\pi_{ls}^{**}}^{\pi_{ls}^{*}} -\lambda_1 e_{lS} + \lambda_2 e_{sL} d\pi_{ls} d\pi_{sl}$$
$$+ \int\int_{\pi_{ls}^{*}} -\lambda_1 e_{lL} + \lambda_2 e_{sL} d\pi_{ls} d\pi_{sl} = 0. \tag{D-2}$$

The functional forms of the shadow values are calculated for each parameter (π_{ls}, π_{sl}). The functional forms are different depending upon the behaviors of workers, and then the functional forms are characterize for each behaviors.

- For workers who take PSB, the laws of motion are:

$$\dot{e}_{lL} = \gamma p(e_l - e_{lL}) + \pi_{sl}(e_s - e_{sS}) - \pi_{ls} e_{lL}, \tag{D-3}$$
$$\dot{e}_{sS} = (1-\gamma)p(e_s - e_{ss}) + \pi_{ls}(e_l - e_{lL}) - \pi_{sl} e_{sS}. \tag{D-4}$$

By $\dot{\lambda_1} = -r\lambda_1$ and $\dot{\lambda_2} = -r\lambda_2$ and the optimal conditions are:

$$e_{lL} : e^{-rt}\Delta_l + \lambda_1(p_L + \pi_{ls} + r) + \lambda_2 \pi_{ls} = 0, \tag{D-5}$$
$$e_{sS} : e^{-rt}\Delta_s + \lambda_1 \pi_{sl} + \lambda_2(p_S + \pi_{sl} + r) = 0. \tag{D-6}$$

Then, the shadow values are:

$$\lambda_1 = \frac{-e^{-rt}((p_S + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s)}{(p_S + \pi_{ls} + r)(p_L + \pi_{ls} + r) - \pi_{ls}\pi_{sl}}, \tag{D-7}$$
$$\lambda_2 = \frac{-e^{-rt}((p_L + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l)}{(p_S + \pi_{ls} + r)(p_L + \pi_{ls} + r) - \pi_{ls}\pi_{sl}}. \tag{D-8}$$

- For workers who take SLB, the laws of motions are:

$$\dot{e}_{lL} = \gamma p(e_l - e_{lL}) + \pi_{sl}(e_s - e_{sS}) - (\pi_{ls}) e_{lL}, \tag{D-9}$$
$$\dot{e}_{sS} = \pi_{ls}(e_l - e_{lL}) - (\pi_{sl} + \gamma p) e_{sS}. \tag{D-10}$$

By $\dot{\lambda_1} = -r\lambda_1$ and $\dot{\lambda_2} = -r\lambda_2$, the optimal conditions are:

$$e_{lL} : e^{-rt}\Delta_l + \lambda_1(p_L + \pi_{ls} + r) + \lambda_2 \pi_{ls} = 0, \tag{D-11}$$
$$e_{sS} : e^{-rt}\Delta_s + \lambda_1 \pi_{sl} + \lambda_2(p_L + \pi_{sl} + r) = 0. \tag{D-12}$$

Then, the shadow values are:

$$\lambda_1 = \frac{-e^{-rt}((p_L + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s)}{(p_L + \pi_{ls} + r)(p_L + \pi_{ls} + r) - \pi_{ls}\pi_{sl}}, \tag{D-13}$$
$$\lambda_2 = \frac{-e^{-rt}((p_L + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l)}{(p_L + \pi_{ls} + r)(p_L + \pi_{ls} + r) - \pi_{ls}\pi_{sl}}. \tag{D-14}$$

- For workers who take SSB, the low of motions are:

$$\dot{e}_{lL} = \pi_{sl}(e_s - e_{sS}) - ((1-\gamma)p + \pi_{ls})e_{lL}], \tag{D-15}$$

$$\dot{e}_{sS} = (1-\gamma)p(e_l - e_{lL}) + \pi_{ls}(e_l - e_{lL}) - (\pi_{sl} + \gamma p)e_{sS}. \tag{D-16}$$

By $\dot{\lambda_1} = -r\lambda_1$ and $\dot{\lambda_2} = -r\lambda_2$, the optimal conditions are:

$$e_{lL} : e^{-rt}\Delta_l + \lambda_1(p_S + \pi_{ls} + r) + \lambda_2\pi_{ls} = 0, \tag{D-17}$$

$$e_{sS} : e^{-rt}\Delta_s + \lambda_1\pi_{sl} + \lambda_2(p_S + \pi_{sl} + r) = 0. \tag{D-18}$$

Then, the shadow values are:

$$\lambda_1 = \frac{-e^{-rt}((p_S + \pi_{sl} + r)\Delta_l - \pi_{ls}\Delta_s)}{(p_S + \pi_{ls} + r)(p_S + \pi_{ls} + r) - \pi_{ls}\pi_{sl}}, \tag{D-19}$$

$$\lambda_2 = \frac{-e^{-rt}((p_S + \pi_{ls} + r)\Delta_s - \pi_{sl}\Delta_l)}{(p_S + \pi_{ls} + r)(p_S + \pi_{ls} + r) - \pi_{ls}\pi_{sl}}. \tag{D-20}$$

By substituting above shadow values into (D-2), the optimal condition of the social planner problem is introduced.

E The Social Planner Problem for Market Tightness

From (D-1), the first order condition for θ is:

$$\begin{aligned} k =& \int\int^{\pi_{ls}^{**}} [-\lambda_1\gamma p' e_{lS} + \lambda_2\gamma p' e_{sS}]d\pi_{ls}d\pi_{sl} + \int\int_{\pi_{ls}^{**}}^{\pi_{ls}^{*}} [-\lambda_1\gamma p' e_{lS} - \lambda_2(1-\gamma)p' e_{sL}]d\pi_{ls}d\pi_{sl} \\ &- \int\int_{\pi_{ls}^{*}} [\lambda_1(1-\gamma)p' e_{lL} - \lambda_2(1-\gamma)p' e_{sL}]d\pi_{ls}d\pi_{sl}. \end{aligned} \tag{E-1}$$

This is equivalence as:

$$\begin{aligned} k =& \gamma\Big[\int\int^{\pi_{ls}^{**}} [-\lambda_1 p' e_{lS} + \lambda_2 p' e_{sS}]d\pi_{ls}d\pi_{sl} + \int\int_{\pi_{ls}^{**}}^{\pi_{ls}^{*}} [-\lambda_1 p' e_{lS}]d\pi_{ls}d\pi_{sl}\Big] \\ &+ (1-\gamma)\Big[\int\int_{\pi_{ls}^{**}}^{\pi_{ls}^{*}} [-\lambda_2 p' e_{sL}]d\pi_{ls}d\pi_{sl} + \int\int_{\pi_{ls}^{*}} [\lambda_1 p' e_{lL} - \lambda_2 p' e_{sL}]d\pi_{ls}d\pi_{sl}\Big]. \end{aligned} \tag{E-2}$$

From (D-2), the formula in first brackets equals the formula in the second brackets. Then, the optimal condition for market tightness is:

$$\begin{aligned} k =& \int\int^{\pi_{ls}^{**}} [-\lambda_1 p' e_{lS} + \lambda_2 p' e_{sS}]d\pi_{ls}d\pi_{sl} + \int\int_{\pi_{ls}^{**}}^{\pi_{ls}^{*}} [-\lambda_1 p' e_{lS}]d\pi_{ls}d\pi_{sl} \\ =& \int\int_{\pi_{ls}^{**}}^{\pi_{ls}^{*}} [-\lambda_2 p' e_{sL}]d\pi_{ls}d\pi_{sl} + \int\int_{\pi_{ls}^{*}} [\lambda_1 p' e_{lL} - \lambda_2 p' e_{sL}]d\pi_{ls}d\pi_{sl}. \end{aligned} \tag{E-3}$$

By substituting shadow values (D-19) and (D-20) into (E-3), the optimal condition of the social planner problem is introduced.

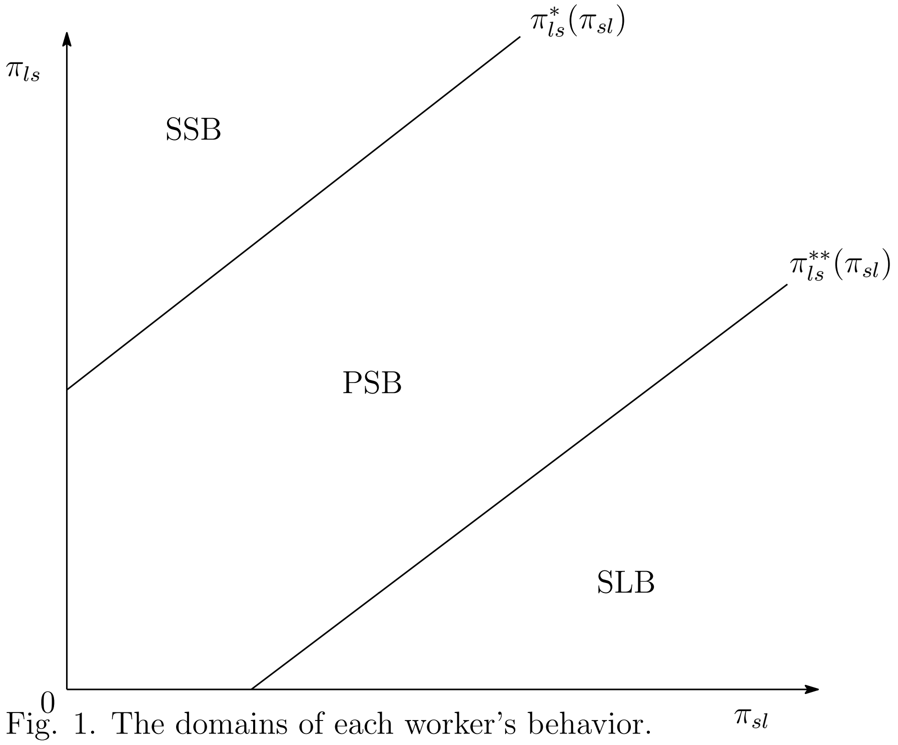

Fig. 1. The domains of each worker's behavior.

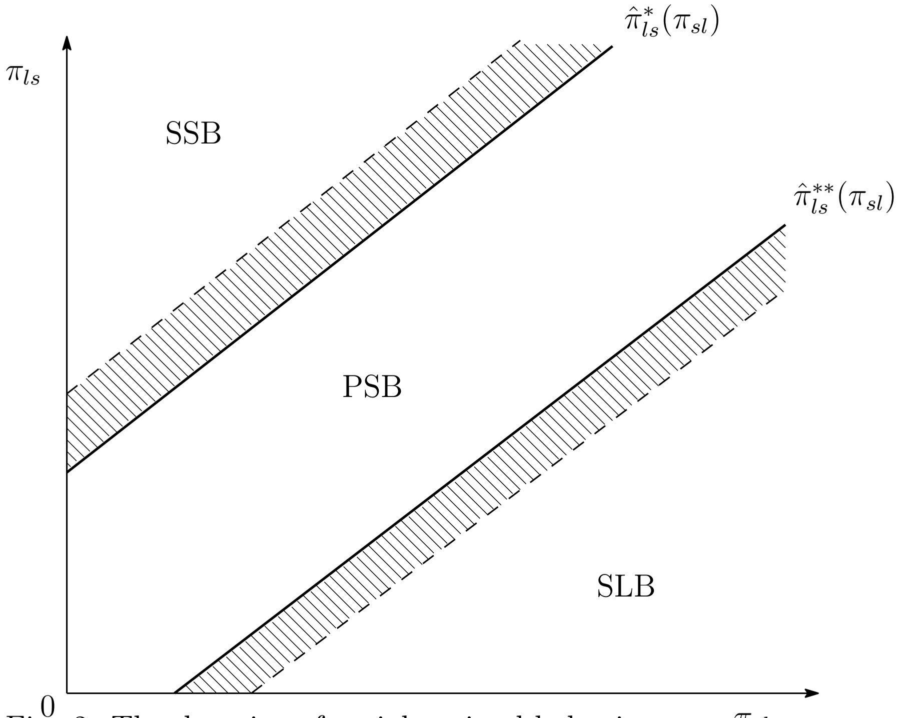

Fig. 2. The domains of social optimal behaviors.