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Abstract

How many members should committees consist of? This paper addresses this question in view of imperfect information and coordination behavior among the members, which is a new approach alternative to introducing information acquisition cost. First, using a simple model, I show that the existence of the coordination motive dismisses Condorcet's (1785) suggestion and the finite optimal size of the committee is determined. Second, I provide an application of the mechanism to monetary policy committees in a basic New Keynesian model. This example will inspire other applications to policy issues in the dynamic stochastic general equilibrium framework.

Keywords: committee, Condorcet jury theorem, coordination, higher order beliefs monetary policy

JEL Classification: D71; D84; E58

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1 Introduction

How many members should committees consist of? This paper addresses this question in view of imperfect information and coordination behavior among the members, which is a new approach alternative to introducing information acquisition cost. While enlarging the committee promotes information aggregation effect, it also foments inefficient coordination behavior among the members by reducing the power of the individuals in the whole of the committee. This ensures the existence of the optimal size of the committee.

The economic theory of committee decision-making has been developing rapidly in recent years. The background of this stream is not only the progress of game theory but also that decision-making by committee has played an important role in the actual economic activities. Many countries traditionally adopt the jury systems and their design problems have been one of the central subjects of the academic and practical arguments in committee design. As the latest event, Japan brought the citizen judge system into effect in 2009 and its design problem was and will be discussed hard. Another outstanding example is the establishment of the monetary policy committees in many countries. Bank of England and Bank of Japan founded the Monetary Policy Committee in 1997 and the Policy Board in 1998 respectively and the central banks of the other countries one after another. Besides, the governments of many countries traditionally call the well-informed persons to the committees for the important policy issues such as tax reforms and the big firms in general hold the meetings to make decisions on the important matters for their business. Following this trend, the demand for the committee design is growing day by day.

The problem of optimal committee size is one of the important issues in the area of group decision-making. The most fundamental argument is whether or not we can enrich the performance of the committees (unlimitedly) by increasing the number of the committee members (infinitely). A famous answer for it is Condorcet's (1785) jury theorem. The theorem asserts that when the committee members vote honestly by use of their own information, enlarging the committee always raises the probability that the committee makes an appropriate decision and it converges to one as the committee size goes to infinity. This implies that finite optimal sizes of committees do not exist. The result is intuitive in a sense but there exist several critical arguments against it in recent literature. They dispute Condorcet's assertion, mainly focusing on that information acquisition of the committee members is costly in the real world and this affects the members' behavior as follows. [1] When the committee size is large, the committee members are apt to avoid paying the information acquisition cost and free-ride on the information that the other

[1] Gerling et al. (2005) provides a brief survey on this topic.

members provide because their contribution to the decision of the committee is small relatively to the information acquisition cost. Thus, Condorcet jury theorem can fail to hold under the existence of information acquisition cost.

In this paper, I take an approach different from the existing studies to discuss on the optimal committee size. I focus on the role of coordination behavior among the committee members with noisy common information. This is motivated as follows. In the case of the jury systems, although individual names of jurors are not disclosed, trials by courts are held publicly and their sayings are documented and reported. When the individual jurors face uncertainty, they may avoid distant voting from the general tendency in the jury. Besides, many organizations have instituted the rules of information disclosure in recent years. In particular, disclosure of public information on the policy issues of the government and central bank has usually been a legal mandate in many developed countries. How does such an institutional trend make a difference in committee decision-making? Transparency may generate the incentive of the committee members to coordinate with the other members because the minutes are often opened to the public under transparency and hence the individual member faces an accountability problem. That is, when the information on the decision-making process in the committee is disclosed to the outsiders, the inefficiency of each member's vote, which is revealed ex post, is also known to them and it affects the individual valuation or reputation of each member. Unless the common information of all the members is perfectly correct, the coordination behavior can bring an inefficient decision.

There is another reason for taking the approach other than information acquisition cost. Although the assumption of the existence of information acquisition cost is intuitively plausible, it is questionable whether the assumption fits with design of committees, for example, the jury systems. Indeed, jurors usually question the accused in the courts but they fundamentally rely on the material and circumstantial evidence provided by the prosecutors and the defense. That is, in general, jurors do not get information drudgingly. I think that this is one of the limits of introducing information acquisition cost in committee design. Also, the model with costly information does not fit for the committees consisting of experts. This is because experts are generally well-informed about the issues concerned in advance and detail information such as data is often provided not by the committee members but the staff. Typical examples are the monetary policy committees and the other policy boards. In fact, Toshiro Muto, who is a deputy governor of Bank of Japan, said that he did not sense the effect of the free-rider problem of information provision in the meetings according to his experience as an insider of the Policy Board.[2]

[2]See Muto (2007).

The setting of the present paper is an attempt to find an alternative framework which is suitable to the case where the committee consists of experts.

The main findings of this paper are as follows. When the committee members have the incentive to coordinate with the other members, enlarging the committee size foments the coordination behavior. The reasoning is that each member realizes that her power to the final decision of the committee is reduced and has to bury herself so as not to stand out in all the committee members. Thus, while a large committee can absorb idiosyncrasy of noisy decentralized information strongly, it brings a large coordination loss. The optimal size of the committee is the size which brings the best trade-off between the positive and negative effects above and it generally exists in a wide parameter region. The optimal committee size decreases in the potential coordination motive and the precision of common information since they increase the dependency of the committee members on noisy common information. Thus, this paper provides a new explanation for the existence and properties of the optimal committee size.

Another contribution of this paper to the literature is that the benchmark model has an explicit application to a concrete economic problem. I extend the benchmark model to a macroeconomic model for monetary policy analysis which starts from decision-making by the monetary policy committee and investigate its optimal size. In the last decade, many countries established the formal committee for decision-making on monetary policy. So that, in recent arguments of monetary policy, the institutional design problems of monetary policy committee are regarded as important matters and the optimal committee size is one of them. [3] This paper is the first paper which analyzes the optimal size of the monetary policy committee in a formal model of the modern framework for monetary policy analysis. Since central bank transparency is also a remarkable feature of modern monetary policy as mentioned above, this paper's approach is motivated along the trend of central banking.

There is enormous literature on Condorcet jury theorem. Here, I briefly review several studies relevant to the present paper. It is known that Condorcet jury theorem does not hold under some kinds of strategic voting since sincere voting, which is one of crucial assumption for the theorem, is inconsistent with equilibrium. Austen-Smith and Banks (1996) show that sincere voting is not attained in equilibrium when the individuals take their being pivotal into consideration. Feddersen and Pesendorfer (1998) show that the probability of false accusation can be increasing in the jury size under unanimity rule. Several studies suggest that Condorcet jury theorem also fails to hold when information acquisition of the individuals is costly. [4] Mukhopadhaya (2003) assumes the environment

[3] Blinder (2007) provides a brief survey on this issue.

[4] It is known that there is a case where the asymptotic efficiency as in Condorcet jury theorem can

where the individuals have identical preferences and their information acquisition is costly. By a numerical method, he shows that welfare can be lower in mixed-strategy equilibria of large committees than pure-strategy equilibria of small committees. Koriyama and Szentes (2009) assume the environment similar to Mukhopadhaya (2003) and show that the optimal committee size is bounded. They also show that the inefficiency of oversized committees is smaller than that of the undersized committees. Thus, the recent literature mainly focuses on information acquisition cost. The framework of the present paper can treat the cases of experts committees to which the existence of information acquisition cost is not suitable.

There are a few studies on optimal size of monetary policy committees. Sibert (2006) conjectures that the free-rider problem of information acquisition can play a role for the discussion on the issue. [5] However, it seems disputable in the standpoint of practice of decision-making in the monetary policy committees as Muto (2007) suggests. This paper provides an alternative approach to study optimal size of monetary policy committees and an intuitively plausible answer for the problem in a formal model in modern framework for monetary policy analysis. Some empirical studies find the facts on monetary policy committees. In particular, Berger and Nitsch (2008) find the fact that inflation volatility is U-shaped in the size of the monetary policy committees. This shows that the structure of the monetary policy committees affects the economic outcome in actual. The result of the present paper explains the fact above by showing the two effects of enlarging the monetary policy committee: the positive effect of information aggregation and the negative effect of coordination.

In the rest of this paper is organized as follows. Section 2 provides a simple microeconomic model which describes the substantial mechanism for the determination of the optimal committee size. Section 3 applies the mechanism in Section 2 to a design of the monetary policy committee in a modern macroeconomic model and provides some policy implications. Section 4 concludes. All proofs are given in the Appendix.

2 A Simple Model

This section provides a simple microeconomic model which describes the process of decision-making and determination of the finite optimal size of the committee. It is very abstract but can grasp clearly the role of coordination behavior in decision-making

hold even if information acquisition is costly. Martinelli (2006) shows that if there is only variable cost for obtaining precision of the signals, then the probability that the committee makes a correct decision converges to one as the committee size goes to infinity.

[5] On the free-rider problem of information as a public good, for example, see Li (2001).

by the committee.

2.1 Setup

I set up a benchmark model. The committee consists of N (ex-ante) homogeneous members. It is seated to pursue a target $\theta \in \mathbb{R}$ on behalf of an organization in the background. [6] For example, juries are called to judge criminal suits reasonably in the cause of (social) justice and monetary policy committees are organized to make an appropriate decision on monetary policy for society's benefit. The target θ is interpreted as an underlying state or the committee's optimal response to it. For instance, θ is the truth of the case in the trials. [7] In the model of monetary policy by committee which I provide in the next section, the counterpart of θ is a level of nominal interest rate set in optimal discretionary policy under perfect information. [8]

Each member of the committee has uninformative flat prior about θ over the real line but receives common and private signals on θ. [9] The common signal is of the form such as

$$y = \theta + \eta,$$

where the noise term η is normally distributed with mean zero and variance α^{-1}. Each member knows that the realization of y and the distribution of η are common and known to everyone. The common signal is interpreted as a content of a staff report or well-balanced recognition among the committee members especially as experts. The private signal of an arbitrary member j is also of the standard form such as

$$x_j = \theta + \varepsilon_j,$$

where the noise term ε_j is mutually independent and normally distributed with mean zero and variance β^{-1}. She knows that the distribution is common to every member

[6] I believe that this is a natural motivation for the establishements of the actual committees in many cases according to concrete application (such as the example in section 3) although there are probably counter examples.

[7] Most of literature on problems of juries adopt two state models in which $\theta =$ 'innocent' or $\theta =$ 'guilty'. The model of continuous state admits the case where the jury also participates in the determination of the appropriate punishment as the citizen judge system in Japan.

[8] In a basic New Keynesian model, nominal interest rate set in optimal discretionary policy is linear in demand shock and cost shock. In this example, (innovations of) these shocks are the underlying states. For detail, see the next section.

[9] This assumption on the prior about θ is intended for analytical ease. Although this is not so natural according to the application given in the next section, no substantial difference arises from considering proper prior. For a detail discussion, see the supplementary note.

but does not know the realization of the others' private signals. The private signals represent the members' individual views on the target which are generally distinct and not communicated to the others.

Next, I set the payoff structure of the committee. Although the committee itself is seated for making a decision near to the true θ under imperfect information, each committee member pursues her own objectives. In this simple model, I assume that each member j votes $a_j \in \mathbb{R}$ so as to maximize her own payoff function

$$u_j = -(1-r)(a_j - \theta)^2 - r(a_j - \bar{a})^2, \tag{1}$$

where $r \in [0, 1)$ and $\bar{a} = \frac{1}{N}\sum_{k=1}^{N} a_k$. The meaning of the payoff function above is as follows. Each member j has two goals. One is to hit the true target and the other is not to remove her vote from the average of all. That is, while she honestly tries to contribute to an appropriate decision, she also seeks coordination with the other members even though it makes the performance of the committee worse. Thus, I interpret parameter r as the degree of coordination motive of the members. Note that I assume that the objective of establishing the committee is to grasp the true target and make the decision as correct as possible. [10] Since the coordination motive distorts the members' use of information, it generates only a loss for the performance of the committee.

However, there are some reasons for considering such coordination motive. Usual committees are established for better decision-making by choosing the delegations from the large organizations. For example, firms hold committee meetings to make decisions on big bargains or selections of recruits, governments summon well-informed persons committees for various policy issues and the central banks have the formal policy boards for decision-making on monetary policy. In the cases of policy issues, since each member's saying or voting in the meeting is often released to the public, she will be at least partially motivated to coordinate with the other members. In the cases of the firms, although the records are rarely opened formally, what the members said in the meeting usually spreads from nowhere or can be speculated by the outsiders. Therefore, I interpret r also as a measure of the indirect effect of transparency on decision-making in the committee.

All the votes are aggregated by a specific voting rule and it becomes the final decision of the committee. For analytical ease, I assume that the voting rule of the benchmark case is the arithmetic mean:

$$\hat{a} = \frac{1}{N}\sum_{j=1}^{N} a_j. \tag{2}$$

[10] I set the performance measure of the committee along this line soon later.

This rule is quite simple but enough for grasping the basic mechanism this paper suggests. Of course, the approach I will take is applicable to the case of more realistic voting rule. In section 2.4, I analyze the case of the median-voting rule:

$$\hat{a} = \underset{1\leq j\leq N}{\text{med}}\{a_j\}.$$

I will show that the basic properties of the model do not change in that case.

I finally set a performance measure of the committee. Since in this paper I assume that the committee is seated to make accurate decisions for the benefit of the organization in the background, a natural measure of the committee's performance is

$$W = -(\hat{a} - \theta)^2. \tag{3}$$

2.2 Equilibrium

I derive the Bayesian Nash equilibrium of the model. The first order condition of each member j's problem is

$$a_j = \frac{1-r}{1-\frac{r}{N}} E_j(\theta) + \frac{(1-\frac{1}{N})r}{1-\frac{r}{N}} E_j(\bar{a}),$$

where the symbol E_j represents a mathematical expectation conditioned on information available to member j. Let $\delta = \frac{(1-\frac{1}{N})r}{1-\frac{r}{N}}$. Then, the FOC above is rewritten as

$$a_j = (1-\delta)E_j(\theta) + \delta E_j(\bar{a}). \tag{4}$$

Note that δ is increasing in N. That is, each member places a higher weight on the average of all the votes relative to the target as the committee becomes larger. What does cause it? Since the number of the committee members is finite, each member can partially control the average of all the votes: see (1). When the committee size is small, she does not have to care about the others' behavior extremely because her power in the committee is relatively large. As the committee becomes large, the power fades away and she must seek harder for coordination with the other members. In short, the massiveness of the group drives the individuals to bury themselves in it.

Equation (4) provides the conjecture that the equilibrium strategy is linear in common and private signals. [11] In fact, the following proposition supports for it together with the uniqueness property.

[11] For a heuristic derivation of equilibrium, put $a_j = (1-\gamma)y + \gamma x_j$. Here γ is an undetermined coefficient. Substituting this into (4) and comparing the coefficients of both sides, the response coefficient γ given in Proposition 1 is obtained.

Proposition 1 *There exists a unique equilibrium strategy such that for all* $y, x_j \in \mathbb{R}$*,*

$$a_j = (1-\gamma)y + \gamma x_j,$$

where $\gamma = \frac{(1-r)\beta}{(1-\frac{r}{N})(\alpha+\beta) - r(1-\frac{1}{N})(\frac{\alpha}{N}+\beta)}$.

Proof.
See Appendix A.

This equilibrium strategy brings the essential mechanism for the main result of the present paper. The next assertion shows it.

Corollary 1 *The response coefficient* γ *to private signal is decreasing in* N*. Given* α*,* β *and* r*, it lies in the half-open interval* $\left(\frac{(1-r)\beta}{\alpha+(1-r)\beta}, \frac{\beta}{\alpha+\beta}\right]$.

Proof.
See Appendix B.

Corollary 1 asserts that each member's dependency of the common signal increases as the size of the committee becomes larger. It results from the relationship between each member's control of decision-making in the committee and coordination behavior which I explained above. When the committee becomes larger, each member becomes more sensitive to common signal to adjust her voting to the others' more precisely. If the committee consists of only one person, the coordination motive vanishes and his behavior accords with that of the basic statistical decision-making: $\gamma|_{N=1} = \frac{\beta}{\alpha+\beta}$. As the committee size goes to infinity, the response coefficient γ converges to $\frac{(1-r)\beta}{\alpha+(1-r)\beta}$. That is, the equilibrium strategy corresponds to that of the finite-players version of Morris and Shin's (2002) beauty contest game. In the finite-players version of Morris and Shin's (2002) beauty contest game, each player j has the same informational structure as the model of the present paper and her utility function is [12]

$$-(1-r)(a_j-\theta)^2 - r\left(a_j - \frac{1}{N-1}\sum_{k\neq j} a_k\right)^2.$$

[12] As an alternative setting, we may assume that her utility function is

$$-(1-r)(a_j-\theta)^2 - \frac{r}{N-1}\sum_{k\neq j}(a_j-a_k)^2.$$

The solution under this utility function is the same as the case of the function in the text although the social welfare function changes.

The second objective of each player is the average of the others' actions, which does not include her own action. In this case, each player can not control the average and hence has to care about the others' behavior more greatly than the case where she can do it. Therefore, the basic beauty contest game can be regarded as a limiting case of the coordination game in this paper.

2.3 Optimal Size of the Committee

By (3) and Proposition 1, the decision of the committee is

$$\hat{a} = (1-\gamma)y + \gamma \frac{\sum_{k=1}^{N} x_k}{N}. \tag{5}$$

Thus, substituting (5) into (3), the expected performance of the committee is

$$\begin{aligned} E(W|\theta) &= -E\left[\left((1-\gamma)(\theta+\eta) + \gamma \frac{\sum_{k=1}^{N}(\theta+\varepsilon_k)}{N} - \theta\right)^2\right] \\ &= -E\left[\left((1-\gamma)\eta + \gamma \frac{\sum_{k=1}^{N}\varepsilon_k}{N}\right)^2\right] \\ &= -(1-\gamma)^2\alpha^{-1} - \gamma^2 \frac{\beta^{-1}}{N}. \end{aligned}$$

I investigate the relationship between the size of the committee and its performance. Considering the continuation of $E(W|\theta)$ with respect to N, I obtain

$$\frac{\partial E(W|\theta)}{\partial N} = 2(1-\gamma)\alpha^{-1}\frac{\partial \gamma}{\partial N} - 2\gamma \frac{\beta^{-1}}{N} \cdot \frac{\partial \gamma}{\partial N} + \gamma^2 \beta^{-1} N^{-2}. \tag{6}$$

The meaning of (6) is clear in view of Corollary 1. Enlarging the committee has three effects on its performance. Note that, since $\frac{\partial \gamma}{\partial N} < 0$ by Corollary 1, the first term is negative and the second and third terms are positive. The first term represents the indirect negative effect due to the stronger coordination behavior. The second term is the indirect positive effect owing to the decrease of the members' dependency on the noisy private signals. The third term is the direct positive effect from the decrease of the volatility due to the noisy private signals by averaging larger samples.

The next proposition provides a necessary and sufficient condition for the existence of the optimal committee size under the average-voting rule.

Proposition 2 *Under the average-voting rule, there exists a finite optimal size of the committee if and only if the parameter set satisfies* $r > \frac{1}{5}$ *and* $\frac{\beta}{\alpha} < \frac{5r-1}{(1-r)^2}$.

Proof.
See Appendix C.

The parameter condition above means that the degree of coordination motive, r, is not so weak and the precision of common (private, resp.) information relative to private (common) information is large (small).

This result is intuitively plausible. When the coordination motive is strong, each member depends highly on common information to approximate her own voting to the others'. Besides, when the relative precision of common information is larger, each member also places a higher weight on common information to hit the true target more accurately. [13] The negative effect of enlarging the committee is amplified in such a case. Then, this dominates the positive effects in the limit since the marginal contribution of averaging reduces as the committee becomes larger.

In fact, the expected performance $E(W|\theta)$ is single-peaked in the committee size N under the necessary and sufficient condition for the existence of the optimal size. [14]

[Figure 1 about here.]

Figure 1 gives a numerical example of this relationship. [15] When the committee size is small, the direct positive effect of averaging is very large and it (and the indirect positive effect) dominates the negative effect. However, as the committee size becomes larger, the direct positive effect becomes smaller and the negative effect becomes relatively larger. Thus, the expected performance has the single peak in the committee size. According to the discussion above, I obtain the following characterization of the optimal committee size.

Corollary 2 *Suppose that the parameter set satisfies the necessary and sufficient condition for the existence of the optimal committee size under the average-voting rule:* $r > \frac{1}{5}$ *and* $\frac{\beta}{\alpha} < \frac{5r-1}{(1-r)^2}$. *Then, the optimal size* N^* *is given by the following equation.*

$$N^* = \operatorname*{argmax}_{N \in \{N_-(\tilde{N}), N_+(\tilde{N})\}} E(W|\theta),$$

where $\tilde{N} \in \mathbb{R}$ *is the solution of the equation*

$$\frac{\gamma}{1-\gamma} \cdot \frac{1}{N} - \frac{\gamma^2}{2(1-\gamma)} \Big(\frac{\partial \gamma}{\partial N} N^2 \Big)^{-1} = \frac{\beta}{\alpha}$$

[13] This mechanism necessarily works since I assume $r < 1$.
[14] A formal proof of this fact is given in the necessity part of the proof of Proposition 2.
[15] To depict Figure 1, I set $r = 0.4$, $\alpha = 1.25$ and $\beta = 1$.

with respect to $N \in \mathbb{R}$, $N_-(\tilde{N})$ is the maximal integer which is not larger than $\tilde{N}$ and $N_+(\tilde{N})$ is the minimal integer which is not smaller than $\tilde{N}$.

Proof.
It is obtained immediately from the proof of Proposition 2. See Appendix C.

Note that $\tilde{N}$ depends only on r and the precision ratio $\frac{\beta}{\alpha}$ because γ and $\frac{\partial \gamma}{\partial N}$ does so. [16] Since $\tilde{N}$ is the single peak of the continuation of $E(W|\theta)$ with respect to N, N^* is mainly determined by them. However, rigorously speaking, N^* depends on not the precision ratio but the pair (α, β) in general. This is because $E(W|\theta)$ can not be written as the function of r and $\frac{\beta}{\alpha}$ and it is impossible to decide which of $N_-(\tilde{N})$ and $N_+(\tilde{N})$ the optimal size N^* is equal to only with the values of r and $\frac{\beta}{\alpha}$.

Next, I investigate two relationships between the parameters and the optimal size. Although it is difficult to obtain them in analytical ways due to discreteness of the committee size, I can find the robust qualitative results below.

First, I fix $\alpha = 1.25$ and $\beta = 1$ as in Figure 1 and calculate the optimal sizes for various values of r in [0.3, 0.99]. [17]

[Figure 2 about here.]

Figure 2 illustrates the result. The optimal committee size is non-increasing in the degree of coordination motive, r. When r is small and hence the coordination motive is weak, the negative effect of enlarging the committee is small. The importance of enhancing the positive effect of information aggregation is then relatively large. Therefore, the optimal size of the committee is very large for small r. As r becomes larger, the optimal size decreases rapidly since the negative effect of coordination acceleratingly swells. Figure 2 provides another interesting fact. It illustrates that the optimal size of the committee is one for sufficiently large r. That is, if the coordination motive is very strong and adding a committee member is too costly, then a single decision-maker can be optimal to choose a correct alternative. When the members bury themselves and follow the incorrect common sense with little criticism, appointing a sincere individual may be a good measure to reach a right conclusion.

Second, I set $r = 0.4$ as in Figure 1 and calculate the optimal sizes for various values of α (β, resp.) in $[1, 5]$ with β fixed to 2 (α fixed to 3).

[Figure 3 about here.]

[16] See the necessity part of the proof of Proposition 2.

[17] When $\alpha = 1.25$ and $\beta = 1$, the existence condition is met for all $0.3 \leq r \leq 0.99$.

Figure 3 illustrates the result. Given the precision of private information (common information, resp.), the optimal committee size is non-increasing (non-decreasing) in the precision of common information (private information). Given the degree of coordination motive, when the precision of common information is large relative to that of private information, each member places a high weight on common information. Then, enlarging the committee strongly foments the coordination behavior among the members. The optimal size is hence non-increasing in α. Contrary, since such a negative effect of enlarging the committee is small when the precision of private information is large, the gain of information aggregation is relatively large. Thus, the optimal size is non-decreasing in β. The result above suggests that we should establish a small committee to reduce the coordination loss when the common sense (or the staff report) on the issue concerned is precise to some extent.

2.4 Median-Voting Rule

In the last of this section, I consider the case where the voting rule is the median-voting rule. The basic properties of the model do not change even under this voting rule. The discussion below ensures the robustness of the results in section 2.3 in a wide class of voting rules.

Since each committee member's behavior is invariant in voting rules, the decision of the committee is

$$\begin{aligned}\hat{a} &= \operatorname*{med}_{1\leq j\leq N}\{(1-\gamma)y+\gamma x_j\} \\ &= \theta+(1-\gamma)\eta+\gamma\operatorname*{med}_{1\leq j\leq N}\{\varepsilon_j\}.\end{aligned}$$

Thus, the expected performance of the committee is

$$E(W|\theta) = -(1-\gamma)^2\alpha^{-1} - \gamma^2 E\left[\left(\operatorname*{med}_{1\leq j\leq N}\{\varepsilon_j\}\right)^2\right].$$

The expectation in the second term of the right-hand side has no analytical expression for finite N. However, since the distribution of $\text{med}_{1\leq j\leq N}\{\varepsilon_j\}$ is approximately the normal distribution with mean 0 and variance $\frac{\pi\beta^{-1}}{2N}$ for sufficiently large N ($\text{med}_{1\leq j\leq N}\{\varepsilon_j\} \propto N(0,\frac{\pi\beta^{-1}}{2N})$), I obtain

$$E(W|\theta) \approx -(1-\gamma)^2\alpha^{-1} - \gamma^2\frac{\pi\beta^{-1}}{2N} \tag{7}$$

for sufficiently large N. [18] Thus, I obtain a sufficient condition for the existence of the finite optimal committee size under the median-voting rule by use of this asymptotic property.

Corollary 3 *Under the median-voting rule, there exists a finite optimal size of the committee if the parameter set satisfies* $r > \frac{\pi}{\pi+8}$ *and* $\frac{\beta}{\alpha} < \frac{(\pi+8)r-\pi}{\pi(1-r)^2}$.

Proof.
See Appendix D.

The intuition for this sufficient condition is similar to Proposition 2 but there is a quantitative difference between the median-voting rule and the average-voting rule. One can show that the parameter region which satisfies the sufficient condition under the median-voting rule is smaller than that of the average-voting rule. Figure 4 illustrates it.

[Figure 4 about here.]

The cause of this result is the difference between statistical properties of mean and median. That is, as seen in $\text{med}_{1\leq j\leq N}\{\varepsilon_j\} \propto N(0, \frac{\pi\beta^{-1}}{2N})$, sample median converges in probability more slowly than sample mean. [19] Therefore, under the median-voting rule, it is more important to promote the positive effect of information aggregation than under the average-voting rule. This makes the existence condition under the median-voting rule stricter.

A necessary condition along the line of Proposition 2 can not be obtained under the median-voting rule because there is no analytical expression of the distribution of sample median for small sample: see the necessity part of the proof of Proposition 2. However, the basic properties under the average-voting rule will be robust since the distribution of sample median approaches that of sample mean very quickly. [20] Moreover, the proofs of Proposition 2 and Corollary 3 imply the following. In general, we can obtain a similar (sufficient) condition for the existence of the optimal committee size when we adopt as

[18] It is known that when n random variables X_i $(i = 1, ..., n)$ identically and independently follow a distribution with median M and density function f, the distribution of $Y_n = \text{med}_{1\leq i\leq n}\{X_i\}$ with large sample is approximately the normal distribution with mean M and variance $\frac{1}{4n(f(M))^2}$. As a special case, when each X_i follow the normal distribution with mean μ and variance σ^2 identically and independently, the distribution of the sample median Y_n is approximately $N(\mu, \frac{\pi\sigma^2}{2n})$ for large n. For a detail explanation, see Kenney and Keeping (1962).

[19] Since $\frac{1}{N}\sum_{j=1}^{N}\varepsilon_j \sim N(0, \frac{\beta^{-1}}{N})$, the variance of sample mean is $\frac{\pi}{2}$ times as small as that of sample median.

[20] For a detail explanation on this fact, see Maritz and Jarret (1978).

a voting rule any statistics which has the consistency as an estimator of the true state with respect to private signals and converges at an order not lower than sample mean. [21]

3 Monetary Policy Committee

In this section, I provide a simple application of the model in Section 1. Following Morimoto (2009) basically, I set up the model for monetary policy analysis which starts from decision-making by the monetary policy committee. For a detail explanation of the model setting, see Morimoto (2009).

3.1 Macroeconomic Model

As the underlying macroeconomic model, I adopt a basic New Keynesian model. The model consists of the two stochastic difference equations

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + u_t, \tag{8}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \tag{9}$$

together with a monetary policy rule. Here, $x_t, \pi_t, i_t, u_t, e_t$ are output gap, inflation rate, nominal interest rate, demand shock and cost shock in period t respectively and β, σ, κ are positive parameters. Parameters β, σ, κ represent the discount rate, constant elasticity of intertemporal substitution and impact of one unit output gap on inflation, respectively. I assume that u_t and e_t follow AR(1) processes in such a way that

$$u_t = \rho_u u_{t-1} + \varphi_t,$$

$$e_t = \rho_e e_{t-1} + \psi_t,$$

where $\rho_u, \rho_e \in [0, 1)$ and the innovations φ_t and ψ_t are normally distributed with mean zero and variances σ_φ^2 and σ_ψ^2 respectively. Once a setting rule of nominal interest rate is specified, macro dynamics of the model economy is determined as sequences of output gap and inflation rate under the policy rule.

As in most works in optimal monetary policy, I adopt the following social loss function as the welfare measure.

$$L \equiv V(\pi) + \lambda V(x), \tag{10}$$

[21] An estimator for a parameter is said to satisfy consistency if it converges in probability to the true value of the parameter.

where $V(\pi)$ and $V(x)$ are asymptotic variances of inflation rate and output gap and λ is the weight that society places on output gap relative to inflation. [22]

3.2 Interest Rate Setting by Committee

Next, I set up the process of decision-making on interest rate setting in the monetary policy committee. The committee consists of N (ex-ante) homogeneous members. The informational structure of the committee is as follows. Each committee member faces information imperfectness about innovations of demand shock and cost shock. For simplicity, I assume that each committee member has improper flat prior about them over the real line. [23] In the end of period $t-1$, each member receives two kinds of signals on innovations of demand shock and cost shock in period t. One is common signal and the other is private signal. Each member $j's$ common signal is of the standard form such as

$$\begin{aligned} \varphi_t^c &= \varphi_t + \mu_t, \\ \psi_t^c &= \psi_t + \nu_t, \end{aligned}$$

where the noise terms of μ_t and ν_t are independently and normally distributed with mean zero and variance σ_μ^2 and σ_ν^2 respectively. There are some sources of common signals of the committee members. The committee members probably have a kind of well-balanced recognition on economic states as macroeconomists. Besides, in most actual central banks, the first step of or one of preparations for the meeting of monetary policy committees is the staff report on the present conditions and future developments of the economies.

Each member j's private signal is of the standard form such as

$$\varphi_t^j = \varphi_t + \varepsilon_t^j,$$

[22] In the usual analysis in the New Keynesian literature, the welfare measure is the second order approximation of the household's utility function. In the basic model, it is proportional to

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda x_t^2\right).$$

In this paper, however, I focus only on the average performance of monetary policy and reset it to

$$\lim_{\beta \to 1} (1-\beta) E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda x_t^2\right) = L.$$

[23] Although it is more natural to assume that each member knows the distribution of innovations, $\varphi_t \sim N(0, \sigma_\varphi^2)$ and $\psi_t \sim N(0, \sigma_\psi^2)$, this does not change the basic properties below and only complicates manipulations. For details, see the supplementary note.

$$\psi_t^j = \psi_t + \eta_t^j,$$

where the noise terms of ε_t^j and η_t^j are independently and normally distributed with mean zero and variance σ_ε^2 and σ_η^2 respectively.

In the meeting of the monetary policy committee, based on her own information, each member j votes the level of nominal interest rate in period t, i_t^j, to maximize her own expected payoff. The function form of the payoff function is assumed to be

$$-r(i_t^j - i_t^*)^2 - (1-r)(i_t^j - \bar{i}_t)^2, \tag{11}$$

where $r \in [0,1)$, $\bar{i}_t = \frac{1}{N}\sum_{j=1}^N i_t^j$ and i_t^* is the nominal interest rate in period t set in optimal discretionary policy under perfect information by a single policy maker. That is, i_t^* is the solution of the following linear-quadratic problem. [24]

$$\begin{aligned}
\min \quad & \pi_t^2 + \lambda x_t^2, \\
\text{s.t.} \quad & x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}) + u_t, \\
& \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.
\end{aligned}$$

Solving this problem, I obtain the analytical expression of i_t^* such that

$$i_t^* = \sigma u_t + \Phi e_t,$$

where $\Phi = \frac{\lambda^c \rho_e + (1-\rho_e)\sigma\kappa}{\lambda^c(1-\beta\rho_e)+\kappa^2}$.

Nominal interest rate in period t, i_t, is determined by aggregating the voting rates with a specific rule. As the benchmark case, I assume that the voting rule is the arithmetic mean:

$$i_t = \frac{1}{N}\sum_{j=1}^N i_t^j. \tag{12}$$

This is quite simple but sufficient for the purpose of this paper. In the actual institutions, however, the majority rule is often adopted and hence it is natural to consider the case of the median-voting rule:

$$i_t = \operatorname*{med}_{1 \le j \le N}\{i_t^j\}. \tag{13}$$

I will show that as in the simple model of the previous section, the basic result does not change under the median-voting rule.

[24]For a detail explanation of this issue, see Walsh (2003).

3.3 Equilibrium Dynamics and Macroeconomic Volatility

Now let us see the equilibrium strategy of the subgame in the committee. The first order condition of member j's problem is

$$i_t^j = (1-\delta)E_{t-1}^j(i_t^*) + \delta E_{t-1}^j(\bar{i}_t),$$

where E_{t-1}^j is a mathematical expectation conditioned on information available to member j in the end of period $t-1$. Since

$$E_{t-1}^j(i_t^*) = \sigma\left[\rho_u u_{t-1} + \frac{\sigma_\mu^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\mu^{-2}}\varphi_t^c + \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\mu^{-2}}\varphi_t^j\right] + \Phi\left[\rho_e e_{t-1} + \frac{\sigma_\nu^{-2}}{\sigma_\eta^{-2}+\sigma_\nu^{-2}}\psi_t^c + \frac{\sigma_\eta^{-2}}{\sigma_\eta^{-2}+\sigma_\nu^{-2}}\psi_t^j\right],$$

I obtain the following proposition immediately from Proposition 1.

Proposition 3 *There exists a unique equilibrium strategy of the form such that*

$$i_t^j = \sigma[\rho_u u_{t-1} + \gamma_u\varphi_t^j + (1-\gamma_u)\varphi_t^c] + \Phi[\rho_e e_{t-1} + \gamma_e\psi_t^j + (1-\gamma_e)\psi_t^c], \tag{14}$$

where $\gamma_u = \frac{(1-r)\sigma_\varepsilon^{-2}}{(1-\frac{r}{N})(\sigma_\mu^{-2}+\sigma_\varepsilon^{-2}) - r(1-\frac{1}{N})(\frac{\sigma_\mu^{-2}}{N}+\sigma_\varepsilon^{-2})}$ *and* $\gamma_e = \frac{(1-r)\sigma_\eta^{-2}}{(1-\frac{r}{N})(\sigma_\nu^{-2}+\sigma_\eta^{-2}) - r(1-\frac{1}{N})(\frac{\sigma_\nu^{-2}}{N}+\sigma_\eta^{-2})}$.

Substituting (14) into (12), I obtain the following equilibrium nominal interest rate.

$$\begin{aligned} i_t &= \sigma[u_t + \gamma_u\tilde{\varepsilon}_t + (1-\gamma_u)\mu_t] + \Phi[e_t + \gamma_e\tilde{\eta}_t + (1-\gamma_e)\nu_t] \\ &= i_t^* + \sigma[\gamma_u\tilde{\varepsilon}_t + (1-\gamma_u)\mu_t] + \Phi[\gamma_e\tilde{\eta}_t + (1-\gamma_e)\nu_t], \end{aligned} \tag{15}$$

where $\tilde{\varepsilon}_t = \frac{1}{N}\sum_{j=1}^N \varepsilon_t^j$ and $\tilde{\eta}_t = \frac{1}{N}\sum_{j=1}^N \eta_t^j$. The second and third terms of (15) represent the inefficiency of interest rate setting due to imperfect information and coordination behavior among the committee members.

Macro dynamics of the model economy is given by (8), (9) and (15). Note that since the relevant state variables in period t are $e_t, \tilde{\varepsilon}_t, \tilde{\eta}_t, \mu_t$ and ν_t, equilibrium output gap and inflation rate is linear in them. Thus, by the method of undetermined coefficients, I obtain the following equilibrium output gap and inflation rate. [25]

$$\begin{aligned} x_t &= -\frac{\kappa}{\lambda^c(1-\beta\rho_e)+\kappa^2}e_t - \left[\gamma_u\tilde{\varepsilon}_t + \frac{\gamma_e\Phi}{\sigma}\tilde{\eta}_t + (1-\gamma_u)\mu_t + \frac{(1-\gamma_e)\Phi}{\sigma}\nu_t\right], && (16) \\ \pi_t &= \frac{\lambda^c}{\lambda^c(1-\beta\rho_e)+\kappa^2}e_t - \kappa\left[\gamma_u\tilde{\varepsilon}_t + \frac{\gamma_e\Phi}{\sigma}\tilde{\eta}_t + (1-\gamma_u)\mu_t + \frac{(1-\gamma_e)\Phi}{\sigma}\nu_t\right]. && (17) \end{aligned}$$

[25] For the detail of the calculation, see Appendix E.

The bracketed parts of the equilibrium output gap and inflation rate are the inefficient economic fluctuations due to the inefficient interest rate setting by the committee. Note that the second term of (17) is κ times as large as that of (16). This means that the inefficient interest rate setting brings the inefficient output gap fluctuation, $-[\gamma_u \tilde{\varepsilon}_t + \frac{\gamma_e \Phi}{\sigma}\tilde{\eta}_t + (1-\gamma_u)\mu_t + \frac{(1-\gamma_e)\Phi}{\sigma}\nu_t]$, and it hits on inflation through the aggregate supply relation, the New Keynesian Phillips curve.

To find equilibrium social loss, let us calculate the asymptotic variances of the output gap and inflation rate. After some calculations, I obtain

$$\begin{aligned}
V(x) &= \left[\frac{\kappa}{\lambda^c(1-\beta\rho_e)+\kappa^2}\right]^2 \frac{\sigma_\psi^2}{1-\rho_e^2} + \frac{1}{N}\left[\gamma_u^2\sigma_\varepsilon^2 + \left(\frac{\gamma_e\Phi}{\sigma}\right)^2\sigma_\eta^2\right] \\
&\quad + \left[(1-\gamma_u)^2\sigma_\mu^2 + \left(\frac{(1-\gamma_e)\Phi}{\sigma}\right)^2\sigma_\nu^2\right], \qquad (18) \\
V(\pi) &= \left[\frac{\lambda^c}{\lambda^c(1-\beta\rho_e)+\kappa^2}\right]^2 \frac{\sigma_\psi^2}{1-\rho_e^2} + \frac{\kappa^2}{N}\left[\gamma_u^2\sigma_\varepsilon^2 + \left(\frac{\gamma_e\Phi}{\sigma}\right)^2\sigma_\eta^2\right] \\
&\quad + \kappa^2\left[(1-\gamma_u)^2\sigma_\mu^2 + \left(\frac{(1-\gamma_e)\Phi}{\sigma}\right)^2\sigma_\nu^2\right]. \qquad (19)
\end{aligned}$$

The first terms of (18) and (19) are due to cost shock, one of the economic fundamentals. The second (third, resp.) terms are due to noisiness of private (common) information of the committee members, which is one of the non-fundamentals. By (10), (18) and (19), the social loss in equilibrium is

$$\begin{aligned}
L &= \left[\left(\frac{\lambda^c}{\lambda^c(1-\beta\rho_e)+\kappa^2}\right)^2 + \lambda\left(\frac{\kappa}{\lambda^c(1-\beta\rho_e)+\kappa^2}\right)^2\right]\frac{\sigma_\psi^2}{1-\rho_e^2} \\
&\quad + (\kappa^2+\lambda)\left[\frac{1}{N}\left(\gamma_u^2\sigma_\varepsilon^2 + \left(\frac{\gamma_e\Phi}{\sigma}\right)^2\sigma_\eta^2\right) + \left((1-\gamma_u)^2\sigma_\mu^2 + \left(\frac{(1-\gamma_e)\Phi}{\sigma}\right)^2\sigma_\nu^2\right)\right].
\end{aligned}$$

The social loss L seems somewhat complicated but its meaning is clear. The first term is equal to the social loss under optimal discretionary policy under perfect information. Since I adopt optimal policy under discretion as the optimality concept, it is not relevant to the performance of the monetary policy committee. The second term is the social loss generated by the inefficient interest rate setting due to imperfect information and coordination behavior among the members. Therefore, the second term of L should be regarded as the performance measure of the monetary policy committee in this model. Note that it corresponds just to $E(W)$ in the previous section.

3.4 Optimal Size of MPC

In the rest of this section, I investigate an existence condition of the optimal size of the monetary policy committee and its property. As mentioned above, the optimal size is the size $N \in \mathbb{N}$ which maximizes the performance measure of the committee:

$$P \equiv -\frac{1}{N}\left[\gamma_u^2\sigma_\varepsilon^2 + \left(\frac{\gamma_e\Phi}{\sigma}\right)^2\sigma_\eta^2\right] - \left[(1-\gamma_u)^2\sigma_\mu^2 + \left(\frac{(1-\gamma_e)\Phi}{\sigma}\right)^2\sigma_\nu^2\right].$$

Note that when the response coefficients γ_u and γ_e are independent of the committee size N, P is increasing in N and the first term of P converges to zero as N goes to infinity. This is a variant of Condorcet's assertion. Although the inefficiency of interest rate setting does not disappear because of noisy common information, the idiosyncratic noise of private information is perfectly absorbed by averaging the large sample in the limit. [26]

In general cases, γ_u and γ_e depend on N and hence P is not necessarily increasing in N. So that, the optimal size of the monetary policy committee can exist. The next proposition provides both of necessary and sufficient conditions separately.

Proposition 4 *The following statements on the existence of the optimal size of the monetary policy committee hold under the average-voting rule.*

1. *There exists a finite optimal size of the monetary policy committee if the parameter set satisfies that* $r > \frac{1}{5}$, $\frac{\sigma_\varepsilon^{-2}}{\sigma_\mu^{-2}} < \frac{5r-1}{(1-r)^2}$ *and* $\frac{\sigma_\eta^{-2}}{\sigma_\nu^{-2}} < \frac{5r-1}{(1-r)^2}$.
2. *There exists a finite optimal size of the monetary policy committee only if the parameter set satisfies that* $r > \frac{1}{5}$ *and* $\frac{\sigma_\varepsilon^{-2}}{\sigma_\mu^{-2}} < \frac{5r-1}{(1-r)^2}$ *or* $\frac{\sigma_\eta^{-2}}{\sigma_\nu^{-2}} < \frac{5r-1}{(1-r)^2}$.

Proof.
See Appendix F.

Because this model includes two economic shocks, demand and cost shock, it is difficult to find exactly a necessary and sufficient condition along the line of Proposition 2. [27] However, the necessary condition above is close to the sufficient condition. This is because the mechanism for the existence of the optimal size is similar to that of the simple model in the previous section. Note that according to Figure 4, if $r > \frac{1}{5}$, the condition that $\frac{\sigma_\varepsilon^{-2}}{\sigma_\mu^{-2}} < \frac{5r-1}{(1-r)^2}$ and $\frac{\sigma_\eta^{-2}}{\sigma_\nu^{-2}} < \frac{5r-1}{(1-r)^2}$ is normally satisfied because common information is interpreted as well-balanced recognition among the economist or the staff report and hence it does

[26] See Corollary 1 and 7 of Morimoto (2009).

[27] See the proofs of Proposition 4 and Corollary 4.

not seem far more imprecise than the members' private information. Therefore, it can be said that the monetary policy committees should not be so large under central bank transparency and a reputation effect if we consider a source of the coordination motive as such.

The properties of the optimal size of the monetary policy committee are basically similar to those of the simple model. So that, I do not report numerical examples here. First, the optimal size N^* is non-increasing in r. That is, the stronger what foments the coordination behavior among the committee members is, the smaller the monetary policy committees should be. In the standpoint mentioned above, it is optimal to promote efficient use of the members' information by holding the monetary policy committees to small groups when central bank transparency and a reputation effect is large. Second, given the precision of private information, the optimal size is non-increasing in the precision of common information. In the context of the monetary policy committee, it suggests that the committee size should be small when the staff report or common understanding of general economists on the present and future economic states is reliable to some extent.

Next, I provide a sufficient condition for the existence of the optimal size of the monetary policy committee under the median-voting rule, which is similar to the case of the average-voting rule as in the simple model.

Corollary 4 *If the parameter set satisfies that* $r > \frac{\pi}{\pi+8}$, $\frac{\sigma_\varepsilon^{-2}}{\sigma_\mu^{-2}} < \frac{(\pi+8)r-\pi}{\pi(1-r)^2}$ *and* $\frac{\sigma_\eta^{-2}}{\sigma_\nu^{-2}} < \frac{(\pi+8)r-\pi}{\pi(1-r)^2}$, *then there exists a finite optimal size of the monetary policy committee.*

Proof.
See Appendix G.

As in the simple model in the previous section, the parameter condition for the existence of the optimal committee size under the median voting rule is stricter than that of the average-voting rule. It suggests that the the effect of information aggregation is more valuable for the monetary policy committees which adopt the majority rule.

3.5 A Positive Implication

Finally, I provide a positive implication of the model for the actual monetary policy committees. Using a data set on the characteristics of the monetary policy committees in more than 30 countries from 1960 through 2000, Berger and Nitsch (2008) report that inflation volatility is U-shaped in the size of the monetary policy committee. In the present paper, the inflation volatility exhibits a similar behavior as in the case of the simple model since the the part of variace of inflation which depends on the committee size is

$$\frac{\kappa^2}{N}\left[\gamma_u^2\sigma_\varepsilon^2+\left(\frac{\gamma_e\Phi}{\sigma}\right)^2\sigma_\eta^2\right]+\kappa^2\left[(1-\gamma_u)^2\sigma_\mu^2+\left(\frac{(1-\gamma_e)\Phi}{\sigma}\right)^2\sigma_\nu^2\right].$$

Since Berger and Nitsch (2008) also report that similar results are obtained about output growth, one of the real terms, I conjecture that the property results from a feature of the functions of committee decision-making for economic stabilization. The present paper gives an explanation for the property in terms of the positive effect of information aggregation and the negative effect of fomenting the inefficient coordination which an increase of the committee size brings to the performance of the monetary policy committee.

4 Conclusion

How many members should the committees consist of? The present paper provides an answer for this question in committee design, focusing on coordination behavior among the members based on higher order beliefs. The approach in this paper is an alternative to incorporating information acquisition costs into the payoff structures, which is usually adopted in literature. This paper also analyzes in a formal model the optimal size of monetary policy committees, which is one of the most important issues of monetary policy design.

There are a few remaining problems. The first one is to construct a theoretical foundation for the members' coordination motive. It seems very significant in the area of committee design. Especially, I think transparency and reputation can play an important role for it. The second one is to find applications of the mechanism given in this paper to other economic problems. Considering the structure of this paper's model, the mechanism will be applicable to the models in which the optimal action of the committee under perfect information is a linear function of states. I believe that such situations are not rare in the economic phenomena of our interest.

Appendices

Appendix A: Proof of Proposition 1

By the iterated substituion of F.O.C. (4), I obtain [28]

$$
\begin{aligned}
a_j &= (1-\delta)E_j(\theta) + (1-\delta)\delta E_j(\bar{E}(\theta)) + (1-\delta)\delta^2 E_j(\bar{E}^2(\theta)) + \cdots \\
&= (1-\delta)\sum_{s=0}^{\infty} \delta^s E_j(\bar{E}^s(\theta)). \qquad (20)
\end{aligned}
$$

To calculate this infinite series, I use the following lemma.

Lemma 1 *For all j and s,*

$$E_j(\bar{E}^s(\theta)) = (1-\mu_s)y + \mu_s x_j, \qquad (21)$$

where $\mu_s = \frac{\beta(\frac{\alpha}{N}+\beta)^s}{(\alpha+\beta)^{s+1}}$.

Proof of Lemma 1.

I prove it by induction. Choose arbitrary member j. The assertion obviously holds when $s = 0$. Suppose that it holds for an arbitrary s. Then, by (21), I obtain

$$
\begin{aligned}
\bar{E}^{s+1}(\theta) &= \frac{1}{N}\sum_{k=1}^{N} E_k(\bar{E}^s(\theta)) \\
&= (1-\mu_s)y + \mu_s \frac{\sum_{k=1}^{N} x_k}{N}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
E_j(\bar{E}^{s+1}(\theta)) &= (1-\mu_s)y + \mu_s E_j\Big(\frac{\sum_{k=1}^{N} x_k}{N}\Big) \\
&= (1-\mu_s)y + \mu_s\Big(\frac{1}{N}x_j + \frac{N-1}{N}\cdot\frac{\alpha y + \beta x_j}{\alpha+\beta}\Big) \\
&= \Big[1 - \Big(1 - \frac{N-1}{N}\cdot\frac{\alpha}{\alpha+\beta}\Big)\mu_s\Big]y + \mu_s\Big(\frac{1}{N} + \frac{N-1}{N}\cdot\frac{\beta}{\alpha+\beta}\Big)x_j.
\end{aligned}
$$

By $1 - \frac{N-1}{N}\cdot\frac{\alpha}{\alpha+\beta} = \frac{1}{N} + \frac{N-1}{N}\cdot\frac{\beta}{\alpha+\beta} = \frac{\frac{\alpha}{N}+\beta}{\alpha+\beta}$, I obtain

$$
\begin{aligned}
E_j(\bar{E}^{s+1}(\theta)) &= \Big(1 - \frac{\frac{\alpha}{N}+\beta}{\alpha+\beta}\mu_s\Big)y + \mu_s\frac{\frac{\alpha}{N}+\beta}{\alpha+\beta}x_j \\
&= (1-\mu_{s+1})y + \mu_{s+1}x_j.
\end{aligned}
$$

[28]The symbol $\bar{E}^s(\theta)$ denotes an s-th order average expectation. That is, for an arbitrary $s \in \{1, 2, ...\}$, $\bar{E}^s(\theta) = \frac{1}{N}\sum_{k=1}^{N} E_k\big(\bar{E}^{s-1}(\theta)\big)$.

This completes the proof of Lemma 1.

Substituting (21) into (20), I obtain

$$\begin{aligned} a_j &= (1-\delta)\sum_{s=0}^{\infty}\delta^s\Big[(1-\mu_s)y+\mu_s x_j\Big] \\ &= \Big[1-(1-\delta)\sum_{s=0}^{\infty}\delta^s\mu_s\Big]y+(1-\delta)\Big(\sum_{s=0}^{\infty}\delta^s\mu_s\Big)x_j. \end{aligned}$$

After some algebraic manipulations, I obtain [29]

$$(1-\delta)\sum_{s=0}^{\infty}\delta^s\mu_s=\gamma,$$

where γ is defined in Proposition 1. *Q.E.D.*

Appendix B: Proof of Corollary 1

The denominator of γ can be reduced to

$$(1-r)\alpha+\Big[1-r\Big(\frac{2}{N}-\frac{1}{N^2}\Big)\Big]\beta.$$

Considering its continuation with respect to N, I obtain

$$\frac{\partial}{\partial N}\Big(\frac{2}{N}-\frac{1}{N^2}\Big)=-2N^{-3}(N-1)\leq 0.$$

This shows that the denominator of γ is increasing in N and positive. [30] Therefore, γ is decreasing in N.

It is immediately obtained that $\gamma=\frac{\beta}{\alpha+\beta}$ when $N=1$ and $\lim_{N\to\infty}\gamma=\frac{(1-r)\beta}{\alpha+(1-r)\beta}$. *Q.E.D.*

Appendix C: Proof of Proposition 2

(*Sufficiency*)

Since $E(W|\theta)$ converges as N goes to infinity, a finite optimal size of the committee exists if $\frac{\partial E(W)}{\partial N}<0$ for sufficiently large N. I derive an explicit expression of this sufficient condition.

[29] Note that the infinite series $\sum_{s=0}^{\infty}\delta^s\mu_s$ converges since $\sum_{s=0}^{\infty}\delta^s\mu_s=\frac{\beta}{\alpha+\beta}\sum_{s=0}^{\infty}\left(\frac{(1-\frac{1}{N})r}{1-\frac{r}{N}}\cdot(\frac{\alpha}{N}+\beta)\right)^s$.

[30] It is larger than or equal to $(1-r)(\alpha+\beta)>0$.

Equation (6) can be reduced to

$$\frac{\partial E(W|\theta)}{\partial N} = 2(1-\gamma)\beta^{-1}\frac{\partial \gamma}{\partial N}\left[\frac{\beta}{\alpha} - \left(\frac{\gamma}{1-\gamma}\cdot\frac{1}{N} - \frac{\gamma^2}{2(1-\gamma)}\left(\frac{\partial \gamma}{\partial N}N^2\right)^{-1}\right)\right]. \tag{22}$$

Note that $\frac{\partial \gamma}{\partial N} < 0$ for all N by Corollary 1 and that the second term of the bracketed part of (22) converges to zero as N goes infinity. Thus, if

$$\frac{\beta}{\alpha} > -\lim_{N\to\infty}\frac{\gamma^2}{2(1-\gamma)}\left(\frac{\partial \gamma}{\partial N}N^2\right)^{-1}, \tag{23}$$

then $\frac{\partial E(W|\theta)}{\partial N} < 0$ for sufficiently large N, which ensures the existence of a finite optimal size of the committee.

Let us find the parameter condition equivalent to (23). After some calculations, I obtain

$$\lim_{N\to\infty}\frac{\partial \gamma}{\partial N}N^2 = -\frac{2r(1-r)\alpha\beta}{[\alpha+(1-r)\beta]^2}.$$

By $\lim_{N\to\infty}\gamma = \frac{(1-r)\beta}{\alpha+(1-r)\beta}$, I obtain

$$\lim_{N\to\infty}\frac{\gamma^2}{2(1-\gamma)}\left(\frac{\partial \gamma}{\partial N}N^2\right)^{-1} = -\frac{\beta}{\alpha}\cdot\frac{1-r}{4r}\cdot\left[1+(1-r)\frac{\beta}{\alpha}\right].$$

Thus, (23) if and only if

$$1 > \frac{1-r}{4r}\cdot\left[1+(1-r)\frac{\beta}{\alpha}\right], \tag{24}$$

which is equivalent to that $r > \frac{1}{5}$ and $\frac{\beta}{\alpha} < \frac{5r-1}{(1-r)^2}$.

(*Necessity*)

I will show that $E(W|\theta)$ is monotonically increasing in N unless the parameter set satisfies that $r > \frac{1}{5}$ and $\frac{\beta}{\alpha} < \frac{5r-1}{(1-r)^2}$. Define the function $f : (1,\infty) \to \mathbb{R}$ by

$$f(N) = \frac{\gamma}{1-\gamma}\cdot\frac{1}{N} - \frac{\gamma^2}{2(1-\gamma)}\left(\frac{\partial \gamma}{\partial N}N^2\right)^{-1}, \quad \text{for all} \quad N \in (1,\infty).$$

Note that $f(N)$ is the second term of the bracketed part of (22). Since γ is decreasing in N by Corollary 1, the first term of $f(N)$ is decreasing in N. Since

$$\frac{\partial \gamma}{\partial N}N^2 = -\frac{2r(1-r)(1-\frac{1}{N})\alpha\beta}{\left[(1-\frac{r}{N})(\alpha+\beta) - r(1-\frac{1}{N})(\frac{\alpha}{N}+\beta)\right]^2},$$

by use of the definition of γ, I obtain

$$-\frac{\gamma^2}{2(1-\gamma)}\Big(\frac{\partial\gamma}{\partial N}N^2\Big)^{-1}=\frac{(1-r)^2\beta^2}{1-\gamma}\cdot\frac{1}{2r(1-r)(1-\frac{1}{N})\alpha\beta}.$$

Since γ is decreasing in N by Corollary 1, the denominator of the right-hand side above is increasing in N. This implies that the second term of $f(N)$ is also decreasing in N. Therefore, f is a decreasing function.

Note that $\lim_{N\to 1}\frac{\partial E(W)}{\partial N}>0$ by $\lim_{N\to 1}f(N)=+\infty$. Thus, since f is monotonically decreasing, $\frac{\partial E(W)}{\partial N}>0$ for all N unless

$$\frac{\beta}{\alpha}>\lim_{N\to\infty}f(N).$$

Immediately from the calculation in the necessity part, this condition is equivalent to that $r>\frac{1}{5}$ and $\frac{\beta}{\alpha}<\frac{5r-1}{(1-r)^2}$. *Q.E.D.*

Appendix D: Proof of Corollary 3

Since equation (7) holds for sufficiently large N, according to the proof of Proposition 2, the next inequality is a sufficient condition for the existence of a finite optimal size.

$$\frac{\beta}{\alpha}>\frac{\pi}{2}\lim_{N\to\infty}\frac{\gamma^2}{2(1-\gamma)}\Big(\frac{\partial\gamma}{\partial N}N^2\Big)^{-1}. \tag{25}$$

Hence, similar to (24), inequality (25) is equivalent to

$$1>\frac{\pi}{2}\cdot\frac{1-r}{4r}\Big[1+(1-r)\frac{\beta}{\alpha}\Big], \tag{26}$$

which is equivalent to that $r>\frac{\pi}{\pi+8}$ and $\frac{\beta}{\alpha}<\frac{(\pi+8)r-\pi}{\pi(1-r)^2}$. *Q.E.D.*

Appendix E: Derivation of (16), (17), (18) and (19)

Macroeconomic dynamics of the artificial economy is given by the following system of stochastic difference equations.

$$x_t = E_t x_{t+1}-\frac{1}{\sigma}(i_t-E_t\pi_{t+1})+u_t, \tag{27}$$

$$\pi_t = \beta E_t\pi_{t+1}+\kappa x_t+e_t, \tag{28}$$

$$i_t = \sigma[u_t+\gamma_u\tilde{\varepsilon}_t+(1-\gamma_u)\mu_t]+\Phi[e_t+\gamma_e\tilde{\eta}_t+(1-\gamma_e)\nu_t]. \tag{29}$$

Since the relevant state variables in period t are $e_t, \tilde{\varepsilon}_t, \tilde{\eta}_t, \mu_t$ and ν_t, the solution will be of the form

$$\begin{aligned} x_t &= A_x e_t + B_x \tilde{\varepsilon}_t + C_x \tilde{\eta}_t + D_x \mu_t + E_x \nu_t, &(30)\\ \pi_t &= A_\pi e_t + B_\pi \tilde{\varepsilon}_t + C_\pi \tilde{\eta}_t + D_\pi \mu_t + E_\pi \nu_t, &(31)\end{aligned}$$

where A_k, B_k, C_k, D_k and $E_k\,(k = x, \pi)$ are undetermined coefficients. Substituting (30),(31) into (27),(29) and then (29) into (27) and comparing the coefficients of both sides, I obtain

$$\begin{aligned} A_x &= \rho_e A_x - \frac{\Phi - \rho_e A_\pi}{\sigma},\ B_x = -\gamma_u,\ C_x = -\frac{\gamma_e \Phi}{\sigma}, &(32)\\ D_x &= -(1-\gamma_u),\ E_x = -\frac{(1-\gamma_e)\Phi}{\sigma}. \end{aligned}$$

Substituting (30),(31) into (28) and comparing the coefficients of both sides,

$$A_\pi = \beta\rho_e A_\pi + \kappa A_x + 1,\ B_\pi = \kappa B_x,\ C_\pi = \kappa C_x,\ D_\pi = \kappa D_x,\ E_\pi = \kappa E_x. \quad (33)$$

Solving the first equations of (32) and (33), A_x and A_π turn out to be

$$A_x = -\frac{\kappa}{\lambda^c(1-\beta\rho_e)+\kappa^2}, \quad A_\pi = \frac{\lambda^c}{\lambda^c(1-\beta\rho_e)+\kappa^2}.$$

Thus, I obtain equilibrium output gap and inflation rate in period t in the text.

Finally, I calculate asymptotic variances of output gap and inflation rate. By $e_t = \rho_e e_{t-1} + \psi_t$, asymptotic variance of e_t is $\frac{\sigma_\psi^2}{1-\rho_e^2}$. Besides, since $e_t, \varepsilon_t^j, \eta_t^j, \mu_t$ and ν_t are mutually independent, each covariance of them is zero. Noting the two facts above, I find asymptotic variance of output gap in the text. Similarly, I can calculate asymptotic variance of inflation rate in the text.

Appendix F: Proof of Proposition 4

(1. The Sufficient Condition)

I prove it in the same way as the proof of Proposition 3. By the definition of the performance measure of the monetary policy committee, the counterpart to (22) is

$$\begin{aligned} \frac{\partial P}{\partial N} &= 2\Bigg\{(1-\gamma_u)\sigma_\varepsilon^2 \frac{\partial \gamma_u}{\partial N}\left[\frac{\sigma_\varepsilon^{-2}}{\sigma_\mu^{-2}} - \frac{\gamma_u}{1-\gamma_u}\cdot\frac{1}{N} + \frac{\gamma_u^2}{2(1-\gamma_u)}\left(\frac{\partial \gamma_u}{\partial N}N^2\right)^{-1}\right] \\ &\quad + (1-\gamma_e)\sigma_\eta^2 \frac{\partial \gamma_e}{\partial N}\left(\frac{\Phi}{\sigma}\right)^2\left[\frac{\sigma_\eta^{-2}}{\sigma_\nu^{-2}} - \frac{\gamma_e}{1-\gamma_e}\cdot\frac{1}{N} + \frac{\gamma_e^2}{2(1-\gamma_e)}\left(\frac{\partial \gamma_e}{\partial N}N^2\right)^{-1}\right]\Bigg\}. \quad (34)\end{aligned}$$

Hence, a sufficient condition for that $\frac{\partial L}{\partial N} > 0$ for sufficiently large N is

$$\frac{\sigma_\varepsilon^{-2}}{\sigma_\mu^{-2}} > \lim_{N\to\infty} \frac{\gamma_u^2}{2(1-\gamma_u)} \left(\frac{\partial \gamma_u}{\partial N} N^2\right)^{-1}$$

and

$$\frac{\sigma_\eta^{-2}}{\sigma_\nu^{-2}} > \lim_{N\to\infty} \frac{\gamma_e^2}{2(1-\gamma_e)} \left(\frac{\partial \gamma_e}{\partial N} N^2\right)^{-1}.$$

Similar to the proof of Proposition 3, this is equivalent to that $r > \frac{1}{5}$, $\frac{\sigma_\varepsilon^{-2}}{\sigma_\mu^{-2}} < \frac{5r-1}{(1-r)^2}$ and $\frac{\sigma_\eta^{-2}}{\sigma_\nu^{-2}} < \frac{5r-1}{(1-r)^2}$.

(2. The Necessary Condition)

According to the proof of Proposition 2, if both of the first and the second terms of (35) converge to some non-negative numbers, then the optimal size does not exists. Thus, for the existence of the optimal size, it is necessary for at least one of them to converge to a negative number. Similar to the proof of Proposition 3, this is equivalent to $r > \frac{1}{5}$ and $\frac{\sigma_\varepsilon^{-2}}{\sigma_\mu^{-2}} < \frac{5r-1}{(1-r)^2}$ or $r > \frac{1}{5}$ and $\frac{\sigma_\eta^{-2}}{\sigma_\nu^{-2}} < \frac{5r-1}{(1-r)^2}$, which can be reduced to $r > \frac{1}{5}$ and $\frac{\sigma_\varepsilon^{-2}}{\sigma_\mu^{-2}} < \frac{5r-1}{(1-r)^2}$ or $\frac{\sigma_\eta^{-2}}{\sigma_\nu^{-2}} < \frac{5r-1}{(1-r)^2}$. *Q.E.D.*

Appendix G: Proof of Corollary 4

Note that $\text{med}_{1\le j\le N}\{\varphi_t^j\} = \varphi_t + \text{med}_{1\le j\le N}\{\varepsilon_t^j\}$ and $\text{med}_{1\le j\le N}\{\psi_t^j\} = \psi_t + \text{med}_{1\le j\le N}\{\eta_t^j\}$. Thus, by (13) and (14), nominal interest rate under median-voting rule is

$$i_t = i_t^* + z_t + \sigma(1-\gamma_u)\mu_t + \Phi(1-\gamma_e)\nu_t,$$

where $z_t = \text{med}_{1\le j\le N}\{\sigma\gamma_u\varepsilon_t^j + \Phi\gamma_e\eta_t^j\}$.

By the same way as in Appendix E, I obtain the following equilibrium output gap and inflation rate.

$$\begin{aligned} x_t &= -\frac{\kappa}{\lambda^c(1-\beta\rho_e)+\kappa^2} e_t - \left[\frac{1}{\sigma} z_t + (1-\gamma_u)\mu_t + \frac{(1-\gamma_e)\Phi}{\sigma}\nu_t\right], \\ \pi_t &= \frac{\lambda^c}{\lambda^c(1-\beta\rho_e)+\kappa^2} e_t - \kappa\left[\frac{1}{\sigma} z_t + (1-\gamma_u)\mu_t + \frac{(1-\gamma_e)\Phi}{\sigma}\nu_t\right]. \end{aligned}$$

The asymptotic variances of the equilibrium output gap and inflation rate are

$$\begin{aligned} V(x) &= \left[\frac{\kappa}{\lambda^c(1-\beta\rho_e)+\kappa^2}\right]^2 \frac{\sigma_\psi^2}{1-\rho_e^2} + \frac{1}{\sigma^2}V(z_t) + \left[(1-\gamma_u)^2\sigma_\mu^2 + \left(\frac{(1-\gamma_e)\Phi}{\sigma}\right)^2\sigma_\nu^2\right], \\ V(\pi) &= \left[\frac{\lambda^c}{\lambda^c(1-\beta\rho_e)+\kappa^2}\right]^2 \frac{\sigma_\psi^2}{1-\rho_e^2} + \frac{\kappa^2}{\sigma^2}V(z_t) + \kappa^2\left[(1-\gamma_u)^2\sigma_\mu^2 + \left(\frac{(1-\gamma_e)\Phi}{\sigma}\right)^2\sigma_\nu^2\right], \end{aligned}$$

which give the social loss in equilibrium by (11). Since

$$z_t \propto N\left(0,\ \frac{\pi(\sigma^2\gamma_u^2\sigma_\varepsilon^2 + \Phi^2\gamma_e^2\sigma_\eta^2)}{2N}\right),$$

I obtain that

$$\begin{aligned}
V(x) &\approx \left[\frac{\kappa}{\lambda^c(1-\beta\rho_e)+\kappa^2}\right]^2 \frac{\sigma_\psi^2}{1-\rho_e^2} + \frac{\pi}{2}\cdot\frac{1}{N}\left[\gamma_u^2\sigma_\varepsilon^2 + \left(\frac{\gamma_e\Phi}{\sigma}\right)^2\sigma_\eta^2\right] \\
&\quad + \left[(1-\gamma_u)^2\sigma_\mu^2 + \left(\frac{(1-\gamma_e)\Phi}{\sigma}\right)^2\sigma_\nu^2\right], \\
V(\pi) &\approx \left[\frac{\lambda^c}{\lambda^c(1-\beta\rho_e)+\kappa^2}\right]^2 \frac{\sigma_\psi^2}{1-\rho_e^2} + \frac{\pi}{2}\cdot\frac{\kappa^2}{N}\left[\gamma_u^2\sigma_\varepsilon^2 + \left(\frac{\gamma_e\Phi}{\sigma}\right)^2\sigma_\eta^2\right] \\
&\quad + \kappa^2\left[(1-\gamma_u)^2\sigma_\mu^2 + \left(\frac{(1-\gamma_e)\Phi}{\sigma}\right)^2\sigma_\nu^2\right]
\end{aligned}$$

for sufficiently large N.

Therefore, according to the proof of Corollary 2 and Proposition 4, I obtain the result of Corollary 4. [31] *Q.E.D.*

[31] See Appendix D and F.

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A Supplementary Note for "Coordination Behavior and Optimal Committee Size"*

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Abstract

This is a supplementary note for Morimoto (2010) "Coordination Behavior and Optimal Committee Size". Here, I treats the case where each committee member has proper prior on underlying state θ.

Keywords: beauty contest, proper prior, Condorcet jury theorem
JEL Classification: D71; D84; E58

1 Incorporating Proper Prior into the Simple Model

Along the line of Morris and Shin (2002), Morimoto (2010) assumes each member's improper flat prior about the underlying state over the real line. However, it is more natural to adopt some informative prior about the underlying state according to applications to committees of experts. I analyze such a case here and show that the results of Morimoto (2010) do not change. For a detail of the notations and model setting, see Morimoto (2010).

Suppose that the underlying state θ is normally distributed with mean zero and variance $\omega^{-1} > 0$ and each committee member knows this.

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1.1 A Derivation of the Unique Equilibrium Strategy

Recall that the first order condition for each member problem is

$$a_j = (1-\delta)E_j(\theta) + \delta E_j(\bar{a}). \tag{1}$$

First, I find the equilibrium strategy by a heuristic method. Since each member's estimation of the underlying state is partially anchored by the prior $\theta \sim N(0, \omega^{-1})$, it becomes

$$E_j(\theta) = \frac{\omega \times 0 + \alpha y + \beta x_j}{\omega + \alpha + \beta} = \frac{\alpha}{\omega + \alpha + \beta} y + \frac{\beta}{\omega + \alpha + \beta} x_j. \tag{2}$$

Note that $\frac{\alpha}{\omega+\alpha+\beta} + \frac{\beta}{\omega+\alpha+\beta} < 1$. Thus, I can conjecture that the equilibrium strategy is of the form such that

$$a_j = \gamma_c y + \gamma_p x_j, \tag{3}$$

where γ_c and γ_p are undetermined coefficients such that $\gamma_c + \gamma_p < 1$. Substituting (2) and (3) into (1), after some calculations, the left-hand size of (1) turns out to be

$$\begin{aligned} & \left[\frac{(1-\delta)\beta}{\omega + \alpha + \beta} + \delta \gamma_c + \delta \frac{\beta}{\omega + \alpha + \beta} \left(1 - \frac{1}{N}\right) \gamma_p \right] y \\ + & \left[\frac{(1-\delta)\alpha}{\omega + \alpha + \beta} + \frac{\delta}{N} \gamma_p + \delta \frac{\alpha}{\omega + \alpha + \beta} \left(1 - \frac{1}{N}\right) \gamma_p \right] x_j. \end{aligned}$$

Thus, comparing the coefficients of both the sides of (1) and using the definition of δ, I obtain

$$\gamma_c = \frac{(1 - 2rN^{-1} + rN^{-2})\alpha}{\left(1 - \frac{r}{N}\right)[\beta + \omega + \alpha] - r\left(1 - \frac{1}{N}\right)\left[\beta + \frac{\omega+\alpha}{N}\right]}, \tag{4}$$

$$\gamma_p = \frac{(1-r)\beta}{\left(1 - \frac{r}{N}\right)[\beta + \omega + \alpha] - r\left(1 - \frac{1}{N}\right)\left[\beta + \frac{\omega+\alpha}{N}\right]}. \tag{5}$$

Second, I formally show that the strategy above is the unique equilibrium strategy. Substituting (1) recursively, I obtain

$$a_j = (1-\delta) \sum_{s=0}^{\infty} \delta^s E_j(\bar{E}^s(\theta)). \tag{6}$$

Thus, I next calculate the s-th order belief $E_j(\bar{E}^s(\theta))$. For given $s = 0, 1, ...$, put the s-th order belief of arbitrary member j as follows.

$$E_j(\bar{E}^s(\theta)) = \mu_s y + \nu_s x_j. \tag{7}$$

Then, since $\bar{E}^{s+1}(\theta) = \mu_s y + \nu_s \frac{1}{N}\sum_{k=1}^{N} x_k$, I obtain

$$\begin{aligned} E_j(\bar{E}^{s+1}(\theta)) &= \mu_s y + \nu_s\Big(\frac{1}{N}x_j + \frac{N-1}{N}\cdot\frac{\alpha y + \beta x_j}{\omega+\alpha+\beta}\Big) \\ &= \Big[\mu_s + \frac{\alpha}{\omega+\alpha+\beta}\Big(1-\frac{1}{N}\Big)\nu_s\Big]y + \frac{\frac{\omega+\alpha}{N}\beta}{\omega+\alpha+\beta}\nu_s x_j. \end{aligned}$$

Therefore, the coefficients of higher order beliefs satisfy the simultaneous difference equation

$$\begin{aligned} \mu_{s+1} &= \mu_s + \frac{\alpha}{\omega+\alpha+\beta}\Big(1-\frac{1}{N}\Big)\nu_s, \\ \nu_{s+1} &= \frac{\frac{\omega+\alpha}{N}\beta}{\omega+\alpha+\beta}\nu_s, \end{aligned}$$

with the initial value $(\mu_0, \nu_0) = (\frac{\alpha}{\omega+\alpha+\beta}, \frac{\beta}{\omega+\alpha+\beta})$. The solution of this equation is

$$\mu_s = \frac{\alpha}{\omega+\alpha+\beta}\Big[1+\Big(1-\frac{1}{N}\Big)\frac{\beta}{\omega+\alpha+\beta}\cdot\frac{1-\tau^s}{1-\tau}\Big], \quad \text{for} \quad s = 0, 1, ..., \tag{8}$$

$$\mu_s = \frac{\beta}{\omega+\alpha+\beta}\tau^s, \quad \text{for} \quad s = 0, 1, ..., \tag{9}$$

where $\tau = \frac{\frac{\omega+\alpha}{N}+\beta}{\omega+\alpha+\beta}$. Substituting (7), (8) and (9) into (6) and using the definition of δ, I obtain the unique equilibrium strategy of the form given by (3), (4) and (5).

I finally check that the relationship between the committee size and the equilibrium strategy.

$$\frac{\partial \gamma_c}{\partial N} = \frac{2r(1-r)N^{-2}(1-N^{-1})\alpha\beta}{\Big[(1-2rN^{-1}-N^{-2})(\omega+\alpha)+(1-r)\beta\Big]^2} > 0, \tag{10}$$

$$\frac{\partial \gamma_p}{\partial N} = -\frac{2r(1-r)N^{-2}(1-N^{-1})(\omega+\alpha)\beta}{\Big[(1-2rN^{-1}-N^{-2})(\omega+\alpha)+(1-r)\beta\Big]^2} < 0. \tag{11}$$

$$\lim_{N\to\infty}\gamma_c = \frac{\alpha}{\omega+\alpha+(1-r)\beta}, \quad \gamma_c|_{N=1} = \frac{\alpha}{\omega+\alpha+\beta}, \tag{12}$$

$$\lim_{N\to\infty}\gamma_p = \frac{(1-r)\beta}{\omega+\alpha+(1-r)\beta}, \quad \gamma_p|_{N=1} = \frac{\beta}{\omega+\alpha+\beta} \tag{13}$$

1.2 Proper Prior and the Existence of Optimal Size

Next, I investigate the existence condition for the optimal size of the committee. Since the distribution of θ is given in this case, it is natural to consider the unconditional performance of the committee:

$$\begin{aligned} E(W) &= -E\Big[\Big(\gamma_c(\theta+\eta)+\gamma_p\frac{1}{N}\sum_{k=1}^{N}(\theta+\varepsilon_k)-\theta\Big)^2\Big] \\ &= -E\Big[\Big(\gamma_c\eta+\gamma_p\frac{1}{N}\sum_{k=1}^{N}\varepsilon_k-(1-(\gamma_c+\gamma_p))\theta\Big)^2\Big] \\ &= -\gamma_c^2\alpha^{-1}-\gamma_p^2\frac{\beta^{-1}}{N}-[1-(\gamma_c+\gamma_p)]^2\omega^{-1}. \end{aligned} \tag{14}$$

The third term of (14) is generated by the weak response to common and private signals due to the prior of mean zero. Differentiating $E(W)$ with respect to N, I obtain

$$\frac{\partial E(W)}{\partial N}=-2\gamma_c\alpha^{-1}\frac{\partial\gamma_c}{\partial N}-2\gamma_p\frac{\beta^{-1}}{N}\cdot\frac{\partial\gamma_p}{\partial N}+\gamma_p^2\beta^{-1}N^{-2}+2[1-(\gamma_c+\gamma_p)]\omega^{-1}\frac{\partial(\gamma_c+\gamma_p)}{\partial N}.$$

The first, second and third terms are the same effects as those of the benchmark model. The forth term is the negative effect from that each member is more strongly anchored by the prior of mean zero. [1] Since

$$\begin{aligned} \frac{\partial E(W)}{\partial N} &= 2\gamma_c\beta^{-1}\frac{\partial\gamma_p}{\partial N}\Big[-\frac{\beta}{\alpha}\cdot\frac{\partial\gamma_c/\partial N}{\partial\gamma_p/\partial N}-\Big(\frac{\gamma_p}{\gamma_c}\cdot\frac{1}{N}-\frac{\gamma_p^2}{2\gamma_c}\Big(\frac{\partial\gamma_p}{\partial N}N^2\Big)^{-1} \\ &\qquad -[1-(\gamma_c+\gamma_p)]\frac{1}{\gamma_c}\cdot\frac{\beta}{\omega}\cdot\frac{\partial(\gamma_c+\gamma_p)/\partial N}{\partial\gamma_p/\partial N}\Big)\Big], \end{aligned}$$

along the line of the proof of Proposition 2, the following inequality is a sufficient condition for the existence of the optimal committee size. [2]

$$\lim_{N\to\infty}\Big\{-\frac{\beta}{\alpha}\cdot\frac{\partial\gamma_c/\partial N}{\partial\gamma_p/\partial N}\Big\}>\lim_{N\to\infty}\Big\{-\frac{\gamma_p^2}{2\gamma_c}\Big(\frac{\partial\gamma_p}{\partial N}N^2\Big)^{-1}+[1-(\gamma_c+\gamma_p)]\frac{1}{\gamma_c}\cdot\frac{\beta}{\omega}\cdot\frac{\partial(\gamma_c+\gamma_p)/\partial N}{\partial\gamma_p/\partial N}\Big\}.$$

By (10), (11), (12) and (13), this is equivalent to

$$\frac{\beta+(1-r)^{-1}\omega}{\alpha}<\frac{5r-1}{(1-r)^2}. \tag{15}$$

1.3 The Case of Median-voting Rule

Incorporating the proper prior, the sufficient condition for the existence of the optimal size under median-voting rule in Corollary 3 should be modified in the same way. Similar

[1] By (10) and (11), I obtain $\frac{\partial(\gamma_c+\gamma_p)}{\partial N}=-\frac{2r(1-r)N^{-2}(1-N^{-1})\omega\beta}{\left[\left(1-2rN^{-1}-N^{-2}\right)(\omega+\alpha)+(1-r)\beta\right]^2}<0$.

[2] Similar to the proof of Proposition 2, it can be easily checked that this is also a necessary condition.

to section 2.4 in the main text, since

$$E(W) \approx -\gamma_c^2\alpha^{-1} - \frac{\pi}{2}\gamma_p^2\frac{\beta^{-1}}{N} - [1-(\gamma_c+\gamma_p)]^2\omega^{-1}$$

for sufficiently large N under the median-voting rule, a sufficient condition for the existence of the optimal size is

$$\lim_{N\to\infty}\left\{-\frac{\beta}{\alpha}\cdot\frac{\partial\gamma_c/\partial N}{\partial\gamma_p/\partial N}\right\} > \lim_{N\to\infty}\left\{-\frac{\pi}{2}\cdot\frac{\gamma_p^2}{2\gamma_c}\Big(\frac{\partial\gamma_p}{\partial N}N^2\Big)^{-1} + [1-(\gamma_c+\gamma_p)]\frac{1}{\gamma_c}\cdot\frac{\beta}{\omega}\cdot\frac{\partial(\gamma_c+\gamma_p)/\partial N}{\partial\gamma_p/\partial N}\right\}.$$

This is equivalent to

$$\frac{\beta+(1-r)^{-1}\omega}{\alpha} < \frac{(\pi+8)r-\pi}{\pi(1-r)^2}. \tag{16}$$

2 The Model of Monetary Policy Committee

Next, I modify the model of monetary policy committee in section 3 of the main text along the line of the modification of the simple model. In this model, each member of the monetary policy committee has prior such that $\varphi_t \sim N(0,\sigma_\varphi^2)$ and $\psi_t \sim N(0,\sigma_\psi^2)$.

2.1 The Case of Average-voting Rule

Similar to section 1.2, the sufficient condition and necessary condition for the existence of the optimal size in Proposition 4 are modified to '$r > \frac{1}{5}$, $\frac{\sigma_\varepsilon^{-2}+(1-r)^{-1}\sigma_\varphi^{-2}}{\sigma_\mu^{-2}} < \frac{5r-1}{(1-r)^2}$ and $\frac{\sigma_\eta^{-2}+(1-r)^{-1}\sigma_\psi^{-2}}{\sigma_\nu^{-2}} < \frac{5r-1}{(1-r)^2}$' and '$r > \frac{1}{5}$ and $\frac{\sigma_\varepsilon^{-2}+(1-r)^{-1}\sigma_\varphi^{-2}}{\sigma_\mu^{-2}} < \frac{5r-1}{(1-r)^2}$ or $\frac{\sigma_\eta^{-2}+(1-r)^{-1}\sigma_\psi^{-2}}{\sigma_\nu^{-2}} < \frac{5r-1}{(1-r)^2}$', respectively.

2.2 The Case of Median-voting Rule

Similar to section 1.3, the sufficient condition for the existence of the optimal size in Corollary 4 is modified to '$r > \frac{\pi}{\pi+8}$, $\frac{\sigma_\varepsilon^{-2}+(1-r)^{-1}\sigma_\varphi^{-2}}{\sigma_\mu^{-2}} < \frac{(\pi+8)r-\pi}{\pi(1-r)^2}$ and $\frac{\sigma_\eta^{-2}+(1-r)^{-1}\sigma_\psi^{-2}}{\sigma_\nu^{-2}} < \frac{(\pi+8)r-\pi}{\pi(1-r)^2}$'.

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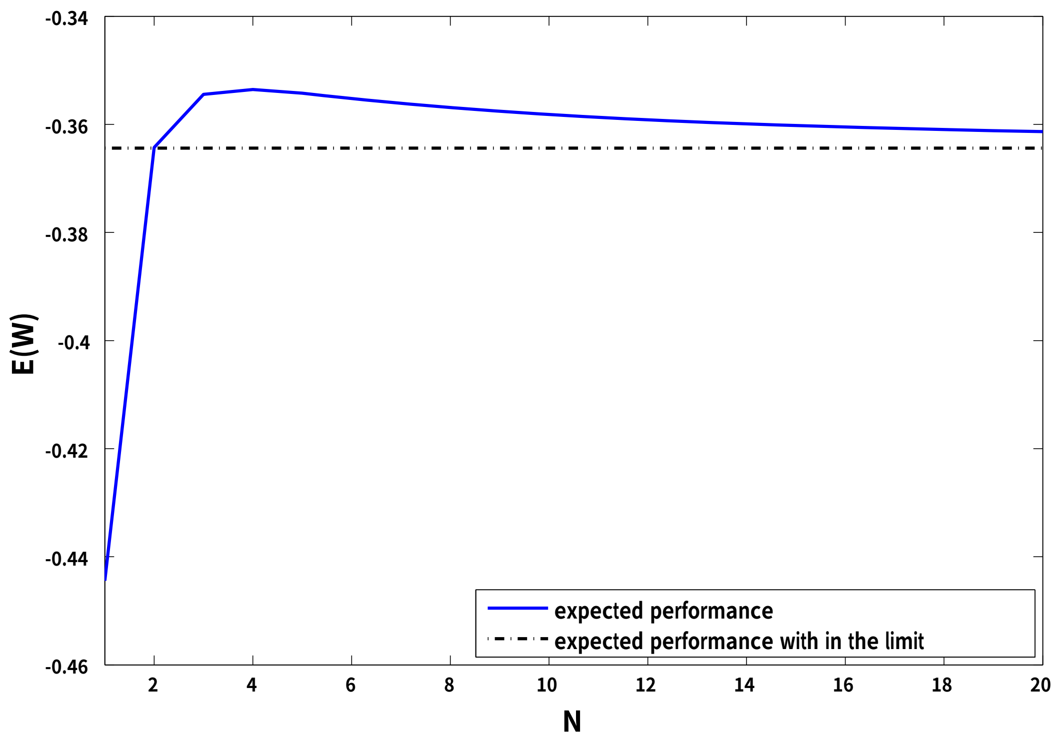

Figure 1: The relationship between committee size and expected performance in the simple model

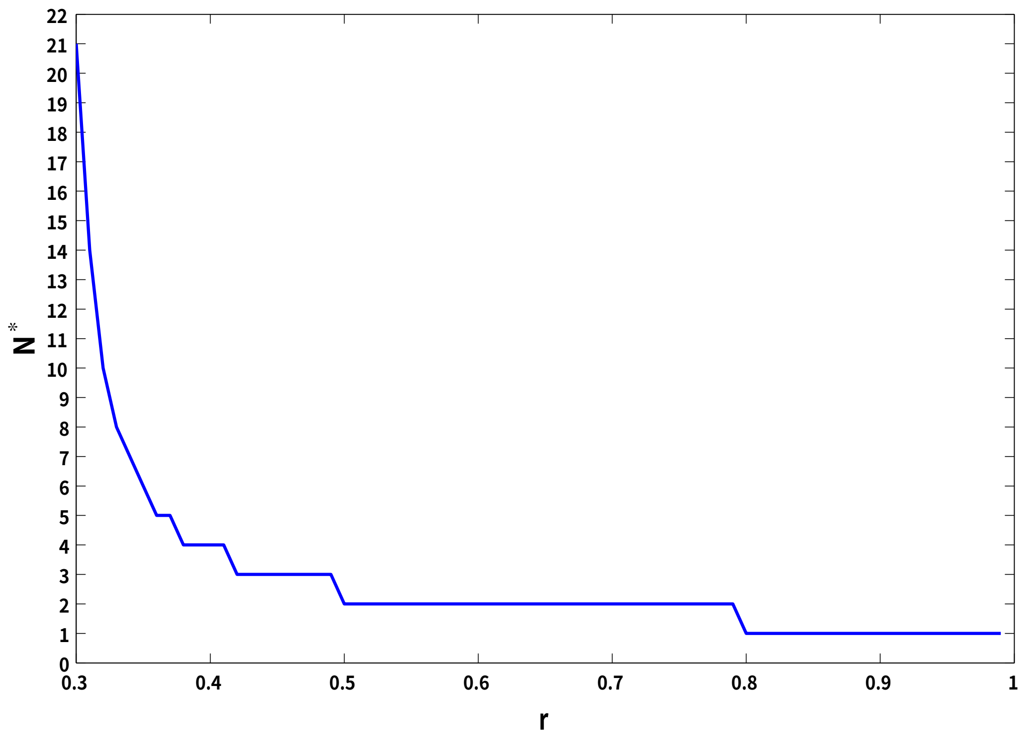

Figure 2: The relationship between degree of coordination motive and optimal size in the simple model

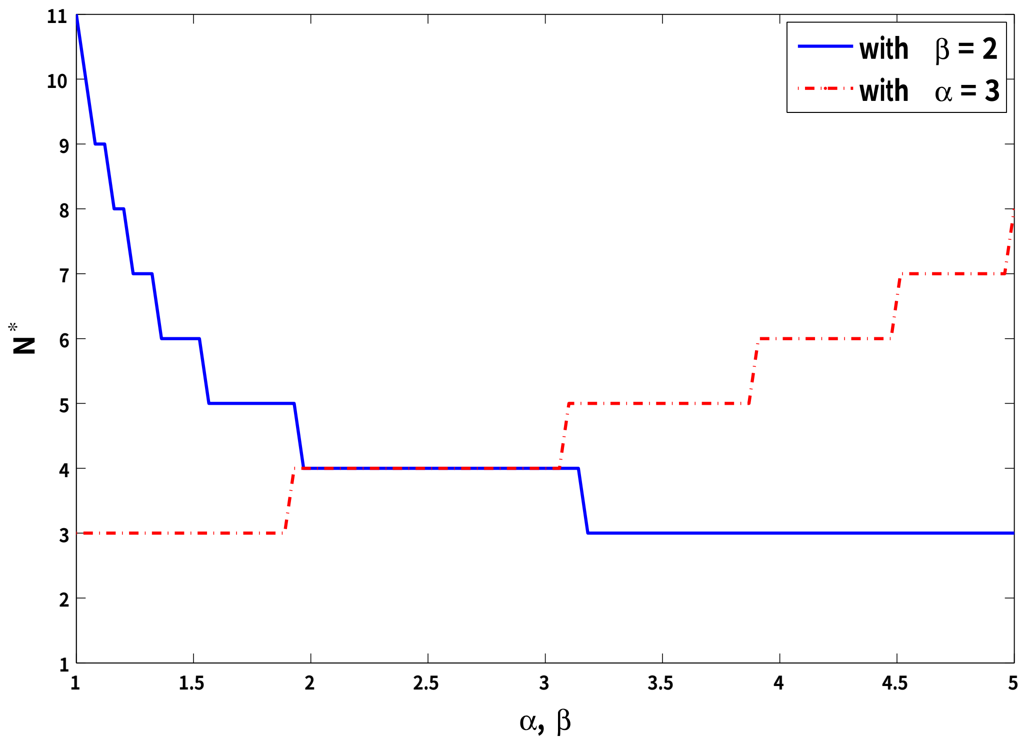

Figure 3: The relationship between precisons of common and private signals and optimal size in the simple model

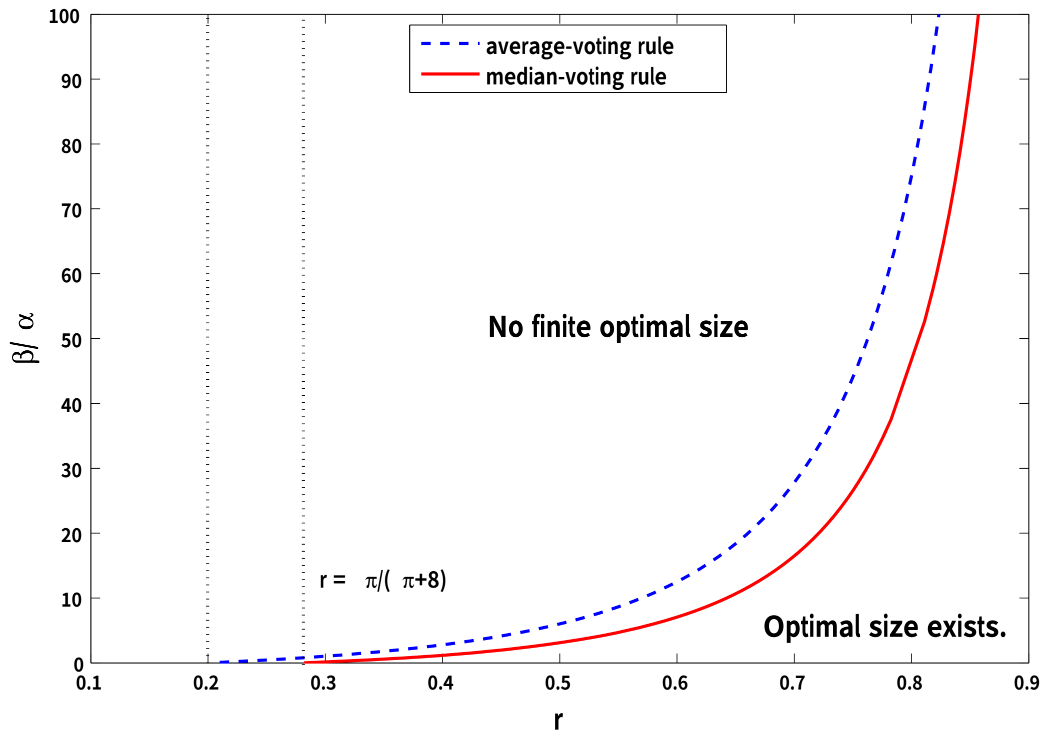

Figure 4: The regions below the dashed and solid curves are the parameter regions which satisfy the sufficient conditions for the exsistence of optimal committee size in the simple model under the average and median voting rules respectively. The former contains the latter.