Discussion Papers In Economics And Business

Public Capital, Public Pension, and Growth

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Abstract

This paper constructs an endogenous growth model with overlapping generations, whose engine of economic growth is productive public capital. The government faces a trade-off in public policy between public investment and social security provision because of its budget constraint. Larger public investment accelerates economic growth. On the other hand, larger public investment reduces the social security provision. This may reduce the consumption stream of agents. We first show that when the government aims at growth maximization, it chooses no social security provision. However, we also show that the growth-maximizing policy does not maximize welfare levels of each generation on the balanced growth path. Early generations may demand social security provision because the benefits from economic growth caused by an acceleration of public investment are relatively small. In contrast, future generations may require no social security provision but a large amount of public capital. Additionally, by setting the tax rate below the level that maximizes the growth rate, the government can make the welfare levels of all generations from the initial state on the balanced growth path better off. Moreover, in an economy facing an aging population, an increase in the social security provision to the old rather than an increase in public investment can be preferable from the viewpoint of social welfare.

JEL classification: E62, H54, H55
Keywords: public capital, social security, overlapping generations

1 Introduction

There are many theoretical studies on how publicly provided infrastructure affects economic growth, for example, Barro (1990), Barro and Sala-i-Martin (1992), Futagami et al. (1993), Turnovsky (1997), and Yakita (2008); however, they do not consider the public pension. [1] Most developed countries have aging populations and some studies have examined social security policy in an aging economy, for example, Pecchenino and Pollard (1997), Pecchenino and Utendorf (1999), and Yakita (2001). If the rapid aging of the

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[1]Yakita (2008) uses an overlapping generation model in contrast to the other studies that are representative agent models.

population in most developed countries is considered, then public policy that focuses only on public investment does not fully capture the issues in developed countries. Therefore, incorporating a social security system into such a growth model with public capital is worth investigating.

The present analysis is based on an overlapping generation model, in which the growth engine is public capital accumulation, such as Yakita (2008).[2] We assume that the government imposes a distortionary tax on income and allocates its tax revenue between public investment and social security provision. Therefore, the government controls both the tax rate and the expenditure share between public investment and social security provision. In this model, the social security provision has two effects on the economy.[3] Firstly, because the social security provision reduces public investment to satisfy the government's budget, it lowers the accumulation of public capital. Secondly, the social security provision decreases the amount of young agents' saving because the social security provision increases the agent's income after retirement and thus reduces the incentive to save. Hence, the social security provision also lowers the accumulation of private capital.

This paper has two main objectives. One is investigating the policy that maximizes the balanced growth rate. The other is examining welfare effects on the balanced growth path when the government controls the tax rate and the expenditure share.

The findings of this paper on growth-maximizing policies are as follows. As described above, the social security provision lowers the accumulation of both public capital and private capital. Thus, when there is no social security provision, the growth rate is maximized by setting the tax rate at the elasticity of output with respect to public capital.[4] We next show that, although the social security provision lowers economic growth, there exists a tax rate that maximizes the growth rate of the economy in the presence of social security provision. This tax rate becomes smaller as the social security provision becomes larger. This finding implies that an economy with a large social security provision can adopt a small tax rate to maximize economic growth.

We next establish the following results regarding social welfare. The public policy that maximizes the growth rate, that is, the policy in which there is no expenditure on social security and the tax rate is set at the elasticity of output with respect to public capital, does not maximize the welfare level of every generation. Reducing the tax level from that under the growth-maximizing policy improves the welfare levels of all generations from the initial state on the balanced growth path. In terms of the allocation of tax revenue between public investment and social security, there exists intergenerational conflict. Early generations may prefer the social security provision to be larger when they are old because the benefits stemming from higher economic growth caused by an acceleration of public investment are relatively small. In contrast, for future generations, benefits from economic growth driven by the accumulation of public capital are relatively high. Therefore, they demand no social security provision.

When the government conducts policies that take into account the weighted sum of

[2]Yakita (2008) does not include social security but incorporates the maintenance of public capital.

[3]It is well known that pay-as-you-go social security systems hinder economic growth in an economy with no public capital, only private capital. See, for example, de la Croix and Michel (2002) Chapter 3 and Pecchenino and Pollard (1997).

[4]Barro (1990) and Futagami et al. (1993) show that, when the government allocates the tax revenue into only public investment, the growth rate is maximized by setting the tax rate at the elasticity of output with respect to public capital.

each generation's welfares on the balanced growth path and if it puts heavier weight on future generations, large public investment and small social security provision should be carried out. If it puts heavier weight on early generations, small public investment and the large social security provision should be carried out. Furthermore, we find in numerical examples that, in the process of population aging, the expenditure share of the social security provision that maximizes the social welfare increases as population aging occurs. Population aging reduces the social security benefit per one old agent. As a result, this increases the savings of agents and reduces the social welfare. A rise in the expenditure on social security will prevent agents from saving excessively.

This paper is organized as follows. Section 2 develops the model and characterizes the equilibrium. Section 3 explores how the tax rate and the expenditure share affects the growth rate on the balanced growth path. Section 4 discusses the welfare effects of each generation on the balanced growth path. Section 5 discusses the case in which a benevolent government controls the policy instruments and takes every generation on the balanced growth path into consideration and derives implications of public policy in an aging economy. Section 6 gives concluding remarks.

2 The model

We consider a two period overlapping-generations model in which lifetime uncertainty is incorporated. The economy comprises identical agents, a large number of symmetric firms, and a government. Agents live for at most two periods. A cohort born in period t is called generation t. Therefore, there exist two generations in period t ($t \geq 0$); that is, generation t (as a young generation) and generation $t-1$ (as an old generation). The economy begins its operation in period 0 and those who are old in period 0 are called the initial old.

2.1 Production

All outputs in this economy are homogeneous and produced by a large number of symmetric firms installing the private capital stock and labor for the production. The production technology of firm j is represented as:

$$Y_{jt} = K_{jt}^{\alpha}(h_t L_{jt})^{1-\alpha}, (0 < \alpha < 1) \tag{1}$$

where Y_{jt}, K_{jt} and L_{jt} are the output level, private capital stock, and labor employed by firm j, and h_t is the labor productivity in period t. Perfect competition prevails in the good and the factor markets. Thus, denoting the rental price of private capital and the wage rate as q_t and w_t, the profit maximizing conditions of firm j are given by:

$$\alpha \left(\frac{K_{jt}}{L_{jt}}\right)^{\alpha-1} h_t^{1-\alpha} - q_t = 0, \tag{2}$$

$$(1-\alpha) \left(\frac{K_{jt}}{L_{jt}}\right)^{1-\alpha} h_t^{1-\alpha} - w_t = 0. \tag{3}$$

We assume that the public capital stock provides a positive externality in production. Specifically, we specify h_t as follows:

$$h_t = \frac{G_t}{L_t}. \tag{4}$$

From (2) and (3), the capital–labor ratio becomes the same for all firms, that is, $K_{jt}/L_{jt} = K_t/L_t$ and $Y_{jt}/K_{jt} = Y_t/K_t$ hold in the equilibrium. Here, K_t is the aggregate capital stock and L_t is the total labor supply of the economy as follows: $K_t = \sum_j K_{jt}$ and $L_t = \sum_j L_{jt}$. We assume that the private capital stock fully depreciates in each period. Substituting (4) into (1), we obtain the aggregate production function as follows:

$$Y_t = K_t^{\alpha} G_t^{1-\alpha}. \tag{5}$$

Profit maximizing conditions (2) and (3) can be rewritten as:

$$q_t = \alpha \left(\frac{K_t}{G_t}\right)^{\alpha-1}, \tag{6}$$

$$w_t = (1-\alpha)\left(\frac{K_t}{G_t}\right)^{\alpha}\left(\frac{G_t}{L_t}\right). \tag{7}$$

2.2 Government

In this economy, the government pursues both the accumulation of the public capital stock, which increases labor productivity, and a social security policy based on a pay-as-you-go pension system. The government imposes a flat rate of income tax τ on wage and interest income and allocates its tax revenue between public investment and public pension payments. Let us denote public investment and public pension payments in period t as E_t and P_t, respectively. The government budget constraint in period t becomes:

$$E_t + P_t = \tau Y_t. \tag{8}$$

The government allocates a constant ratio, $\phi \in (0, 1]$ of its tax revenue to public investment and the rest of the tax revenue to public pension payments. Thus, we obtain:

$$E_t = \phi\tau Y_t, \tag{9}$$

$$P_t = (1-\phi)\tau Y_t. \tag{10}$$

The evolution of the public capital stock is given by:

$$G_{t+1} - (1-\delta_G)G_t = E_t = \phi\tau Y_t, \tag{11}$$

where δ_G is the depreciation rate of public capital.

2.3 Agents

Agents live for at most two periods. Some agents die at the end of the first period of life. We denote the probability that agents are alive in the second period of life by $\lambda \in (0,1)$. In each period t, N_t persons are born, so that the size of generation t is expressed as N_t in period t (when young) and λN_t in period $t+1$ (when old). Therefore, the total population in period t is $\lambda N_{t-1} + N_t$. We assume that the size of the newly born cohort in each period is constant over time, that is, $N_{t-1} = N_t = N$.

Agents are endowed with one unit of labor and they supply it inelastically to firms in their young period, and retire when old. Hence, total labor supply equals the population size, that is, $L_t = N$. They derive utility from their own consumption when young and old. Therefore, the lifetime expected utility of a representative agent in generation t is represented as:

$$\ln c_t + \lambda\rho \ln d_{t+1}, \tag{12}$$

where c_t and d_{t+1} denote consumption levels in young and old ages, respectively, and $\rho \in (0,1)$ is the discount factor.

The agents allocate their labor income w_t between consumption c_t and saving s_t, and the payment of taxes is a proportion of his/her wage τw_t. Thus, the budget constraint for a young agent of generation t is given by:

$$(1-\tau)w_t = c_t + s_t. \tag{13}$$

We assume there exists an actuarially fair insurance company in this paper, as used in Yaari (1965) and Blanchard (1985). The company collects funds and invests them in firms. The company repays returns on the investment to the insured households still living in old age. When the insurance company collects from each young agent ($A_t = a_t N_t$ in the aggregate) in period t, it obtains the total proceeds $q_t A_t$ in period $t+1$. Because only λN_t old people are alive in period $t+1$, each of them receives $(q_{t+1}/\lambda)a_t$ from the insurance company because of perfect competition between the companies. Because the government imposes tax on this return, the net return for agents is $[(1-\tau)q_{t+1}/\lambda]a_t$. On the other hand, if the young agents in period t invest a_t in firms directly, they will receive $(1-\tau)q_{t+1}a_t$ whether they are alive or dead in period $t+1$. As we assume that households have no bequest motive, they accept the insurance contract, which yields a higher interest rate than self-investment. In addition to the return on the annuity assets, agents receive social security benefits from the government in old age. Let p_{t+1} be a social security benefit for one agent of generation t when all agents survive in their retired period; then, total social security benefits $p_{t+1}N_t$ must be equivalent to the government's expenditure on public pension payment P_{t+1} in period $t+1$. However, because the public pension payment P_{t+1} is actually distributed among agents who are alive in old age λN_t, an agent can receive p_{t+1}/λ if he/she is alive in his/her second period. Thus, the budget constraint of each old agent is as follows:

$$\frac{(1-\tau)q_{t+1}s_t}{\lambda} + \frac{p_{t+1}}{\lambda} = d_{t+1}. \tag{14}$$

Each agent of generation t maximizes (12) subject to (13) and (14), where w_t, q_{t+1}, p_{t+1}, and τ are given. The first-order condition results in the following:

$$d_{t+1} = \rho(1-\tau)q_{t+1}c_t. \tag{15}$$

From (13), (14), and (15) we obtain the following saving function:

$$s_t = \frac{\lambda\rho(1-\tau)}{1+\lambda\rho}w_t - \frac{p_{t+1}}{(1+\lambda\rho)(1-\tau)q_{t+1}}. \tag{16}$$

This saving function shows that a higher level of wage leads to higher savings, whereas a higher level of social security benefit reduces savings. A higher interest rate lowers the value of social security benefits evaluated in youth, and thus enhances savings.

2.4 Dynamics

The equilibrium condition in the capital market is given by:

$$K_{t+1} = s_t N. \tag{17}$$

Substituting (16) into (17) yields:

$$K_{t+1} = \frac{\lambda\rho(1-\tau)}{1+\lambda\rho}w_t N - \frac{p_{t+1}N}{(1+\lambda\rho)(1-\tau)q_{t+1}}. \tag{18}$$

From (5) and (10), the total social security benefit of generation t paid by the government in period $t+1$ is given by:

$$p_{t+1}N = P_{t+1} = (1-\phi)\tau Y_{t+1} = (1-\phi)\tau\left(\frac{K_{t+1}}{G_{t+1}}\right)^{\alpha} G_{t+1}. \tag{19}$$

Substituting (6), (7), and (19) into (18), we obtain:

$$K_{t+1} = \frac{\lambda\rho(1-\tau)(1-\alpha)}{1+\lambda\rho}\left(\frac{K_t}{G_t}\right)^{\alpha} G_t - \frac{(1-\phi)\tau\left(\frac{K_{t+1}}{G_{t+1}}\right)^{\alpha} G_{t+1}}{(1+\lambda\rho)(1-\tau)\alpha\left(\frac{K_{t+1}}{G_{t+1}}\right)^{\alpha-1}}. \tag{20}$$

Dividing both sides of (20) by K_t and that of (11) by G_t, we obtain the dynamic system of this economy as follows:

$$\frac{K_{t+1}}{K_t} = \frac{\lambda\rho(1-\tau)^2(1-\alpha)\alpha}{(1+\lambda\rho)(1-\tau)\alpha+(1-\phi)\tau}x_t^{\alpha-1}, \tag{21}$$

$$\frac{G_{t+1}}{G_t} = \phi\tau x_t^{\alpha} + 1 - \delta_G, \tag{22}$$

where $x_t \equiv \frac{K_t}{G_t}$. We obtain following dynamic system of x by dividing both sides of (21) by (22).

$$x_{t+1} = \frac{\lambda\rho(1-\tau)^2(1-\alpha)\alpha}{(1+\lambda\rho)(1-\tau)\alpha+(1-\phi)\tau}\frac{x_t^{\alpha}}{\phi\tau x_t^{\alpha}+1-\delta_G}. \tag{23}$$

In the steady state, x satisfies the following:

$$\frac{\lambda\rho(1-\tau)^2(1-\alpha)\alpha}{(1+\lambda\rho)(1-\tau)\alpha+(1-\phi)\tau}(x^*)^{\alpha-1}=\phi\tau(x^*)^\alpha+1-\delta_G. \tag{24}$$

Proposition 1 *There is a unique x^* that attains a balanced growth path along which private capital, public capital, and aggregate output grow at the same rate: $K_{t+1}/K_t = G_{t+1}/G_t = Y_{t+1}/Y_t = \gamma$. The steady state value x^* is uniquely determined by* (24) *and is globally stable.*

Proof.

Equation (23) satisfies the following conditions.

$$\frac{dx_{t+1}}{dx_t}=\psi\frac{(1-\delta_G)\alpha x_t^{\alpha-1}}{(\phi\tau x_t^\alpha+1-\delta_G)^2}>0$$

$$\frac{d^2x_{t+1}}{(dx_t)^2}=-\psi\frac{(1-\delta_G)\alpha x_t^{\alpha-2}[\phi\tau x_t^\alpha(\alpha+1)+(1-\alpha)(1-\delta_G)]}{(\phi\tau x_t^\alpha+1-\delta_G)^3}<0$$

$$\lim_{x_t\to 0}\frac{dx_{t+1}}{dx_t}=\infty$$

$$\lim_{x_t\to \infty}\frac{dx_{t+1}}{dx_t}=0$$

$$\psi\equiv\frac{\lambda\rho(1-\tau)^2(1-\alpha)\alpha}{(1+\lambda\rho)(1-\tau)\alpha+(1-\phi)\tau}.$$

Thus, (23) intersects the 45° line at $x^*(>0)$ uniquely and the steady state is globally stable as depicted in Figure 1. When $x_{t+1}=x_t=x^*$, (24) holds and it implies $K_{t+1}/K_t = G_{t+1}/G_t$ from (21) and (22). Q.E.D.

[figure1]

3 The policy maximizing the long-run growth rate

In this section, we investigate the following questions. What should be the amount of tax to maximize the balanced growth rate? How should the government divide tax revenue between investment in public capital and public pension payments so as to maximize the balanced growth rate? To solve this policy set (τ,ϕ), we firstly analyze the expenditure share of public investment ϕ for a given tax rate that maximizes the balanced growth rate.

Secondly, for a given expenditure share, we find the tax rate that maximizes the growth rate.

3.1 Choice of the expenditure share ϕ

From (22), the balanced growth rate γ becomes:

$$\gamma = \phi\tau(x^*)^\alpha + 1 - \delta_G. \tag{25}$$

By using (24) and (25), we obtain the effect of changes in the expenditure share of public investment on the balanced growth rate. Appendix A provides the derivation of the equation below.

$$\frac{d\gamma}{d\phi} = \frac{\tau(x^*)^\alpha}{\phi\tau(x^*)^\alpha + (1-\alpha)(1-\delta_G)} \left\{ 1 - \alpha + \frac{\alpha\phi\tau}{(1+\lambda\rho)(1-\tau)\alpha + (1-\phi)\tau} \right\} \gamma. \tag{26}$$

Because (26) takes a positive value, the long run growth rate is maximized when $\phi = 1$. Hence, when the government allocates all of the tax revenue to public investment, it can maximize the balanced growth rate. The reason for this result is quite simple. The social security provision reduces public investment to satisfy the government's budget, and thus lowers the accumulation of public capital. In addition, the social security provision decreases the amount of savings by young agents because social security decreases the insentive of young agents to save. Therefore, the social security provision also lowers the accumulation of private capital. We can examine the effect of changes in the expenditure share of public investment on the balanced growth rate.

3.2 Choice of the tax rate with social security

In this subsection, we focus on the tax rate, taking $\phi \in (0, 1)$ as given, and investigate the effects of changes in the tax rate on the balanced growth rate. From (24) and (25), we obtain the effect of changes in the tax rate on the balanced growth rate. Appendix B provides the derivation of the equation below.

$$\frac{d\gamma}{d\tau} = \Omega[((1+\lambda\rho)\alpha - (1-\phi))\tau^2 - ((1+\lambda\rho)(2-\alpha)\alpha + 2\alpha(1-\phi))\tau + (1+\lambda\rho)(1-\alpha)\alpha]. \tag{27}$$

$$\Omega \equiv \frac{\phi(x^*)^{2\alpha}\lambda\rho(1-\alpha)(1-\tau)\alpha}{[\phi\tau(x^*)^{\alpha+1} + (1-\alpha)(1-\delta_G)x^*][(1+\lambda\rho)(1-\tau)\alpha + (1-\phi)\tau]^2} > 0.$$

To explore the relationship between τ and γ, we must analyze the following quadratic function $f(\tau)$:

$$f(\tau) = [(1+\lambda\rho)\alpha - (1-\phi)]\tau^2 - [(1+\lambda\rho)(2-\alpha)\alpha + 2\alpha(1-\phi)]\tau + (1+\lambda\rho)(1-\alpha)\alpha. \tag{28}$$

The growth rate γ increases with τ if τ is small enough so that $f(\tau) > 0$ and decreases with τ if τ is large enough so that $f(\tau) < 0$, and the tax rate τ^* that maximizes the growth rate corresponds to the condition; $f(\tau) = 0$.

Proposition 2 *(i)There exists a tax rate $\tau^* \in (0,1)$ for any $\phi \in (0,1)$ that maximizes the balanced growth rate in an economy with a social security system. This tax rate is given by:*

$$\tau^* = \frac{1}{2[(1+\lambda\rho)\alpha - (1-\phi)]}\Big[(1+\lambda\rho)(2-\alpha)\alpha + 2\alpha(1-\phi) \\ -\sqrt{[(1+\lambda\rho)(2-\alpha)\alpha + 2\alpha(1-\phi)]^2 - 4[\alpha(1+\lambda\rho) - (1-\phi)](1+\lambda\rho)(1-\alpha)\alpha}\Big]. \quad (29)$$

(ii)The tax rate τ^* is monotonically increasing in the expenditure share ϕ and, when $\phi = 1$, τ^* takes the largest value $1-\alpha$.

Proof.

(i) We will show that the tax rate τ^* given by (29) exists within (0,1) for $\forall\phi \in (0,1)$ and maximizes the balanced growth rate.

The quadratic function of (28) satisfies the following condition:

$$f(0) = (1+\lambda\rho)(1-\alpha)\alpha > 0,$$
$$f(1) = -(1-\phi)(1+\alpha) < 0.$$

Hence, the tax rate τ^* exists within (0,1) for $\forall\phi \in (0,1)$

(ii)Differentiating $f(\tau)$ with respect to ϕ, we obtain:

$$\frac{\partial f(\tau)}{\partial\phi} = \tau(1+2\alpha\tau) > 0.$$

Therefore, when ϕ increases, $f(\tau)$ shifts upward so that τ^* increases. Q.E.D.

In Subsection 3.1, we have shown that an increase in the ratio of public investment raises the balanced growth rate. Therefore, the tax rate $\tau^*|_{\phi=1} = 1-\alpha$ leads to the highest growth rate. Thus, the growth-maximizing policy is given by $(\tau, \phi) = (1-\alpha, 1)$. This corresponds to the result of Barro (1990) and Futagami, et al. (1993). They show that, when the government sets the tax rate on income at the elasticity of output with respect to public capital, the balanced growth rate takes its maximum value. In the economy with a social security transfer to old generations, when the government allocates a larger share of its tax revenue to public investment, the tax rate that maximizes the balanced growth rate becomes higher.[5]

[5] This result is similar to that in Section 5 in Barro (1990). Barro (1990) shows that, when the government expenditure finances some service that increases household's utility, the growth-maximizing share of productive government spending is smaller. In our analysis, social security provision is a kind of service that enters into the household's utility in the second period.

4 Welfare level of each generation

In this section, our task is to investigate the welfare effects of each generation on the balanced growth path when the government controls its policy (τ, ϕ). We show that the growth-maximizing policy does not maximize welfare levels of generations on the balanced growth path. In particular, we find that, when the government aims at growth maximization, it chooses no social security provision. Although social security reduces the benefits of economic growth, it increases consumption after retirement. We show that some expenditure on social security provision improves the welfare of the early generations. However, later generations lose some of the benefits of economic growth if the government chooses a large expenditure on social security provision.

Because on the balanced growth path, the wage rate w^* and the rental price of private capital q^* are constant over time, the consumption by the young agents in period t is written as follows:

$$\begin{aligned} c_t =& \frac{1-\tau}{1+\lambda\rho} w^* + \frac{p_{t+1}}{(1+\lambda\rho)(1-\tau)q^*} \\ =& \frac{(1-\tau)(1-\alpha)}{(1+\lambda\rho)N} (x^*)^\alpha \left\{ 1 + \frac{\lambda\rho(1-\phi)\tau}{(1+\lambda\rho)(1-\tau)\alpha + (1-\phi)\tau} \right\} [\phi\tau(x^*)^\alpha + 1 - \delta_G]^t G_0, \end{aligned} \tag{30}$$

where G_0 denotes the initial level of public capital. From (15), the consumption by old agents of generation t becomes as follows:

$$d_{t+1} = \rho(1-\tau)q^* c_t. \tag{31}$$

Substituting (30) and (31) into (12) yields the utility of generation t on the balanced growth path.

$$\begin{aligned} u_t =& \lambda\rho(\ln\rho + \ln\alpha) + (1+\lambda\rho)\ln(1-\alpha)\frac{G_0}{N} \\ &+ (1+\lambda\rho)\alpha + \lambda\rho(\alpha-1)\ln x^* + (1+2\lambda\rho)\ln(1-\tau) \\ &+ (1+\lambda\rho)\left\{ \ln[(1-\tau)\alpha + (1-\phi)\tau] + \ln[(1+\lambda\rho)(1-\tau)\alpha + (1-\phi)\tau] \right\} \\ &+ (1+\lambda\rho)t\ln\gamma. \end{aligned} \tag{32}$$

The first and second terms are common to all generations for $t = 0, \cdots, \infty$ but the last term is different between generations. The last term represents the benefit from accumulated private capital and public capital from an initial state along the balanced growth path and it is increasing in t. Therefore, later generations obtain more benefits from economic growth.

Differentiating (32) with respect to τ and evaluating it at $(\tau, \phi) = (1-\alpha, 1)$, we obtain:

$$\frac{du_t}{d\tau}|_{(\tau,\phi)=(1-\alpha,1)} = -\frac{1+\lambda\rho}{(1-\alpha)\alpha} < 0. \tag{33}$$

Hence, lowering τ from $1-\alpha$ increases the welfare levels of all generations from the initial state along the balanced growth path.

Next, we investigate how an increase in the social security provision affects the utility of each generation with the tax rate set at $1-\alpha$. To get a clear result we evaluate this effect at $\phi = 1$, that is, no provision of public pension. Taking the derivative of (32) with respect to ϕ, and evaluating it at $(\tau, \phi) = (1-\alpha, 1)$, we obtain:

$$\frac{du_t}{d\phi}|_{(\tau,\phi)=(1-\alpha,1)} = [\lambda\rho(\alpha-1)+\alpha(1+\lambda\rho)]\frac{1}{x^*}\frac{dx^*}{d\phi}|_{(\tau,\phi)=(1-\alpha,1)} - \frac{\lambda\rho(1-\alpha)}{\alpha^2} + (1+\lambda\rho)t\frac{1}{\gamma}\frac{d\gamma}{d\phi}|_{(\tau,\phi)=(1-\alpha,1)}. \tag{34}$$

When the government decreases ϕ from 1, there are the following effects on the welfare of generation t. The first term depends on the sign of $dx/d\phi$. This term indicates that a change in the ratio of private capital to public capital because of a fall in ϕ affects the agent's consumption in both his/her young and old periods. The second term reflects the following two opposite effects because of a fall in ϕ . First, it has positive effects on consumption because of an increase of the social security benefit. On the other hand, it also has negative effects on consumption caused by a fall in the growth rate of both private capital and public capital because of a decrease in public investment. Because the second term takes a negative value, this indicates that the positive effects dominate the positive effects. The first term and the second term are common to all generations. However, the last term differs between generations. The last term causes the differences in the policy effects on each generation's welfare. The last term indicates that economic growth exerts positive effects on the wage income and the social security benefit of the agents alive today. Because the last term is increasing in t, an increase in the social security provision (a decrease in ϕ) negatively affects the welfare level of the later generations.

However, it is difficult to evaluate (34) analytically because the sign of $dx/d\phi$ remains ambiguous. Therefore, we present numerical examples to show differences in the welfare effects between generations. We set the parameters as follows: $(\alpha, \rho, \lambda, \delta_G, \tau)$=(0.3,0.6,0.8,0.01,0.7). Figure 2 indicates the relationship between the welfare levels of generations from zero to five when the tax rate is fixed at $1-\alpha = 0.7$. An increase in the social security provision (smaller public investment) raises the welfare levels of generations close to the initial generation; however, it reduces those of the later generations. Therefore, there is a conflict between generations as to the level of the social security provision.

[figure 2]

5 Social welfare

In this section, we make the assumption that the government is benevolent and its objective is to improve the social welfare by taking all generations on the balanced growth path into consideration. This objective function consists of the weighted sum of each generation's welfare on the balanced growth path. Let θ denote the time discount factor of the benevolent government, which evaluates all future generations' utilities at the initial period 0 on the balanced growth path. A higher value of θ means that government is more concerned about future generations.

$$U = \sum_{t=0}^{\infty} \theta^t \left(\ln c_t + \frac{\lambda\rho}{\theta} \ln d_t \right). \tag{35}$$

On the balanced growth path, the initial old generation's consumption level d_0 is written as follows:

$$d_0 = \frac{(1-\tau)q^* s_{-1}}{\lambda} + \frac{p_0}{\lambda} = \frac{(1-\tau)\alpha + (1-\phi)\tau}{\lambda} (x^*)^{\alpha} \left(\frac{G_0}{N} \right), \tag{36}$$

Substituting (30), (31), and (36) into (35), we obtain:

$$\begin{aligned} U =& D_1 + \frac{1+2\lambda\rho}{1-\theta} \ln(1-\tau) + \frac{\lambda\rho(\alpha-1) + \alpha(1+\lambda\rho)}{1-\theta} \ln x^* \\ &+ \frac{1+\lambda\rho}{1-\theta} \{ \ln[(1-\tau)\alpha + (1-\phi)\tau] - \ln[(1+\lambda\rho)(1-\tau)\alpha + (1-\phi)\tau] \} \\ &+ \frac{\theta(1+\lambda\rho)}{(1-\theta)^2} \ln \gamma \\ &+ \underbrace{D_2 + \frac{\lambda\rho\alpha}{\theta} \ln x^* + \frac{\lambda\rho}{\theta} \ln[(1-\tau)\alpha + (1-\phi)\tau]}_{\text{the welfare of the initial old}}. \end{aligned} \tag{37}$$

$$D_1 \equiv \frac{1}{1-\theta}(\lambda\rho \ln \rho + \lambda\rho \ln \alpha) + \frac{1+\lambda\rho}{1-\theta} \ln \frac{(1-\alpha)G_0}{N}, D_2 \equiv \frac{\lambda\rho}{\theta} \ln \frac{G_0}{\lambda N}.$$

Let us calculate the welfare effect of ϕ by using a numerical example. We set the parameters as follows: $(\alpha, \rho, \lambda, \delta_G, \tau, \theta) = (0.3, 0.6, 0.8, 0.01, 0.7, 0.6)$. The relationship between the social welfare U and the expenditure share: ϕ can be calculated in Figure 3.

[Figure 3]

The expenditure share that maximizes the social welfare shown in Figure 3 when the tax rate is fixed at $1 - \alpha = 0.7$ is contingent on how highly the government values the welfare levels of future generations. If θ takes a relatively large value, the government cares about later generations and, hence, they accelerate public investment to enhance economic growth, that is, ϕ takes a larger value. If θ takes a relatively small value, the government chooses higher expenditure on social security provision to increase early generations' welfare levels.

We next study how population aging influences the welfare effects of ϕ. In this model, population aging can be expressed as an increase in λ. Here, we use the same parameter as before. We calculate social welfare when $\lambda = 0.6$ and 0.8 respectively, and obtain the effects of population aging on social welfare.

[figure 4]

Figures 4-(i) and 4-(ii) respectively show the relationship between social welfare U and expenditure share ϕ when $\lambda = 0.6$ and 0.8. From these figures, as aging progresses from $\lambda = 0.6$ to 0.8, the expenditure share ϕ that maximizes the social welfare for a given tax rate decreases. This fact indicates that the government should expand the expenditure on social security in accordance with population aging.

The reason for this is as follows. Population aging reduces the social security benefit per old agent, so that each agent is likely to save more. Consequently, this increase in the saving of agents reduces social welfare. Hence, enhancing social security provision prevents agents from saving excessively.

6 Concluding remarks

This paper investigates public policy when the government allocates tax revenue between public investment and social security payments in an endogenous growth model with a growth engine of productive public capital. We obtain the following results. Firstly, there is a unique transition path converging to the steady-state equilibrium. Secondly, in terms of growth-maximizing policies, when there is no social security provision, setting the tax rate at the elasticity of output with respect to public capital as in Barro (1990) and Futagami et al. (1993) leads to the highest growth. With social security provision, there is a tax rate to maximize the balanced growth rate for each expenditure share of social security provision and, if the economy adopts a larger amount of social security, higher tax revenue is necessary to maximize the growth rate. Thirdly, as for welfare analysis, the policy that maximizes the growth rate does not maximize social welfare. One way to improve social welfare is to reduce the tax rate below the elasticity of output with respect to public capital. This increases the welfare levels of all generations from the initial state along the balanced growth path. The other way is introducing social security expenditure. However, this causes intergenerational conflict. A larger social security system increases the welfare level of the generations close to the initial because the loss of consumption stemming from higher economic growth is smaller; however, for future generations, it becomes larger. Therefore, the future generations prefer a smaller the social security system. Finally, as population aging occurs, agents increase saving and reduce consumption because of a fall in social security provision per an agent when old; hence, the government may increase social security expenditure in accordance with population aging.

A Appendix

Appendix A

Differentiating γ with respect to ϕ yields

$$\frac{d\gamma}{d\phi} = \tau(x^*)^\alpha \left(1 + \frac{\phi\alpha}{x^*}\frac{\partial x^*}{\partial \phi}\right). \tag{A.1}$$

By totally differentiating (24), we obtain the following derivative of x^* with respect to ϕ :

$$\frac{\partial x^*}{\partial \phi} = \frac{-\tau(x^*)^\alpha \left\{x^* - \frac{\lambda\rho(1-\tau)^2(1-\alpha)\alpha}{[(1+\lambda\rho)(1-\tau)\alpha+(1-\phi)\tau]^2}\right\}}{\phi\tau(x^*)^\alpha + (1-\alpha)(1-\delta_G)}. \tag{A.2}$$

There are the following two effects. A rise in the expenditure share of public investment ϕ leads to an acceleration of the accumulation of public capital. This effect reduces x^*. On the other hand, the rise in ϕ means a reduction in social security benefits for agents, which enhances saving and private capital accumulation. This effect increases x^*. Therefore, a rise in ϕ increases not only the public capital stock but also the private capital stock. If the increase in the private capital stock is larger (smaller) than that of the public capital stock, (A.2) takes a positive (negative) value.

Substituting (A.2) into (A.1), we obtain (26).

Appendix B

$$\frac{d\gamma}{d\tau} = \tau(x^*)^\alpha \left(1 + \frac{\tau\alpha}{x^*}\frac{\partial x^*}{\partial \tau}\right). \tag{A.3}$$

Totally differentiating (24) yields:

$$\frac{\partial x^*}{\partial \tau} = \frac{-(x^*)^\alpha \left\{\phi x^* + \frac{\lambda\rho(1-\tau)(1-\alpha)\alpha[(1+\lambda\rho)(1-\tau)\alpha+(1-\phi)(1+\tau)]}{[(1+\lambda\rho)(1-\tau)\alpha+(1-\phi)\tau]^2}\right\}}{\phi\tau(x^*)^\alpha + (1-\alpha)(1-\delta_G)} < 0. \tag{A.4}$$

Substituting (A.4) into (A.3), we obtain (27).

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Figure1 **Dynamics of** x

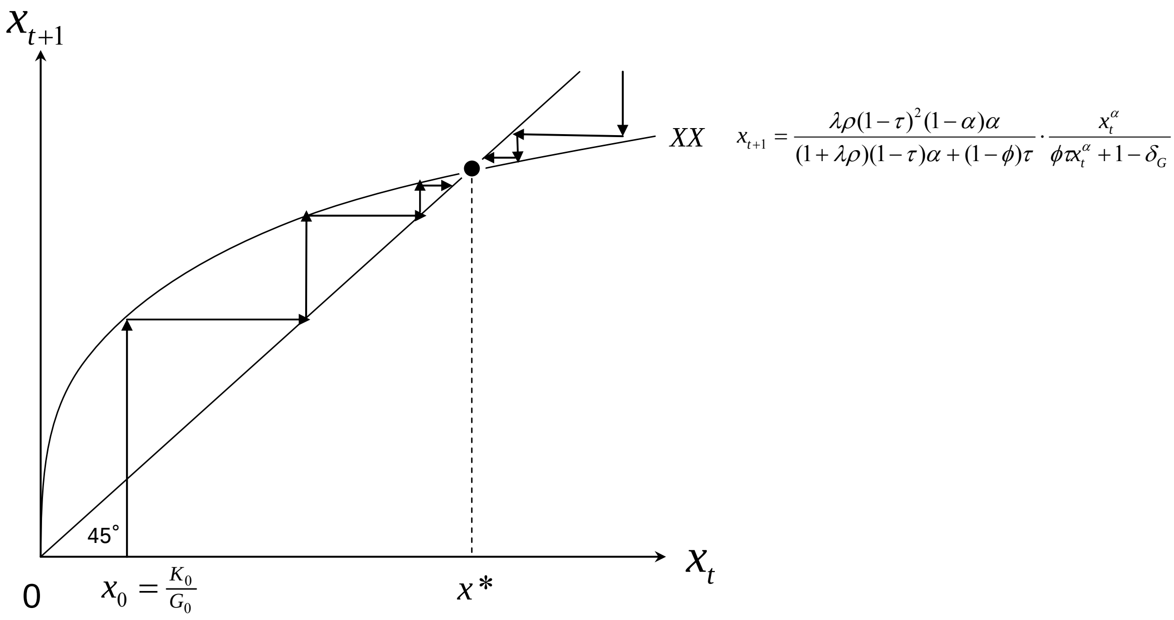


Figure 2 **The effects of ϕ on distinctive generations' utility**

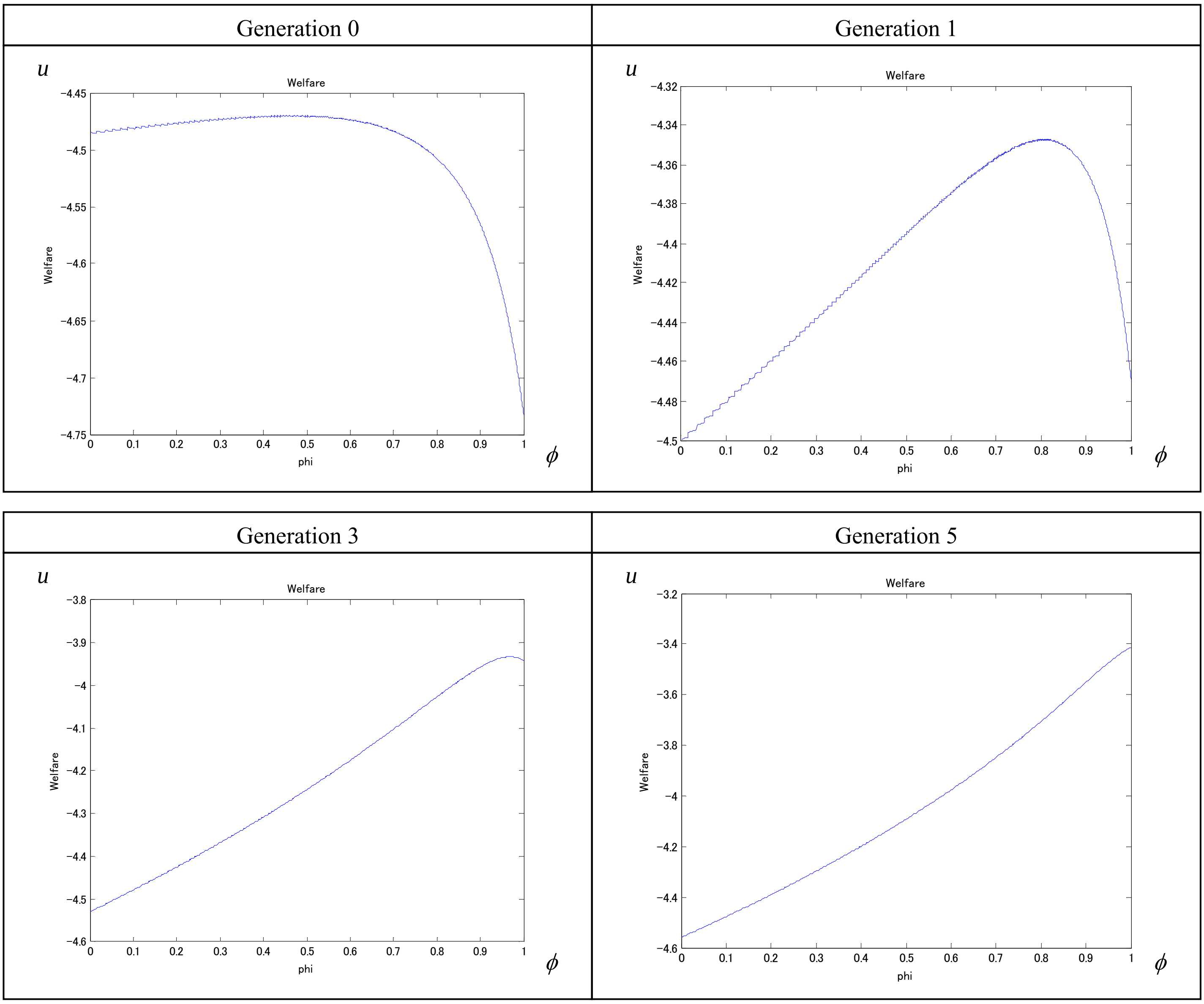
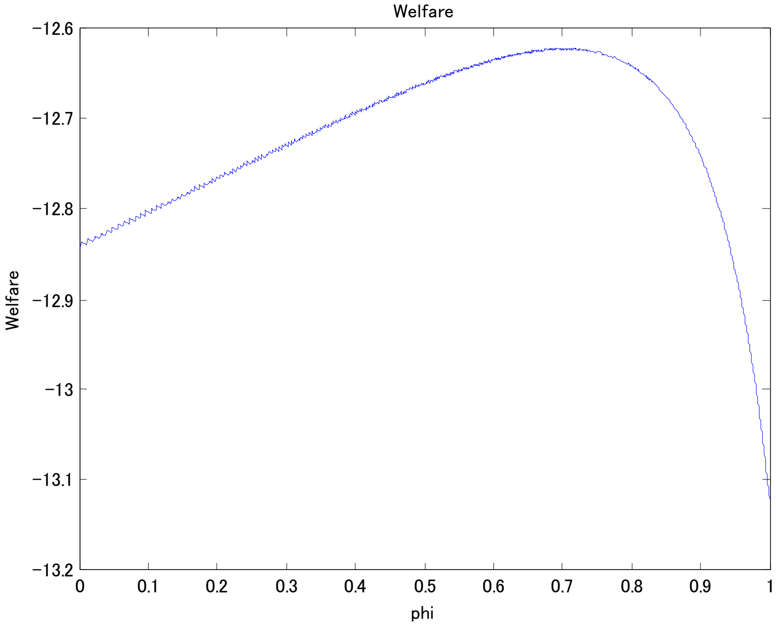


Figure 3 **The relationship between ϕ and U**

Social welfare: U

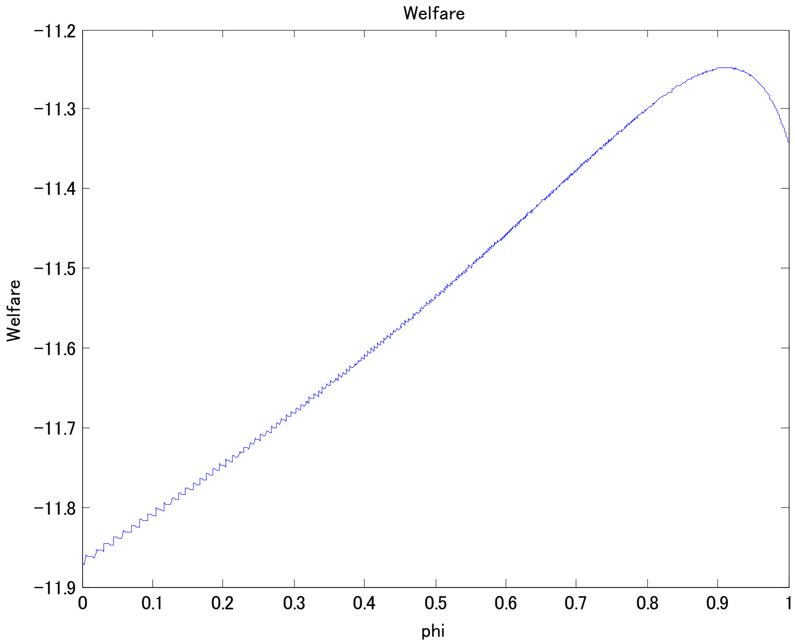


ϕ

Figure 4 **The welfare effects of ϕ in an aging economy**

(i) $\lambda = 0.6$

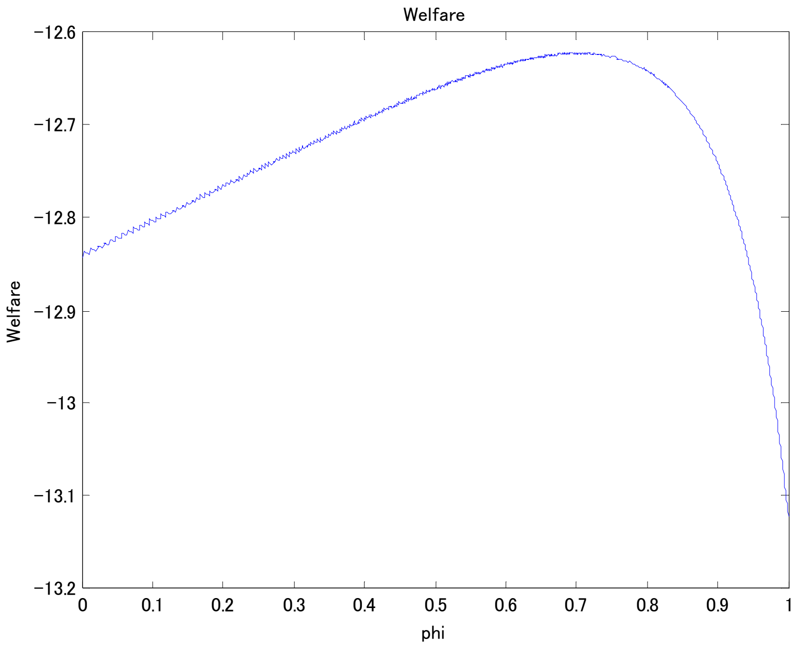
Social welfare: *U*



ϕ

(ii) $\lambda = 0.8$

Social welfare: *U*



ϕ