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Abstract

This study develops an on-the-job search model involving spatial structure. In this model, workers are either employed and commuting frequently to a central business district (CBD) or unemployed and commuting less frequently to the CBD to search for a job. When an unemployed worker succeeds in off-the-job search, the quality of the job match is determined stochastically: a good match yields high-productivity whereas a bad match yields low-productivity. Although a high-productivity worker does not search for a new job, a low-productivity worker decides whether to conduct an on-the-job search, which would require additional commuting to the CBD. Analysis of this model demonstrates that in equilibrium, the relocation path of workers corresponds to their career path, while welfare analysis demonstrates that such a spatial structure distorts firms' decision regarding the posting of vacancies.

JEL classification: J64; R14; R23
Keywords: City structure; On-the-job search; Unemployment; Efficiency; Relocation and career paths;

1 Introduction

There is no doubt that urban areas currently play a dominant role as areas of employment and residence throughout the world.[1] In many cities, people live in suburbs and commute from there to business districts. How does the structure of a city and commuting relate to job creation and unemployment within it? Recently, several studies have addressed this question by combining urban and labor economics. Some of these studies have succeeded in providing answers to a certain part of this question by developing search and matching models within urban structures.[2]

Wasmer and Zenou [15] introduced a monocentric city structure a la Alonso [1] into a job search model based on the key assumption that workers' search intensity is negatively affected by access to jobs that are concentrated within a Central Business District (CBD). Such an assumption leads to multiple urban configurations in equilibrium, including that of unemployed worker residing close to jobs or that of unemployed workers residing distant from jobs, with the

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[1] United Nations [14] reports that in 2000, 76 percent of the population in developed countries and 39 percent of the population in developing countries resided and worked in urbanized areas.

[2] There are also studies that have addressed this question by using other types of models and have yielded interesting findings. For instance, Zenou [17] adopted an efficiency wage model involving an urban structure. He found that firms do not recruit workers who reside at an excessive distance from the place of work because their productivity is lower than that of workers residing more proximate to the place of work.

later configuration being consistent with the well-known spatial mismatch hypothesis.[3] Smith and Zenou [13] demonstrated that when search intensity is determined endogenously, another type of configuration is obtained in equilibrium: there are two areas where unemployed workers reside, one of which is close to the CBD and the other of which is distant from the CBD. Employed workers live in between these two areas.

Crampton [3] analyzed worker's simultaneous decision regarding job search and residential areas. Rouwendal [9] demonstrated the possibility of excessive commuting due to the existence of information asymmetry in the job search process. Sato [12] analyzed how city structure affects workers' job-acceptance behavior and the labor market by introducing city structure into a search model with workers' decision to accept a job offer. He found that reductions in the urban costs of living such as commuting costs, increase the likelihood that job seekers will accept job offers. Zenou [19] proposed a spatial search model in which both job creation and job destruction are endogenous. He demonstrated that in equilibrium, workers with high productivity and wages live close to jobs, have low per distance commuting costs, and pay high land rents. He also showed that higher per distance commuting costs and higher unemployment benefits lead to more job destruction. Zenou [20] developed a spatial search model in which firms post wages. He found by simulation that when workers have different value imputed to leisure and different equilibrium wages, a reduction in commuting costs for all workers reduces the unemployment rate of high-wage workers and the profit of all firms while increasing the wages of all workers and the proportion of firms paying the high wage.[4][5]

This study focuses on a different significant aspect of job search, namely, on-the-job search, i.e., we introduce the possibility that employed workers search for a new job. It is often observed that employed workers search for a new job when they are unsatisfied with their current jobs. We also observe that people change their locations when they land a new job and obtain higher wage income. This study explores the possible interactions between such a career path in the labor market and a relocation path in the urban land market. To do so, the monocentric city model a la Alonso [1] is introduced in the on-the-job search model developed by Pissarides [7], in which workers are either employed or unemployed. An employed worker frequently commutes to the CBD to work while an unemployed worker commutes less frequently to the CBD in order to search for a job. When an unemployed worker succeeds in obtaining a job (i.e. succeeds in off-the-job search), the quality of the job match is determined stochastically: a good match yields high productivity whereas a bad match leads to low productivity.

Although high productivity workers do not search for a new job, low productivity workers decide whether or not she/he looks for a new job. On-the-job search requires additional commuting to the CBD, which leads to the following urban configuration: currently employed job seekers (on-the-job searchers) live closest to the CBD, unemployed workers (off-the-job searchers) live most distant from the CBD, and employed workers not searching for a new position (non-job-searchers) live in between the former two. In this model, spatial relocation corresponds to a career path, with the career path from unemployment to employment or from on-the-job search

[3]First stated by Kain [5], the spatial hypothesis posits that job decentralization to the suburbs without the residential movement of African Americans has led to a high unemployment rate and low wages paid in inner-city neighborhoods, where African Americans are concentrated. Since then, a large number of empirical studies testing this hypothesis have offered much evidence for its support (see Preston and McLafferty [8] for a survey of recent empirical studies).

[4]For a comprehensive survey of this literature, see Zenou [18], among others.

[5]Sato [10] [11] also developed a job search model involving a monocentric city structure. However, he assumed that because all types of workers have the same commuting costs and the same level of housing consumption, the residential location of a particular type of worker cannot be determined. Therefore, he focused on the effects of the overall cost of living, which depends on the population size of the city, on the local labor market. Sato [10] investigated the relationship between wage and unemployment differences among different cities in inter-urban equilibrium. Sato [11] found a link between agglomeration economies and the worker-firm matching process, as did Wheeler [16] in a separate study using a different model. Sato [11] provided the conditions of the matching process necessary for the existence of agglomeration economies as well.

to a good-match entailing relocation from the outskirts of a city to an inner part of the city or from the innermost area of the city to the suburbs.

In this spatial equilibrium, the wages of low productivity workers are lower when they live closer to the CBD. In the model proposed here, residing proximate to the CBD signals that a worker is an on-the-job seeker, which decreases the bargaining position of the worker and reduces her/his wages. Spatial structure also impacts the efficiency of the properties of the model. This model proposes that whereas decision regarding an on-the-job search is efficient, firms' decision regarding job vacancies is distorted even under the Hosios condition (Hosios [4]), which contrasts with the findings of related studies of spatial off-the-job search models, such as those of Wasmer and Zenou [15] and Zenou [18].

This paper is structured as follows. Section 2 introduces the basic structure of the model. Section 3 demonstrates the existence of and the properties of a unique equilibrium. Section 4 addresses issues regarding welfare and Section 5 presents the study conclusions.

2 Model

Consider a closed, linear, and monocentric city whose land is owned by absentee landlords. It has one CBD, the location of which is approximated by one point and within which all firms are assumed to be exogenously located.[6] A continuum of risk-neutral workers of size N live within the city, each of which exits the labor market according to a random Poisson process with the exogenous aggregate rate $\delta > 0$. Such an exit is replaced by an entry of a new worker, who first searches for a job as an unemployed worker. This assumption keeps the total number of workers constant. Although identical ex ante, the workers become heterogeneous after entering the labor market due to the occurrence of stochastic events, with u of N being unemployed workers and $N - u$ being employed workers. While unemployed workers search for a job, employed workers earn wage income that depends on their productivity y, which is either high or low and determined randomly upon the job match, with a good match yielding productivity y_h and a bad match yielding productivity y_l. A subscript h (l) represents that the variable is related to a good (bad) match. We assume that $y_h > y_l$ and $\Pr(y = y_l) = \Pr(y = y_h) = 1/2$. Moreover, it is assumed that y_l is sufficiently large such that even if a match yields low productivity, the worker prefers to be employed rather than unemployed, and her/his employer does not dismiss her/him. We assume that on-the-job search is possible, such that a low-productivity worker can search for a new job while working.

In the city, all workers occupy the same amount of land (normalized to 1) outside the CBD. We assume that the density of land is 1, implying that x units of housing are located within a distance x of the CBD.

In this paper, it is assumed that the sole commuting cost is that of the time expended in doing so. Each worker is endowed with one unit of time, that she/he uses for working, searching, commuting and leisure. Assume that an employed worker must expend a fixed amount of time h_w (> 0) on work activities and that the job search requires a fixed amount of time h_s (> 0). Moreover, we assume that the commuting time to the CBD is $\sigma_w x$ for an employed worker and $\sigma_u x$ for an unemployed worker. Here, commuting is necessary for an unemployed worker to be interviewed. We assume that $h_w > h_s$ and $\sigma_w \geq \sigma_u$: working at a job requires more time and frequent commuting than does searching for a job. A worker is assumed to obtain the instantaneous utility $u_l(l)$ from leisure, where $u_l' > 0$ and $u_l'' < 0$. The time for leisure is given by $l_u = 1 - h_s - \sigma_u x$ for an unemployed worker, $l_e^1 = 1 - h_w - h_s - \sigma_w x$ for an employed worker who is searching for a new position (i.e., an on-the-job searcher), $l_e^0 = 1 - h_w - \sigma_w x$ for an employed worker who is not searching.

[6] This model of a centralized city can easily be modified to describe a decentralized city by locating all firms within a suburban business district that is located at one end of a linear city. Such a modification would not alter the results of this study.

Based on these assumptions, the decrease in utility resulting from living more distantly from the CBD can be expressed as follows:

$$\frac{\partial u_l(l_u)}{\partial x} = -\sigma_u u_l'(1 - h_s - \sigma_u x),$$
$$\frac{\partial u_l(l_e^1)}{\partial x} = -\sigma_w u_l'(1 - h_w - h_s - \sigma_w x),$$
$$\frac{\partial u_l(l_e^0)}{\partial x} = -\sigma_w u_l'(1 - h_w - \sigma_w x).$$

Assuming that $h_w > h_s$ and $\sigma_w \geq \sigma_u$ implies that $l_u > l_e^0 > l_e^1$ for a given x. Because $\sigma_u u_l'(l_u) < \sigma_w u_l'(l_e^0) < \sigma_w u_l'(l_e^1)$ is readily known from $u'' < 0$, the marginal cost of commuting is the highest for on-the-job searchers and the lowest for unemployed workers, as expressed by the followings:

$$0 > \frac{\partial u_l(l_u)}{\partial x} > \frac{\partial u_l(l_e^1)}{\partial x} > \frac{\partial u_l(l_e^0)}{\partial x}.$$

For the sake of analytical tractability, we adopt the simplest way of describing commuting cost. Specifically, commuting cost is described by linear functions of the distance x from the CBD such that it is τx for an employed worker not searching for a new job, $(1+s^*)\tau x$ for an on-the-job searcher, and $s\tau x$ for an unemployed worker, where τ, s^* and s are exogenous and satisfy that $\tau > 0$ and $0 < s^* \leq s < 1$.[7][8]

Because of these assumptions, as is demonstrated in Section 3, the equilibrium urban configuration is such that on-the-job searchers reside most proximate to the CBD, unemployed workers reside most distant from the CBD, and employed workers not searching for a job reside between the first two groups. The cost of living in the city is the sum of the residential land rent $R(x)$ and the commuting cost: it is $R(x) + \tau x$ for an employed worker not searching for a job, $R(x) + (1 + s^*)\tau x$ for an on-the-job searcher, and $R(x) + stx$ for an unemployed worker. Although workers are identical ex ante, they are heterogeneous in terms of place of residence x, employment status (employed or unemployed), productivity (high or low), and job search activity (no search, on-the-job search, or off-the-job search). Following Burdett and Mortensen [2] and Pissarides [7], on-the-job search by firms is assumed to be impossible, i.e., firms cannot search for a new worker for an already filled job.

2.1 Matching framework

There are v firms that have vacancies and search for workers. Each of them posts one vacancy, which can be filled by only one worker. Let e_s denote the number of on-the-job searchers and ϕ denote the efficiency of their on-the-job search. We normalize the efficiency of off-the-job search to one. Job matches are generated by a Poisson process with the aggregate rate of $M = m(u + \phi e_s, v)$, where $u + \phi e_s$ represents the number of job searchers in terms of efficiency units. $m(\cdot, \cdot)$, which is defined on $\mathbf{R}_+ \times \mathbf{R}_+$, is assumed to be strictly increasing in both arguments, twice differentiable, strictly concave, and homogeneous of degree one. We also assume that $m(\cdot, \cdot)$ satisfies $0 \leq M \leq \min[u + \phi e, v]$, $m(u + \phi e_s, 0) = m(0, v) = 0$ and the Inada condition for both arguments.

[7] More precisely, it is assumed that the value of leisure measured is given by

$$u_l(l_u) = b - s\tau x,$$
$$u_l(l_e^1) = \flat - (\tau + s^*)x,$$
$$u_l(l_e^0) = \flat - \tau x,$$

where b and $\flat$ are constants that represent the value of leisure at location $x = 0$. We normalize $\flat$ to zero.

[8] s and s^* can be interpreted as indicators of search intensity. For a discussion on the endogenous determinant of search intensity, see Smith and Zenou [13] and Wasmer and Zenou [15].

Job matching occurs at the rate of $p(\theta) = M/(u + \phi e_s) = m(1, \theta)$ for each unemployed workers, $\phi p(\theta)$ for each on-the-job searchers, and $q(\theta) = M/v = m(1/\theta, 1)$ for a firm seeking to fill a vacancy, where θ is the measure of labor market tightness defined as $\theta = v/(u + \phi e_s)$.From the assumptions regarding $m(\cdot, \cdot)$, we obtain that $p(\theta)(u+\phi e) = q(\theta)v$, $\mathsf{d}p/\mathsf{d}\theta > 0$ and $\mathsf{d}q/\mathsf{d}\theta < 0$ for any $\theta \in (0, +\infty)$. We can also see that $\lim_{\theta\to 0} p = 0$, $\lim_{\theta\to\infty} p = \infty$, $\lim_{\theta\to 0} q = \infty$ and $\lim_{\theta\to\infty} q = 0$. If $\phi = 0$, this model becomes a model without the presence of on-the-job search. For the sake of descriptive simplicity, an on-the-job searcher is assumed to move to a new job if the new job provides her/him with the same or a larger asset value as does the current job. As is standard in on-the-job search models, it is assumed that firms, upon meeting a worker, do not know whether she/he is an on-the-job searcher or an off-the-job searcher.

2.2 Value functions

Let $W_j(x)$, $U(x)$, $J_j(x)$ and V denote the asset value of an employed worker, an unemployed worker, a firm with a filled job, and a firm with a vacancy, respectively. j represents the productivity of a match (i.e., high (h) or low (l), $j \in \{h, l\}$). x describes the worker's residential location ($x \in [0, \infty)$), which is the distance to the CBD. In $J_j(x)$, x refers to the residential location of a worker employed by the firm.

The asset value of an unemployed worker who lives at x is, therefore, given by

$$(r + \delta)U(x) = b + \frac{p(\theta)}{2} \sum_{j\in\{h,l\}} \left(W_j^{\max} - U(x)\right) - R(x) - s\tau x, \tag{1}$$

where $W_j^{\max} = \max_{x\in[0,\infty]} W_j(x)$. r, b, and $R(x)$ represent the discount rate, the value of leisure, and the market land rent, respectively, with r and b being positive constants and $R(x)$ being determined endogenously. It is assumed that even a bad match yields a value sufficiently lager than the value of leisure. The first term of (1) is the instantaneous utility of being unemployed while the second term is the expected capital gain from off-the-job search. An unemployed worker is matched to a job at the rate of $p(\theta)$, with the match yielding high (low) productivity at a probability of 1/2 (1/2), leading to an asset value of $W_h^{\max}$ ($W_l^{\max}$). The third and fourth terms are the land rent and the location-dependent cost of off-the-job search, respectively. As previous explained, it is assumed that an unemployed worker will accept any type of job when she/he has a chance of making a match.

The asset value of an employed worker with productivity j is described as

$$\begin{aligned}(r + \delta)W_j(x) &= w_j(x) + \lambda(U^{\max} - W_j(x)) \\ &\quad + \max_{i\in\{0,1\}} \left\{ i \left(\frac{\phi p(\theta)}{2} \sum_{j'\in\{h,l\}} \max\left[W_{j'}^{\max} - W_j(x), 0\right] - s^*\tau x \right) \right\} - R(x) - \tau x,\end{aligned} \tag{2}$$

where $U^{\max} = \max_{x\in[0,\infty]} U$, $w_j(x)$ is the wage and λ is the exogenous job separation rate. The first and second terms are the instantaneous utility obtained from the wage income and the capital loss from job separation, respectively. The third term represents the expected net return from on-the-job search, which is the expected capital gain $(\phi p(\theta)/2) \sum_{j'} \max \left[W_{j'}^{\max} - W_j(x), 0\right]$ from on-the-job search minus the cost $s^*\tau x$ of it. If the net return is non-negative, an employed worker will search for a new job ($i = 1$), but will not do so if it is negative ($i = 0$).

While it is assumed that firms do not know whether employees are searching for a new job, it is assumed that firms know where employees live. Based on the knowledge of its employees' residential location and productivity, a firm can makes the belief regarding whether its employee is doing an on-the-job search. The asset values of firms are given by

$$rJ_j(x) = y_j - w_j(x) + (\lambda + \delta + i^* \phi p(\theta))(V - J_j(x)), \tag{3}$$

$$rV = -c + \frac{q(\theta)}{2} \sum_{j \in \{h,l\}} (J_j(x) - V), \tag{4}$$

where y_j is the output of a match yielding productivity j, c is the cost of posting a vacancy, and i^* is the firm's belief regarding whether an employee is looking for a new job. As later discussed in more detail, such a belief must be consistent with the relationship among the productivity of the match, the residential location of the employee, and her/his job search status. In (3), the first and second terms describe instantaneous profits while the third term represents capital loss from job destruction resulting from job separation (λ) of the employee, her/his exit (δ) from the labor market, and her/his job turnover ($i^* \phi p(\theta)$). In (4), the first and second terms are the (instantaneous) cost of recruitment and the capital gain from filling a vacancy.

Equilibrium is determined by (i) the outcome of wage bargaining (Section 2.3), (ii) labor market conditions (the free entry of firms and on-the-job search decision of employees, as described in Section 2.4), (iii) land market conditions (locational arbitrage and bid rent, as described in Section 2.5) and (iv) steady state conditions.

2.3 Wage determination

Following Pissarides [7], wages are assumed to be determined by the first-order condition of the Nash bargaining conducted between a firm and an employee:

$$(1 - \beta)(W_j(x) - U^{\max}) = \beta(J_j(x) - V). \tag{5}$$

This relationship can also be expressed as $W_j(x) - U^{\max} = \beta(W_j(x) + J_j(x) - U^{\max} - V)$. As β represents the labor share of the total surplus, it indicates the bargaining power of a worker. Note that as employee's job searching status is private information, it does not affect her/his wages directly, but does so indirectly because it is determined by her/his residential location, which is public information and a factor considered in wage bargaining.

2.4 Labor market conditions

As will be formally described in Section 2.5, it is assumed that workers can move freely within the city and face no costs of relocation, implying that the asset value of a worker is the same across locations by locational arbitrage, as is expressed in (6):

$$W_j(x) = W_j \quad \text{and} \quad U(x) = U, \quad \forall x. \tag{6}$$

Moreover, from the wage determination rule (5), we obtain

$$J_j(x) = J_j, \quad \forall x. \tag{7}$$

Assuming the free entry and exit of firms, when the value V of posting a vacancy is positive, more firms will post vacancies, and when V is negative, some firms will stop posting vacancies. In equilibrium, V is driven to zero. Combined with $V = 0$ and (7), (4) leads to the following free entry condition:

$$\frac{2c}{q(\theta)} = \sum_{j \in \{h,l\}} J_j. \tag{8}$$

From (5), (6) and (7), the value (1) of an unemployed worker becomes

$$(r + \delta)U = b + \frac{\beta p(\theta)}{2(1 - \beta)} \sum_{j \in \{h,l\}} (J_j - V) - R(x) - s\tau x,$$

which, combined with $V = 0$ and (8), leads to

$$(r + \delta)U = b + \frac{\beta c\theta}{1-\beta} - R(x) - s\tau x. \tag{9}$$

Similarly, (6) and (7) implies that the value (2) of an employed worker becomes

$$\begin{aligned}(r + \delta)W_j &= w_j(x) + \lambda(U - W_j) \\ &+ \max_{i\in\{0,1\}} \left[i \left(\frac{\phi p(\theta)}{2} \sum_{j'\in\{h,l\}} \max\left[W_{j'} - W_j, 0\right] - s^*\tau x \right) \right] - R(x) - \tau x.\end{aligned} \tag{10}$$

Whether an employed worker searches for a new job depends on whether her/his return from on-the-job search, expressed as $\phi p(\theta)/2 \sum_{j'\in\{h,l\}} \max\left[W_{j'} - W_j, 0\right]$, is larger than the cost of doing so, expressed as $s^*\tau x$. Because $\sum_{j'} \max\left[W_{j'} - W_h, 0\right]$ is always zero, we readily know that a high-productivity worker will not search for a new job. In contrast, a low-productivity worker will search for a job if and only if

$$\frac{\phi p(\theta)}{2}(W_h - W_l) \geq s^*\tau x. \tag{11}$$

This equation, the left hand side of which expresses the expected capital gain from on-the-job search and the right hand side is the cost of doing so, is very similar to the equation describing on-the-job search conditions presented in Pissarides [7]. The difference between the two is that in (11), the search cost includes the spatial factor whereas the equation in [7] does not do so. As is often assumed in search models, workers (firms) are assumed to regard θ as given, and hence, $p(\theta)$ (θ and $q(\theta)$) as given, which implies that they take the left hand side of (11) as given. This implies that a threshold x^* exists such that a low-productivity worker who lives at $x \leq x^*$ searches for a new job, where x^* is defined by

$$x^* = \frac{\phi p(\theta)(W_h - W_l)}{2s^*\tau}. \tag{12}$$

Firms know the productivity and location of their workers, and use this information in forming their belief i^*. In equilibrium, a firm forms a belief that only unemployed workers and low-productivity workers with $x \leq x^*$ are searching for a job. Letting $w_l^1(x)$ and $w_l^0(x)$ denote the wages of a low productivity worker who is searching for a new job and that of a low productivity worker who is not, respectively, leads to the following lemma.

Lemma 1 *The equilibrium wages satisfy that* $w_h(x) = w_h$, $w_l^1(x) = w_l^1$, *and* $w_l^0(x) = w_l^0$, $\forall x$. *Furthermore, the on-the-job search reduces wages by* $\phi p(\theta)J_l$, *i.e.,* $w_l^0 = w_l^1 + \phi p(\theta)J_l$.

Proof See Appendix A.

Intuitively, the asset value of a firm matched with a worker who is searching a new job is lower than that of a firm matched with a worker who is not by the expected loss $\phi p(\theta)J_l$ caused by job turnover. The wage differentials compensate for such differentials in the asset value.

2.5 Land market conditions

Based on the assumption that workers can change their locations of residence without incurring costs, there is no incentive for workers to relocate in equilibrium. Therefore, as shown in (6), in equilibrium, all employed workers enjoy the same level of value ($W_j(x) = W_j^{\max} = W_j$) and all unemployed workers enjoy the same level of value ($U(x) = U^{\max} = U$). These two conditions and the wage determination rule (5) imply that the value of a firm is also independent of the location of its employee ($J_j(x) = J_j$) as is described in (7).

In order to determine the equilibrium location of workers, we use the concept of bid rents, defined as the maximum land rent at location x that each type of worker is willing to pay in order to reach her/his respective level of equilibrium utility (in this paper, asset value). Note that in such a framework, the equilibrium asset value of a firm and that of its employee do not depend on whether the employee is doing on-the-job search. This is due to the adjustment through land rent and wages. From the definition of bid rents and consideration of Lemma 1 and equations (12), (9) and (10), the bid rents Ω of unemployed and employed worker can be expressed as follows:

Unemployed workers:

$$\Omega_u(x) = b - (r+\delta)U - s\tau x, \tag{13}$$

Employed low-productivity workers:

$$\Omega_l(x) = \begin{cases} \Omega_l^1(x) \equiv w_l^1 + \lambda U - (r+\delta+\lambda+\phi p(\theta)/2)W_l + \phi p(\theta)W_h/2 - (1+s^*)\tau x & \text{if} \quad x < x^* \\ \Omega_l^0(x) \equiv w_l^0 + \lambda U - (r+\delta+\lambda)W_l - \tau x \quad \text{if} \quad x \geq x^* \end{cases}, \tag{14}$$

Employed high-productivity workers:

$$\Omega_h(x) = w_h + \lambda U - (r+\delta+\lambda)W_h - \tau x. \tag{15}$$

Differentiating (13) and (15) with respect to x, we obtain

$$\frac{\partial \Omega_u}{\partial x} = -s\tau, \qquad \frac{\partial \Omega_h}{\partial x} = -\tau.$$

$\Omega_l(x)$ is a relatively complex function of x whose slope is described by

$$\begin{aligned} \frac{\partial \Omega_l^0}{\partial x} &= -(1+s^*)\tau, \qquad \text{if} \quad x \leq x^*, \\ \frac{\partial \Omega_l^1}{\partial x} &= -\tau, \qquad \text{if} \quad x > x^*. \end{aligned}$$

$\Omega_l(x)$ is discontinuous at x^* and shifts upwards by $\phi p(\theta)J_l$, reflecting the decreases in wages resulting from the firm's belief that a low-productivity worker who lives at $x \leq x^*$ is searching for a new job and the difference in the bid rent balancing the difference in wages. Put differently, from the viewpoint of workers, low productivity employees can commit themselves to not searching for a job by living farther away from the CBD than x^*. The decline in bid rents reflects the value of their commitment.

Figure 1 summarizes the above arguments regarding bid rents.

[Figure 1 around here]

In this figure, the horizontal and vertical axis represent the distance from the CBD and the bid rent, respectively, and the market land rent is the upper envelope of the bid rent curves. From this figure, the equilibrium locational pattern in the model can be derived as follows:

Proposition 1 *In equilibrium, the residential area is separated into four areas. The area closest to the CBD is occupied by low-productivity workers searching for a new job, the area second closest is occupied by high-productivity workers, the third closest area by both high-productivity workers and low-productivity workers not searching for a new job, and the fourth closest (the most distant) area by unemployed workers.*

The underlying mechanism of this proposition is similar to that of Wasmer and Zenou [15]: workers locate in descending order of the losses that they incur from living distant from the CBD. Because $\Omega_l(x)$ is discontinuous at x^*, there is an area where only high-productivity workers live between two areas where low productivity workers live.

Let e_l denote the number of low-productivity workers. Because high-productivity workers don't look for a new job, there are only two possibilities: either (i) only a proportion of low-productivity workers search for a new job ($e_s < e_l$) or (ii) all low-productivity workers search for a new job ($e_s = e_l$). Based on the possibility of these situations, the bid rents in Figure 1 satisfy the followings:

$$\Omega_l^1(e_s) = \Omega_h(e_s) \quad \text{and} \quad \Omega_h(N-u) = \Omega_l^0(N-u) = \Omega_u(N-u) \qquad \text{if} \quad e_s < e_l,$$
$$\Omega_l^1(e_s) = \Omega_h(e_s) \quad \text{and} \quad \Omega_h(N-u) = \Omega_u(N-u) \qquad \text{if} \quad e_s = e_l.$$

Equations from (13) to (15) together with $\Omega_l^1(e_s) = \Omega_h(e_s)$ and $\Omega_h(N-u) = \Omega_l^0(N-u)$ yield

$$w_h - w_l^1 - (r+\delta+\lambda)(W_h - W_l) + \frac{\phi p(\theta)}{2}(W_l - W_h) + s^*\tau e_s = 0, \tag{16}$$
$$w_h - w_l^0 - (r+\delta+\lambda)(W_h - W_l) = 0. \tag{17}$$

Substituting (17) into (16), we obtain

$$w_l^0 - w_l^1 + \frac{\phi p(\theta)}{2}(W_l - W_h) + s^*\tau e_s = 0.$$

From (12) and Lemma 1, this equation determines the number e_s of on-the-job searchers as

$$e_s = \min\left[\frac{\phi p(\theta)(W_h - W_l - 2J_l)}{2s^*\tau}, e_l\right] = \min\left[x^* - \frac{\phi p(\theta) J_l}{s^*\tau}, e_l\right] \tag{18}$$

In equilibrium with the interior solution $e_s < e_l$, a certain proportion of the low-productivity workers are crowded out by high-productivity workers from the interval $[x^* - \phi p(\theta)J_l/(s^*\tau), x^*]$, where they would have searched for a new job if they had resided there. Put differently, $x^* - e_s$ represents the number of low-productivity workers who are crowded out from on-the-job search. Such a crowding out effect does not exist in the non-spatial on-the-job search model a la Pissarides [7]. Hereafter, we assume the interior solution (i.e., $e_s < e_l$), whose sufficient condition will be provided in Lemma 5.

3 Equilibrium

We focus on a steady state equilibrium in which the inflow into unemployment is equal to the outflow from it:

$$\delta N + \lambda(N-u) = (p(\theta)+\delta)u,$$

which yields

$$\frac{u}{N} = \frac{\delta+\lambda}{\delta+\lambda+p(\theta)}. \tag{19}$$

Lemma 2 *For a given market tightness θ, the unemployment rate does not depend on the city size N.*

The free-entry condition (8) can be rewritten as

$$
\begin{aligned}
\underbrace{\frac{c}{q(\theta)}}_{\text{expected total search costs}} &= \frac{1}{2}\left(J_h + J_l\right) \\
&= \frac{1}{2}\frac{y_l - w_l^0 + y_h - w_h}{r+\delta+\lambda} \\
&= \frac{1}{r+\delta+\lambda}\left(\underbrace{\frac{y_h + y_l}{2}}_{\text{expected output}} - \underbrace{\frac{w_h + w_l^0}{2}}_{\text{expected wage}}\right)
\end{aligned}
\tag{20}
$$

which connects market tightness θ to wages w_h and w_l^0.

Wages w_h and w_l^0 can be derived from asset value functions. Equations (3) and (10) can be rewritten as the followings by using $V = 0$:

$$
\begin{aligned}
(r+\delta+\lambda)J_h &= y_h - w_h, \\
(r+\delta+\lambda)J_l &= y_l - w_l^0 \qquad \text{if} \quad x > x^*, \\
(r+\delta+\lambda)(W_h - U) &= w_h - (r+\delta)U - \tau x - R(x), \\
(r+\delta+\lambda)(W_l - U) &= w_l^0 - (r+\delta)rU - \tau x - R(x) \qquad \text{if} \quad x > x^*.
\end{aligned}
\tag{21}
$$

Substituting these equations and (9) into (5), we obtain[9]

$$
\begin{aligned}
w_h &= \beta(y_h + c\theta) + (1-\beta)\left[b + (1-s)\tau(N-u)\right], \\
w_l^0 &= \beta(y_l + c\theta) + (1-\beta)\left[b + (1-s)\tau(N-u)\right].
\end{aligned}
\tag{22}
$$

In these equations, wages are increasing functions of market tightness θ. For a given θ, the wages increase with an increase in productivity and the value of leisure, in accordance with the findings of Pissarides [7], as well as with an increase in the commuting cost τ and $N-u$, and decrease with an increase in the search frequency of unemployed workers s, all of which are common characteristics of spatial search models (see Zenou [18]). Lemma 1 and (22) yield the wages of an on-the-job searcher w_l^1:

$$
\begin{aligned}
w_l^1 &= \beta(y_l + c\theta) + (1-\beta)\left[b + (1-s)\tau(N-u)\right] - \phi p(\theta) J_l \\
&= \beta(y_l + c\theta) + (1-\beta)\left[b + (1-s)\tau(N-u)\right] - \phi p(\theta)\frac{(1-\beta)\left[y_l - b - (1-s)\tau(N-u)\right] - \beta c\theta}{r+\delta+\lambda},
\end{aligned}
$$

which is a decreasing function of on-the-job search efficiency ϕ for a given θ.

The expected wage is obtained from (22) as follows:

$$
\frac{w_h + w_l^0}{2} = \frac{\beta(y_h + y_l)}{2} + \beta c\theta + (1-\beta)\left[b + (1-s)\tau(N-u)\right]. \tag{23}
$$

Equations (19), (20) and (23) are used to determine u, θ and $(w_h + w_l^0)/2$, and other variables (i.e. w_l^1, e_s) are well determined from them.

[9] From (5) and $V = 0$, it is readily shown that workers prefer to be employed rather than unemployed if and only if $J_j > 0$ ($j \in \{h, l\}$). From (21) and (22), the sufficient condition for this inequality to hold true is given by

$$
y_l - w_l^0 > 0,
$$

which can be rewritten as

$$
(1-\beta)\left[y_l - b - (1-s)\tau(N-u)\right] > \beta c\theta.
$$

Under this condition, $J_h > J_l > 0$ holds true, that is, even a bad match gives positive profits to a firm, implying that any type of match is maintained under this condition.

Substituting (19) into (23) yields

$$\frac{w_h + w_l^0}{2} = \frac{\beta(y_h + y_l)}{2} + \beta c\theta + (1-\beta)\left[b + (1-s)\tau N\left(1 - \frac{\delta+\lambda}{\delta+\lambda+p(\theta)}\right)\right], \qquad (24)$$

from which it can be observed that the market tightness θ affects the expected wage via two channels. In one channel, a higher θ implies the existence of better outside opportunities, thus giving workers a better bargaining position. In the other channel, a higher θ leads to a higher employment rate, which increases the total commuting costs of employed workers and, by increasing the relative attractiveness of outside opportunities, increases wages.

Figure 2 depicts (20) and (24) with the horizontal axis representing the market tightness θ, vertical axis representing the expected wage $(w_h + w_l^0)/2$. The the downward-sloping curve describes (20) and the upward-sloping curve describes (24).

[Figure 2 around here]

From the Inada condition regarding matching technology, the two curves always has the unique intersection, implying that the model has the unique steady state equilibrium.

Proposition 2 *There exists the unique steady state equilibrium of the model.*

Using Figure 2, the effects of a change in exogenous variables can be determined,

Lemma 3 *The labor market tightness θ decreases and the expected wage $(w_h + w_l^0)/2$ increases as the city size N or the commuting cost τ increases, and as the commuting frequency s of off-the-job search increases.*

As described in (24), an increase in τ or N (a decrease in s) increases the expected wages for a given θ, which reduces both the profit of posting a vacancy and θ.

This change in θ, in turn, affects the threshold location x^* of on-the-job searchers and the number e_s of on-the-job searchers. To examine this, we need to know the expression of $W_h - W_l$. From (21), we know that

$$\begin{aligned} W_h - W_l &= \frac{w_h - w_l^0}{r+\delta+\lambda} \\ &= \frac{\beta(y_h - y_l)}{r+\delta+\lambda}. \end{aligned}$$

Substituting this equation into (12), we obtain

$$x^* = \frac{\phi p(\theta)\beta(y_h - y_l)}{2s^*\tau(r+\delta+\lambda)} > 0. \qquad (25)$$

Lemma 4 *There is always a non-empty interval $[0, x^*]$ such that low-productivity workers who live within it look for a new job. The threshold of the interval x^* decreases with an increases in the city size N, the commuting cost τ and the commuting frequency s^* of on-the-job search, and increases with an increase in the commuting frequency s of off-the-job search.*

As is observed directly from (25), an increase in s^* or τ leads to a higher cost of on-the-job search and a decrease in the threshold x^*. An increase in N, τ or s indirectly affects x^* by changing θ.

From (21), it can be observed that $J_l = \left(y_l - w_l^0\right)/(r+\delta+\lambda)$. Substituting this equation as well as (22) and (25) into (18), we obtain

$$
\begin{aligned}
e_s &= \frac{\phi p(\theta)}{s^*\tau(r+\delta+\lambda)}\left[\frac{\beta\left(y_h - y_l\right)}{2} - y_l + w_l^0\right] \qquad (26)\\
&= \frac{\phi p(\theta)}{s^*\tau(r+\delta+\lambda)}\left\{\frac{\beta\left(y_h + y_l\right)}{2} - y_l + \beta c\theta + (1-\beta)\left[b + (1-s)\tau(N-u)\right]\right\},
\end{aligned}
$$

which demonstrates the existence of three possible cases described in Lemma 5:

Lemma 5 *In equilibrium, (i) a certain proportion of low-productivity workers search for a new job:*

$$
e_s = \frac{\phi p(\theta)}{s^*\tau(r+\delta+\lambda)}\left[\frac{\beta\left(y_h - y_l\right)}{2} - y_l + w_l^0\right], \qquad \text{if} \quad w_l^0 > y_l - \frac{\beta\left(y_h - y_l\right)}{2},
$$

(ii) no employed worker looks for a new job:

$$
e_s = 0, \qquad \text{if} \quad w_l^0 \leq y_l - \frac{\beta\left(y_h - y_l\right)}{2},
$$

(iii) all low-productivity workers seek for a new job:

$$
e_s = e_l, \qquad \text{if} \quad \frac{\phi p(\theta)}{s^*\tau(r+\delta+\lambda)}\left[\frac{\beta\left(y_h - y_l\right)}{2} - y_l + w_l^0\right] \geq e_l.
$$

The remainder of this paper focuses on the most interesting case (i). From (26), the following proposition can be derived:

Proposition 3 *The number e_s of on-the-job searchers decreases as the commuting frequency s^* of on-the-job search or the commuting cost τ increases, whereas the city size N and the commuting frequency s of off-the-job search have an ambiguous effect on e_s.*

An increase in N or τ, or a decrease in s affects e_s through two channels: (i) by leading to a decrease in θ, which as shown in Lemma 3, or (ii) by leading to an increase in wage w_l^0, as confirmed by Lemma 3 and (22). Such changes in θ and w_l^0 have the following impacts on x^* and the crowding out effect $x^* - e_s$, which is given by

$$
\begin{aligned}
x^* - e_s &= \frac{\phi p(\theta) J_l}{s^*\tau} \\
&= \frac{\phi p(\theta)\left(y_l - w_l^0\right)}{s^*\tau\left(r+\delta+\lambda\right)}.
\end{aligned}
$$

From this and (25), we know that a decrease in θ lowers both x^* and $x^* - e_s$ whereas an increase in w_l^0 has no impacts on x^* but reduces $x^* - e_s$. Hence, an increase in w_l^0 leads to an increase in e_s, while a decrease in θ has an ambiguous effect on e_s. An increase in N or a decrease in s affects e_s only through these two channels, implying that the effects of N and s are ambiguous. An increase in τ has another negative direct effect on e_s for any θ and this effect dominates the other effects. An increase in s^* has only the direct negative effect on e_s.

We next provide the comparative steady-state analysis regarding the importance of on-the-job search in the economy. To see this, we focus on the proportion of on-the-job searchers among all workers and among low-productivity workers. From (19) and (26), the proportion of on-the-job searchers among all workers becomes as

$$
\frac{e_s}{N} = \frac{\phi p(\theta)}{s^*\tau(r+\delta+\lambda)}\left\{\frac{1}{N}\left[\frac{\beta\left(y_h + y_l\right)}{2} - y_l + \beta c\theta\right] + (1-\beta)\left[\frac{b}{N} + \frac{(1-s)\tau p(\theta)}{\delta+\lambda+p(\theta)}\right]\right\},
$$

from which the following proposition can be derived:

Proposition 4 *An increase in the city size N, the commuting frequency s^* of on-the-job search, or the commuting cost τ leads to a decrease in the proportion of on-the-job searchers among all workers, whereas an increase in the commuting frequency s of off-the-job search has an ambiguous effect on it.*

According to Proposition 3, an increase in s^* or τ leads to a decrease in e_s, implying a lower e_s/N. Although the effect of N or s on e_s is ambiguous, the effect of N on the denominator of e_s/N is prominent and thus e_s/N decreases as N increases.

The steady-state condition for the number e_l of low-productivity workers becomes

$$\frac{up(\theta)}{2} = (\lambda + \delta)e_l + \frac{\phi p(\theta) e_s}{2},$$

where the left hand represents the inflow to the pool of low-productivity workers from the pool of unemployed workers and the right hand side represents the outflow of workers due to the exogenous shocks and job turnover. Combining this equation with (19) yields

$$e_l = \frac{p(\theta)}{2}\left(\frac{N}{\delta + \lambda + p(\theta)} - \frac{\phi e_s}{\delta + \lambda}\right).$$

The steady-state condition for the number e_h of high productivity workers is given by

$$\frac{up(\theta)}{2} + \frac{\phi p(\theta) x^*}{2} = (\lambda + \delta)e_h,$$

where the left hand side represents the inflow of low-productivity and unemployed workers into the pool of high-productivity workers and the right hand side represents the outflow of high-productivity workers solely due to exogenous shocks.[10] Combining this equation with (19) yields

$$e_h = \frac{p(\theta)}{2}\left(\frac{N}{\delta + \lambda + p(\theta)} + \frac{\phi e_s}{\delta + \lambda}\right).$$

Hence, we now have the proportion of each type of employed workers among all workers:

$$\begin{aligned} \frac{e_l}{N} &= \frac{p(\theta)}{2(\delta + \lambda)}\left(\frac{u}{N} - \frac{\phi e_s}{N}\right), \\ \frac{e_h}{N} &= \frac{p(\theta)}{2(\delta + \lambda)}\left(\frac{u}{N} + \frac{\phi e_s}{N}\right). \end{aligned} \tag{27}$$

When the proportion of on-the-job searchers e_s/N increases, job turnover increases, raising the proportion of high-productivity workers and lowering the proportion of low-productivity workers.

Finally, the proportion of on-the-job searchers among the low-productivity workers can be derived from (27) as follows:

$$\begin{aligned} \frac{e_s}{e_l} &= \frac{2(\delta + \lambda)e_s}{p(\theta)(u - \phi e_s)} \\ &= \frac{2(\delta + \lambda)(\delta + \lambda + p(\theta))e_s}{p(\theta)\left[(\delta + \lambda)N - (\delta + \lambda + p(\theta))\phi e_s\right]}. \end{aligned}$$

which readily leads to the following proposition:

Proposition 5 *An increase in the commuting frequency s^* of on-the-job search decreases the proportion of on-the-job seekers among the low-productivity workers. So does an increase in the city size N or in the commuting cost τ if $(\delta + \lambda + p(\theta))\, \phi p(\theta) > (\delta + \lambda)^2$.*

[10] Note that no high-productivity workers look for a new position.

Although an increase in s^* leads to a decrease in e_s, it does not affect θ, implying that an increase in s^* leads to a decrease in the proportion e_s/e_l. Having observed that an increase in τ or N leads to a decrease in both $p(\theta)$ and e_s/N, it can now be observed that a decrease in e_s/N always leads to a decrease in e_s/e_l. From the partial differentiation of e_s/e_l with respect to $p(\theta)$, the following can be derived to demonstrate that an increase in N or τ decreases the proportion of on-the-job searchers among low-productivity workers, as stated in Proposition 5:

$$\frac{\partial e_s/e_l}{\partial p(\theta)} > 0 \Leftrightarrow (\delta + \lambda + p(\theta))\,\phi p(\theta) > (\delta + \lambda)^2.$$

When the number of each type of worker can be determined, the market land rent function can be derived as follows:

$$R = \begin{cases} (1+s)\tau(e_s - x) + \tau(N - u - e_s) + s\tau u & \text{if} \quad x \in (0, e_s) \\ \tau(N - u - x) + s\tau u & \text{if} \quad x \in [e_s, N-u] \\ s\tau(N-u) & \text{if} \quad x \in (N-u, N] \end{cases}.$$

4 Efficiency

This section derives the first and second best optimal allocations and compares them to the equilibrium allocation described in the previous section, using the social surplus SS as the criterion of welfare.

Second-best optimal allocation

We start from the analysis of the second-best optimal allocation, in which social planners can determine workers' locations and whether they look for a new job although the market tightness is determined by the free-entry condition. As shown in Appendix B, the second-best optimal allocation requires that workers live in the same areas in which they would do so in the equilibrium. Hence, the welfare function (the social surplus) is given by

$$SS = \int \exp[-rt] \Bigg[\underbrace{y_h e_h + y_l e_l + bu}_{\text{output}} \underbrace{- c\theta(u + \phi e_s^{**})}_{\text{search costs}} \underbrace{- \int_0^{e_s^{**}} (1+s^*)\tau x dx - \int_{e_s^{**}}^{N-u} \tau x dx - \int_{N-u}^{N} s\tau x dx}_{\text{commuting costs}} \Bigg] dt,$$

where e_s^{**} is the number of on-the-job searchers as determined by the social planner. SS consists of the output from matches, search costs, and commuting costs. The social planner's problem in the second-best optimal allocation is then defined as

$$\max_{e_s^{**}} SS \tag{28}$$

$$\text{s.t.} \quad \frac{\mathrm{d}e_h}{\mathrm{d}t} = \frac{up(\theta)}{2} + \frac{\phi p(\theta)}{2} e_s^{**} - (\delta + \lambda)e_h,$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \delta N + \lambda(N-u) - (\delta + p(\theta))u,$$

$$\frac{c}{q(\theta)} = \frac{1}{r + \delta + \lambda} \left\{ (1-\beta) \left[\frac{y_h + y_l}{2} - b - (1-s)\tau(N-u) \right] - \beta c\theta \right\},$$

where the first constraint describes the law of motion of the number of high productivity workers, the second constraint describes the law of motion of the number of unemployed workers, and the third constraint is the free-entry condition. Here, it is assumed that unemployment is socially undesirable, or more precisely, that the shadow price of unemployment is negative.[11] Without this assumption, the social planner may prefer leaving workers unemployed rather than inducing them to form bad matches and shut bad matches down.

As shown in Appendix C, the optimal number of on-the-job searchers is given by

$$e_s^{**} = \frac{\phi}{s^* \tau} \left[\frac{p(\theta)(y_h - y_l)}{2\,(r+\delta+\lambda)} - c\theta \right], \tag{29}$$

which satisfies the following property:

Proposition 6 *For any θ, the equilibrium number of on-the-job searchers is equal to the optimal number (i.e., $e_s = e_s^{**}$ must hold true).*

Proof See Appendix D.

In a standard on-the-job search model a la Pissarides [7], the number of on-the-job searchers in equilibrium is larger than that in the optimum. Here, spatial crowding out by high-productivity workers reduces the number of on-the-job seekers, leading to the attainment of the second best allocation, i.e., there is no distortion in decision regarding on-the-job search.

Put differently, the equilibrium decision regarding on-the-job search that is described in (18) maximizes the total surplus of forming a match with low-productivity workers, with the total surplus given by

$$\begin{aligned}
&(r+\delta)(W_l(x) + J_l(x) - U) \\
&= \begin{cases} y_l - (r+\delta)U + \lambda(U - W_l - J_l) & \text{if the employed worker} \\ \quad + \phi p(\theta)\,(W_h - W_l - 2J_l)\,/2 - s^* \tau x & \text{looks for a new job} \\ & \\ y_l - (r+\delta)U + \lambda(U - W_l - J_l) & \text{otherwise} \end{cases}.
\end{aligned}$$

Therefore, as long as $\phi p(\theta)(W_h - W_l - 2J_l)/2$ is larger than the search cost $s^* \tau x$, on-the-job search increases the total surplus, implying that the equilibrium decision regarding on-the-job search maximizes the surplus. In the absence of spatial structure, low-productivity (bad matched) workers initiate on-the-job search if the search cost is smaller than $\phi p(\theta)(W_h - W_l)/2$, which yields a larger number of on-the-job searchers than is optimal.

4.1 First-best optimal allocation

Having demonstrated that decision regarding on-the-job search is efficient, the next question is whether the free-entry condition leads to the optimal number of vacancies. This consideration is addressed by comparing the equilibrium to the first-best optimal allocation, in which the social planner can choose the market tightness θ as well as the number of on-the-job searchers e_s^{**}. The corresponding problem can be expressed as:

$$\begin{aligned}
&\max_{e_s^{**},\theta} SS \\
\text{s.t.}\quad &\frac{\mathrm{d}e_h}{\mathrm{d}t} = \frac{up(\theta)}{2} + \frac{\phi p(\theta)}{2} e_s^{**} - (\delta+\lambda)e_h, \\
&\frac{\mathrm{d}u}{\mathrm{d}t} = \delta N + \lambda(N - u) - (\delta + p(\theta))u.
\end{aligned}$$

As shown in Appendix E, the optimality condition is given by

$$\begin{aligned}
c\theta = {} & \frac{\theta p'(\theta)}{r+\delta+\lambda+p(\theta) - u\theta p'(\theta)/(u+\phi e_s^{**})} \\
& \times \left[\left(r+\delta+\lambda+\frac{\phi e_s^{**} p(\theta)}{u+\phi e_s^{**}} \right) \frac{y_h - y_l}{2(r+\delta+\lambda)} + [y_l - b - (1-s)\tau(N-u)] \frac{u}{u+\phi e_s^{**}} \right].
\end{aligned} \tag{30}$$

[11] See Appendix C.

In the absence of on-the-job search, as shown in Wasmer and Zenou [15] and Zenou [18], even with consideration of the spatial structure, the Hosios condition (Hosios [4]) ensures the equilibrium to be optimal, i.e., the free-entry condition (20) coincides to (30) when $\beta = -\theta q'(\theta)/q(\theta)$ $(= 1 - \theta p'(\theta)/p(\theta))$ holds true: by substituting $\phi = 0$ and $\beta = 1 - \theta p'(\theta)/p(\theta)$ into (20) and (30), we obtain[12]

$$c\theta = \frac{\theta p'(\theta)}{r + \delta + \lambda + p - \theta p'(\theta)} \left[\frac{y_h + y_l}{2} - b - (1 - s)\tau(N - u) \right]. \tag{31}$$

However, in the presence of on-the-job search, the optimal θ under the Hosios condition is given by (30) although (20) under the Hosios condition becomes (31), which determines the equilibrium tightness. As shown in Appendix F, the right hand side of (30) is smaller than the right hand side of (20) for a given θ. In contrast, the left hand side is the same for both the equilibrium and the optimum. These loci are described in Figure 3.

[Figure 3 around here]

From Figure 3, we can readily see that the equilibrium value of θ is larger than the optimal value of θ.

Proposition 7 *In the presence of on-the-job search, vacancies are over-provided in the equilibrium under the Hosios condition.*

As social benefits derived from making a new job match is smaller for on-the-job search than for off-the-job search, the optimal value of the market tightness θ decreases when the on-the-job search is possible. In contrast, the equilibrium value of θ is determined independently of whether $\phi > 0$, which is the result of spatial arbitrage among employed workers.

5 Concluding remarks

This study investigated the nature and impacts of the interaction between the structure of a city and the on-the-job search. In equilibrium, workers relocate within a city correspondingly to their career turnover. Unemployed workers relocate from the outskirts of city to the inner regions of the city when they land a job. Employed workers must determine whether to search for a new job. If an employed worker searches, she/he lives most proximate to the CBD. If not, she/he resides inbetween the unemployed workers and on-the-job searchers. Thus, the model proposed in this paper provides a tool to analyze the simultaneous determination of career and relocation paths. Efficiency analysis demonstrated that although the decision to post a job vacancy is distorted by the introduction of the spatial structure, decision regarding on-the-job search is efficient.

A few comments are in order. First, in the model presented here, the spatial dimension of on-the-job search appears only on the cost side of a job search. However, the spatial concentration of job searchers and vacancies may lead to agglomeration economies (Sato [10] and Wheeler [16]), the importance of which has already been noted by Marshall [6]. As is always true for economics analysis, balanced consideration of the costs and benefits must be given, and therefore examination of the effects of introducing the benefits of on-the-job search within a city is warranted. Second, the model is a closed city model in which the population size of the city is exogenous. However, if the benefits of concentration of job-seekers and vacancies outweighs the costs of it, a large city will attract more workers and firms. It is therefore particularly important to investigate the resulting distribution of economic activities in a multi-city world. Finally, for the sake of analytical tractability, this model considered only two levels of productivity, high

[12] Note that $e_s = 0$ when $\phi = 0$.

and low, which led to the development of an over-simplified wage distribution based on only three possible values. Future research efforts should introduce a continuous wage distribution a la Burdett and Mortensen [2]. All these are important directions in the future research.

Appendix A: *Proof of Lemma 1*

Start with the wages paid to a good-match worker. Because $i^* = 0$ for J_h, (3), $V = 0$ and (7) imply that

$$w_h(x) = w_h \equiv y_h - (\lambda + \delta + r)J_h, \quad \forall x.$$

Similar arguments for the wages paid to a bad-match worker imply that

$$\begin{aligned} w_l^1(x) = w_l^1 &\equiv y_l - (\lambda + \delta + r + \phi p(\theta))J_l, \quad \forall x. \\ w_l^0(x) = w_l^0 &\equiv y_l - (\lambda + \delta + r)J_l, \quad \forall x. \end{aligned}$$

Hence, we obtain the results:

$$w_l^0 - w_l^1 = \phi p(\theta) J_l.$$

Appendix B: *The location patterns in the first and second best optimal allocation*

This appendix shows that in the optimal allocation, on-the-job searchers live closet to the CBD, employed non-job searchers live next closest to the CBD, and unemployed workers live most distant from the CBD.

Consider two workers A and B, whose commuting costs are τ_A and τ_B, respectively. Assume that $\tau_A > \tau_B$ and denote their locations by x_A and x_B, respectively. We show by contradiction that $x_A < x_B$ must hold true in the optimal allocation.

Suppose now that $x_A > x_B$ holds true in the optimal allocation and let Δ denote $x_A - x_B > 0$. Then, by interchanging their location, the total commuting costs in the economy changes by $-\tau_A\Delta + \tau_B\Delta = (\tau_B - \tau_A)\Delta < 0$. This contradicts the definition of the optimal allocation. Hence, $x_A < x_B$ must hold true, i.e., workers with higher commuting costs must locate closer to the CBD in the optimal allocation, which is the same location pattern as that described in Proposition 1.

Appendix C: *Derivation of the second-best optimal allocation*

As the free-entry condition in (28) determines θ as a function of u, it can be denoted as $\theta(u)$. Using this expression, the welfare function can be rewritten as

$$\begin{aligned} SS = \int \exp[-rt] \Bigg[& y_h e_h + y_l e_l + bu - c\theta(u)(u + \phi e_s^{**}) \\ & - \int_0^{e_s^{**}} (1 + s^*)\tau x dx - \int_{e_s^{**}}^{N-u} \tau x dx - \int_{N-u}^{N} s\tau x dx \Bigg] dt, \end{aligned}$$

which implies that (28) becomes

$$\begin{aligned} & \max_{e_s^{**}} SS \\ \text{s.t.} \quad & \frac{\mathsf{d}e_h}{\mathsf{d}t} = \frac{up(\theta(u))}{2} + \frac{\phi p(\theta(u))}{2} e_s^{**} - (\delta + \lambda)e_h, \\ & \frac{\mathsf{d}u}{\mathsf{d}t} = \delta N + \lambda(N - u) - (\delta + p(\theta(u)))u, \end{aligned}$$

Noting that $e_l = N - u - e_h$, the present-value Hamiltonian is defined as

$$H = \exp[-rt]\left[y_h e_h + y_l(N-u-e_h) + bu - c\theta(u)(u+\phi e_s^{**}) - \int_0^{e_s^{**}} (1+s^*)\tau x dx \right.$$
$$\left. - \int_{e_s^{**}}^{N-u} \tau x dx - \int_{N-u}^{N} s\tau x dx\right] + \mu_g\left[\frac{p(\theta(u))}{2}(u+\phi e_s^{**}) - (\delta+\lambda)e_h\right]$$
$$+ \mu_u\left[\delta N + \lambda(N-u) - (\delta + p(\theta(u)))u\right].$$

Note here that the control variable is e_s^{**} and the state variables are e_h and u. The optimality conditions are

$$\begin{aligned} e_s^{**}:\quad & 0 = \frac{\partial H}{\partial e_s^{**}}, \\ e_h:\quad & \frac{\mathsf{d}\mu_g}{\mathsf{d}t} = -\frac{\partial H}{\partial e_h}, \\ u:\quad & \frac{\mathsf{d}\mu_u}{\mathsf{d}t} = -\frac{\partial H}{\partial u}. \end{aligned} \tag{32}$$

By solving the three equations in (32), we obtain

$$\begin{aligned} 0 &= \exp[-rt](-c\theta\phi - s^*\tau e_s^{**}) + \mu_g\frac{\phi p(\theta)}{2}, \\ \mu_g &= \exp[-rt]\frac{y_h - y_l}{r+\delta+\lambda}, \\ \mu_u &= -\exp[-rt]\frac{(r+\delta+\lambda)\left[y_l - b + c\theta - (1-s)\tau(N-u)\right] - p(\theta)(y_h-y_l)/2}{(r+\delta+\lambda)(\lambda+r+\delta+p(\theta))}, \end{aligned} \tag{33}$$

which determine the optimal number of on-the-job searchers:

$$e_s^{**} = \frac{\phi}{s^*\tau}\left[\frac{p(\theta)(y_h-y_l)}{2\,(r+\delta+\lambda)} - c\theta\right].$$

The regularity condition that the social planner prefers employment to unemployment is then given by

$$\mu_u < 0 \qquad \Leftrightarrow \qquad \frac{p(\theta)(y_h-y_l)}{2(r+\delta+\lambda)} < y_l - b + c\theta - (1-s)\tau(N-u), \tag{34}$$

and we assume that this inequality holds true.

Appendix D: *Proof of Proposition 6*

Substituting the free entry condition in (28) into (29), yields

$$\begin{aligned} e_s^{**} &= \frac{\phi p(\theta)}{s^*\tau\,(r+\delta+\lambda)}\left\{\frac{y_h - y_l}{2} - (1-\beta)\frac{y_h+y_l}{2} + \beta c\theta + (1-\beta)\left[b + (1-s)\tau(N-u)\right]\right\} \\ &= \frac{\phi p(\theta)}{s^*\tau(r+\delta+\lambda)}\left\{\frac{\beta\,(y_h+y_l)}{2} - y_l + \beta c\theta + (1-\beta)\left[b+(1-s)\tau(N-u)\right]\right\}, \end{aligned}$$

which is equal to (26).

Appendix E: *Derivation of the first-best optimal allocation*

In the first-best optimal allocation, the social planner can choose the market tightness θ as well as the number of on-the-job searchers e_s^{**}. The corresponding problem can be expressed as:

$$\begin{aligned} &\max_{e_s^{**},\theta} SS \\ \text{s.t.}\quad & \frac{\mathsf{d}e_h}{\mathsf{d}t} = \frac{up(\theta)}{2} + \frac{\phi p(\theta)}{2}e_s^{**} - (\delta+\lambda)e_h, \\ & \frac{\mathsf{d}u}{\mathsf{d}t} = \delta N + \lambda(N-u) - (\delta + p(\theta))u, \end{aligned}$$

The present-value Hamiltonian is defined as

$$H = \exp[-rt]\left[y_h e_h + y_l(N-u-e_h) + bu - c\theta(u+\phi e_s^{**}) - \int_0^{e_s^{**}}(1+s^*)\tau x dx \right.$$
$$\left. - \int_{e_s^{**}}^{N-u}\tau x dx - \int_{N-u}^{N} s\tau x dx\right] + \mu_g\left[\frac{p(\theta)}{2}(u+\phi e_s^{**}) - (\delta+\lambda)e_h\right]$$
$$+ \mu_u\left[\delta N + \lambda(N-u) - (\delta + p(\theta))u\right].$$

The optimality conditions are

$$e_s^{**}: \quad 0 = \frac{\partial H}{\partial e_s^{**}},$$
$$\theta: \quad 0 = \frac{\partial H}{\partial \theta},$$
$$e_h: \quad \frac{\mathsf{d}\mu_g}{\mathsf{d}t} = -\frac{\partial H}{\partial e_h},$$
$$u: \quad \frac{\mathsf{d}\mu_u}{\mathsf{d}t} = -\frac{\partial H}{\partial u}.$$

A new condition now exists

$$\theta: e^{-rt}c = \frac{\mu_g p'(\theta)}{2} - \frac{\mu_u p'(\theta)u}{u+\phi e_s^{**}}.$$

Solving the optimality condition yields

$$\frac{c}{p'(\theta)} = \frac{y_h - y_l}{2(r+\delta+\lambda)} + \frac{1}{r+\delta+\lambda+p(\theta)}\left[y_l - b + c\theta - (1-s)\tau(N-u) - \frac{p(\theta)(y_h-y_l)}{2(r+\delta+\lambda)}\right]\frac{u}{u+\phi e_s^{**}},$$

Appendix F: *The loci of the right hand sides of (30) and (31) in Figure 3*

Note that (30) becomes (31) when $\phi = 0$. When $\phi > 0$, (30) can be rewritten as

$$c\theta = \frac{\theta p'(\theta)}{r+\delta+\lambda+p(\theta) - \theta p'(\theta)(1-A)} \times \left\{(r+\delta+\lambda+p(\theta)A)\frac{y_h-y_l}{2(r+\delta+\lambda)} + (1-A)\left[y_l - b - (1-s)\tau(N-u)\right]\right\}, \tag{35}$$

where $A = \phi e_s^{**}/(u+\phi e_s^{**})$. Note that $\partial A/\partial\phi > 0$ for a given θ. Let Φ denote the right hand side of (35). For a given θ, a change in A affects Φ in the following manner:

$$\frac{\partial\Phi}{\partial A} = \frac{\theta p'(\theta)}{r+\delta+\lambda+p(\theta)-\theta p'(\theta)(1-A)}\left\{\frac{p(\theta)(y_h-y_l)}{2(r+\delta+\lambda)} - [y_l - b - (1-s)\tau(N-u)] - \Phi\right\}.$$

Because (34) is assumed to hold, $\partial\Phi/\partial A < 0$ is obtained, and hence $\partial\Phi/\partial\phi = (\partial\Phi/\partial A)(\partial A/\partial\phi) < 0$ for a given θ. This fact, combined with the fact that (30) becomes (31) when $\phi = 0$, demonstrates that the right hand side of (30) is smaller than that of (31) for any θ when $\phi > 0$, as described in Figure 3.

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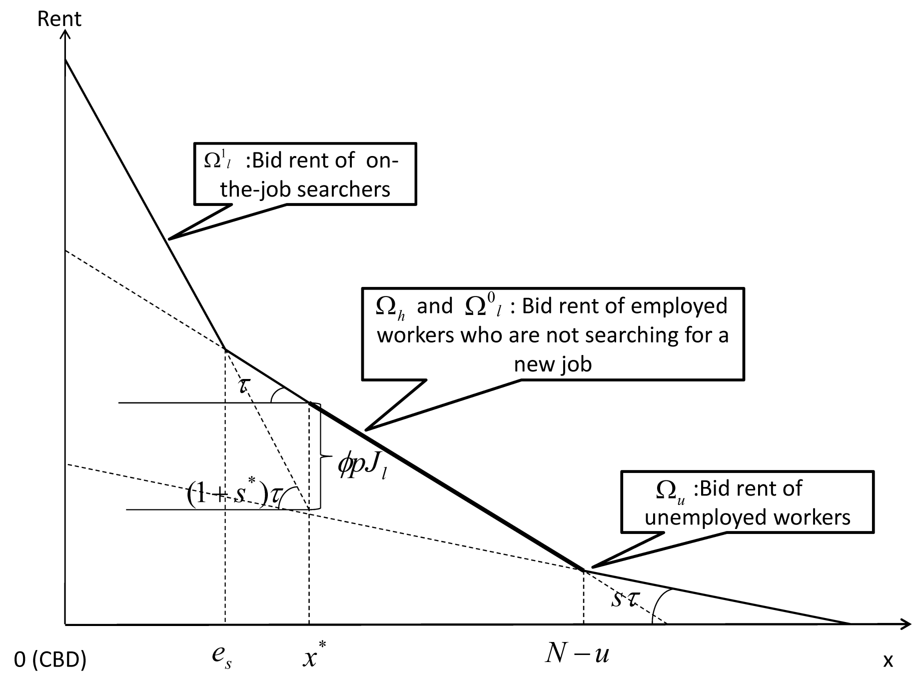

Figure 1: Bid rent curves

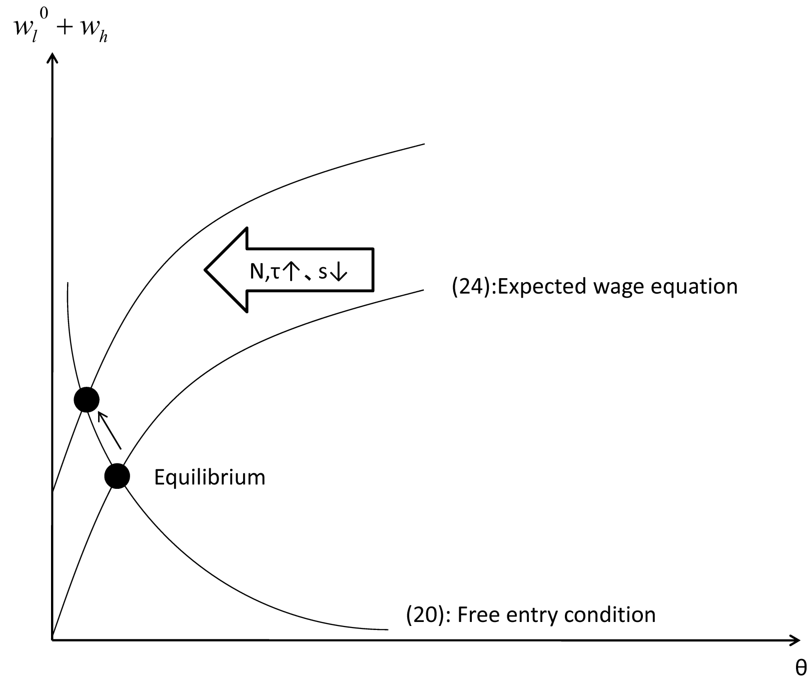

Figure 2: Equilibrium of the model

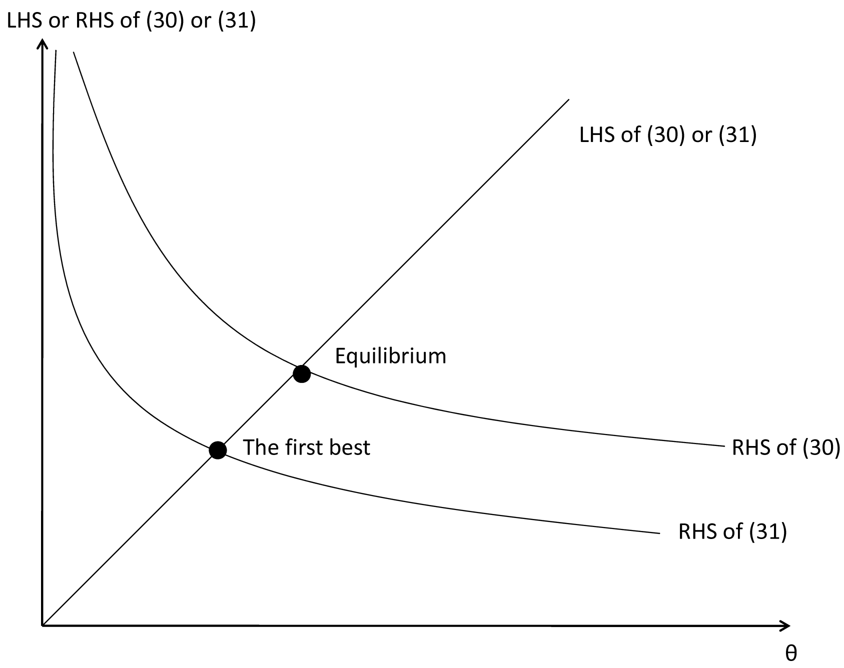

Figure 3: The value of market tightness: Equilibrium v.s. the first best