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**Abstract**

I investigate the decision-making process of an owner of abandoned farmland that is currently restricted to agricultural use but will be available for nonagricultural use in the future. I find that a slight probability of land conversion greatly increases the land value and discourages the owner from cultivating the land. I also observe that a small gap in the anticipation of land conversion prevents the owner from selling or leasing the land to a more efficient farmer.

**JEL Classifications Code:** G13, Q15, R14.
**Keywords:** real option, abandoned farmland, land development, land conversion.

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# 1 Introduction

Recently, there has been increasing concern about whether food production can keep pace with population growth in the future ([6]). Promoting the effective use of agricultural land is critical in resolving the issue. However, abandoned farmland that will not be cultivated for several years has become more prevalent in Japan. In fact, Japan's total area of abandoned farmland has almost tripled in the last two decades.

The purpose of this paper is to investigate the problem of abandoned farmland and offer suggestions for promoting its effective use. I investigate the decision-making process of an owner of abandoned land using the real options framework. The real options approach, in which option pricing theory is applied to real investment problems, is useful in evaluating land development in the face of uncertainty about cash flows from the development ([14, 1, 2, 5]). For instance, [1, 2] apply real options models to land conversion decisions, whereas [5] investigates land development timing with an alternative land use choice in terms of a max-option. Several papers, including [12, 11, 10], provide empirical evidence for real options valuations. Although the previous studies assume that the owner optimizes the development (or conversion) timing, this is not the case with Japan's abandoned farmland. In most cases, abandoned farmland is currently restricted to agricultural use but will be available for nonagricultural use with environmental and regulatory changes at some point in the future.

This paper models the decision-making process of an owner of abandoned farmland as follows. An opportunity that enables development for nonagricultural use comes as an exponential distribution. Before the arrival of the conversion opportunity, the owner has the option to cultivate the land with sunk costs at an arbitrary time. As soon as the land conversion opportunity arrives, the owner will immediately sell the land to a developer for residential or commercial use. This assumption is consistent with the fact that the price of land for nonagricultural use is much higher than that of land for agricultural use. To maintain brevity, I assume that cash flows from agricultural and nonagricultural land use follow bi-dimensional geometric Brownian motions.

Although the problem is expressed as a complex bi-dimensional optimal stopping problem, I reduce it to a standard one-dimensional problem and derive a closed-form solution under plausible assumptions. The value of abandoned land is derived as a sum of the values of the cultivation option and prospective land conversion. Further, I calibrate the model with Japan's land price data from 2005. Results show that a slight probability of conversion greatly increases the value of abandoned land and discourages the owner from cultivating the land. I argue that, in order to promote agricultural use of land, the government needs a zoning ordinance that completely removes owners' anticipation of land conversion. It seems that owners' abandonment of farmland is primarily caused by the insufficient restriction.

In addition, I explore why an owner of abandoned farmland neither sell nor lease the land to a more efficient farmer. With the same anticipation of land conversion, an inefficient owner would sell or lease the land to a more experienced farmer who is eager to cultivate it. However, in Japan's current system, less productive farmers are more likely to receive permission for land conversion. As a result, the anticipation of land conversion differs between inefficient and efficient farmers. I demonstrate that only a small gap in the anticipation prevents the owner of abandoned land from selling or leasing the land to a more productive farmer. As a policy implication, I argue that the government needs a unified guideline for all farmers, even if it does not completely remove the anticipation of land conversion due to the prospective Trans-Pacific Partnership (TPP) which might force the government to deregulate the conversion of agricultural land.

# 2 Abandoned farmland in Japan

Abandoned farmland is defined as farmland that has been left idle for more than one year and will not be cultivated for several years. It is inefficient from the perspective of land use and has negative consequences, such as increasing the prevalence of harmful insects and the illegal dumping of waste, on the community. Recently, a proliferation in the amount of abandoned farmland has become a serious issue in Japan. Japan's total area of abandoned farmland increased from about $130,000$ hectares in 1985 to about $386,000$ hectares in 2005. Typically, owners of abandoned farmland are heirs to their parents' land. They have jobs in urban areas and do not intend to cultivate the inherited land. Many individuals who are engaged in agriculture are aging, which will aggravate the problem of inheritance. Tax savings and the conversion of agricultural land are major reasons why owners of abandoned farmland neither sell nor lease their abandoned land. In this paper, I concentrate on the problem of land conversion. According to [13] who investigates the case of rice land in Japan, the price of land converted for nonagricultural use is almost five times higher than the original price.

In Japan, the Agricultural Land Act controls the conversion of farmland to protect agricultural land. In principle, owners need to receive permission from the prefectural governor (or the minister of agriculture in cases involving more than 4 hectares) when they are ready to covert agricultural land for other use. Agricultural land is classified into five categories which determine the permission guidelines. For the purpose of improving the food self-sufficiency rate, almost 90% of farmland is classified into a category that is zoned by the Prefectural Programs for Establishment of Agricultural Promotion Regions. The conversion of farmland in this category for nonagricultural use is supposed to be impossible "in principle." However, the guideline, unlike zoning ordinances in the European Union, is not executed rigorously. Actually, for some obscure reason, owners are sometimes permitted to convert farmland in this category for nonagricultural use. For example, in 2005, 0.31% of rice land was converted for nonagricultural use ([13]). In addition, in Japan's current system, the possibility of obtaining permission for land conversion is higher if a farmer is less efficient (e.g., those with small-scale and unproductive farms). This ambiguous restriction increases owners' anticipation of converting abandoned farmland for nonagricultural use.

The TPP is the multilateral free-trade agreement that aims to integrate the economies of the Asia-Pacific region. The Japanese government is currently facing a difficult decision regarding whether to join the TPP. Although entry into the TPP is required to maintain the competence of the Japanese manufacturing industry, the elimination of tariffs on key agricultural products by the TPP has created serious concerns among farmers. The agriculture ministry claims that the elimination of agricultural tariff greatly increases the number of imports of cheaper farm products from the Asia-Pacific region, forcing many small-scale farms to close. Naturally, the conversion of agricultural land will be greatly deregulated if Japan decides to join the TPP. This uncertainty about the TPP also increases owners' anticipation of land conversion in the future.

# 3 Real options valuation

Consider an owner of abandoned farmland. The land is restricted to agricultural use at the initial time, but an opportunity that enables nonagricultural development arrives as an exponential distribution (Poisson arrival) with intensity $\lambda$. Note that rare events such as regulatory and environmental changes are usually modeled as Poisson arrivals in the real options literature (e.g., [7]). Cultivating abandoned land for agricultural use requires initial investment costs of $I_1(>0)$, while developing the land for nonagricultural use requires initial investment costs of $I_2(>0)$. The land generates risk-adjusted cash flows $X_1(t)$ by agricultural use and $X_2(t)$ by nonagricultural use. The risk-adjusted cash flows $X(t)=(X_1(t),X_2(t))$ are random and follow geometric Brownian motions

$$\mathrm{d}X_i(t)=\mu_i X_i(t)\mathrm{d}t+\sigma_i X_i(t)\mathrm{d}B_i(t),\ \ X_i(0)=x_i(>0)$$

where $B_1(t),B_2(t)$ are Brownian motions with correlation coefficient $\rho$. Constants $\mu_i$ and $\sigma_i(>0)$ denote the risk-adjusted growth rate and volatility, respectively. Mathematically, the model is built on the filtered probability space $(\Omega,\mathcal{F},P;\mathcal{F}_t)$ generated by $(B_1(t),B_2(t))$. The set $\mathcal{F}_t$ represents the set of available information at time $t$, and the owner optimizes the policy under this information. The risk-free rate is a constant $r(>0)$. For convergence, $r>\mu$ is assumed. Refer to [4] for the economic rationale for this assumption.

## 3.1 Model solution

This section provides a real options valuation for abandoned farmland. The value of abandoned (cultivated) farmland that is available for nonagricultural purpose is denoted by $F_0(X(t))$ ($F_1(X(t))$) for the state variables $X(t)$. For $X(0) = x$, the value of abandoned farmland is expressed as the following optimal stopping problem:

$$\begin{aligned} V(x;\lambda) \quad &:= \quad \sup_{\tau\in\mathcal{T}} \mathbb{E}[\int_0^\infty \{1_{\{\tau<y\}} \left( \int_\tau^y \mathrm{e}^{-rt} X_1(t)\mathrm{d}t - \mathrm{e}^{-r\tau} I_1 + \mathrm{e}^{-ry} F_1(X(y)) \right) \\ &\qquad + 1_{\{\tau\geq y\}} \mathrm{e}^{-ry} F_0(X(y))\} \lambda \mathrm{e}^{-\lambda y} \mathrm{d}y], \end{aligned} \tag{1}$$

where $\mathcal{T}$ denotes the set of all stopping times. In (1), $\tau$ means the time of cultivation, while $y$ means the time when the land permission conversion is provided. At $t = y$, the value of land becomes $F_0(X(y))$ for $y \leq \tau$ and $F_1(X(y))$ for $y > \tau$. Formally, $F_0(X(t))$ is the value of the option to develop the land for either agricultural or nonagricultural use, whereas $F_1(X(t))$ consists of a current cash flow from the cultivated land and the option value of the nonagricultural development. Both $F_0(X(t))$ and $F_1(X(t))$ may include the value of the option to switch between agricultural and nonagricultural land use in the future. It is impossible to derive any closed-form solution to the bi-dimensional problem (1), although several studies, including [3, 8, 9], showed the properties of solutions to multi-dimensional optimal stopping problems.

As mentioned in the previous section, in reality, the price of land converted for nonagricultural use is almost five times higher than that of land for agricultural use ([13]). When permitted to convert agricultural land to nonagricultural use, most owners immediately sell the land to residential or commercial developers. Considering this fact, I assume that

$$\frac{x_1}{r-\mu_1} - I_1 << \frac{x_2}{r-\mu_2} - I_2 \tag{2}$$

and

$$F_0(X(t)) = F_1(X(t)) = \mathbb{E}[\int_t^\infty \mathrm{e}^{-r(s-t)} X_2(s)\mathrm{d}s - I_2 \mid \mathcal{F}_t] = \frac{X_2(t)}{r-\mu_2} - I_2. \tag{3}$$

Assumption (3) means that land is developed for nonagricultural use as soon as the land conversion permission is provided. This is a good approximation of the reality. In addition, by (3) I can reduce problem (1) to a standard one-dimensional optimal stopping problem and derive a closed-form solution as follows.

**Proposition 1**

$$V(x;\lambda) = V_1(x_1;\lambda) + V_2(x_2;\lambda), \tag{4}$$

where

$$V_1(x_1;\lambda) := \begin{cases} \left( \dfrac{x_1^*(\lambda)}{r+\lambda-\mu_1} - I_1 \right) \left( \dfrac{x_1}{x_1^*(\lambda)} \right)^\beta & (0 < x_1 < x_1^*(\lambda)) \\ \dfrac{x_1}{r+\lambda-\mu_1} - I_1 & (x_1 \geq x_1^*(\lambda)), \end{cases} \tag{5}$$

$$x_1^*(\lambda) := \frac{\beta(r+\lambda-\mu_1)I_1}{\beta-1} \tag{6}$$

$$\beta := \frac{1}{2} - \frac{\mu_1}{\sigma_1^2} + \sqrt{\left(\frac{\mu_1}{\sigma_1^2} - \frac{1}{2}\right)^2 + \frac{2(r+\lambda)}{\sigma_1^2}}$$

$$V_2(x_2;\lambda) := \frac{\lambda x_2}{(r-\mu_2)(r+\lambda-\mu_2)} - \frac{\lambda I_2}{r+\lambda}. \tag{7}$$

**Proof** By (3) I can simplify (1) as follows:

$$\begin{aligned}
& V(x;\lambda) \\
= \; & \sup_{\tau\in\mathcal{T}} \mathbb{E}[\int_0^\infty \left\{ 1_{\{\tau<y\}} \left( \int_\tau^y \mathrm{e}^{-rt} X_1(t)\mathrm{d}t - \mathrm{e}^{-r\tau} I_1 \right) + \mathrm{e}^{-ry} \left( \frac{X_2(y)}{r-\mu_2} - I_2 \right) \right\} \lambda \mathrm{e}^{-\lambda y} \mathrm{d}y] \\
= \; & \sup_{\tau\in\mathcal{T}} \mathbb{E}[\int_\tau^\infty \left( \int_\tau^y \mathrm{e}^{-rt} X_1(t)\mathrm{d}t - \mathrm{e}^{-r\tau} I_1 \right) \lambda \mathrm{e}^{-\lambda y} \mathrm{d}y] + \frac{\lambda x_2}{(r-\mu_2)(r+\lambda-\mu_2)} - \frac{\lambda I_2}{r+\lambda} \\
= \; & \underbrace{\sup_{\tau\in\mathcal{T}} \mathbb{E}[\mathrm{e}^{-(r+\lambda)\tau} \left( \frac{X_1(\tau)}{r+\lambda-\mu_1} - I_1 \right)]}_{=V_1(x_1;\lambda) \text{ one-dimensional problem}} + \underbrace{\frac{\lambda x_2}{(r-\mu_2)(r+\lambda-\mu_2)} - \frac{\lambda I_2}{r+\lambda}}_{=V_2(x_2;\lambda)}. \qquad (8)
\end{aligned}$$

To obtain (8), I used

$$\begin{aligned}
& \mathbb{E}[\int_\tau^\infty \left( \int_\tau^y \mathrm{e}^{-rt} X_1(t)\mathrm{d}t - \mathrm{e}^{-r\tau} I_1 \right) \lambda \mathrm{e}^{-\lambda y} \mathrm{d}y] \\
= \; & \mathbb{E}[\int_0^\infty \left( \int_\tau^{y+\tau} \mathrm{e}^{-rt} X_1(t)\mathrm{d}t - \mathrm{e}^{-r\tau} I_1 \right) \lambda \mathrm{e}^{-\lambda(y+\tau)} \mathrm{d}y] \qquad (9) \\
= \; & \mathbb{E}[\int_0^\infty \left( \int_\tau^{y+\tau} \mathrm{e}^{-rt} X_1(t)\mathrm{d}t \right) \lambda \mathrm{e}^{-\lambda(y+\tau)} \mathrm{d}y] - \mathbb{E}[\mathrm{e}^{-(r+\lambda)\tau} I_1] \\
= \; & \mathbb{E}[\int_0^\infty \frac{\mathrm{e}^{-r\tau} X_1(\tau)(1-\mathrm{e}^{-(r-\mu_1)y})}{r-\mu_1} \lambda \mathrm{e}^{-\lambda(y+\tau)} \mathrm{d}y] - \mathbb{E}[\mathrm{e}^{-(r+\lambda)\tau} I_1] \qquad (10) \\
= \; & \mathbb{E}[\mathrm{e}^{-(r+\lambda)\tau} \frac{X_1(\tau)}{r+\lambda-\mu_1}] - \mathbb{E}[\mathrm{e}^{-(r+\lambda)\tau} I_1],
\end{aligned}$$

where I used the change of variables in (9), while I obtained (10) by the tower property and the strong Markov property of $X_1(t)$. Note that, in (8), the optimal stopping problem $V_1(x_1;\lambda)$ depends only on $X_1(t)$. This enables us to derive the value function (5) and the threshold (6) in closed forms (e.g., [4]). The proof is complete. □

Proposition 1 shows that the value of abandoned farmland, $V(x;\lambda)$, is composed of two values: the value of the option to cultivate the land, $V_1(x_1;\lambda)$, and the value of prospective land conversion, $V_2(x_2;\lambda)$. The owner cultivates the land which is restricted to agricultural use when the cash flow from farm products, $X_1(t)$, increases above the threshold $x_1^*$. The decision on the cultivation timing is independent of the dynamics of the cash flow by nonagricultural use, $X_2(t)$. When the intensity $\lambda$ goes to 0, $V(x;\lambda)$ converges to $V_1(x;0)$. On the other hand, when $\lambda$ goes to $\infty$, $V(x;\lambda)$ converges to $x_2/(r-\mu_2) - I_2$, which is the value of the immediate development for nonagricultural use. By (2), (4), and (6) I have $\partial V(x;\lambda)/\partial\lambda > 0$ and $\mathrm{d}x_1^*(\lambda)/\mathrm{d}\lambda > 0$.

## 3.2 Model calibration

In the following numerical example, I consider a typical case. The owner has another job and owns 50 ares of farmland, which has been abandoned for many years. In 2005, the average price of 50 ares of rice land was approximately 27 million yen (the exchange rate average was 1 yen $\approx$ 0.009 dollars in that year), whereas the average price of land that was permitted to convert for nonagricultural use was 130 million yen ([13]). I set the initial investment cost as $I_1 = 15$ million yen which consists of three parts:

$I_{1E} = 5$ million yen: Extra costs stemming from abandoned land. Preliminary work, such as restoring soil, is required to convert abandoned land into farmland.

$I_{1O} = 8$ million yen: Ordinary initial costs of cultivation. Farming equipment, such as a farm tractor, requires high initial costs.

$I_{1I} = 2$ million yen: Costs stemming from the owner's inefficiency. The owner is less efficient than an average farmer because he/she is not accustomed to farming.

Plausible parameter values of $r = 0.07, \mu_1 = 0.03$, and $\sigma_1 = 0.2$ are assumed. To fit the fact that the average price of farmland was 27 million yen, I set the value of $x_1$ as a solution to

$$\frac{x_1}{r - \mu_1} - I_{1O} = 27.$$

Therefore, I have $x_1 = 1.4$, which means that the annual income reaches approximately 1.4 million yen from the cultivated farmland. This value is consistent with the average income from farm products at a small farm. Note that an owner of a small farm frequently has another job. To fit the fact that the average price of converted land was 130 million yen, I set

$$\frac{x_2}{r - \mu_2} - I_2 = 130. \tag{11}$$

However, it is difficult to estimate $x_2$ and $I_2$ individually because they differ across land uses. To avoid this calibration challenge, I set the plausible parameter value of $\mu_2 = 0$. In this case, I do not need to specify $x_2$ and $I_2$ because by (7) and (11) I have

$$V_2(x_2; \lambda) = \frac{\lambda}{r + \lambda}\left(\frac{x_2}{r} - I_2\right) = \frac{130\lambda}{0.07 + \lambda}.$$

For expositional purposes, I define $p := -\log(1 - \lambda)$, which denotes the probability that the land conversion permission will be provided within one year. As mentioned in Section 2, in 2005, 0.31% of rice land was converted for other use. This percentage is not equal to $p$ because not all farmers applied for land conversion permission. In addition, the percentage depends greatly on the regions and farm scales. For instance, more than 1% of race land was converted around urban areas, such as Tokyo, Kanagawa, and Osaka. In this paper, I present the results with varying levels of $p$ from 0 to 0.05(= 5%).

Figure 1 shows the land value $V(x; \lambda)$, value of the cultivation option, $V_1(x_1; \lambda)$, value of prospective land conversion, $V_2(x_2; \lambda)$, and threshold $x_1^*(\lambda)$. A slight probability of land conversion greatly increases $V(x;\lambda)$ and $x_1^*(\lambda)$. For example, compared to $V(x;\lambda) = 20.1, V_1(x_1;\lambda) = 20.1, V_2(x_2;\lambda) = 0$, and $x_1^*(\lambda) = 1.54$ million yen for $p = 0$, I have $V(x;\lambda) = 30.2, V_1(x_1;\lambda) = 13.95, V_2(x_2;\lambda) = 16.25$, and $x_1^*(\lambda) = 1.93$ million yen for $p = 0.01 = (1\%)$. Only a small probability of land conversion makes $V_2(x_2;\lambda)$ higher than $V_1(x_1;\lambda)$. Note that, the nonagricultural value (11) does not influence $x_1^*(\lambda)$, although it influences $V(x;\lambda)$. In conclusion, in order to promote agricultural use of land, the government needs a zoning ordinance that completely removes owners' anticipation of land conversion. The problem of abandoned farmland primarily arises from Japan's insufficient restriction.

Next, I consider why inefficient owners of abandoned land neither sell nor lease the land to farmers who are experienced in managing large farmland. Assume that an efficient farmer requires an investment cost of $I_1 := I_{1E} + I_{1O} = 13$ million yen, which is lower than $I_1 = I_{1E} + I_{1O} + I_{1I} = 15$ million yen for the inefficient owner. Figure 2 compares the cases of $I_1 = 15$ and $I_1 = 13$. For $I_1 = 13$, due to the efficiency, the land value $V(x;\lambda)$ is higher and the cultivation threshold $x_1^*(\lambda)$ is lower. Then, with the same anticipation of land conversion, the owner would sell or lease the land to the effective farmer who is more eager to cultivate the land. However, this is not the case. In Japan's current system, $p$ is higher for the inefficient owner than the efficient farmer. As mentioned in Section 2, the possibility of obtaining land conversion permission is greater for less efficient farmers. In addition, productive farmers, such as agricultural corporations, rarely attempt to convert agricultural land for other use. The left panel in Figure 2 shows that, with only a small gap in $p$, the land value for the inefficient owner increases far beyond that of the efficient farmer. This prevents the inefficient owner from selling or leasing the land. Accordingly, I argue that the government needs a unified guideline for all farmers, even if it does not completely remove the influence of owners' anticipation of land conversion due to uncertainty about the TPP.

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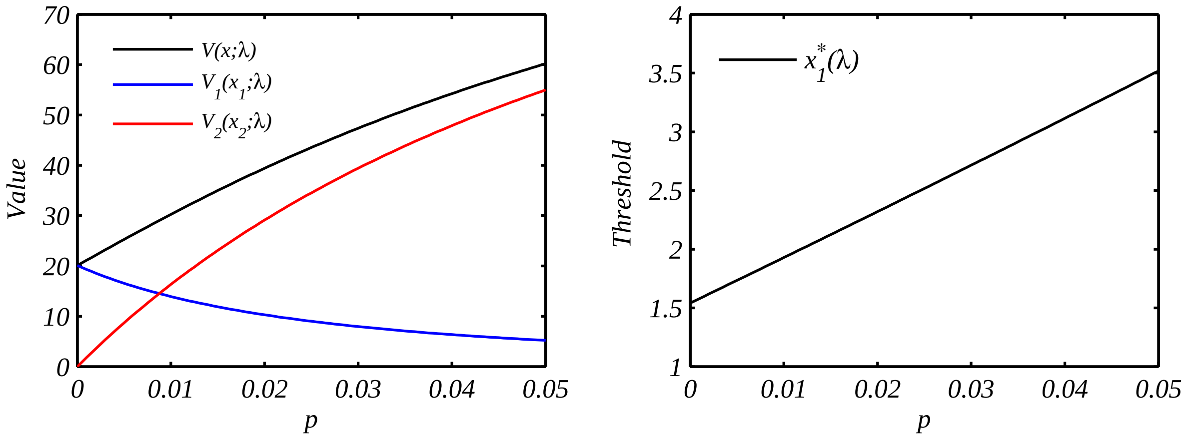


Figure 1: The left panel plots $V(x;\lambda)$, $V_1(x_1;\lambda)$, and $V_2(x_2;\lambda)$. The right panel plots $x_1^*(\lambda)$.

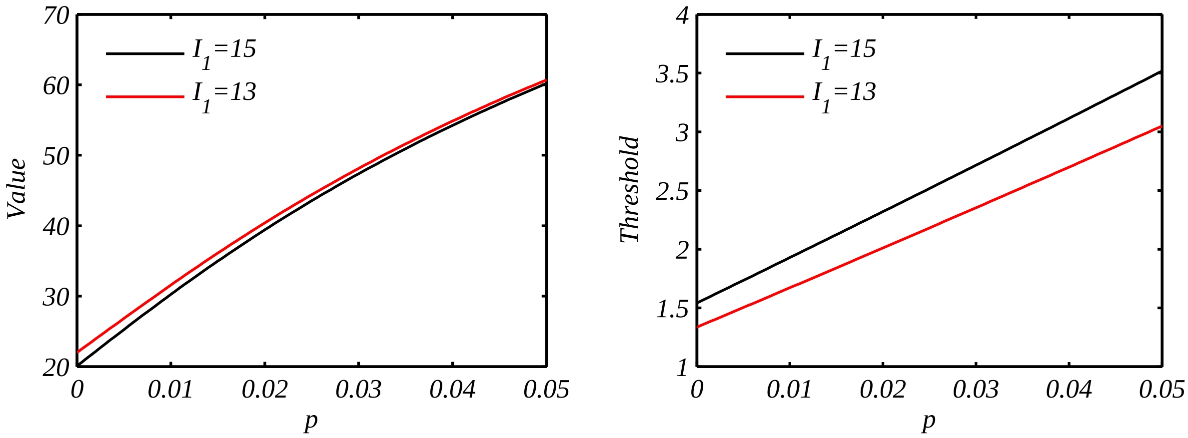


Figure 2: The left panel plots $V(x;\lambda)$ for $I_1 = 15$ and $I_1 = 13$. The right panel plots $x_1^*(\lambda)$ for $I_1 = 15$ and $I_1 = 13$.

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